# $K^L_{\mu 3}$ decay: A first evidence of Right-Handed Quark Currents ?

E.Passemar* [a]

[a]Groupe de Physique Théorique, IPN, Université de Paris Sud-XI, F-91406 Orsay, France
email: passemar@ipno.in2p3.fr

The experimental results published by KTeV and the preliminary results from NA48 concerning the slope of the $K\pi$ scalar form factor suggest a significant discrepancy with the prediction of the Callan-Treiman low energy theorem once interpreted within the Standard Model. In this talk, we will show how this discrepancy could be explained as a first evidence of the direct coupling of right-handed quarks to W as suggested by certain type of effective electroweak theories.

## 1. Slope of the scalar $K\pi$ form factor.

The hadronic matrix element associated with $K^0_{\mu 3}$ decay is given by

$$\begin{aligned}&\langle \pi^-(p')|\bar{s}\gamma_\mu u|K^0(p)\rangle = \\ &(p'+p)_\mu\ f_+^{K^0\pi^-}(t) + (p-p')_\mu\ f_-^{K^0\pi^-}(t)\ ,\end{aligned} \tag{1}$$

where $t = (p'-p)^2$. The vector form factor $f_+^{K^0\pi^-}(t)$ represents the P-wave projection of the crossed channel matrix element $\langle 0|\bar{s}\gamma_\mu u|K\pi\rangle$, whereas the S-wave projection is described by the scalar form factor

$$f_S^{K^0\pi^-}(t) = f_+^{K^0\pi^-}(t) + \frac{t}{m_K^2 - m_\pi^2} f_-^{K^0\pi^-}(t)\ . \tag{2}$$

In the sequel we consider the normalized scalar form factor

$$f(t) = \frac{f_S^{K^0\pi^-}(t)}{f_+^{K^0\pi^-}(0)}\quad ,\quad f(0) = 1\ . \tag{3}$$

The experimental measurements usually concern the slope $\lambda_0$ and/or the curvature $\lambda_0'$ of the form factor considering a Taylor expansion

$$f(t) = 1 + \lambda_0 \frac{t}{m_\pi^2} + \frac{1}{2}\lambda_0'\left(\frac{t}{m_\pi^2}\right)^2 + \dots\ . \tag{4}$$

A linear fit leads to the following values of the slope

$$\lambda_0^{lin} = 0.01372 \pm 0.00131 \quad [\text{KTeV [2]}]\ , \tag{5}$$

$$\lambda_0^{lin} = 0.0120 \pm 0.0017 \quad [\text{NA48 [3]}]\ . \tag{6}$$

These have to be compared with the theoretical prediction of the Standard Model (SM). This can be done by matching the dispersive representation of the scalar form factor with the Callan-Treiman (CT) low energy theorem [4] which predicts the value of $f(t)$ at the Callan-Treiman point $t = \Delta_{K\pi} = m_{K^0}^2 - m_{\pi^+}^2$ in the SU(2)×SU(2) chiral limit. One has

$$\mathrm{C} \equiv f(\Delta_{K\pi}) = \frac{F_{K^+}}{F_{\pi^+}}\frac{1}{f_+^{K^0\pi^-}(0)} + \Delta_{CT} \tag{7}$$

where the CT correction, $\Delta_{CT} \sim \mathcal{O}\Big(\frac{m_{u,d}}{4\pi F_\pi}\Big)$, is not enhanced by chiral logarithms or by small denominators arising from the $\pi^0$-$\eta$ mixing in the final state[2]. This correction has been estimated within Chiral Perturbation Theory (ChPT) at next to leading order (NLO) in the isospin limit [5] with the result : $\Delta_{CT}^{NLO} = -3.5\ 10^{-3}$ . (8)

Assuming the SM couplings, the experimental results for the branching ratios (BR) $\mathrm{Br}\big(K^+_{l2}(\gamma)/\pi^+_{l2}(\gamma)\big)$ [6], for $|f_+^{K^0\pi^-}(0)V^{us}|$ [7] and for $V^{ud}$ [8] allow to write

$$\begin{aligned}\mathrm{C_{SM}} &= \left|\tfrac{F_{K^+}V^{us}}{F_{\pi^+}V^{ud}}\right|\tfrac{1}{|f_+^{K^0\pi^-}(0)V^{us}|}|V^{ud}| + \Delta_{CT} \\ &= B_{exp} + \Delta_{CT}\ ,\end{aligned} \tag{9}$$

with $B_{exp} = 1.2440 \pm 0.0039$ . In the following, the relevant quantity will be

$$\ln\mathrm{C_{SM}} = 0.2183 \pm 0.0031 + \Delta_{CT}/B_{exp}\ . \tag{10}$$

Since we know the value of $f(t)$ at two points at low energy: at $t=0$, Eq. (3), and at $t = \Delta_{K\pi}$, Eq. (7), one can write a dispersion relation with two subtractions for $\ln(f(t))$. Assuming that $f(t)$ has no zero, one obtains

$$f(t) = \exp\Big[\frac{t}{\Delta_{K\pi}}(\ln\mathrm{C} - \mathrm{G(t)})\Big]\ ,\quad \text{where} \tag{11}$$

$$\begin{aligned}\mathrm{G(t)} &= \frac{\Delta_{K\pi}(\Delta_{K\pi}-t)}{\pi} \\ &\times \int_{t_{K\pi}}^{\infty}\frac{ds}{s}\frac{\phi(s)}{(s-\Delta_{K\pi})(s-t-i\epsilon)}\ ,\end{aligned} \tag{12}$$

*In collaboration with V.Bernard, M.Oertel, J.Stern [1].

[2]This is not the case for the charged K decay mode where an extra contribution due to the $\pi^0$-$\eta$ mixing in the final state is involved.

$t_{K\pi}$ is the threshold of $\pi K$ scattering and $\phi(t)$ is the phase of $f(t)$. According to Brodsky-Lepage, $f(t)$ vanishes as $\mathcal{O}(1/t)$ for large t [9], implying that $\phi(t) \stackrel{t\mapsto\infty}{\longmapsto} \pi$. G(t) can be decomposed into two parts:
$\mathrm{G}(t) = \mathrm{G}_{K\pi}(\Lambda,t) + \mathrm{G}_{as}(\Lambda,t) \pm \delta\mathrm{G}(t)$. The first part, $\mathrm{G}_{K\pi}(\Lambda,t)$, corresponds to the integration region $t_{K\pi} \le s \le \Lambda$ where the $\pi K$ S-wave is still observed to be elastic ($\Lambda \simeq 2.77$ GeV$^2$ [10], [11]). In this region, $\phi(t)$ equals to the $I = 1/2$ S-wave scattering phase shift according to Watson's theorem. The scattering phase has been inferred from experimental data [10] solving the Roy-Steiner equations [11]. The second part, $\mathrm{G}_{as}(\Lambda,t)$, is the asymptotic contribution to the integral, Eq. (12), for $s > \Lambda$. There, we replace $\phi(t)$ by its asymptotic value $\pi$. We include the possible deviation from this asymptotic estimate into the uncertainty. Thanks to the two subtractions, the integral in Eq. (12) converges very rapidly and $\mathrm{G}_{K\pi}(\Lambda,t)$ dominates. $\delta\mathrm{G}(t)$ contains the two sources of uncertainties arising from the two parts of $\mathrm{G}(t)$ as discussed in details in [1]. The resulting function $\mathrm{G}(t)$ is shown in Fig. 1.

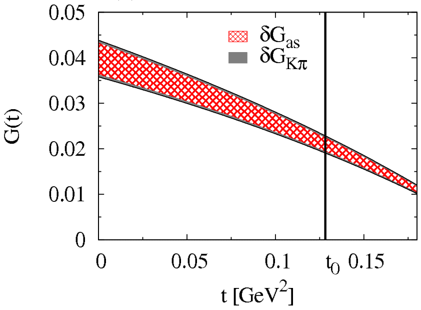


Figure 1. G(t) with the uncertainties $\delta\mathrm{G}_{as}$ and $\delta\mathrm{G}_{K\pi}$ added in quadrature.

Using the exact parametrization, Eq. (11), the linear slope and the curvature are given by

$$\lambda_0 = m_\pi^2\ f'(0) = \frac{m_\pi^2}{\Delta_{K\pi}}(\ln\mathrm{C} - \mathrm{G}(0)) \ , \qquad (13)$$

$$\begin{aligned} \lambda_0' &= m_\pi^4\ f''(0) = \lambda_0^2 - 2\tfrac{m_\pi^4}{\Delta_{K\pi}}\mathrm{G}'(0) \\ &= \lambda_0^2 + (4.16 \pm 0.50) \times 10^{-4} \ . \end{aligned} \qquad (14)$$

Taking the value of $\ln\mathrm{C}_{\mathrm{SM}}$, Eq. (10), we obtain[3]:

$$\lambda_0 = 0.01524 \pm 0.00044 + 0.0686\ \Delta_{CT} \ , \qquad (15)$$

to be compared with the experimental result of KTeV, Eq. (5), and the preliminary one of NA48, Eq. (6). With the estimate of $\Delta_{CT}^{NLO}$, Eq. (8), the KTeV result is still compatible with the theoretical prediction, whereas the NA48 result requires $\Delta_{CT} \le -2.2\ 10^{-2}$, i.e. at least six times larger in absolute value than the estimate of Eq. (8). Moreover these measurements do not take into account the effect of the **positive curvature** $\lambda_0'$, Eq. (14), in a proper way. For this reason, they should be actually interpreted as representing an **upper bound** for $\lambda_0 = m_\pi^2\ f'(0)$, Eq. (15), since the curvature is necessary positive. This, concerning the NA48 result at least, accentuates the discrepancy between the experimental measurements and the SM prediction of $\lambda_0$. The actual value of $\lambda_0$ should be confirmed by the direct measurement of lnC using the exact dispersive parametrization, Eq. (11). This parametrization is very powerful since one parameter, lnC, allows a measurement of both the slope and the curvature of $f(t)$. In this way, one avoids the problem of the strong correlations as shown by Eq. (14) that appears in the extraction of the slope and the curvature using the quadratic parametrization, Eq. (4).

[3]This, in principle, increases the precision of the NLO ChPT result, $\lambda_0 = 0.017 \pm 0.004$ [5].

## 2. A first evidence of right-handed quark currents (RHCs) ?

We now point out how a possible discrepancy between the SM prediction of the linear slope $\lambda_0$ and its measurements could be interpreted as a manifestation of physics beyond the SM. We refer to the framework of the "low energy effective theory" (LEET) developed in [12]. It is constructed by ordering all the vertices invariant under a suitable symmetry group according to their infrared (chiral) dimension $d$: $\mathcal{L}_{eff} = \Sigma_{d\ge2}\mathcal{L}_d$, with the operators that behave as $\mathcal{L}_d = \mathcal{O}(p^d)$ in the low energy (LE) limit $p \ll \Lambda \sim 3$ TeV. The LEET is not renormalized and unitarized in the usual sense, but order by order in the LE expansion. In the LEET, as in other extensions of the SM, the heavy states beyond the SM present at high energy ($E > \Lambda$) decouple. However the higher local symmetries originally associated to them that contain the SM gauge group as a sub-group do not decouple at LE: they survive and become non linearly realized restricting the interaction vertices of $\mathcal{L}_{eff}$. This higher symmetry, $S_{nat}$, can be inferred [12] from the SM itself: $S_{nat}$ is required to select at leading order (LO) ($\mathcal{O}(p^2)$) the higgs-less vertices of the SM and nothing else. The minimal symmetry group that satisfies this condition is[4] $S_{nat} = [SU(2) \times SU(2)]^2 \times U(1)_{B-L} \times Z_2$ . The reduction of this higher symmetry $S_{nat}$ to $SU(2)_L \times U(1)_Y$ is done via spurions [12]. Higher terms in $\mathcal{L}_{eff}$ are suppressed according to their infrared dimension $d$ and the number of spurions that is needed to restore the invariance under $S_{nat}$. At LO ($\mathcal{O}(p^2)$), we recover the SM couplings without a physical scalar with fermions masses generated by spurions. The first and the most important effects of new physics appear at NLO, before the loops and oblique corrections which only arise at NNLO. At NLO, there are only two operators instead of the 80 operators of mass dimension D = 6 characteristic of the usual decoupling scenario. These two operators modify the couplings of fermions to W and Z. The charged current (CC) lagrangian becomes $\mathcal{L}_{CC} = \tilde{g}[l_\mu + \frac{1}{2}\bar{\mathrm{U}}(\mathcal{V}_{eff}\gamma_\mu + \mathcal{A}_{eff}\gamma_\mu\gamma_5)\mathrm{D}]W^\mu + hc$ where $\mathrm{U} = \begin{pmatrix} u \\ c \\ t \end{pmatrix}$, $\mathrm{D} = \begin{pmatrix} d \\ s \\ b \end{pmatrix}$, and $\mathcal{V}_{eff}$, $\mathcal{A}_{eff}$ are complex $3 \times 3$ effective coupling matrices. In the SM, $\mathcal{V}_{eff} = -\mathcal{A}_{eff} = V_{CKM}$ , where $V_{CKM}$ is the unitary flavour mixing matrix, whereas at NLO right-handed quark currents (RHCs) are present. Indeed,

$$\begin{aligned} \mathcal{V}^{ij}_{eff} &= (1+\delta)V^{ij}_L + \epsilon V^{ij}_R + \mathrm{NNLO}\ , \\ \mathcal{A}^{ij}_{eff} &= -(1+\delta)V^{ij}_L + \epsilon V^{ij}_R + \mathrm{NNLO}\ , \end{aligned} \qquad (16)$$

with $V_L$ and $V_R$ two unitary flavor mixing matrices coming from the diagonalization of the mass matrix of U and D quarks; $\delta$ and $\epsilon$ are small parameters originating from spurions which have been estimated [12] of the order of one percent. $l_\mu$ in $\mathcal{L}_{CC}$ stands for the usual V-A leptonic current since the discrete symmetry $Z_2$ forbids leptonic charged RHCs. These new couplings, Eq. (16), affect the reexpression of Eq. (7) in terms of measurable BR leading to $\mathrm{C} = B_{exp}\ r + \Delta_{CT}$ , where $B_{exp}$ has the same value as the one defined in Eq. (9). However in the presence of RHCs, it reads $B_{exp} = \left|\frac{F_K + \mathcal{A}^{us}_{eff}}{F_\pi + \mathcal{A}^{ud}_{eff}}\right| \frac{1}{|f_+^{K^0\pi^-}(0)\mathcal{V}^{us}_{eff}|}|\mathcal{V}^{ud}_{eff}|$ and an additional factor $r$ appears. It is given in terms of RHCs effective couplings

$$r = \left|\frac{\mathcal{A}^{ud}_{eff}\mathcal{V}^{us}_{eff}}{\mathcal{V}^{ud}_{eff}\mathcal{A}^{us}_{eff}}\right| = 1 + 2(\epsilon_S - \epsilon_{NS}) + \mathcal{O}(\epsilon^2)\ , \qquad (17)$$

where

$$\epsilon_{NS} = \epsilon\ \mathrm{Re}\Big(\frac{V^{ud}_R}{V^{ud}_L}\Big),\quad \epsilon_S = \epsilon\ \mathrm{Re}\Big(\frac{V^{us}_R}{V^{us}_L}\Big) \qquad (18)$$

represent the strengths of $\bar{u}d$ and $\bar{u}s$ RHCs, respectively. Hence Eq. (10) can be rewritten as

$$\ln\mathrm{C} = 0.2183 \pm 0.0031 + \Delta\epsilon\ , \qquad (19)$$

with $\Delta\epsilon = \Delta_{CT}/B_{exp} + 2(\epsilon_S - \epsilon_{NS}) + \mathcal{O}(\epsilon^2)$, a combination of the RHCs couplings, $\Delta\epsilon_0 = 2(\epsilon_S - \epsilon_{NS})$, and the CT correction, $\Delta_{CT}/B_{exp}$. As mentioned before, the experimental measurements published so far only give an upper bound for $\lambda_0$ and hence for lnC and for $\Delta\epsilon$. Comparing the experimental results of $\lambda_0$, Eq. (5) and Eq. (6), with Eq. (13) and using Eq. (19), we obtain an upper bound estimate for $\Delta\epsilon$[5]:

$$\Delta\epsilon_{max} = -0.0178 \pm 0.0161 \quad [\mathrm{KTeV}]\ , \qquad (20)$$

$$\Delta\epsilon_{max} = -0.0379 \pm 0.0205 \quad [\mathrm{NA48}]\ . \qquad (21)$$

Hence the result coming from the KTeV measurement, Eq. (20), can still be interpreted, to a certain extend, within the SM with $\Delta\epsilon = \Delta^{NLO}_{CT}/B_{exp}$. The result coming from the measurement of NA48, Eq. (21), seems more difficult to interpret as a pure CT correction, since that would require $|\Delta_{CT}| \geq 6\ |\Delta^{NLO}_{CT}|$. A non zero value of RHCs, $\Delta\epsilon_0$, can provide an explanation. Using the published experimental measurements based on the linear parametrization of $f(t)$, we can visualize the effect of RHCs. For this purpose, we define the effective slope $\lambda_{eff}(t) = \frac{m_\pi^2}{t}\Big[f(t) - 1\Big]$. For each fixed t, $\lambda_{eff}(t)$ is a function of lnC Eq. (11) or of $\Delta\epsilon$. For the extreme cases $t = 0$ and $t = t_0 = (m_{K^0} - m_{\pi^+})^2$, these two curves are displayed in Fig. 2 together with the range of $\lambda_0$ given by the NA48 measurement. Since $\lambda_{eff}(t)$ is increasing with $t$, the measured value $\lambda^{lin}_0$ is between $\lambda_{eff}(0) = m_\pi^2\ f'(0)$ and $\lambda_{eff}(t_0)$. Consequently the true value of $\Delta\epsilon$ should be somewhere in the gray stripped region

[4] The discrete symmetry $Z_2$ ($\nu_R \to -\nu_R$) forbids the Dirac masses of neutrinos and at the same time the leptonic charged RHCs. Consequently the stringent constraints, which come from polarization measurements in $\mu$, $\tau$ and $\beta$ decays and occur in left-right symmetric models, are automatically satisfied.

[5] The resulting uncertainty is the quadratic sum of the uncertainties on $\lambda^{lin}_0$, on $B_{exp}$ and on G(t).

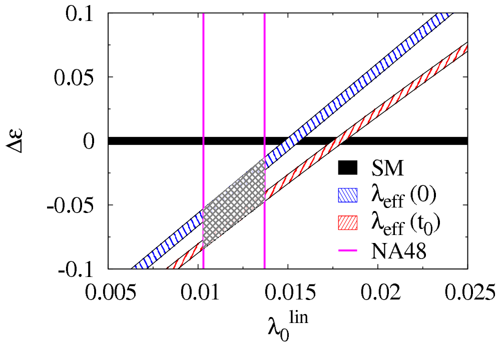


Figure 2. Impact of NA48 data on RHCs. Horizontal line: SM, $\Delta\epsilon = \Delta_{CT}^{NLO}/B_{exp} = \pm\ 0.0028$, vertical lines: NA48 measurement of $\lambda_0$. Top stripped curve: $\lambda_0^{lin} = \lambda_{eff}(0)$ and bottom stripped curve: $\lambda_0^{lin} = \lambda_{eff}(t_0)$ with uncertainties from BR and from G(t) added in quadrature.

shown in Fig. 2 suggesting $\Delta\epsilon = -0.05 \pm 0.03$. A similar figure in the case of KTeV can be found in [1] and leads to $\Delta\epsilon = -0.03 \pm 0.03$. One should wonder whether such a "large" effect of RHCs can be generated by genuine spurions parameters $\delta$ and $\epsilon$ of the size of one percent [12]. Since the left-handed mixing matrix is very close (equal at LO) to the CKM matrix, we can neglect the $V_L^{ub}$ element. Hence $V_L^{ud}$ is of order one and $V_L^{us} \sim 0.22$. As the unitarity of $V_R$ forces $|V_R^{ud}| \leq 1$, $|\epsilon_{NS}|$ can hardly exceed $\epsilon$ and therefore $|\epsilon_{NS}| \sim \epsilon \sim 1\%$. On the other hand, because $V_L^{us}$ is suppressed, $\epsilon_S$ is enhanced unless $V_R^{us}$ is suppressed too. If the hierarchy in the right-handed sector is inverted, then $|\epsilon_S|$ can reach a value of order 4.5 $\epsilon$ and thus $|\Delta\epsilon_0|$ could be as large as nine percent.

## 3. Conclusion.

In this talk, we have pointed out a possible discrepancy between the SM and the experimental measurements of the slope of the scalar form factor in $K_{\mu 3}^L$ decay. In order to establish this discrepancy more precisely, we need an accurate direct measurement of lnC[6], avoiding the ambiguity attached to the slope measurement based on the use of theoretically flawed assumptions. If this disagreement with the SM is confirmed, it could be interpreted in the framework of the LEET as a manifestation of physics beyond the SM by direct right-handed couplings of fermions to W. These new couplings, if they exist, have to appear in every process involving charged currents[7]. However they are not easy to disentangle from the extraction of the fundamental observables of QCD at low energy (form factors, $\alpha_S$, quark masses...). There are not many processes beyond the one discussed in this talk in which the possible enhancement of $\epsilon_S$ could be tested. It is not the case for the hadronic $\tau$ decays, the $\nu$ ($\bar{\nu}$) DIS off valence quarks or $K^0 - \bar{K}^0$ mixing. Constraints arising from the CP violation sector could be interesting to study in this framework.

[6]This could allow a matching with the two loops computation of $K_{l3}$ and the first model independent extraction of $V^{us}$.

**Acknowledgements :** I would like to thank the organizers for this enriching conference.

## REFERENCES

1. V. Bernard, M. Oertel, E. Passemar and J. Stern, Phys. Lett. B **638** (2006) 480
2. T. Alexopoulos *et al.* [KTeV Collaboration], Phys. Rev. D **70** (2004) 092007.
3. A. Winhart [NA48 Collaboration], PoS HEP2005 (2006) 289; R. Wanke [NA48 collaboration], talk given at ICHEP06, Moscow.
4. R. F. Dashen and M. Weinstein, Phys. Rev. Lett. **22** (1969) 1337.
5. J. Gasser and H. Leutwyler, Nucl. Phys. B **250** (1985) 517.
6. M. Jamin, J. A. Oller and A. Pich, arXiv:hep-ph/0605095.
7. T. Alexopoulos *et al.* [KTeV Collaboration], Phys. Rev. Lett. **93** (2004) 181802; A. Lai *et al.* [NA48 Collaboration], Phys. Lett. B **602** (2004) 41; F. Ambrosino *et al.* [KLOE Collaboration], Phys. Lett. B **632** (2006) 43.
8. W. J. Marciano and A. Sirlin, Phys. Rev. Lett. **96** (2006) 032002.
9. G. P. Lepage and S. J. Brodsky, Phys. Lett. B **87** (1979) 359.
10. D. Aston *et al.*, Nucl. Phys. B **296** (1988) 493.
11. P.Buettiker, S.Descotes-Genon and B. Moussallam, Eur. Phys. J. C **33** (2004) 409.
12. J. Hirn and J. Stern, Phys. Rev. D **73** (2006) 056001; J. Hirn and J. Stern, Eur. Phys. J. C **34** (2004) 447.
13. V. Bernard, M. Oertel, E. Passemar, and J. Stern, in preparation.

[7]For a complete analysis of CC interactions see [13].