$K_{\mu 3}^{L}$ decays: A stringent test of charged right handed currents.

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Outline

1. Framework

2. How to probe Right-Handed Quark Currents ?

3. $K_{\mu 3}^{L}$ Decays: A stringent test of right-handed quark currents.

4. Conclusion and outlook.

1. Framework: Low Energy Effective Theory

- Low Energy Effective Theory **[Hirn & Stern'05]** : a non-decoupling theory a la χPT.

- Low Energy degrees of freedom:
  - all observed particles
  - gauge group $SU(2)_W \otimes U(1)_Y$

- At Higher Energy: new symmetries and particles.

- Particles decouple at low energy and symmetries are hidden i.e. non linearly realised.

- Low Effective Energy Theory based on $S_{nat} \supset SU(2)_W \otimes U(1)_Y$ non-linearly realised higher symmetry (not all the local symmetries realised through gauge fields) .

- Reduction of $S_{nat} \to SU(2)_W \otimes U(1)_Y$ via gauge invariant constraints (spurions) ⇨ small expansion parameters κ (parameters of explicit symmetry breaking ~ mass of quarks in χPT)

- Systematic Low Energy expansion in momenta and explicit symmetry breaking parameters.

  $\mathcal{L} = \mathcal{L}(p^2\,\kappa^0) + \mathcal{L}(p^2\,\kappa^2) + \mathcal{L}(p^4\,\kappa^0) + \ldots,\quad \mathbf{p} << \Lambda = 4\pi f \sim 3\ \mathrm{TeV}$

  ⇨ scheme to classify the non-standard operators.

- Renormalizability order by order.

- LO: $\mathcal{O}\left(p^2\kappa^0\right)$ Standard Model without a Higgs.
- NLO: $\mathcal{O}\left(p^2\kappa^2\right)$ only 2 operators ⇨ new couplings of fermions to W.

$$\mathcal{L}_{CC} = \tilde{g}\left[\mathbf{1}_\mu + \frac{1}{2}\bar{U}\left(\mathcal{V}_{eff}\gamma_\mu + \mathcal{A}_{eff}\gamma_\mu\gamma_5\right)D\right]W^\mu + h.c\ , \quad U = \begin{pmatrix} u \\ c \\ t \end{pmatrix},\ D = \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

  - $\mathbf{1}_\mu$: standard V-A leptonic current.
  - $\mathcal{V}_{eff}$, $\mathcal{A}_{eff}$: 3x3 complex matrices of effective couplings.

- In the SM: $\mathcal{V}_{eff} = -\mathcal{A}_{eff} = V_{CKM}$

- Here there are right-handed quarks currents (RHCs) $\mathcal{V}_{eff} \neq -\mathcal{A}_{eff}$

$$\mathcal{V}_{eff}^{ij} = (1+\delta)\,V_L^{ij} + \varepsilon\, V_R^{ij} + NNLO$$ and $$\mathcal{A}_{eff}^{ij} = -(1+\delta)\,V_L^{ij} + \varepsilon\, V_R^{ij} + NNLO$$

  - **V_L, V_R** : 2 unitary flavor mixing matrixes from the diagonalisation of the mass matrix of U and D quarks.
  - δ, ε : small parameters arising from spurions.

- We consider the light quarks sector : 3 parameters to be determined :
  - modification of the left couplings: δ
  - Right-Handed quark Currents (RHCs):
    - In the non-strange sector: $\boldsymbol{\varepsilon_{NS} = \varepsilon\, \mathrm{Re}\left(\frac{V_R^{ud}}{V_L^{ud}}\right)}$
    - In the strange sector: $\boldsymbol{\varepsilon_S = \varepsilon\, \mathrm{Re}\left(\frac{V_R^{us}}{V_L^{us}}\right)}$
- Problem: we do not measure the electroweak parameters directly but a combination of the EW and low energy QCD parameters.

Interdependence of Electroweak couplings and Low Energy QCD observables

- Example: Decay of the Pion:

$$\Gamma\left[\pi^+ \to \mu^+ \nu(\gamma)\right] \sim \left|F_\pi \mathcal{A}_{eff}^{ud}\right|^2$$

$\Rightarrow$ **We do not measure directly Fπ but a combination of Fπ and $\mathcal{A}_{eff}^{ud}$**

$$F_\pi = F_\pi \left|\mathcal{A}_{eff}^{ud}\right| \left|\frac{1}{\mathcal{V}_{eff}^{ud}}\right| \left|\frac{\mathcal{V}_{eff}^{ud}}{\mathcal{A}_{eff}^{ud}}\right|$$

$\Rightarrow$ $F_\pi = F_\pi^{SM}(1+2\varepsilon_{NS})$

Exp. $(0^+ \to 0^+)$ $1+2\varepsilon_{NS}$

F_π extracted in the SM ($\mathcal{V}_{eff}^{ud} = -\mathcal{A}_{eff}^{ud}$) $\Rightarrow$ F_π^{SM}

2. How to probe RHCs ?

- In the past study of RHCs in the context of L-R Symmetric models. **[Mohapatra, Pati, Senjanovic]**

  ➡ Test of contribution ε^2 ($\nu(\bar{\nu})$ DIS off valence quarks, $K^0 - \bar{K}^0$ mixing) **[Abramowicz et al ’82 , Mishra et al ’92]**

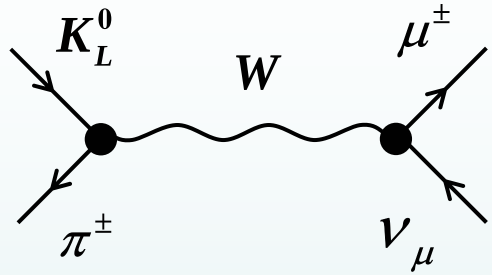
  ➡ Model dependent and final states interactions not considered ($K \to 2\pi / K \to 3\pi$, hyperons decays) **[Donoghue & Holstein ’82 , Bigi & Frere ’82]**

- Strong constraints in the leptonic sector (μ-decays, τ-decays, β-decays) **[Beg et al ’77, Holstein & Treiman ’77, Fetscher & Gerber ’95, Herczeg ’95]...**

  ➡ In the LEET no leptonic RHCs.

3. $K^L_{\mu 3}$ decays: A stringent test of RHCs (effect to 1^{rst} order in ε)

3.1 Introduction.



- $K_L^0 \to \pi^\mp \mu^\pm \nu_\mu$

  The hadronic element :

$$\left\langle \pi^-(p_\pi) \right| \bar{\mathrm{s}}\gamma_\mu \mathrm{u} \left| \mathrm{K}^0(\mathrm{p_K}) \right\rangle = f_+(t)(p_K + p_\pi)_\mu + f_-(t)(p_K - p_\pi)_\mu$$

  → $f_+(t)$, $f_-(t)$: form factors

  → $t = q^2 = (p_\mu + p_{\nu_\mu})^2 = (p_K - p_\pi)^2$

- We consider the scalar form factor : $f_s(t) = f_+(t) + \dfrac{t}{m_K^2 - m_\pi^2} f_-(t)$

  → Normalization : $f(t) = \dfrac{f_s(t)}{f_+(0)},\ f(0) = 1$

3.2 Callan-Treiman relation.

- Callan-Treiman Theorem :

$$C = f(\Delta_{K\pi}) = \frac{F_{K^+}}{F_\pi f_+^{K^0}(0)} + \Delta_{CT}$$

$$\Delta_{K\pi} = m_K^2 - m_\pi^2$$

$$\Delta_{CT} \sim -3.5.10^{-3}$$

[Gasser & Leutwyler]

- Corrections of order m_u, m_d (theorem SU(2) x SU(2))
  → No chiral logarithms $\Delta_{CT} \sim \mathcal{O}\left(\frac{m_{u,d}}{4\pi F_\pi}\right)$
  → No small denominators (π^0 - η mixing $\mathcal{O}\left(\frac{m_d - m_u}{m_s}\right)$) contrary to K^+ decays.
- Corrections $\mathcal{O}(\mathrm{p}^6)$ have to be evaluated.

⇨ Relation which will ultimately test the Standard Model very accurately.

- Experimental measurements :

$$Br\left[\frac{K^+ \to \mu^+ \nu(\gamma)}{\pi^+ \to \mu^+ \nu(\gamma)}\right] \sim \left(\frac{F_K}{F_\pi}\left|\frac{\mathcal{A}_{eff}^{us}}{\mathcal{A}_{eff}^{ud}}\right|\right)^2$$

$$\Gamma\left[K^0 \to \pi^- e^+ \nu(\gamma)\right] \sim \left|f_+^{K^0}(0)\mathcal{V}_{eff}^{us}\right|^2$$

- In the SM : $\mathcal{V}_{eff}^{ij} = -\mathcal{A}_{eff}^{ij} = V_{CKM}^{ij}$
- In theory with Right-handed currents : $\mathcal{V}_{eff}^{ij} \neq -\mathcal{A}_{eff}^{ij}$

3.3 Charged right-handed quark currents.

$$C = f(\Delta_{K\pi}) = \underbrace{\frac{F_K \left|\mathcal{A}^{us}_{eff}\right|}{F_\pi \left|\mathcal{A}^{ud}_{eff}\right| f_+(0) \left|\mathcal{V}^{us}_{eff}\right|}}_{\text{measured BR's}} \underbrace{\left|\mathcal{V}^{ud}_{eff}\right|}_{(0^+ \to 0^+)} \underbrace{\frac{\left|\mathcal{A}^{ud}_{eff}\right|}{\left|\mathcal{V}^{ud}_{eff}\right|} \frac{\left|\mathcal{V}^{us}_{eff}\right|}{\left|\mathcal{A}^{us}_{eff}\right|}}_{\neq 1 \text{ if right-handed currents contribute}} + \Delta_{CT}$$

$$\frac{\left|\mathcal{A}^{ud}_{eff}\right|}{\left|\mathcal{V}^{ud}_{eff}\right|} \frac{\left|\mathcal{V}^{us}_{eff}\right|}{\left|\mathcal{A}^{us}_{eff}\right|} = (1 + 2\varepsilon_S - 2\varepsilon_{NS}) + O(\varepsilon^2)$$

$$\left[\varepsilon_S = \varepsilon \, \mathrm{Re}\left(\frac{V_R^{us}}{V_L^{us}}\right)\right] \qquad \left[\varepsilon_{NS} = \varepsilon \, \mathrm{Re}\left(\frac{V_R^{ud}}{V_L^{ud}}\right)\right]$$

- Experimental results :

$$\rightarrow \left(\frac{F_K}{F_\pi}\left|\frac{\mathcal{A}^{us}_{eff}}{\mathcal{A}^{ud}_{eff}}\right|\right) = 0.27618(48)$$

[Jamin, Oller & Pich'06]

$$\rightarrow f_+^{K_0}(0)\left|\mathcal{V}^{us}_{eff}\right| = 0.21619(55)$$

Average of most recent measurements of NA48, KTeV, KLOE.

$$\rightarrow \left|\mathcal{V}^{ud}_{eff}\right| = 0.97377(26)$$

[Towner & Hardy] ($0^+ \rightarrow 0^+$) updated by **[Marciano & Sirlin '05]**

$$\Rightarrow \ln C = 0.2183 \pm 0.0031 + \Delta\varepsilon$$

with

$$\Delta\varepsilon = \tilde{\Delta}_{CT} + 2(\varepsilon_S - \varepsilon_{NS})$$

0.0031: Experimental uncertainties

$\left[\tilde{\Delta}_{CT} = \frac{\Delta_{CT}}{1.24}\right]$

$2(\varepsilon_S - \varepsilon_{NS})$: expected ~ %

$\Rightarrow$ In order to provide a significative information on right handed currents, lnC should be known at least with 5% accuracy.

Order of magnitude of $2(\varepsilon_S - \varepsilon_{NS})$:

- Unitarity of the first line of the V_L matrix :

$$\left|V_L^{ud}\right|^2 + \left|V_L^{us}\right|^2 + \left|V_L^{ub}\right|^2 = 1$$

(the $\left|V_L^{ub}\right|^2$ term is struck through) V_L close to V_{CKM}

⇒ decoupling of the light quarks: we can introduce the Cabibbo angle θ

⇒ $\mathrm{Re}(V_L^{ud}) = \cos\theta \sim 0.974$
$\mathrm{Re}(V_L^{us}) = \sin\theta \sim 0.227$

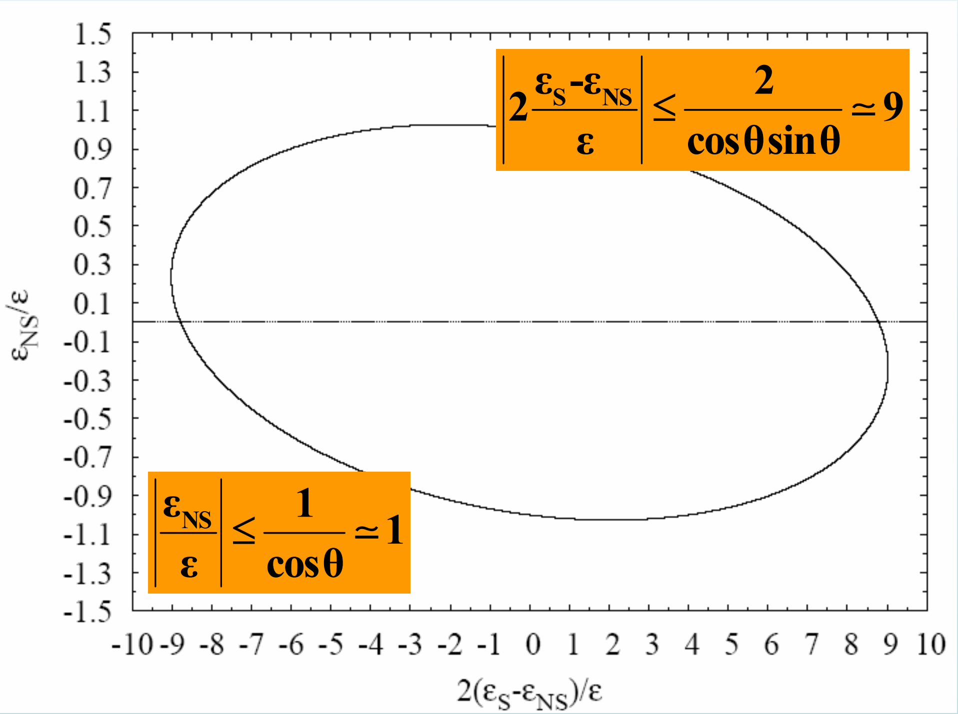
- Unitarity of the first line of the V_R matrix

$$\left(\mathrm{Re}V_R^{ud}\right)^2 + \left(\mathrm{Re}V_R^{us}\right)^2 \le 1$$

$$\varepsilon_{NS} = \varepsilon\,\mathrm{Re}\left(\frac{V_R^{ud}}{V_L^{ud}}\right) \qquad \varepsilon_S = \varepsilon\,\mathrm{Re}\left(\frac{V_R^{us}}{V_L^{us}}\right)$$

$$\left(\frac{\varepsilon_{NS}}{\varepsilon}\right)^2 \cos^2\theta + \left(\frac{\varepsilon_S}{\varepsilon}\right)^2 \sin^2\theta \le 1$$

$$\varepsilon \sim 0.01$$



3.4 How to measure lnC ?
A dispersive representation of the $K\pi$ scalar form factor.

- A dispersion relation with two substractions for ln(f(t)) :

  we know f(t) at two points at low energy :

  → f(0) = 1

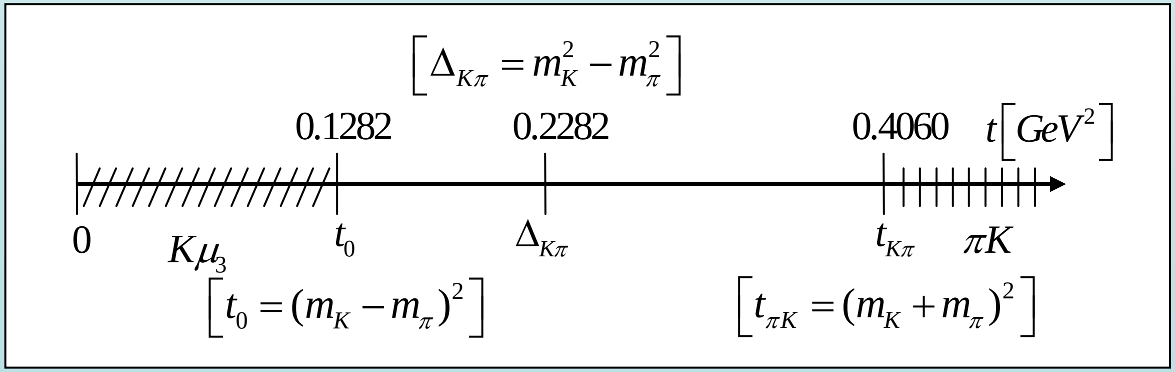
  → f($\Delta_{K\pi}$) = C , Callan-Treiman point

$$\boldsymbol{f(t) = \exp\left[\frac{t}{\Delta_{K\pi}}\left(\ln C - G(t)\right)\right]}$$

with

$$\boldsymbol{G(t) = \frac{\Delta_{K\pi}(\Delta_{K\pi} - t)}{\pi}\int_{t_{\pi K}}^{\infty}\frac{dx}{x}\frac{\phi(x)}{(x-\Delta_{K\pi})(x-t)}}$$

→ ϕ(t) phase of form factor : $\boldsymbol{f(t) = \left|f(t)\right| e^{i\phi(t)}}$

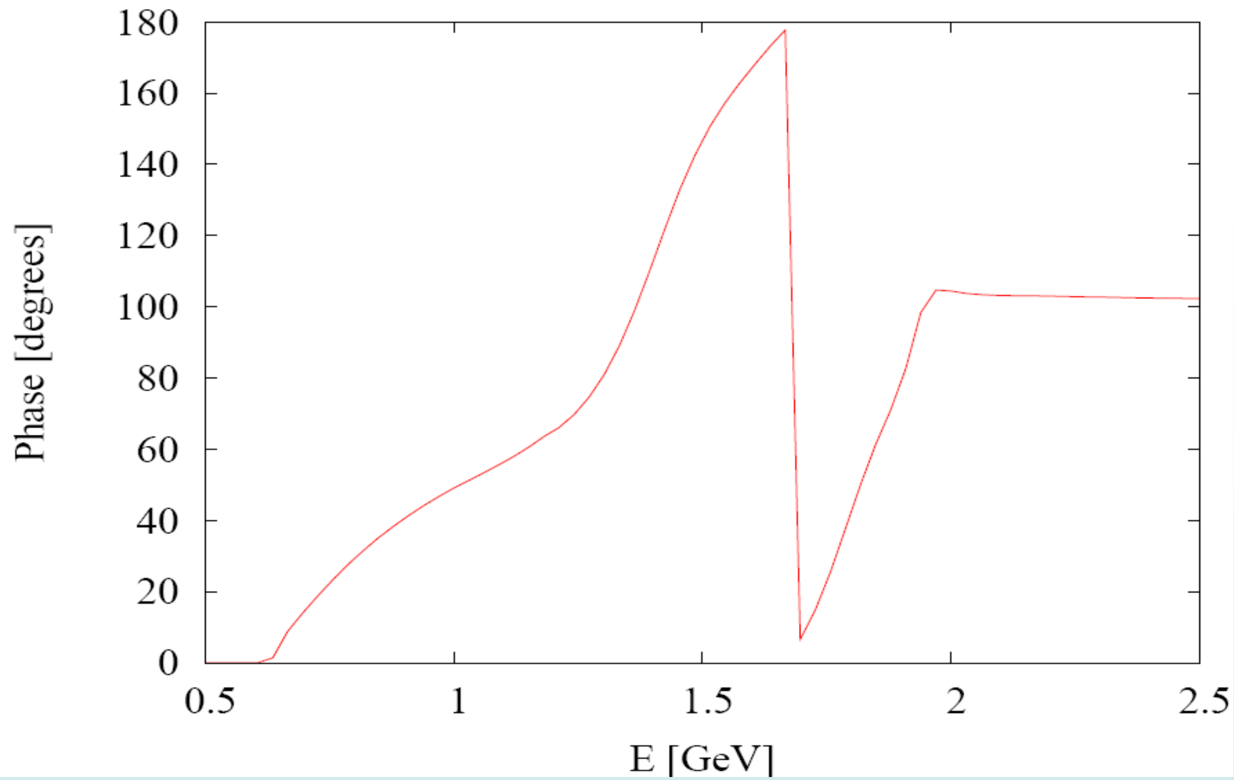


- Advantages :
  - rapid convergence for large t , $\phi(t) \to \pi$
  - At « low energy » $\phi(t) = \delta^{1/2}_{K\pi}(t)$, s wave I=1/2 $K\pi$ scattering phase, well known [**Watson theorem**].

- $K\pi$ scattering phase:
  - experimental input after 1 GeV **[Aston et al]**
  - solving Roy-Steiner equations

  [Buettiker, Descotes, Moussallam '02]

Kπ scattering phase



[Buettiker, Descotes, Moussallam '02]

Elastic up to ~1.5 GeV ⟹ $\phi(t) = \delta_{\pi,K}^{s,\frac{1}{2}}(t)$ for t < 2.77 GeV².

$$G(t) = \frac{\Delta_{K\pi}(\Delta_{K\pi} - t)}{\pi} \int_{t_{\pi K}}^{\Lambda} \frac{dx}{x} \frac{\delta_{\pi K}(x)}{(x - \Delta_{K\pi})(x - t)} + G_{as}(\Lambda, t) \pm \delta G \qquad \Lambda = 2.77 GeV^2$$

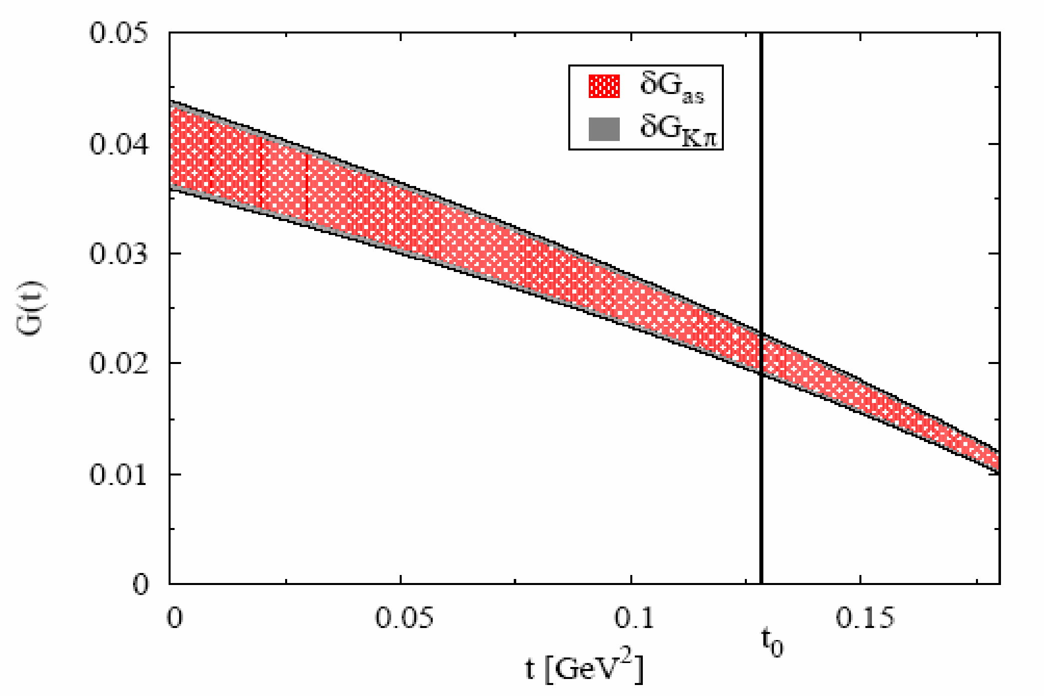
With $$G_{as}(\Lambda, t) = \frac{\Delta_{K\pi}(\Delta_{K\pi} - t)}{\pi} \int_{\Lambda}^{\infty} \frac{dx}{x} \frac{\pi}{(x - \Delta_{K\pi})(x - t)} = \frac{\Delta_{K\pi}}{t} \ln\left(1 - \frac{t}{\Lambda}\right) - \ln\left(1 - \frac{\Delta_{K\pi}}{\Lambda}\right)$$

- Estimate of the uncertainties :
  - →Conservative estimate of $\delta G_{as}(t)$: For $t>2.77\ \mathrm{GeV}^2$, $0 \le \phi(t) \le 2\pi$

$$\left|\delta G_{as}(\Lambda,t)\right| \le \frac{\Delta_{K\pi}}{t}\ln\left(1-\frac{t}{\Lambda}\right)-\ln\left(1-\frac{\Delta_{K\pi}}{\Lambda}\right)$$

$\delta G_{as}(t)<0.0036$ in the physical region

  - →Estimate of $\delta G_{K\pi}(t)$: $\delta G_{K\pi}(t) \le 0.05 \times G_{K\pi}(t)$



G(t) with the uncertainties added in quadrature

G(t) small compared to lnC :
lnC~0.20

3.5 Experimental measurements

- Data available from KTeV, NA48, KLOE.

- **In fixed target experiments** (KTeV, NA48), it is impossible to measure directly the distribution in t (Initial energy of K_L unknown) : For each event, 2 possible values of t.

- Necessity to parameterize the 2 form factors f_+ and f_0 to fit the measured distributions.

- Usually, use of linear parameterization or pole parameterization :

$$f_{lin}(t,\lambda) = 1 + \lambda \frac{t}{m_\pi^2} \qquad f_{pol}(t, m_s) = \frac{m_s^2}{m_s^2 - t}$$

$$\Rightarrow \lambda_{exp} = 0.01372 \pm 0.00131$$ **[KTeV, Phys.Rev.D70 ‘04]**

- Curvature is correlated with the slope and we can't neglect it.

$$f(t) = 1 + \lambda \frac{t}{m_\pi^2} + \frac{1}{2}\lambda'\left(\frac{t}{m_\pi^2}\right)^2 \Rightarrow \lambda' = \lambda^2 - 2\frac{m_\pi^4}{\Delta_{K\pi}}G'(0)$$

$$\lambda' = \lambda^2 + (4.16 \pm 0.50).10^{-4}$$

- Direct use of the dispersive parametrization of f(t)

$$f(t) = \exp\left[\frac{t}{\Delta_{K\pi}}(\ln C - G(t))\right] \Rightarrow \text{extract } \ln C$$

$$\lambda_{\exp} = 0.01372 \pm 0.00131$$

[KTeV, Phys.Rev.D70 '04]

$$f(t) = 1 + \lambda_{eff}(t)\frac{t}{m_\pi^2}$$

NB: Compatible with preliminary results of NA48

$$\lambda_{\exp} = 0.0120 \pm 0.0017$$

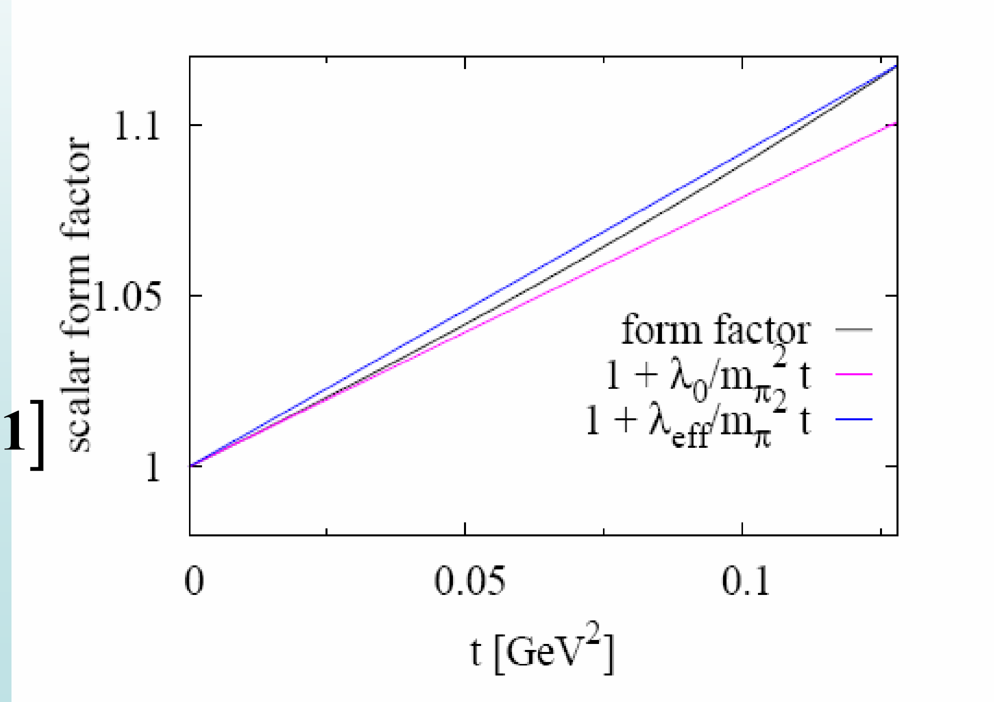
- Extreme estimates of lnC :

$$\rightarrow \lambda_{\exp} = \lambda_{eff}(0) = m_\pi^2 f'(0)$$

$$\ln C_{\max} = \frac{\Delta_{K\pi}}{m_\pi^2}\lambda_{\exp} + G(0)$$

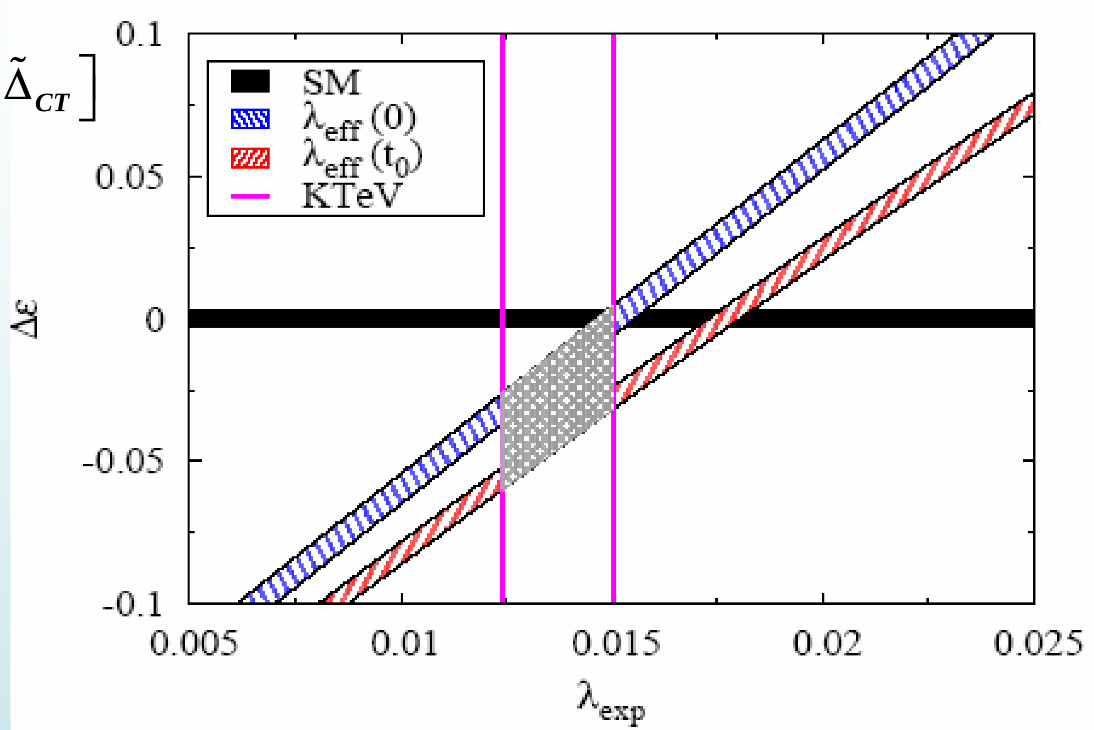
$$\rightarrow \lambda_{\exp} = \lambda_{eff}(t_0) = \frac{m_\pi^2}{t_0}\left[f(t_0) - 1\right]$$

$$\ln C_{\min} = \frac{\Delta_{K\pi}}{t_0}\ln\left[1 + \frac{t_0}{m_\pi^2}\lambda_{\exp}\right] + G(t_0)$$



$\Rightarrow$ $\ln C_{\min} = 0.1748 \pm 0.0141 \pm 0.0016$ and $\ln C_{\max} = 0.2005 \pm 0.0153 \pm 0.0036$

$$\left[\Delta\varepsilon = 2\left(\varepsilon_S - \varepsilon_{NS}\right) + \tilde{\Delta}_{CT}\right]$$



$$\lambda_{\exp} = \overline{\lambda}_{eff} = \frac{1}{t_0}\int_0^{t_0} dt\ \lambda_{eff}(t)$$ $\Rightarrow$ $\ln C = 0.188 \pm 0.015 \pm 0.003 \pm 0.013$

$$\ln C = 0.2183 \pm 0.0031 + \Delta\varepsilon$$ $\Rightarrow$ $\Delta\varepsilon = -0.030 \pm 0.015 \pm 0.003 \pm 0.013$

Preliminary result of NA48 (slope) : $\Rightarrow$ $\Delta\varepsilon = -0.050 \pm 0.019 \pm 0.003 \pm 0.012$

$\Rightarrow$ The uncertainty due to the parametrization largely dominates !

4. Conclusion and outlook

4.1 Conclusion

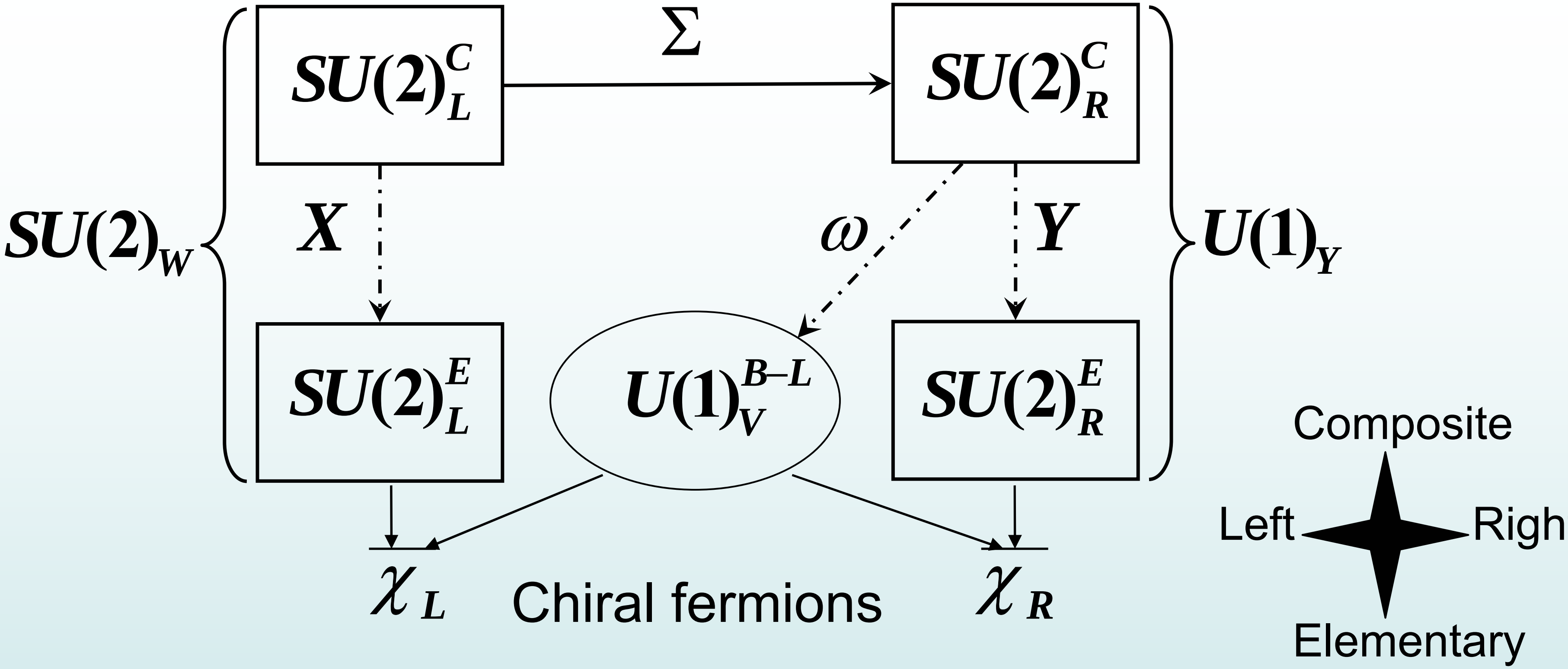
- Study of $K^{L}_{\mu 3}$ ⟹ constraints on right handed quark currents: $(\varepsilon_S - \varepsilon_{NS})$
- Theoretical and experimental accuracies make it very promising.
  ⟹ Uncertainties due to the parametrisation have to be reduced measuring directly lnC !
  - → Ongoing NA48 analysis (2.6 M of events)
  - → Direct measurement of t distribution should be possible with KLOE.
- Once lnC is known ⟹ 2 $O(p^6)$ LECs ⟹ $f_+(0)$ and thus extraction of Vus.

4.2 Outlook

Experimental processes	Parameters extracted	Low Energy/QCD inputs
Nuclear β decays **$0^+ \rightarrow 0^+$**	$\left\|\mathcal{V}_{eff}^{ud}\right\|$	**CVC + nuclear corrections** [Marciano & Sirlin '05]
Ke3 decays KTeV, NA48, KLOE	$\left\|\mathcal{V}_{eff}^{us}\right\|$	**$f_+^{K_0}(0)$: lattice or χPT** [Becirevic et al '05] [Cirigliano et al '05]
Hadronic τ decays **R_V, R_A, R_S,** **Moments** ALEPH, OPAL	ε_{NS}, $\delta + \varepsilon_{NS}$	**OPE** [Brateen et al '92, Lediberder & pich'92….] **$\alpha_S(m_\tau)$, m_q, condensates**
Γ_W LEP, TEVATRON	δ	**Perturbative QCD** **$\alpha_S(m_W)$**
DIS $\nu(\overline{\nu})$ on protons	δ	**Normalized pdf**
$K^L_{\mu 3}$ decays KTeV, NA48, KLOE	$\varepsilon_S - \varepsilon_{NS}$	**$K\pi$ scattering phases** [Buettiker, Descotes, Moussallam' 02] **and Δ_{CT}: χPT**

But also: Heavy quarks, neutral currents, loop effects (CP violation, FCNC….)

Additionnal slide: the LEET



- GB Σ : link $L \longleftrightarrow R$
- Spurions X, Y, ω : link $C \longleftrightarrow E$
- Covariant constraints reducing the symmetry and the physical dof to

$$SU(2) \otimes U(1)_Y \Leftrightarrow \boxed{D_\mu X = D_\mu Y = D_\mu \omega = 0}$$

- $X \sim \xi \quad Y \sim \eta \quad \omega \sim \varsigma$ small expansion parameters :

$$\xi . \eta = \frac{m_{top}}{\Lambda_W} = O(p), \; \varsigma \ll \xi, \eta ... LNV$$