Effective Theory Description of Weak Annihilation in $B \to X_u \ell \nu$ Decays

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The semileptonic $B \to X_u \ell \nu$ decays allow a pretty clean determination of the CKM matrix element $|V_{ub}|$. Nevertheless, the presence of weak annihilation effects near the end-point of the q^2 spectrum introduces uncertainties in the inclusive calculation, requiring the use of non-perturbative techniques like heavy meson chiral perturbation theory and large N_C limit.

1. Introduction

The $B_q \sim b\bar{q}$ meson decay through the inclusive semileptonic channel $B_q \to X_u \ell \nu$ has been shown to be a very clean way to determine $|V_{ub}|$ [2].

The inclusive B_q analysis makes use of the operator product expansion (OPE) [2,3]. The first difference between the B_u^- and $B_{d,s}^0$ decays arises at next-to-next-to-next-to-leading order (N^{3}LO) in the $1/m_b$ expansion. It is due to the topologies where the incoming $b\bar{q}$ pair annihilates into a W–boson (Fig. (1)), giving away an infinitesimal amount of momentum through soft gluon radiation. These contributions are known as weak-annihilation (WA) [4]. They appear as a delta function at the end-point of the spectrum $q^2 = (p_\ell + p_\nu)^2 = m_b^2$. Their determination is rather relevant if the uncertainty on $|V_{ub}|$ is to be reduced up to a few per cent [2].

In order to better isolate the WA, the analysis is carried on within the chiral limit $m_q = 0$.

2. OPE and Weak-Annihilation

At $\mathcal{O}(1/m_b^3)$, the WA in the OPE is given in terms of the bag constants $B_{_{WA}} \equiv B_2 - B_1$ [5]:

$$\Gamma(B_q \to X_u \ell \nu) = ... + \frac{G_F^2\,|V_{ub}|^2}{12\,\pi}\, f_B^2\, m_b^3\, B^q_{_{WA}}\,, \quad (1)$$

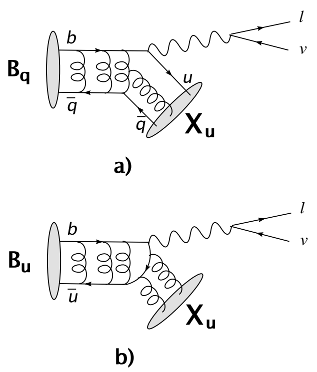

Figure 1. Examples of a) non-annihilating topologies; b) WA diagrams.

with $B^q_{_{WA}} = 0$ in the factorisation limit, independently of the light quark in the B_q meson ($B_2^{d,s} = B_1^{d,s} = 1$ and $B_2^u = B_1^u = 0$). The expected 10% deviations from the factorisation limit provide an estimate of the WA contribution to the branching ratio (BR) [5]:

$$\delta\mathcal{B}_{B_q\to X_u\ell\nu} = 3.9 \times \left(\frac{f_B}{0.2\text{ GeV}}\,|V_{ub}|\right)^2 \left(\frac{B^q_{_{WA}}}{0.1}\right). \quad (2)$$

Other theoretical and experimental estimates provide similar values [6,7].

The OPE calculation also points out relevant aspects about the nature and the scalings of the WA contributions:

–Constant contribution to the width, generated at the end-point, in the non-perturbative region of low hadronic energies $E_X \lesssim 1$ GeV [4].

–The WA operators appear at N^{3}LO in the heavy mass expansion, so their contributions to

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the BR is $\mathcal{O}\left(\Lambda^3_{\rm QCD}/m_b^3\right)$.

–The WA topologies introduce an extra quark loop in the hadronic tensor $T^{\mu\nu}$, being suppressed by $1/N_C$ in the large N_C limit [8].

–The WA $B^- - B^0$ difference is related to the production of chiral singlets in the final state X_u (see Fig. (1)). In the chiral limit, WA is the responsible of the difference between the B^+ and the B^0 widths.

–At N^{3}LO in $1/m_b$ there are other non-WA terms in the OPE (independent of q in B_q) that need to be taken into account and properly disentangled [6].

3. Cut-off on the hadronic energy E_X

In order to isolate the WA effects and remove the $B \to X_c \ell\nu$ background we consider the hadronic energy cut-off $E_X < \Lambda_E$.

Multi-pion production is found to be suppressed both experimentally [9] and theoretically. The analysis of the $B \to \pi\pi\ell\nu$ channel through Heavy Meson Chiral perturbation Theory (HMχPT) [10] finds it widely suppressed with respect to $B \to \pi\ell\nu$ for $\Lambda_E \lesssim 1$ GeV [1].

Hence, only the lightest pseudo-scalar meson production $B \to P\ell\nu$ is considered here. The decay into light vector mesons $B \to V\ell\nu$ introduces small corrections for $\Lambda_E \lesssim 1$ GeV and it can be neglected in first approximation [1].

In the $B_q \to P\ell\nu$ mode, one has $P_X^2 = m_P^2$ and $E_X = E_P$, with the phase-space given by

$$\frac{1}{2}\left(z - \sqrt{z^2 - 4\,r^2}\right) \le \bar{x} \le \frac{1}{2}\left(z + \sqrt{z^2 - 4\,r^2}\right)\,,$$
$$2\,r \quad \le z \le \quad \epsilon\,, \tag{3}$$

where $z = \frac{2E_P}{M_B}$, $\bar{x} = 1 - \frac{2E_\ell}{M_B}$, $\epsilon = \frac{2\Lambda_E}{M_B}$ and $r = \frac{m_P}{M_B}$.

The decay is determined by the matrix element

$$\langle\, P|\bar{u}\gamma^\mu(1-\gamma_5)b|B\,\rangle = (p_B + p_P)^\mu\, f_+ + q^\mu\, f_-\,, \tag{4}$$

although, for $m_\ell = m_\nu = 0$, only the form factor $f_+(z)$ contributes to the width:

$$\frac{1}{\Gamma_0}\,\frac{d\Gamma}{dz} \;=\; |f_+(z)|^2 \;\cdot\; \left[z^2 \,-\, 4\,r^2\right]^{\frac{3}{2}}\,. \tag{5}$$

with $\Gamma_0 = \frac{G_F^2 |V_{ub}|^2 M_B^5}{192\pi^3}$. Integrating the energy on the range $2r \le z \le \epsilon$ provides the corresponding contribution to the width $\Delta\Gamma\left(\epsilon, r_\epsilon\right)$ from this part of the phase-space. It only depends on ϵ and the ratio $r_\epsilon = \frac{2\,r}{\epsilon} = \frac{m}{\Lambda_E}$.

4. HMχPT and large N_C limit

To compute $f_+(z)$ and $\Delta\Gamma\left(\epsilon, r_\epsilon\right)$ we make use of HMχPT. At large N_C, the meson loops are suppressed. The $B \to P\ell\nu$ amplitude is dominated the B^* pole from the tree-level exchange. Its description is provided by the leading HMχPT lagrangian [10]:

$$|f_+(z)|^2_{B_q\to P} \;=\; D_{B_q\to P} \;\cdot\; \frac{f_B^2\, g^2}{2\,F_\pi^2}\;\frac{1}{z^2}\,, \tag{6}$$

with the group factors $D_{B_d\to\pi^+} = D_{B_s\to K^+} = 1$, $D_{B_u\to\pi^0} = \frac{1}{2}$, $D_{B_u\to\eta_8} = \frac{1}{6}$, $D_{B_u\to\eta_0} = \frac{1}{3}$. f_B and $F_\pi \simeq 92.4$ MeV are the B and π decay constants, respectively, and g is the $B\pi B^*$ coupling [10].

In the large N_C and chiral limits, the pseudoscalars (π, K, η_8, η_0) are massless, so $r_\epsilon = 0$. Therefore, the integrated widths coincide:

$$\Delta\Gamma(\epsilon)_{B_d^0,\,B_s^0,\,B_u^+\to X_u\ell\nu} \;=\; \frac{f_B^2 g^2}{4\,F_\pi^2}\;\cdot\;\epsilon^2\,, \tag{7}$$

given by the decays $B_d^0 \to \pi^+$, $B_s^0 \to K^+$ and $B_u^- \to \pi^0, \eta_8, \eta_0$. Notice that at large N_C the η_0 forms the ninth chiral Goldstone boson.

The modifications to the interaction vertices and couplings in the form factors (Eq. (6)) are expected to be of the order of $\frac{1}{N_C}$ and $\frac{\Lambda_{\rm QCD}}{m_b}$.

However, the gaining of mass by the η_0 meson is far more important. It generates large modifications in the available phase-space in Eq. (3) and important kinematical corrections, of the order of $\left(\frac{m_{\eta_0}}{\Lambda_E}\right)^2$. The chiral octet (π, K, η_8) remains massless within the chiral limit whereas the chiral singlet η_0 gains a mass $m_{\eta_0} \sim 900$ MeV due to $1/N_C$ corrections [11]. The available phase-space for the $B_u^- \to \eta_0$ mode gets reduced and the integration of the form factor (Eq. (6)) over the available range of z yields

$$\Delta\Gamma(\epsilon, r_\epsilon)_{B_u\to\eta_0} = \frac{1}{3}\,\times\,\frac{f_B^2 g^2}{4\,F_\pi^2}\times\epsilon^2$$
$$\times\left[\left(1+2r_\epsilon^2\right)\sqrt{1-r_\epsilon^2} + \frac{3r_\epsilon^2}{2}\ln\frac{1+\sqrt{1-r_\epsilon^2}}{1-\sqrt{1-r_\epsilon^2}}\right]\,. \tag{8}$$

The other decay modes remain unaffected.

5. Results and conclusions

By observing the $B^0 - B^-$ difference it is possible to separate the WA from other $1/m_b^3$ effects [6]. The subtraction of the integrated widths $\Delta\Gamma(\epsilon)_{B_q \to X_u \ell\nu}$ gives

$$\frac{[\Delta\Gamma_{B^-} - \Delta\Gamma_{B^0}]}{\Gamma_0} = \frac{f_B^2 g^2}{2F_\pi^2}\frac{m_{\eta_0}^2}{M_B^2}\left\{1 + \ln\frac{r_\epsilon^2}{4}\right\} + ... \quad (9)$$

with the dots standing for contributions suppressed by $\left(\frac{m_{\eta_0}}{\Lambda_E}\right)^2$. This result reproduces all the features in the difference $[\Delta\Gamma_{B^-} - \Delta\Gamma_{B^0}]$ that one expects from the OPE:

–Since $f_B^2 \sim 1/M_B$, suppression by $1/M_B^3$.

–Suppression by $1/N_C$: from the scaling $m_{\eta_0}^2 \sim \frac{1}{N_C}$ with respect to the other energy scales.

–The production of chiral singlets in the final state is responsible of the difference (WA topologies in Fig. (1)).

–As Λ_E grows, the difference $\Delta\Gamma_{B^-} - \Delta\Gamma_{B^0}$ tends to a constant up to a residual term $\ln\left(\frac{m_{\eta_0}}{\Lambda_E}\right)$; the effect is non-perturbative and comes from the end-point of the q^2 spectrum. The removal of this residual log needs the consideration of higher B multiplets, in order to recover the $f_+ \sim \frac{1}{E_P^2}$ behaviour of the form factor at high energies [12]. This have been implemented by considering modified-pole expressions [13].

In Fig. (2), we show the dependence of the difference of branching ratios $[\mathcal{B}_{B_d^0} - \mathcal{B}_{B_u^-}]$ on the cut-off, ϵ. One can see how around $\Lambda_E \sim$1 GeV, the difference stabilises and becomes approximately constant (upto the residual log dependence referred to before), reaching the value

$$[\Delta\mathcal{B}_{B_d^0} - \Delta\mathcal{B}_{B_u^-}] \sim 4 \times \left(\frac{f_B}{0.2\ \text{GeV}}\ \frac{g}{0.5}\,|V_{ub}|\right)^2, \quad (10)$$

in agreement with former estimates [5–7].

This WA estimate shows how it is possible to describe these OPE effects through HMχPT, recovering all the expected features.

The next step is the calculation of the $B \to V\ell\nu$ contribution. The residual $\ln(r_\epsilon)$ dependence in Eq. (9) is expected to be removed through modified pole expressions [13]. The relevance of this effect and the proposed techniques should be taken into account in the analyses of D decays, where WA will be more important.

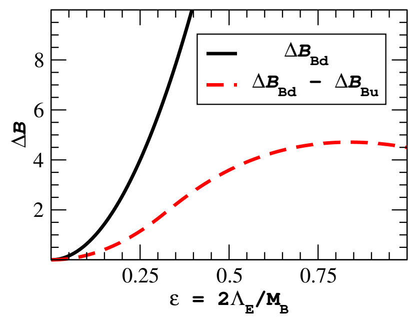

Figure 2. Dependence of the difference of branching ratios $[\Delta\mathcal{B}_{B_d^0} - \Delta\mathcal{B}_{B_u^-}]$ on the cut-off ϵ (dashed curve). For comparison, it is shown together with $\Delta\mathcal{B}_{B_d^0}$ (solid). Expressions are from Eqs. (7)–(9) and $\Delta\mathcal{B}$ is expressed in units of $\left(\frac{f_B}{0.2\ \text{GeV}}\ \frac{g}{0.5}\,|V_{ub}|\right)^2$.

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