Review on Chiral Perturbation Theory

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We introduce the basic ideas of Chiral Perturbation Theory. Then we discuss $K \to 3\pi$/CP asymmetry ($\pi\pi$ scattering lenghts); $K_S \to \gamma\gamma$, $K \to \pi\pi\gamma$. We mention other weak kaon decays close to observation.

1. Chiral Perturbation theory

QCD is non-perturbative at energy scales below 1 GeV and thus symmetry arguments must be invoked in order to be predictive. QCD for massless quarks exhibits the global symmetry $SU(3)_L \otimes SU(3)_R$ and there are strong phenomenological arguments (Goldberger-Treiman relation, ...) that the pion is the Goldstone boson of the broken symmetry $SU(3)_L \otimes SU(3)_R \to SU(3)_V$. Thus χPT [1,2] is an effective field theory based on the following two assumptions: i) the pseudoscalar mesons are the Goldstone bosons (G.B.) of the symmetry above, ii) there is a *(chiral) power counting*, i.e. the theory has small expansion parameters: $p^2/\ \Lambda^2_{\chi SB}$ and/or $m^2/\ \Lambda^2_{\chi SB}$, where p are the external momenta, m the masses of the G.B.'s and $\Lambda_{\chi SB}$ is the chiral symmetry breaking scale: $\Lambda_{\chi SB} \sim 4\pi F_\pi \sim 1.2$ GeV. Being an effective field theory, loops and counterterms are required by unitarity and have to be evaluated order by order [2]. It turns out to be more practical to descibe the chiral fields through a non-linear realization, $U = e^{i\sqrt{2}\Phi/F}$ and $\Phi = \sum_i \lambda_i \phi^i$, λ_i are the Gell-Mann matrices, $F \sim F_\pi$. We can split the lagrangian in a strong ($\Delta S = 0$) and in a weak non-leptonic piece ($\Delta S = 1$): then consider the chiral expansion

$$\mathcal{L}_{\Delta S=0} = \frac{F^2}{4} \underbrace{\langle D_\mu U D^\mu U^\dagger + 2\mathcal{B}(m_q U^\dagger + U m_q^\dagger)\rangle}_{\pi\to l\nu,\ \pi\pi\to\pi\pi} + \sum_i L_i O_i + \cdots \quad (1)$$

where m_q is the quark mass matrix;F and $\mathcal{B}$ parametrize the two order parameters of the spontaneous symmetry breaking of χPT: $F \simeq F_\pi + h.o.$ parametrizes the non-conservation of the axial current, while in the orthodox χPT version, $\mathcal{B}$ leads to the Gell-Mann-Oakes-Renner relation

$$\langle q\bar{q}\rangle = -\mathcal{B}F_\pi^2 \stackrel{\mathcal{B}\neq 0}{\Longrightarrow} m_\pi^2 = \mathcal{B}(m_u + m_d)\left[1 + \mathcal{O}(m_q)\right]$$

Actually, there are speculations that the size of this this condensate is small, then higher dimensional condensates trigger spontaneous symmetry breaking of χPT [3]; consequentely the Gell-Mann-Oakes relation and the pion mass are replaced and involve higher dimensional terms. The second term in (1) represents the strong $\mathcal{O}(p^4)$ (Gasser-Leutwyler) lagrangian.

Perhaps the most beautiful predictions of the strong lagrangian are $\pi\pi$ scattering lenghts [2,4], predicted at $\mathcal{O}(p^2)$ in terms only of the pion decay constant, F_π:

$$a_0 m_\pi \stackrel{\mathcal{O}(p^2)}{=} \frac{7m_\pi^2}{32\pi F_\pi^2} \simeq 0.16 \stackrel{\mathcal{O}(p^4)}{\Longrightarrow} 0.20 \pm 0.01 \quad (2)$$
$$\stackrel{\mathcal{O}(p^6)}{\Longrightarrow} 0.217 \pm 0.005 \stackrel{\mathcal{O}(p^6)+\text{Roy eqs.}}{\Longrightarrow} 0.220 \pm 0.005$$

These extremely precise theoretical predictions [32,4] match the analogous experimental effort to do these measurements (see also refs. of DIRAC and E865 in [4,6]): as we shall see in particular the NA48 effort and devolopment to measure the cusp $K^+ \to 3\pi$[29] is very interesting.

1.1. Strong interaction and $\pi\pi$ scattering lenghts

The determination of the Gasser-Leutwyler coefficients in (1) is very accurate and allows a 2%

*This work was supported in part by the EU Contract No. MRTN-CT-2006-035482, "FLAVIAnet".

measurements of the $\pi\pi$ scattering lenghts in (2) [1,2]. The theoretical understanding of these coefficients is also very advanced: Large N, chiral quark model, Nambu Jona Lasinio model [1,2]; all have been very successful. We want to mention here, also to check its relative role in weak interactions, that the Vector Meson Dominance model [8,9] is very successful. In fact the predictions

$$L_1^{(V)} = L_2^{(V)} = \frac{-L_3^{(V)}}{6} = \frac{L_9^{(V)}}{8} = -L_{10}^{(V+A)} = \frac{F_\pi^2}{16\pi^2}$$

are very much in agreement with experiments and also theoretically appealing since VMD improves the ultra-violet behaviour of form factors, by generating a better matching with QCD [8].

2. The $\Delta S = 1$ chiral lagrangian, $\mathcal{L}_{\Delta S=1}$

Expanding $\mathcal{L}_{\Delta S=1} = \mathcal{L}^2_{\Delta S=1} + \mathcal{L}^4_{\Delta S=1}$

$$G_8 F^4 \underbrace{\langle \lambda_6 D_\mu U^\dagger D^\mu U \rangle}_{K\to 2\pi/3\pi,\gamma\gamma} + \underbrace{G_8 F^2 \sum N_i W_i}_{K^+\to\pi^+\gamma\gamma,\ K\to\pi\pi\gamma}\ , \qquad (3)$$

G_8 is fixed by the $K \to \pi\pi$ amplitudes, while the second term represents the weak $\mathcal{O}(p^4)$ [10, 11] lagrangian: unfortunately both theoretically and phenomenologically the N_i's are very poorly known [12]. Nevertheless, VMD MUST work to improve the matching with QCD in the UV region; indeed there are examples $m_{\pi^+} - m_{\pi^0}$ -electroweak contribution [13], $K_L \to \mu\bar{\mu}$ [14] and as we shall see in $K_L \to \pi^+\pi^-\gamma$. However we are very far from a VMD relation in the weak sector. We mention some cases where we can get some info on various N_i's soon

Table 1

Decay	$\mathcal{L}^4_{\Delta S=1}$ counterterms
$K_S \to \pi^+\pi^-\gamma$	$N_{14} - N_{15} - N_{16} - N_{17}$
$K^\pm \to \pi^\pm\pi^0\gamma$	$N_{14} - N_{15} - N_{16} - N_{17}$
$K^\pm \to \pi^\pm\pi^0\gamma$	$3N_{29} - N_{30}$ magnetic
$K^\pm \to \pi^\pm\gamma\gamma$	$N_{14} - N_{15} - 2N_{18}$

Fortunately there are cases where we do need the N_i's: $K_S \to \gamma\gamma$ has vanishing short-distance contributions and starts at $\mathcal{O}(p^4)$ but with no counterterm structures. This implies that i) we have only a loop contribution and ii) this contribution is scale-independent [15]. The predictions for $B(K_S \to \gamma\gamma)$ is unambigous depending only from $\mathcal{L}^2_{\Delta S=1}$ in (3). This is the *ideal* test of χPT (and in general of effective field theories) at the *quantum level*; we obtain for the branching

$$\begin{array}{ll} \text{TH} \quad (p^4) & 2.1\times 10^{-6} \\ \text{NA48} & (2.78\pm 0.072)\times 10^{-6}. \end{array} \Rightarrow \frac{A^{(6)}}{A^{(4)}} \sim 15\%$$

NA48 result [16] tells us that the χPT predictions works better than the naïve dimensional analysis (NDA) $A^{(6)}/A^{(4)} \sim m_K^2/(4\pi F_\pi)^2$. $A(K_L \to \pi^0\gamma\gamma)$ shares at $\mathcal{O}(p^4)$ the same finiteness properties of $K_S \to \gamma\gamma$, however large $\mathcal{O}(p^6)$ unitarity [17] and VMD contributions [18] affect this amplitude: it is still possible to make a theoretical prediction [19] in agreement with the NA48 and KTeV experiments. In fact reecentely KTeV [20] has reanalyzed the data finding agreement in the width with NA48

$$B(K_L \to \pi^0\gamma\gamma) = 1.30 \pm 0.03 \pm 0.04 \quad \text{KTeV},$$

this could be good news for the direct CP violating contribution to $B(K_L \to \pi^0 e^+e^-)$, since the background CP conserving contribution is proportional to the contribution at low diphoton invarinat mass [21].

3. CP aymmetries in $K^+ \to 3\pi$ and final state interaction

Direct CP violation in charged kaons is subject of extensive researches at NA48/2 [22]. Studying the $K \to 3\pi$ Dalitz distribution in Y, X [1,23]

$$|A(K \to 3\pi)|^2 \sim 1 + g\ Y + j\ X + \mathcal{O}(X^2, Y^2)$$

and determining both charged kaon slopes, $g_\pm$, we can define the slope charge asymmetry:

$$\Delta g/2g = (g_+ - g_-)/(g_+ + g_-). \qquad (4)$$

There are two independent $I = 1$ isospin amplitudes (a, b),

$$A(K^+ \to 3\pi) = a e^{i\alpha_0} + b e^{i\beta_0} Y + \mathcal{O}(Y^2, X^2) \quad (5)$$

with corresponding final state interaction phases, α_0 and β_0. The hope is that Δg in (4) does **NOT** need to be suppressed by a $\Delta I = 3/2$ transition. The strong phases, generated by the $2 \to 2$ rescattering, actually have their own kinematical dependence [24] and can be expressed in terms of the Weinberg scattering lenghts, a_0 and a_2. It is particularly interesting to estimate the Standard Model (SM) size for $\Delta g/2g$, valid if there is a good chiral expansion for the CP conserving/violating a, b amplitudes [1,25]:

$$\frac{\Delta g}{2g} \sim 22\epsilon'(\alpha_0 - \beta_0) \sim 10^{-5}.$$

Ref.[26] evaluates Δg at $\mathcal{O}(p^6)$. The $K^+ \to \pi^+\pi^0\pi^0$ NA48/2 resut [27] and New Physics (NP) scenarios [28]

$$\frac{\Delta g}{2g} \overset{\mathrm{NA48/2}}{=} (1.8 \pm 2.6) \cdot 10^{-5} \qquad \overset{\mathrm{NP}}{\leq} 10^{-4}.$$

can then be compared to the SM. Recentely, a new strategy has been suggested by NA48/2 at CERN to study the $\pi\pi$ scttering lenghts [6,29]: accumulating a lot of $K^+ \to \pi^+\pi^0\pi^0$ decays with an accurate scan in the $\pi^0\pi^0$-invariant mass distribution, $M_{\pi^0\pi^0}$, has found a cusp at $M_{\pi^0\pi^0} = M_{\pi^+\pi^-}$. This result has been nicely investigated by several groups [30–32], which explain the "cusp" as an effect due to the opening of the $\pi^+\pi^-$-threshold. Since the rescattering $\pi^+\pi^- \to \pi^0\pi^0$ is proportional to $a_0 - a_2$ then

$$\left.\frac{d\Gamma(K^+ \to \pi^+\pi^0\pi^0)}{dM_{\pi^0\pi^0}}\right|_{\mathrm{NA48}} \Rightarrow \mathrm{cusp\,for} M_{\pi^0\pi^0} = M_{\pi^+\pi^-} \overset{\mathrm{cusp}}{\Rightarrow} a_0 - a_2.$$

This should give this strong phase difference to a very good accuracy [30]. This is particularly exciting due the intense experimental (DIRAC) and theoretical efforts to determine $a_0 - a_2$.

4. $K \to \pi\pi\gamma$

We can decompose $K(p) \to \pi(p_1)\pi(p_2)\gamma(q)$ amplitudes, according to gauge and Lorentz invariance, in electric (E) and magnetic (M) terms [33,34] In the electric transitions one generally separates the bremsstrahlung amplitude E_B, firmly predicted theoretically by the Low theorem in terms of the non-radiative amplitude and enhanced by the $1/E_\gamma$ behaviour. Summing over photon helicities, there is no interference among electric and magnetic terms: $\mathrm{d}^2\Gamma/(\mathrm{d}z_1\mathrm{d}z_2) \sim |E(z_i)|^2 + |M(z_i)|^2$. At the lowest order, (p^2), one obtains only E_B. Magnetic and electric direct emission amplitudes, appearing at $\mathcal{O}(p^4)$, can be decomposed in a multipole expansion [33,34]).

Table 2 DE_{exp}

$K_S \to \pi^+\pi^-\gamma$	$< 9 \cdot 10^{-5}$	$E1$
$K^+ \to \pi^+\pi^0\gamma$	$(0.44 \pm 0.07)10^{-5}$	$M1, E1$
$K_L \to \pi^+\pi^-\gamma$	$(2.92 \pm 0.07)10^{-5}$	$M1$,VMD

In the Table 2 we show the present experimental status of the DE and the leading multipoles. Due to the recent interesting NA48 data, we discuss $K^+ \to \pi^+\pi^0\gamma$. Due to the $\Delta I = 3/2$ suppression of the bremsstrahlung, interference between E_B and $E1$ and magnetic transitions can be measured. Defining $z_i = p_i \cdot q/m_K^2 \quad z_3 = p_{\scriptscriptstyle K} \cdot q/m_K^2$ and $z_3 z_+ = \frac{m_{\pi^+}^2}{m_K^2} W^2$ we can study of the Dalitz plot distribution in Figure 1

$$\frac{\partial^2\Gamma}{\partial T_c^* \partial W^2} = \frac{\partial^2\Gamma_{IB}}{\partial T_c^* \partial W^2} \left[1 + \frac{m_{\pi^+}^2}{m_K} 2Re\left(\frac{E_{DE}}{eA}\right) W^2 + \frac{m_{\pi^+}^4}{m_K^2} \left(\left|\frac{E_{DE}}{eA}\right|^2 + \left|\frac{M_{DE}}{eA}\right|^2 \right) W^4 \right],$$

where $A = A(K^+ \to \pi^+\pi^0)$; an accurate study of the Dalitz plot in Figure 1 has lead NA48 the preliminary results in **Table 3** [35]

Table 3

NA48		$T_c^* \in [0, 80]$
Frac(DE)	$=$	$(3.35 \pm 0.35 \pm 0.25) \times 10^{-2}$
Frac(INT)	$=$	$(-2.67 \pm 0.81 \pm 0.73) \times 10^{-2}$

This is the first evidence of non-vanishing interference [35] and gives a determination of the counterterm coefficient in Table 1, contributing to $E1$ [10,11]. The magnetic contributions, is by now well established; there are two contributions i) an indirect contributions generated by a pion pole mediating a Wess Zumino Witten (WZW) term and a vertex from $\mathcal{L}^2_{\Delta S=1}$ from (3) and ii) genuine $\mathcal{O}(p^4)$ anomalous-like contributions from $\mathcal{L}^4_{\Delta S=1}$ in Table 1 $N_{28}, .., N_{31}$. These last terms can be obtained from factorization [36]

$$\mathcal{L}_{FM} = 4\, k_F\, G_8 \,\langle\, \lambda \, \frac{\delta S}{\delta \ell_\mu} \frac{\delta S}{\delta \ell^\mu} \,\rangle \; + \; h.c. \,, \qquad (6)$$

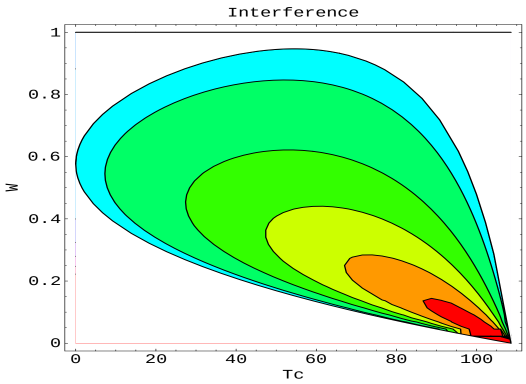

Figure 1. $T_c^* - W$-Dalitz plot. In this contour plot of the interference Branching the red area corresponds to more dense and thus larger contribution

where we consider also the *anomalous current (from the WZW term)*. Then we generate $\mathcal{L}^4_{\Delta S=1}$ in (3) with coefficients

$$a_1 = 8\,\pi^2\, N_{28} \quad , \quad a_2 = 32\,\pi^2\, N_{29}\ ,$$
$$a_3 = \frac{16}{3}\,\pi^2\, N_{30} \quad , \quad a_4 = 16\,\pi^2\, N_{31}\ .$$

The a_i are positive parameters of $\mathcal{O}(1)$. Once we have proven that these terms are there, several dynamical mechanism can generate them; In fact theory [11] and data from $K_L \to \pi^+\pi^-\gamma$ [37,38] point towards a large VMD contributions in these decays. In terms of the counterterms we can write

$$M_L^{(4)} = \frac{eG_8 m_K^3}{2\pi^2 F}(a_2 + 2a_4), \tag{7}$$

$$M_+^{(4)} = -\frac{eG_8 m_K^3}{4\pi^2 F}[2 + 3(2a_3 - a_2)]. \tag{8}$$

It is interesting that the direct emission branching in the table 3, can be obtained by $M_+^{(4)}$ in (8) neglecting the contributions of the a_i's:

M1 $\sim$ (-2 + $3a_2 - 6a_3$)

$\uparrow$ $\qquad$ $\uparrow$

WZW $\qquad$ p^4 CT's (VMD) $\qquad$ $a_i$small?

However if even if for $B(K^+ \to \pi^+\pi^0\gamma)$ maybe a correct interpretation, the fact that the observed B$(K_L \to \pi^+\pi^-\gamma)_{DE}$ implies $a_i \sim O(1)$ calls for a more accurate theoretical investigation [38]. In fact since the presence of a form factor affects the Dalitz plot distribution we think a thorough analysis is required to disentangle as much as possible the interference from possible competing effects in the magnetic amplitudes [39].

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