# Review on Chiral Perturbation Theory

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- Introduction /CHPT
- $K \to 3\pi$/CP asymmetry ($\pi\pi$ scattering lenghts)
- $K_S \to \gamma\gamma$
- $K_L \to \pi^0\gamma\gamma / K^+ \to \pi^+\gamma\gamma$
- $K \to \pi\pi\gamma$
- Conclusions

# Chiral Perturbation Theory

$\chi PT$ effective field theory based on the two assumptions

- $\pi$'s are the Goldstone boson of $SU(3)_L \otimes SU(3)_R \to SU(3)_V$
- *(chiral) power counting* i.e. the theory has a small expansion parameter: $p^2/\ \Lambda^2_{\chi SB}$: $\Lambda_{\chi SB} \sim 4\pi F_\pi \sim 1.2$ GeV

$$\mathcal{L}_{\Delta S=0} = \mathcal{L}^2_{\Delta S=0} + \mathcal{L}^4_{\Delta S=0} + \cdots = \frac{F_\pi{}^2}{4} \overbrace{\langle D_\mu U D^\mu U^\dagger + \chi U^\dagger + U \chi^\dagger \rangle}^{\pi \to l\nu,\ \pi\pi \to \pi\pi,\ K \to \pi..} + \overbrace{\sum_i L_i O_i}^{K \to \pi..} + \cdots$$

Fantastic chiral prediction $A_{\pi\pi} \sim (s - m_\pi^2)/F_\pi{}^2$ Weinberg, Colangelo *et al*

$$\mathcal{L}_{\Delta S=1} = \mathcal{L}^2_{\Delta S=1} + \mathcal{L}^4_{\Delta S=1} + \cdots = G_8 F^4 \underbrace{\langle \lambda_6 D_\mu U^\dagger D^\mu U \rangle}_{K \to 2\pi/3\pi} + \underbrace{G_8 F^2 \sum_i N_i W_i}_{K^+ \to \pi^+ \gamma\gamma,\, K \to \pi l^+ l^-} + \cdots$$

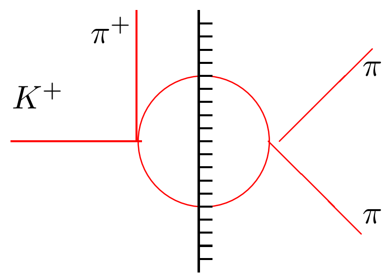
# $K \to 3\pi$: **slope asymm.** $\Delta g/2g = (g_+ - g_-)/(g_+ + g_-)$

- Isospin decomposition, rescattering properties

$$\Downarrow$$

- $A(K^+ \to \pi^+\pi^+\pi^-) \quad = \quad a e^{i\alpha_0} + b e^{i\beta_0} Y + \mathcal{O}(Y^2)$

Final State Interaction




Zeldovich,Grinstein et al
Isidori,Maiani,Pugliese

Compared to $K \to \pi\pi$

- two $\Delta I = 1/2$ transitions ($a$, $b$)
- final state small ( $\alpha_0, \beta_0 \sim 0.1$ )

- CPT relates

$$\begin{aligned} A(K^+ \to \pi^+\pi^+\pi^-) &= a e^{i\alpha_0} + b e^{i\beta_0} Y \\ A(K^- \to \pi^-\pi^-\pi^+) &= a^* e^{i\alpha_0} + b^* e^{i\beta_0} Y \end{aligned}$$

- The asymmetry

  can be evaluated in CHPT

$$\frac{g_+ - g_-}{g_+ + g_-} = \left[\frac{\Im b}{\Re b} - \frac{\Im a}{\Re a}\right] \sin(\alpha_0 - \beta_0) \,,$$

- Only one operator at $\mathcal{O}(p^2)$:

$$G_8 e^{i\phi} \langle \lambda_6 \partial_\mu U \partial^\mu U^\dagger \rangle$$

  $\phi$ $\Delta I = 1/2$ electroweak phase

- Then if we stop at $\mathcal{O}(p^2)$

$$\frac{\Im b}{\Re b} = \frac{\Im a}{\Re a} \Rightarrow \Delta g/2g = 0$$

- However $\mathcal{O}(p^4)$ is necessary in order to reproduce the phenomenological values $\frac{\Delta a}{a} \sim \frac{\Delta b}{b} \sim 30\%$

$$\Downarrow$$

- splitting $a = a^{(2)} + a^{(4)}$ and $b = b^{(2)} + b^{(4)}$

$$\frac{\Delta g}{2g} = \frac{\Im A^0}{\Re A^0} (\alpha_0 - \beta_0) \left( \frac{\Re b^{(4)}}{\Re b^{(2)}} - \frac{\Im b^{(4)}}{\Im b^{(2)}} + \frac{\Im a^{(4)}}{\Im a^{(2)}} - \frac{\Re a^{(4)}}{\Re a^{(2)}} \right)$$

G.D.,Isidori,Paver

- Since
  - $|\frac{\Im A^0}{\Re A^0}| \sim 22\epsilon' \sim 10^{-4}$ $(\alpha_0 - \beta_0) \sim 0.1$
  - to maximize $\Delta g$, we take $\mathcal{O}(p^4) \sim \mathcal{O}(p^2)$

- $\Delta g / 2g \leq 10^{-5}$ NA48 $(1.5 \pm 2.9) \cdot 10^{-4}$

# New Physics

- New Physics: crucial to have large $\Delta g/2g$ is to find an operator which affects $K \to 3\pi$ but not $K \to 2\pi$, limited by the experimental size of $\epsilon'$
- In fact Masiero- Murayama explanation of a possible large size of $\epsilon'$ does the job!

- They suggest that susy (s-)particles, introduce new flavour structures affecting only the $\Delta S = 1$ and not $\Delta S = 2$ interactions

$$\mathcal{H}_{\rm mag} = C_g^+ Q_g^+ + C_g^- Q_g^- + {\rm h.c.}$$

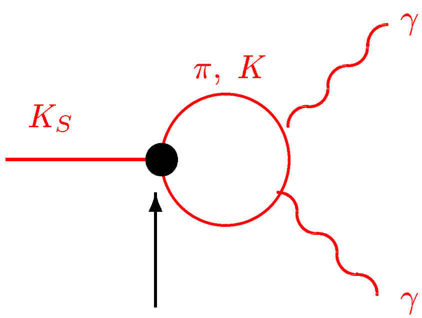
$$Q_g^\pm = \frac{g}{16\pi^2}\left(\bar{s}_L \sigma^{\mu\nu} t^a G^a_{\mu\nu} d_R \pm \bar{s}_R \sigma^{\mu\nu} t^a G^a_{\mu\nu} d_L\right)$$

- $Q_g^+$ is affects only $K \to 3\pi$; $Q_g^-$ only $K \to 2\pi$ G.D,Isidori,Martinelli

- As a result by tuning properly $C_g^\pm$ we can generate large $\Delta g/2g$ $(\leq 10^{-4})$

# $K_S \to \gamma\gamma$

- No short distance contributions, No $O(p^2)$
- Neutral particles $(K_S) \Rightarrow$ No $O(p^4)$ CT : $F_{\mu\nu}F^{\mu\nu}\langle\lambda_6 QU^+QU\rangle$
- Loop contribution finite scale independent and unambigous $\chi$PT prediction




$G_8$ from $K_S \to \pi\pi$

Br$_{\chi\mathrm{PT}}(K_S \to \gamma\gamma)$=2.1·$10^{-6}$

(G.D. and Espriu 86, Goity 87)

$(2.78 \pm 0.072) \cdot 10^{-6}$ (NA48 '02)

- $O(p^6)_{\rm CT}$ $F^{\mu\nu}F_{\mu\nu}\langle\lambda_6 Q^2\mu M U^+\rangle$

  No VMD $\Rightarrow \dfrac{A^{(6)}}{A^{(4)}} \sim \dfrac{m_K^2}{(4\pi F_\pi)^2} \sim 0.2$

  (No terms $\sim \dfrac{m_K^2}{m_\rho^2} \sim 0.4$)

- NA48 $\Rightarrow \dfrac{A^{(6)}}{A^{(4)}} \sim 15\%$

  $\Downarrow$

- The error in the amplitude, is smaller than the naive expectation, 20-30%

## $K_L(p) \to \pi^0(p_3)\gamma(q_1)\gamma(q_2)$

Lorentz + gauge invariance $\Rightarrow$ $M \sim$ $A(y,z)$ $B(y,z)$

| | $A(y,z)$ | $B(y,z)$ |
|---|---|---|
| | $\gamma\gamma$ | $\gamma\gamma$ |
| | $J=0$ | D − wave too |
| | $F^{\mu\nu}F_{\mu\nu}$ | $F^{\mu\nu}F_{\mu\lambda}\partial_\nu K_L \partial^\lambda \pi^0$ |

$y = p\cdot(q_1-q_2)/m_K^2$, $z=(q_1+q_2)^2/m_K^2$

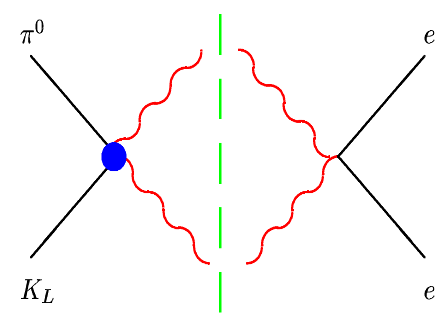
$r_\pi = m_\pi/m_K$

- $\frac{d^2\Gamma}{dydz} \sim z^2|A+B|^2 + \left(y^2 - \left(\frac{(1+r_\pi^2-z)^2}{4} - r_\pi^2\right)\right)^2 |B|^2$ $\quad S, B$
- Different gauge structure $\Rightarrow$ B$\neq$0 at $z \to 0$ (collinear photons).

Crucial role in $K_L \to \pi^0 e^+ e^-$

A suppressed by $m_e/m_K$

B is not




Morozumi et al, Flynn Randall

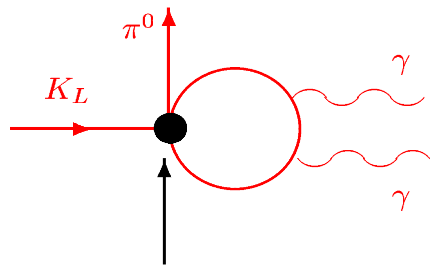
Sehgal Heiliger, Ecker et al., Donoghue et al.

# $K_L \to \pi^0 \gamma\gamma$

- $O(p^4)$ Ecker, Pich, de Rafael; Cappiello,G.D

CT Loop

0




only A **But** $\frac{\Gamma(K_L \to \pi^0 \gamma\gamma)_{p_4}}{\Gamma(K_L \to \pi^0 \gamma\gamma)_{\exp}} \sim \frac{1}{2.5}$

$G_8$ from $K_S \to 2\pi$

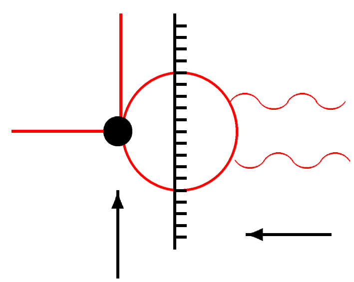
- $O(p^6)$ A, B from:

3 CT′s

$F_{\mu\nu} F^{\mu\alpha} \partial_\alpha K_L \partial^\nu \pi^0$

$F^2 \; \partial K_L \partial \pi^0$

$F^2 \; m_K^2 K_L \pi^0$



Cappiello, G.D., Miragliulo
Cohen, Ecker, Pich

Full description of unitarity cut

$$A(K \to 3\pi) = a + b\, Y + c\, Y^2 + d\, X^2$$

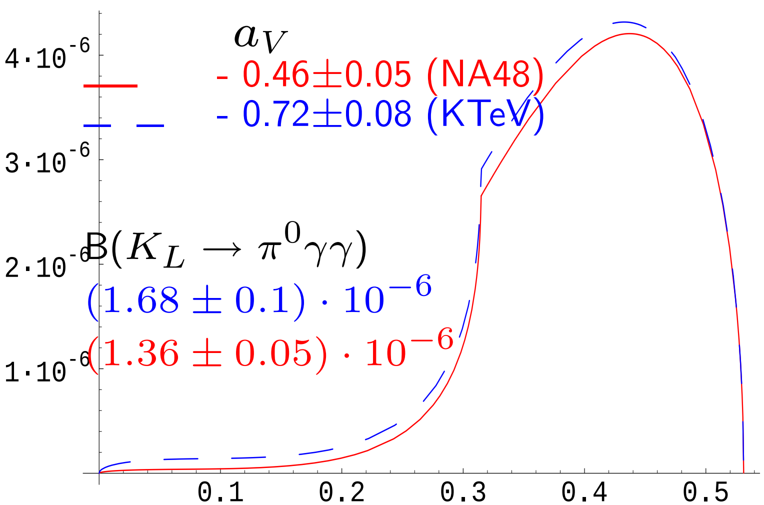
$$A_{\rm CT} = \alpha_1(z - r_\pi^2) + \alpha_2$$
$$B_{\rm CT} = \beta$$

VMD $\Rightarrow$ 1 coupling $a_V$ $\quad(\sim -0.6$ G.D., Portoles$)$

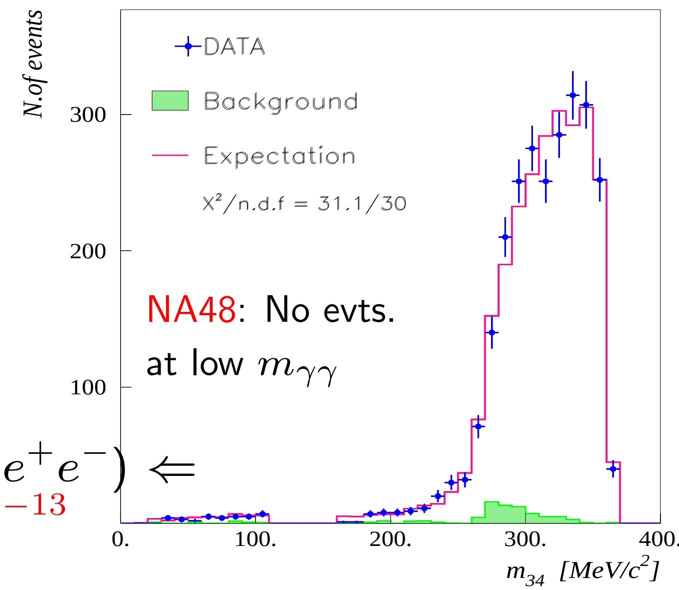
(Ecker, Pich, de Rafael;Sehgal et al.)

$$\alpha_1 = \frac{\beta}{2} = -\frac{\alpha_2}{3} = -4a_V \sim 2 \qquad \text{n.d.a.}\sim 0.2$$

- KTeV and NA48: 1 parameter fit $(a_V)$ with all the unitarity corrections




$B(K_L \to \pi^0 e^+ e^-) \Leftarrow$
$< 5 \cdot 10^{-13}$



# $K^+ \to \pi^+\gamma\gamma$

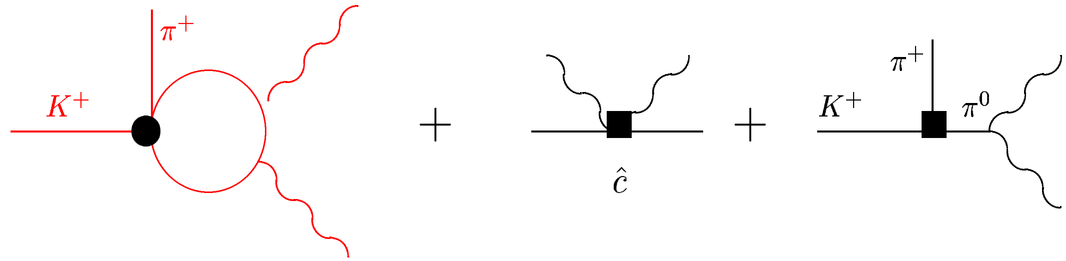
$$\gamma\gamma \quad \text{in} \quad \overbrace{\begin{matrix} F_{\mu\nu}F^{\mu\nu} & F\tilde{F} \\ P=+1 & P=-1 \\ A & C \end{matrix}}^{J=0} \qquad \begin{matrix} J=2 \\ \\ \\ B \end{matrix} \quad + \ \ldots$$

Lorentz + gauge invariance

$$\frac{d^2\Gamma}{dydz} \sim \Big[z^2(|A+B|^2+|C|^2) + \left(y^2 - \left(\frac{(1+r_\pi^2-z)^2}{4} - r_\pi^2\right)\right)^2 |B|^2\Big]$$

# $K^+ \to \pi^+\gamma\gamma$

- $O(p^4)$




Ecker, Pich, de Rafael

In factorization $\hat{c} = 2.3(1 - 2k_f)$

spin-1 contributions (axials)

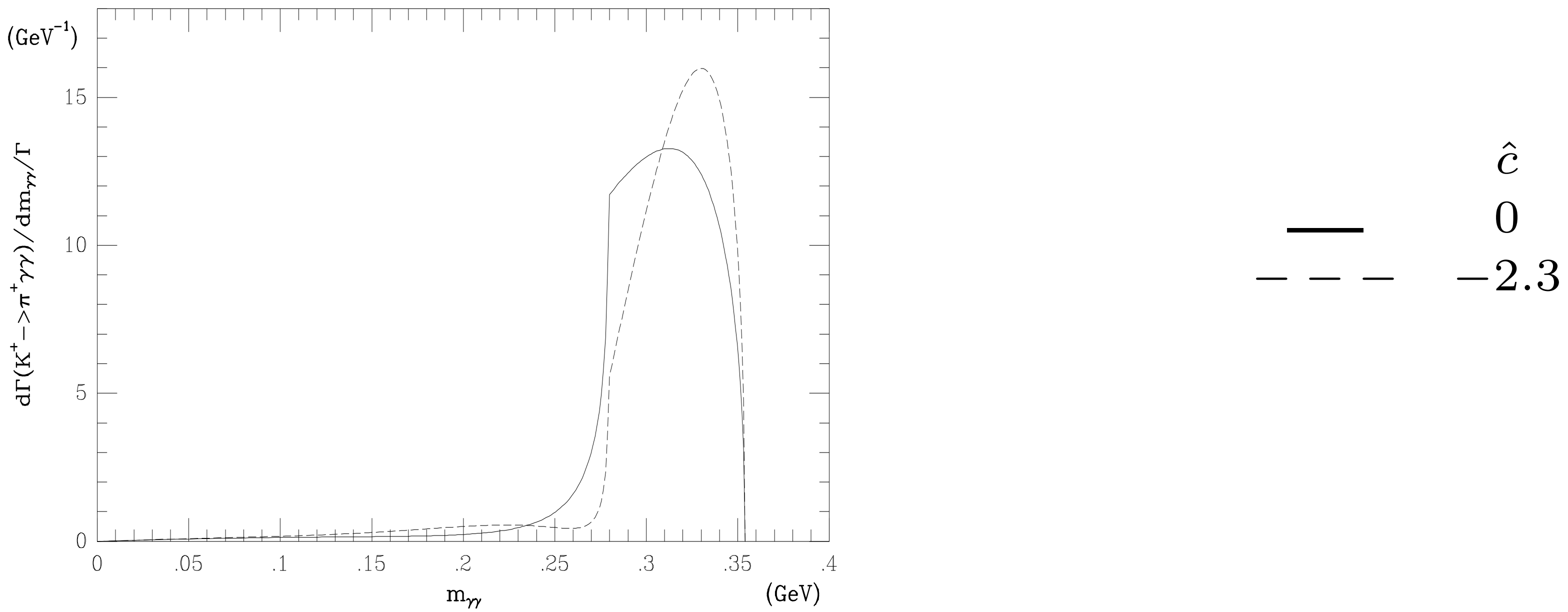
to $\hat{c}$

- $O(p^6)$ G.D., Portoles 96

Unitarity corrections: 30%-40%

$a_{V+}$ negligible

# $K^+ \to \pi^+\gamma\gamma$




BNL 787 (96) got 31 events:
i) confirm $O(p^6)$
ii) Br$\sim$ $(6 \pm 1.6) \cdot 10^{-7}$
iii) $\hat{c} = 1.8 \pm 0.6$

E949 no events at low $m_{\gamma\gamma}$, work for NA48/2, see Smith

$$K(p_K) \to \pi(p_1)\pi(p_2)\gamma(q)$$

- Lorentz + gauge invariance $\Rightarrow$ Electric ($E$) and Magnetic($M$) amplitude

$$A(K \to \pi\pi\gamma) = F^{\mu\nu}\left[E\partial_\mu K\partial_\nu\pi + M\varepsilon_{\mu\nu\rho\sigma}\partial^\rho K\partial^\sigma\pi\right]$$

- Unpolarizated photons

$$\frac{d^2\Gamma}{dz_1 dz_2} \sim |E|^2 + |M|^2$$

$$|E^2| = |E_{IB}|^2 + 2Re(E_{IB}^* E_D) + |E_D|^2$$

$$\downarrow$$

Low Theorem $\Rightarrow E_{IB} \sim \dfrac{1}{E_\gamma^*} +$ c $\qquad$ $E_D$, $M$ chiral tests

# We need FIGHT DE/IB$\sim 10^{-3}$

| | $IB$ | $DE_{exp}$ | |
|---|---|---|---|
| $K_S \to \pi^+\pi^-\gamma$ | $10^{-3}$ | $< 9 \cdot 10^{-5}$ | $E1$ |
| $K^+ \to \pi^+\pi^0\gamma$ | $10^{-4}$ $(\Delta I = \frac{3}{2})$ | $(0.44 \pm 0.07)10^{-5}$ PDG | $M1$,$E1$ |
| $K_L \to \pi^+\pi^-\gamma$ | $10^{-5}$ (CPV) | $(2.92 \pm 0.07)10^{-5}$ KTeVnew | $M1$, VMD |

CPV is only from IB $K_L$ (also measured in $K_L \to \pi^+\pi^-e^+e^-$)

BUT IB suppressed in $K^+$ and $K_L$.

## $K_L \to \pi^+\pi^-\gamma$

M1 transtions clearly measured KTeV (00) with large slope

form factor measured

$$\mathcal{F} = 1 + \frac{a}{1 - \frac{m_k^2}{m_\rho^2} + \frac{2m_K E_\gamma^*}{m_\rho^2}}$$

$E_\gamma^*$ photon energy

KTeV:

- a$=-1.243 \pm 0.057$
- 

| | linear slope | quadratic slope | $\mathcal{F}$ |
|---|---|---|---|
| $\chi^2/DOF$ | $43.2/27$ | $37.6/26$ | $38.8/27$ |

$\Rightarrow$ Large VMD: $\rho-$pole

$$p^4$$

$$\text{Theory} \quad M1 \sim \quad a_2 + 2a_4 \quad +h.o.$$

$$\Downarrow$$

Large VMD in the $a_i$. Not automatic in all spin-1 formulations

[ G.D. Portoles, G.D. Gao]

Consistent with M1 in $K^+ \to \pi^+\pi^0\gamma$ and predictive for spectrum (work for NA48/2)

## $K^+ \to \pi^+\pi^0\gamma$

$$A(K \to \pi\pi\gamma) = F^{\mu\nu}\left[E\partial_\mu K\partial_\nu\pi + M\varepsilon_{\mu\nu\rho\sigma}\partial^\rho K\partial^\sigma\pi\right]$$

$E1$ and $M1$ are measured with Dalitz plot

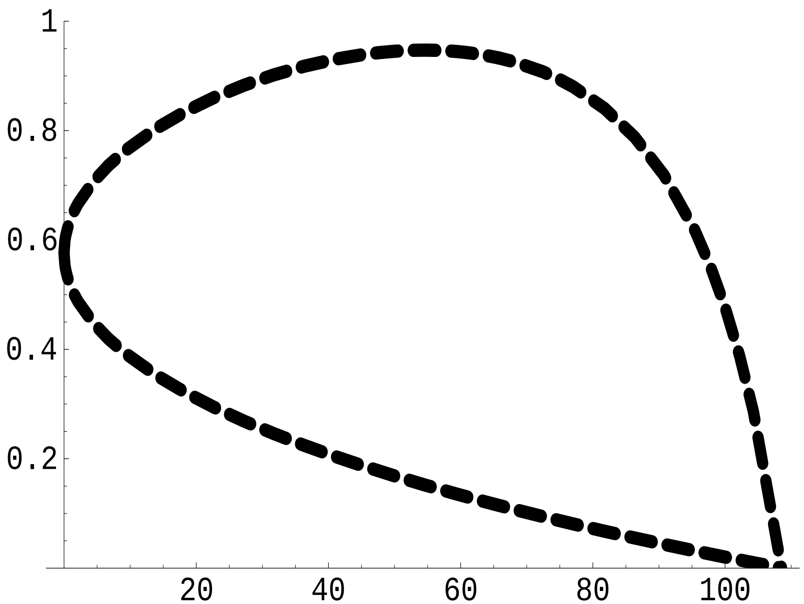
$$\frac{\partial^2\Gamma}{\partial T_c^*\partial W^2} = \frac{\partial^2\Gamma_{IB}}{\partial T_c^*\partial W^2}\left[1 + \frac{m_{\pi^+}^2}{m_K}2Re\left(\frac{E1}{eA}\right)W^2 + \frac{m_{\pi^+}^4}{m_K^2}\left(\left|\frac{E1}{eA}\right|^2 + \left|\frac{M1}{eA}\right|^2\right)W^4\right]$$

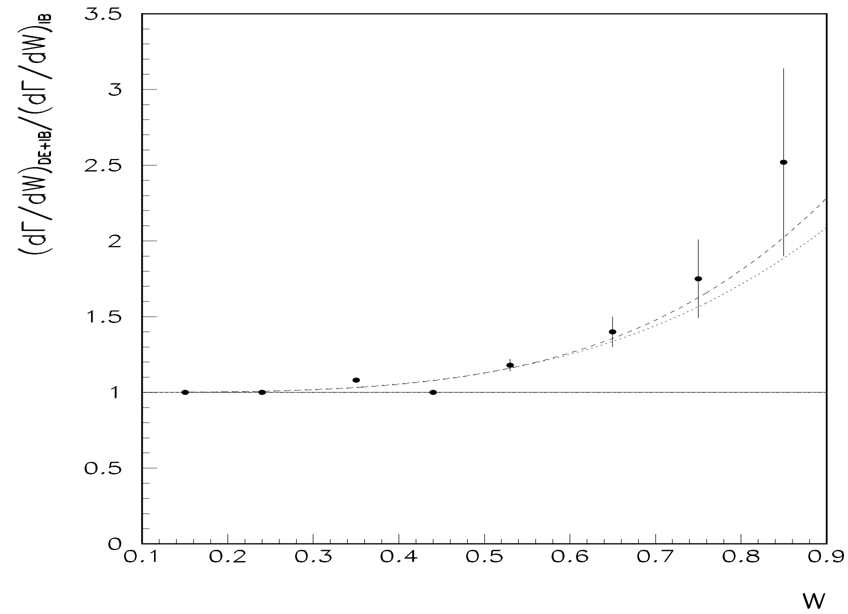
$$W^2 = (q\cdot p_K)(q\cdot p_+)/(m_\pi^2 m_K^2)$$

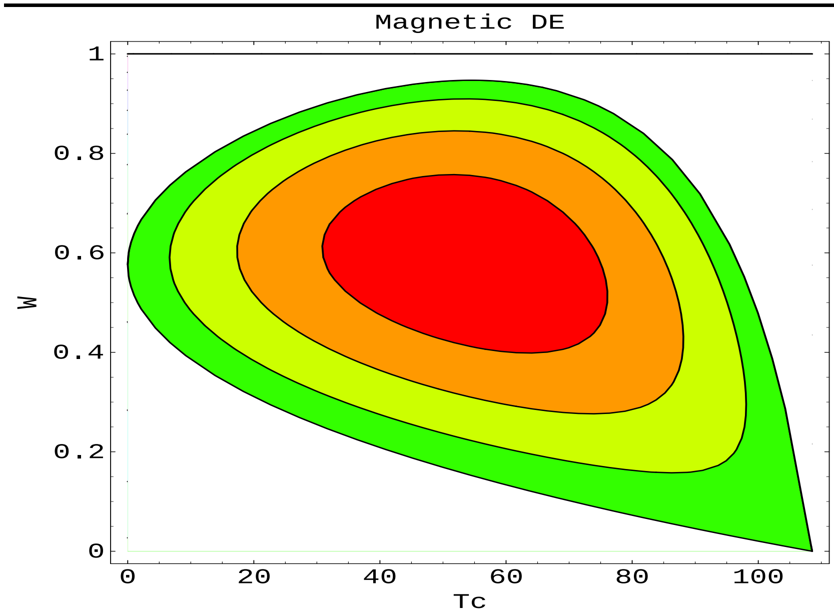
$$A = A(K^+ \to \pi^+\pi^0)$$

# $K^+ \to \pi^+\pi^0\gamma$ $W - T_c$ **Dalitz plot**

Integrating over $T_c$ deviations from IB measured



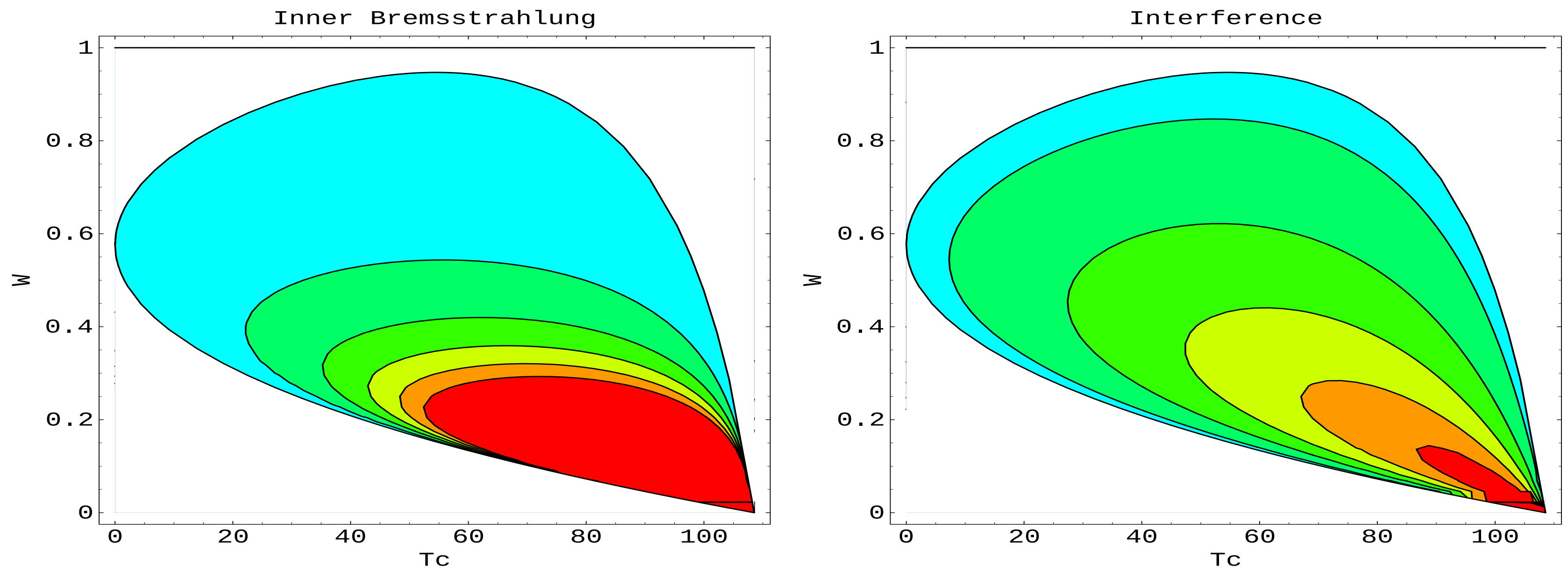



- M1 $\sim$ (-2 + $3a_2 - 6a_3$) -2.5

$\uparrow$ Wess Zumino $\uparrow$ $p^4$ CT's (VMD) $\Downarrow$ $a_i$ small?

Cheng
Bijnens, Ecker, Pich; G.D.,Gao

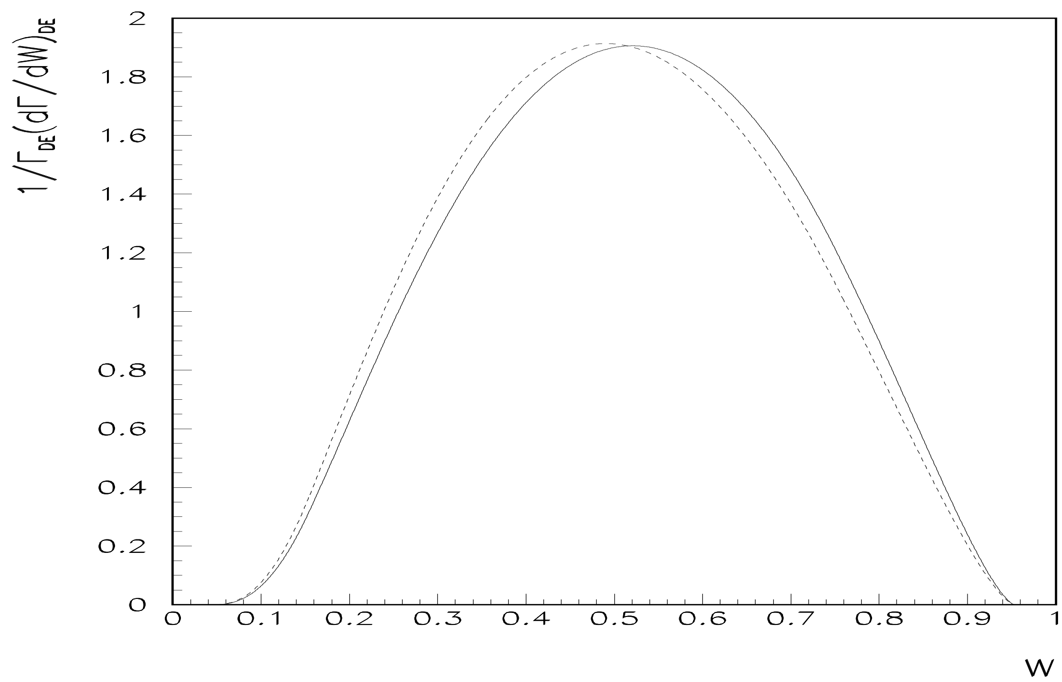
# $K^+ \to \pi^+\pi^0\gamma$




- E787 has measured M1 and Re$\left(\frac{E1}{E_{IB}}\right) \sim (-0.4 \pm 1.6)\%$

$$\downarrow$$

- E1 dominated by CT $\Rightarrow$ E787 constrains models $(k_f < 1)$




- $\frac{d\Gamma}{dW}$ may be crucial to study well the form factor $W^2 = (q \cdot p_K)(q \cdot p_+)/(m_\pi^2 m_K^2)$

- to establish VMD

# CP asymmetry

- In the asymmetry in the slope, $\frac{\partial^2 \Gamma^\pm}{\partial T_c^* \partial W^2}$ select a favourable kin. region (large $W^2$)

- This asymm., $\Omega$, in extensions of SM $\sim \mathcal{O}(10^{-4})$ Colangelo et al.

- SM $\leq \mathcal{O}(10^{-5})$ Paver et al.

- Assuming the expts. are almost seeing the CP conserving $E1$ Statistics seems tough but previous limit (Smith eta l. 76) weak

- Similar analysis for CPV in $K_L$: but time interf. required

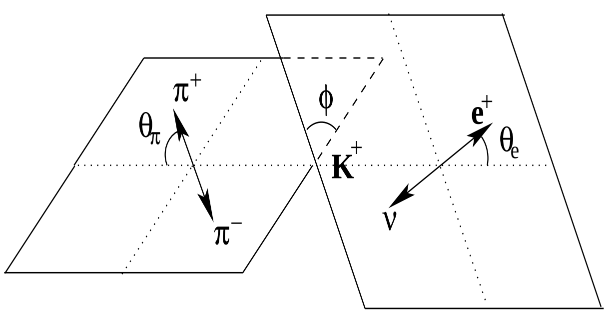
# Conclusions

- $K^+ \to \pi^+\pi^+\pi^-$: CP asymm. Important
- $K^+ \to \pi^+\pi^0\pi^0$: future for 2% measurements scattering lenghts
- $K^+ \to \pi^+\gamma\gamma$ useful to completely determine $K_L \to \pi^0\gamma\gamma$ and VMD test
- $K^+ \to \pi^+\pi^0\gamma$: chiral anomaly-VMD interplay to test
- $K^+ \to \pi^+\pi^0\gamma$: interference interesting to measure, CHECK CP

# $K_{l4}$ and $\pi\pi$ strong phases $\delta_I^l(s)$

$$K^+ \to \pi^+\pi^- l\nu \Longrightarrow \text{form factors} \quad F_i(s) = f_i(s) e^{i\delta_0^0(s)} + ..$$

$$A_{\pi\pi(s)} \sim (s - m_\pi^2)/F_\pi^2 + \quad h.o. \qquad \Leftrightarrow \delta_I^l(s)$$




- Look angular plane asymmetry $\pi\pi \quad l\nu$
- $\delta_I^l(s \sim 4\ m_\pi^2)$, $\pi\pi$ phase shifts near threshold $\Longrightarrow a_I^l$
- $a_0^0$ strongly sensitive to $\langle 0|\bar{q}q|0\rangle$

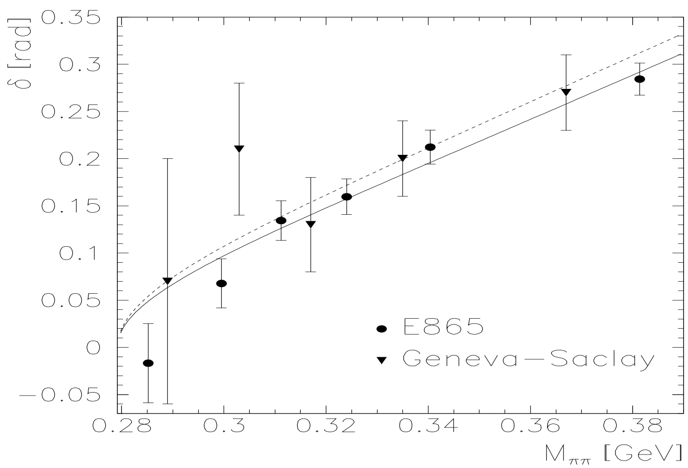
# $a_0^0$: BNL-E865 and THEORY

$$a_0^0 \begin{cases} 0.16 & O(p^2) \\ 0.20 \pm 0.01 & O(p^4) \quad \text{parameter free pre.} \\ 0.220 \pm 0.005 & O(p^6) \quad \text{+analy.+disper.} \end{cases}$$

Weinberg 79

Gasser Leutwyler

Bijnens et. al




BNL-E865 $a_0^0 = 0.216 \pm 0.013$

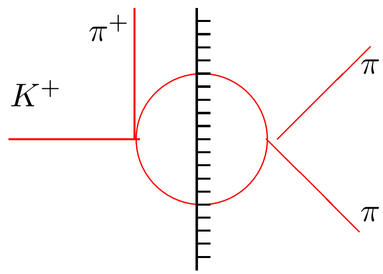
+ChPT+Th [hep-ph/0301040]

Colangelo et al.

## $a_0, a_2$ **from** $K \to 3\pi$ **rescattering;** Cabibbo,Cabibbo-Isidori

- rescattering generates an absorptive contribution proportional to the scattering lenghts $a_0, a_2$

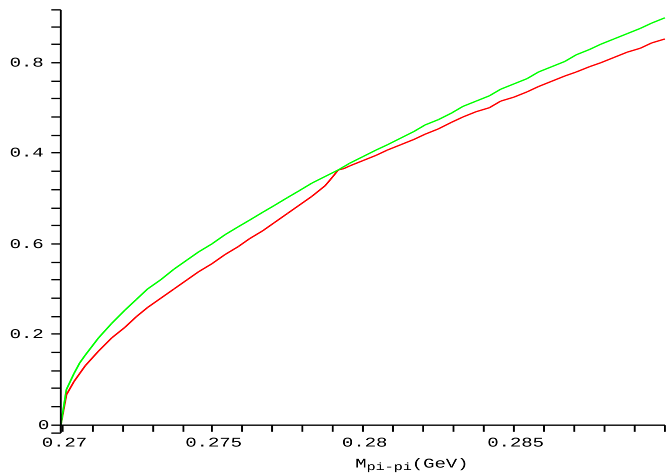
Final State Interaction




Zeldovich,Grinstein et al
Isidori,Maiani,Pugliese

The amplitude T(s) has a critical behaviour near $\pi\pi$ threshold: NA48 good energy resolution$\Longrightarrow a_0, a_2$

## $a_0, a_2$ **Cabibbo,Cabibbo-Isidori**

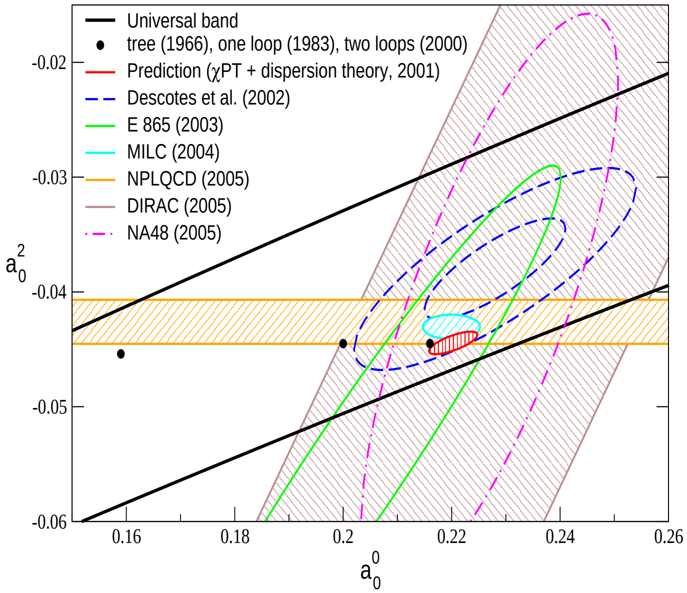



- No cusp with cusp
- cusp: opening of the $\pi^+\pi^-$-threshol
- Rescattering $\pi^+\pi^- \rightarrow \pi$
  proportional to $a_0 - a_2 \Longrightarrow$

$$\left.\frac{d\Gamma(K^+ \rightarrow \pi^+\pi^0\pi^0)}{dM_{\pi^0\pi^0}}\right|_{\mathrm{NA48}} \Rightarrow \mathrm{cusp\,for} M_{\pi^0\pi^0} = M_{\pi^+\pi^-}$$

$$\stackrel{\mathrm{cusp}}{\Rightarrow} a_0 - a_2.$$

# $a_0, a_2$




Colangelo et al.

- Possible impact of a 2% measurement of $a_0 - a_2$