Theoretical issues in rare B decays

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Plan: two aspects of inclusive radiative decays

- The $\bar{B} \longrightarrow X_s\gamma$ Branching Fraction:
  progress towards NNLO

- *The photon–energy spectrum* in $\bar{B} \longrightarrow X_s\gamma$.
  The resummed on-shell perturbative spectrum yields a good approximation!
  Dressed Gluon Exponentiation:
  resummation formalism for inclusive distributions near kinematic thresholds

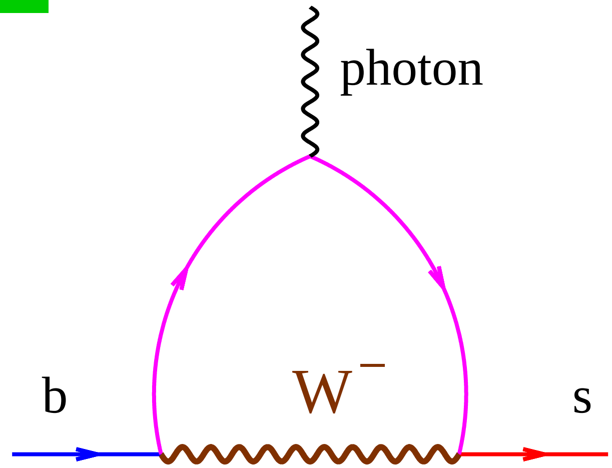
$\bar{B} \longrightarrow X_s\gamma$: motivation

- Rare processes $\Longleftrightarrow$ violation of a symmetry. SM: Flavour Changing Neutral Currents are absent at the classical (tree) level.

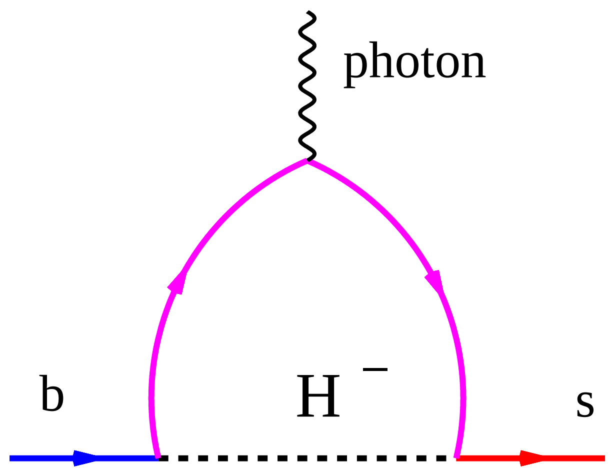
- Measuring flavour–changing transitions: potential for indirect discovery of new physics. At least, strong constraints on physics BSM.

☞ In the SM $b \to s$ transition is suppressed, occurring through loops (penguin diagrams).

☞ In many BSM scenarios the width is different (larger or smaller): $\bar{B} \longrightarrow X_s\gamma$ provides a strong constraint. Currently: good agreement with SM. Uncertainties: $\sim 10\%$.



Standard Model

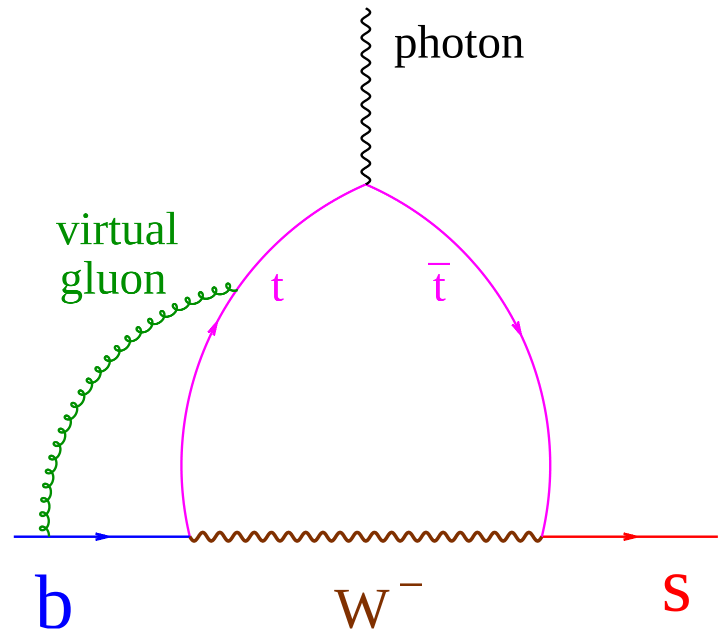


models with charged Higgs

$\bar{B} \longrightarrow X_s\gamma$ BF: hierarchy of scales

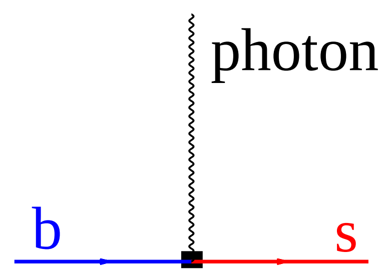
- Heavy particles (W^-, t) are **highly** virtual: available energy $\mathcal{O}(m_b)$, $m_W \gg m_b$.

- Large logarithms $\ln(m_W/m_b)$ appear in the expansion.
  Need to resum powers of $\alpha_s \ln(m_W/m_b)$.
  Most conveniently done using an effective local interaction: integrating out W^-, t.



- To leading order in $1/m_W$ and in α_s:

$$\mathcal{H}_{\text{eff}} = \underbrace{\frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^* C(m_t^2/m_W^2)}_{\text{SM prediction}} \underbrace{\frac{em}{16\pi^2}\, \bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu}}_{O_7}$$



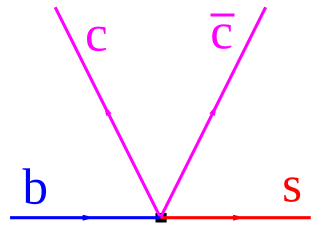
The total BF from the Effective Weak Hamiltonian

- Effective Weak Hamiltonian:

$$\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{8} C_i(\mu) O_i(\mu)$$

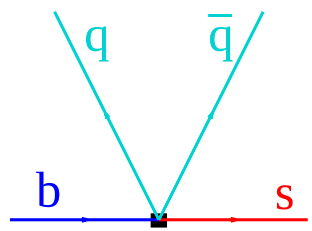
- 3 steps in computing the BF:
  - ☞ Matching to SM: $C_i(m_W)$
  - ☞ $O_i(\mu)$ evolution: $C_i(m_b)$
  - ☞ Matrix elements:

$$G_{ij} = \left\langle b \middle| O_i(\mu) \middle| X_s \gamma \right\rangle \left\langle X_s \gamma \middle| O_j^\dagger(\mu) \middle| b \right\rangle$$



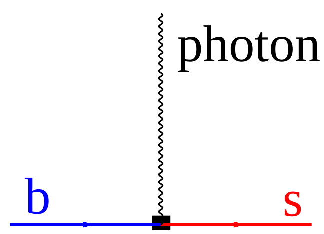
$$O_{1,2} \equiv (\bar{s}_L \Gamma_\mu c_L)\,(\bar{c}_L \tilde{\Gamma}^\mu b_L)$$

$$C_i(m_b) \sim 1$$



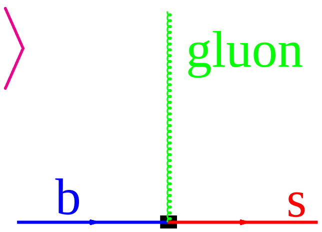
$$O_{3,4,5,6} \equiv (\bar{s}_L \Gamma_\mu b_L) \sum_q (\bar{q} \tilde{\Gamma}^\mu q)$$

$$C_i(m_b) < 0.07$$



$$O_7 \equiv \frac{e\, m_b}{16\pi^2} (\bar{s}_L \sigma^{\mu\nu} b_R)\, F_{\mu\nu}$$

$$C_7(m_b) \simeq -0.3$$



$$O_8 \equiv \frac{g\, m_b}{16\pi^2} (\bar{s}_L \sigma_{\mu\nu} T^a b_R)\, G^a_{\mu\nu}$$

$$C_8(m_b) \simeq -0.15$$

Contributions to the calculation of $\mathcal{B}(\bar{B} \longrightarrow X_s\gamma)$ at NNLO

From Misiak, Moriond 2006:

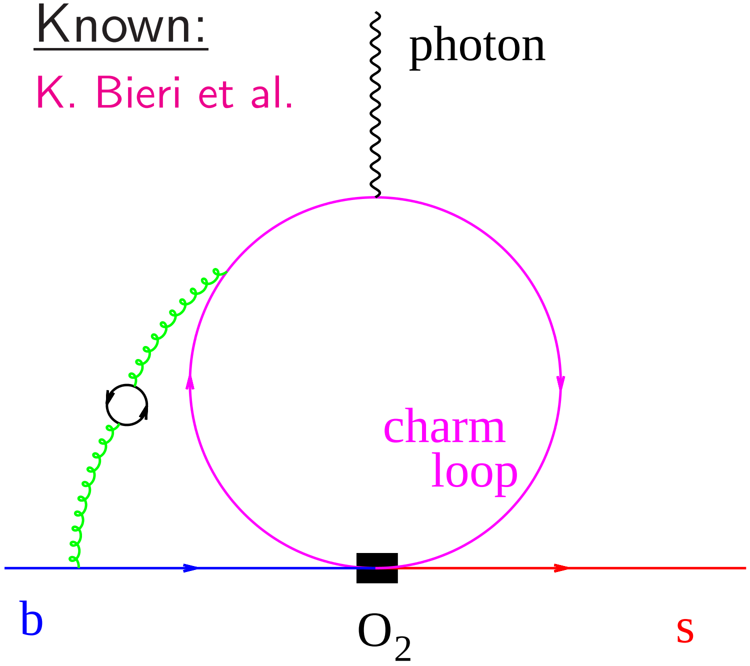
- Three–loop matching for O_7 and O_8:
  M. Steinhauser & M. Misiak, hep-ph/0401041.
- Three–loop mixing in the $(O_1, \ldots, O_6)$ and (O_7, O_8) sectors:
  M. Gorbahn & U. Haisch, hep-ph/0411071;
  M. Gorbahn, U. Haisch & M. Misiak, hep-ph/0504194.
- Four–loop mixing $(O_1, ..., O_6)$ into (O_7, O_8):
  M. Czakon, U. Haisch & M. Misiak, in progress
- Two–loop matrix elements of O_7 and O_8:
  K. Bieri, C. Greub & M. Steinhauser, hep-ph/0302051 (large β_0);
  I. Blokland, A. Czarnecki, M. Misiak, M. Ślusarczyk, F. Tkachov, hep-ph/0506055
  H.M. Asatrian, T. Ewerth, C. Greub, T. Hurth, A. Hovhannisyan, V. Poghosyan, hep-ph/0605009
  K. Melnikov, A. Mitov, hep-ph/0505097 (bremsstrahlung)
  H.M. Asatrian, T. Ewerth, A. Feroglia, C. Greub and P. Gambino, in progress
- Three–loop matrix elements of O_1 and O_2:
  K. Bieri, C. Greub, M. Steinhauser, hep-ph/0302051 (large β_0)
  M. Steinhauser & M. Misiak, in progress (interpolation in m_c)

The ultimate NNLO challenge: 3–loop matrix elements of $O_{1,2}$

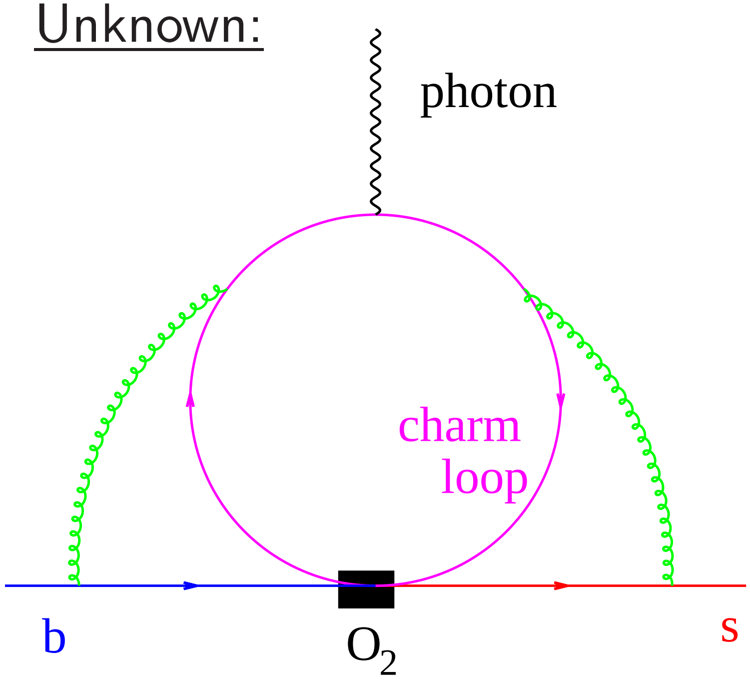
- The interference between O_7 and $O_{1,2}$ begins at $G_{27} \simeq \mathcal{O}(\alpha_s)$.
- Nevertheless, has a big impact on the rate!
- This contribution strongly depends on the charm mass and on the factorization scale.
- Neither the scale of the $\alpha_s(\mu)$ nor that of $m_c(\mu_c)$ can be fixed without NNLO calculation of these matrix elements! Gambino & Misiak (2001)
- Currently, *partial information* is available:

Known:

K. Bieri et al.



Unknown:



The total BF and infrared renormalons

- The result:

$$\mathcal{B}(\bar{B} \longrightarrow X_s\gamma, E > E_0) = \frac{\tau_B \alpha_{\rm em} G_F^2}{32\pi^4} \left|V_{tb}V_{ts}^*\right|^2 \left(m_b^{\overline{\rm MS}}(m_b)\right)^2 m_b^3 \sum_{i,j,\, i\leq j} C_i^{\rm eff}(\mu) C_j^{\rm eff}(\mu) G_{ij}(E_0,\mu)$$

- However $G_{ij}(E_0,\mu)$ and the pole mass m_b have $\mathcal{O}(\Lambda)$ infrared renormalon ambiguities!

- For example, for the 77 matrix element with $\alpha_s(\mu = m_b) = 0.213$:

$$\begin{aligned} G_{77}(E>0,\mu=m_b) &= 1 - 0.83\,\alpha_s(m_b) - 2.34\left(\alpha_s(m_b)\right)^2 - \cdots - \mathcal{O}(\Lambda/m_b) \\ &= 1 - 0.177 - 0.106 - \cdots - \mathcal{O}(\Lambda/m_b) \end{aligned}$$

  while the ratio between the pole mass m_b and $m_b^{\overline{\rm MS}}(m_b)$, raised to power 3, is

$$\begin{aligned} \left(m_b/m_b^{\overline{\rm MS}}(m_b)\right)^3 &= 1 + 1.27\,\alpha_s(m_b) + 4.17\left(\alpha_s(m_b)\right)^2 + \cdots + \mathcal{O}(\Lambda/m_b) \\ &= 1 + 0.27 + 0.19 + \cdots + \mathcal{O}(\Lambda/m_b) \end{aligned}$$

- Both series are non-summable, having $\mathcal{O}(\Lambda/m_b)$ ambiguities.
  These cancel in $\mathcal{B}(\bar{B} \longrightarrow X_s\gamma, E > E_0)$.

The total BF: getting numbers despite renormalons

- $G_{ij}(E_0,\mu)$ and the pole mass m_b have $\mathcal{O}(\Lambda)$ infrared renormalon ambiguities!

$$\mathcal{B}(\bar{B}\longrightarrow X_s\gamma, E>E_0)=\frac{\tau_B\alpha_{\rm em}G_F^2}{32\pi^4}\left|V_{tb}V_{ts}^*\right|^2\left(m_b^{\overline{\rm MS}}(m_b)\right)^2 m_b^3\sum_{i,j,\,i\leq j}C_i^{\rm eff}(\mu)C_j^{\rm eff}(\mu)G_{ij}(E_0,\mu)$$

- Conventional approach: determine the normalization using the semileptonic BF:

$$\begin{aligned}\mathcal{B}(\bar{B}\longrightarrow X_s\gamma, E>E_0) \quad &= \quad \mathcal{B}(\bar{B}\longrightarrow X_c l\bar{\nu})\big|_{\rm exp}\left[\frac{\Gamma(b\to s\gamma)}{\Gamma(b\to cl\nu)}\right]_{\rm LO} f\left(\frac{\alpha_s(m_W)}{\alpha_s(m_b)}\right)\\ &\times \quad \left\{1+\mathcal{O}(\alpha_s)+\mathcal{O}(\alpha_{\rm em})+\mathcal{O}(\alpha_s^2)+\mathcal{O}\left(\frac{\Lambda^2}{m_b^2}\right)+\mathcal{O}\left(\frac{\Lambda^2}{m_c^2}\right)\right\}\end{aligned}$$

So far applied at NLO Gambino & Misiak (2001); Buras et al. (2002)

- Alternative: renormalon resummation of the pole mass and the matrix elements G_{ij}
The method has been applied to the total charmless semileptonic width
(Andersen & Gardi, hep-ph/0509360).

- In both methods, determination of the BF based on partial NNLO is in progress.

$\bar{B} \longrightarrow X_s\gamma$: short– and long–distance dynamics

- Inclusive $\Longrightarrow$ Calculable.
- Short–distance interaction (SM or not)
  determines the total width
  Short–distance QCD dynamics is crucial.
- QCD alone determines the spectrum.
  Both perturbative (hard) and non-perturbative
  (soft) contributions are relevant.

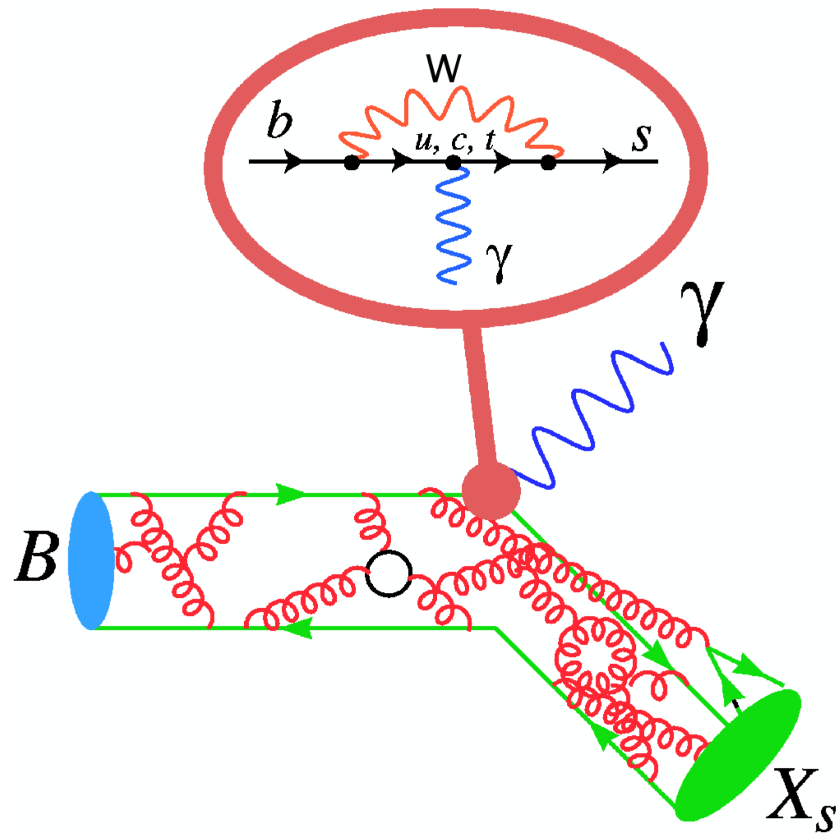
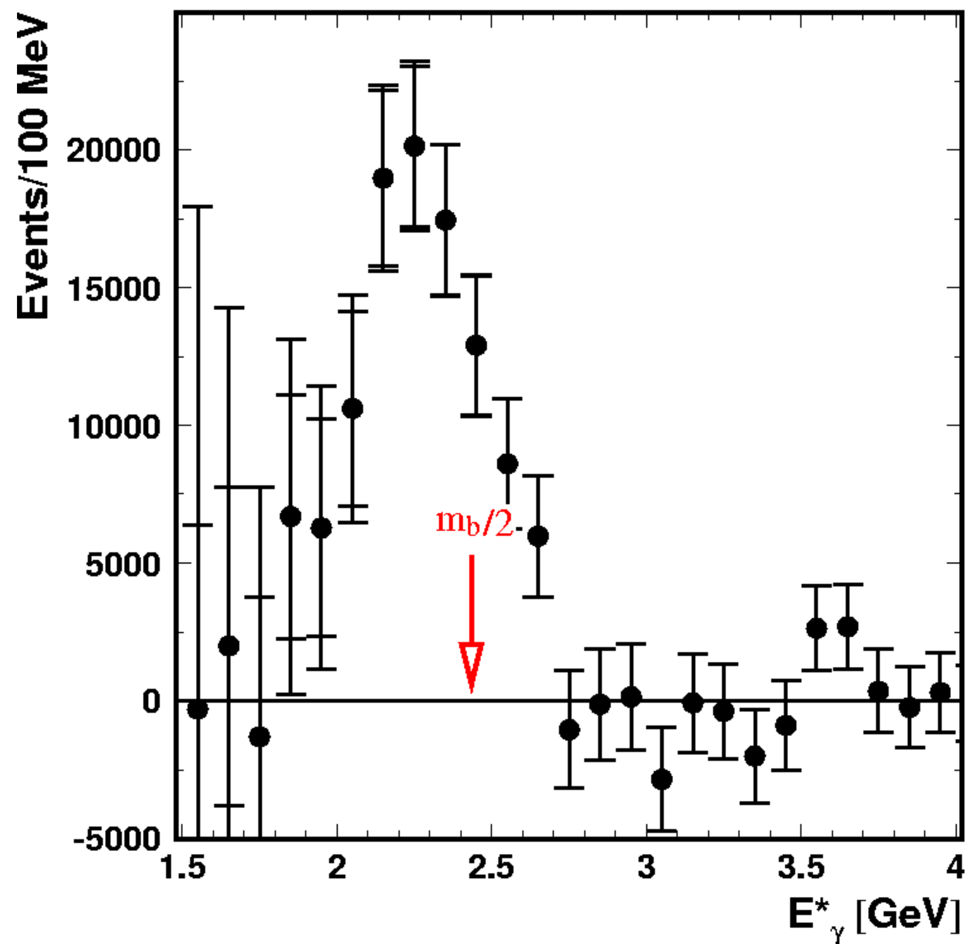


Figure by M. Luke

The photon–energy spectrum in $\bar{B} \longrightarrow X_s\gamma$: experiment

- The photon spectrum has support for $0 < E_\gamma < M_B/2$; it peaks near $m_b/2$.
- **But**, experimentally:
  - ☞ Measurement is possible only for $E_\gamma \gtrsim 1.8$ GeV (background limited).
  - ☞ The B factories will provide accurate measurements for $E_\gamma \gtrsim 2.1$ GeV.

Belle results (Summer 2005)



Theoretical prediction for the $\bar{B} \longrightarrow X_s\gamma$ **spectrum** is essential!

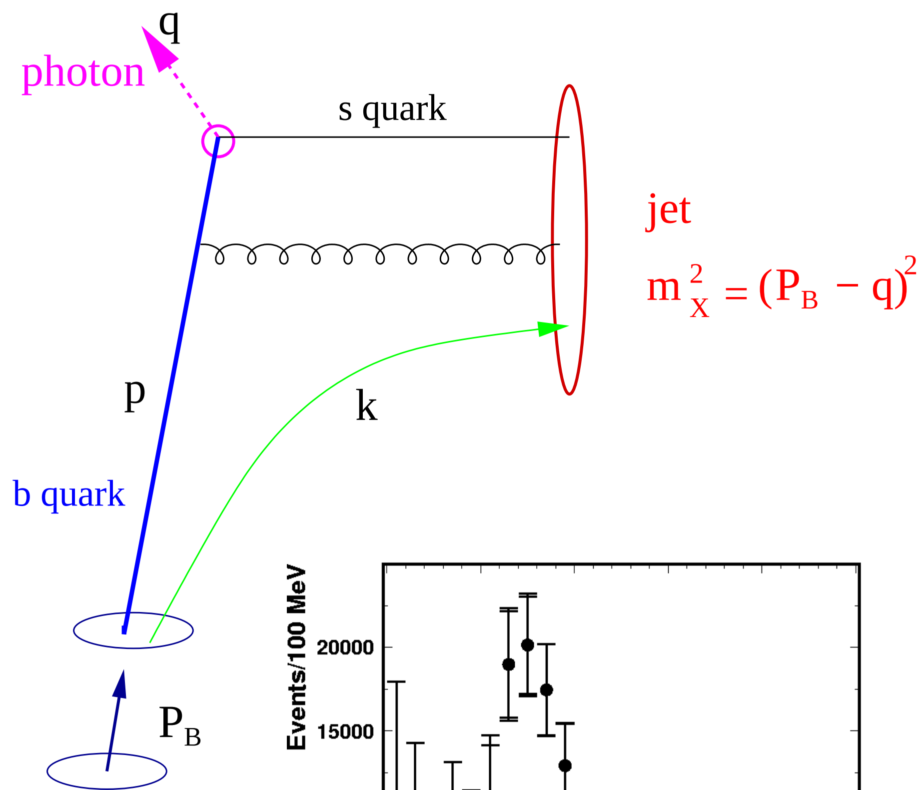
The photon–energy spectrum in $\bar{B} \longrightarrow X_s\gamma$: theory

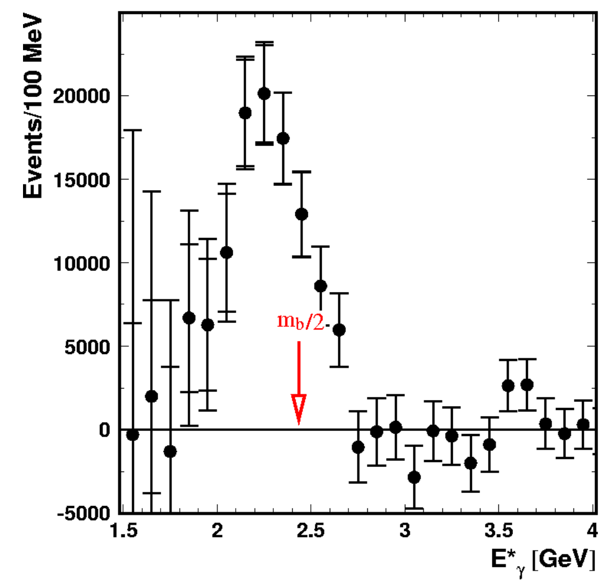
Can we compute the spectrum?

- The physical picture: The "primordial" virtuality of the b quark in the meson generates $\mathcal{O}(\Lambda_{\rm QCD})$ smearing.
  Neubert; Bigi, Shifman, Uraltsev, Vainshtein '93

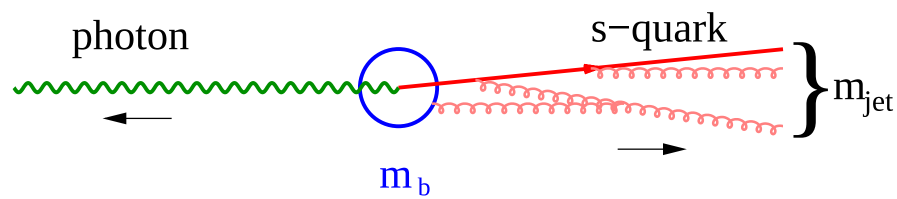
- The common lore: *computing the spectrum in the peak region is beyond the limits of perturbative QCD.*

- In fact *the (resummed) on-shell PT spectrum is a good starting point!*
  ☞ The b quark is close to its mass shell.
  ☞ The on-shell perturbative spectrum is IR and collinear safe
  $\Longrightarrow$ non-perturbative effects can be treated as **power corrections**





The photon–energy spectrum: perturbation theory

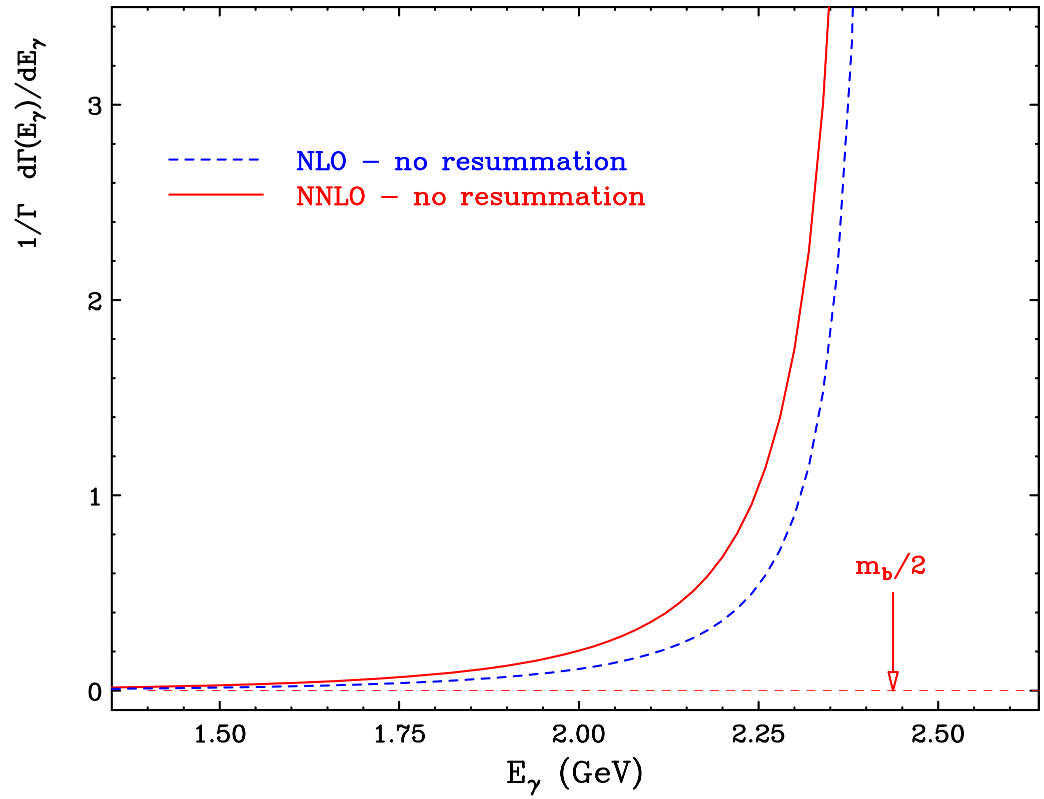


Fixed–order perturbation theory:

- At leading order (LO):

$$\frac{1}{\Gamma}\frac{\Gamma(E_\gamma)}{dE_\gamma} = \delta\left(E_\gamma - \frac{m_b}{2}\right).$$

- The spectrum beyond* LO:
  - support: $E_\gamma < m_b/2$
  - diverges at the exclusive limit, $E_\gamma = m_b/2$ $(m_{\rm jet} = 0)$
  - huge corrections — no stability



*Melnikov & Mitov recently *completed* the NNLO calculation of the normalized O_7 $b \to X_s\gamma$ width.

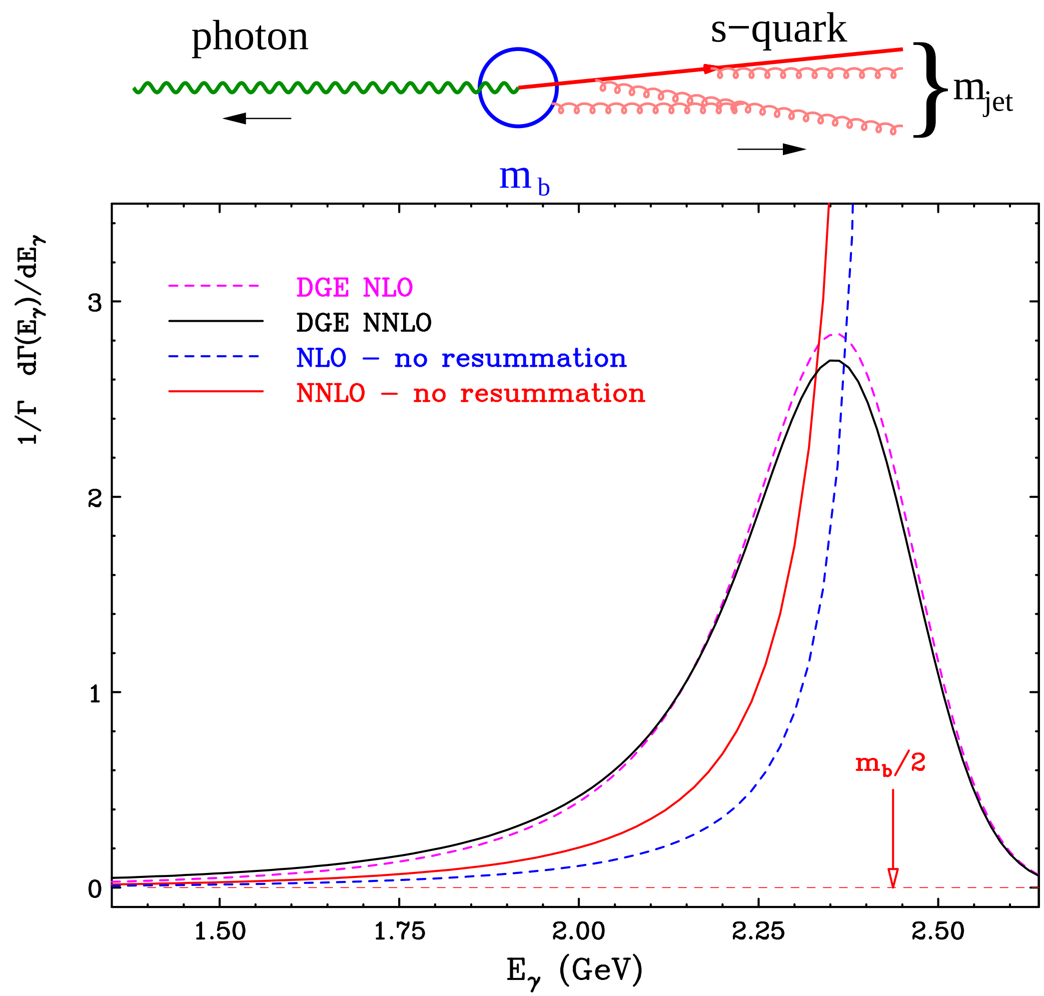
The photon–energy spectrum: resummed perturbation theory

Resummed perturbation theory:

- We understand the origin of large corrections as a combined effect of
  - ☞ renormalons (running coupling)
  - ☞ Sudakov logarithms

  and we can resum the perturbative expansion by Dressed Gluon Exponentiation (DGE)

- The DGE result is qualitatively different from fixed order:
  - ☞ support properties!
  - ☞ stability



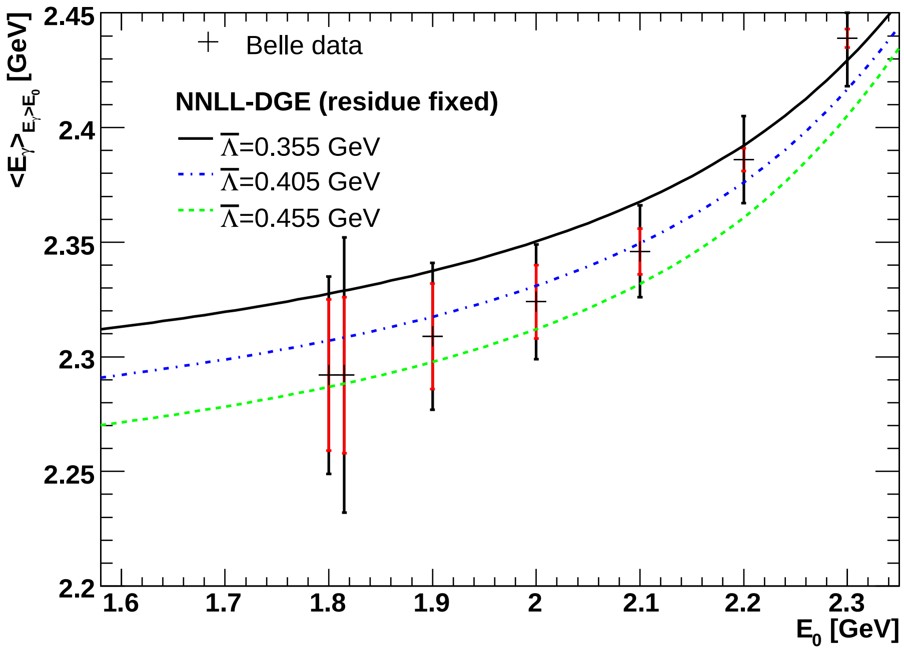
Andersen & Gardi '05

Resummation makes a qualitative difference!

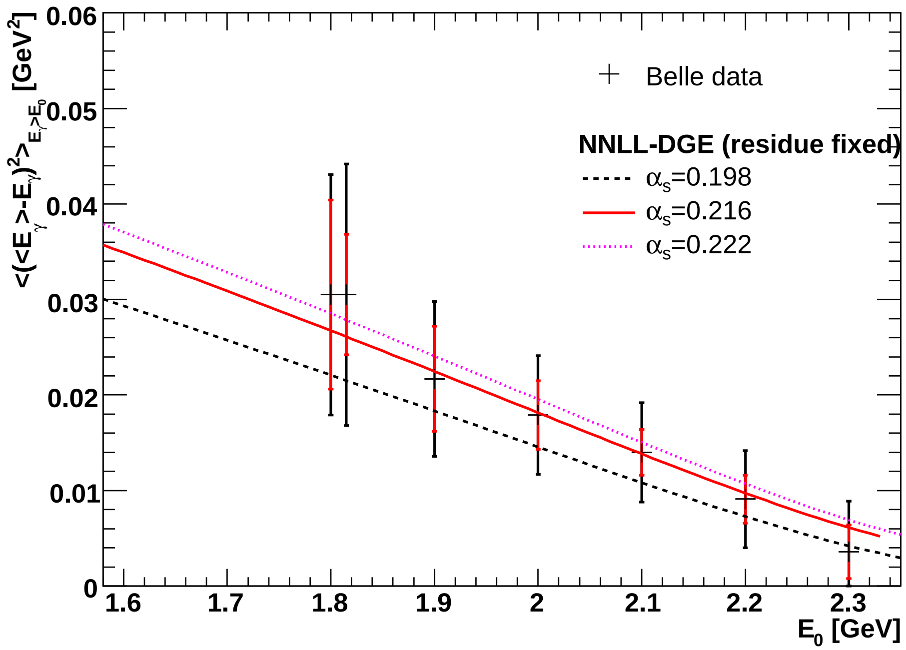
The photon–energy spectrum in $\bar{B} \longrightarrow X_s\gamma$: prediction *vs.* data

Comparison of DGE predictions (no power corrections) to Belle data:

Average energy $(E_\gamma > E_0)$



Variance $(E_\gamma > E_0)$



Conclusion: after resummation, the on-shell approximation is good!

- ☞ It can be reliably used to extrapolate below $E_\gamma \simeq 2.1$ GeV.
- ☞ Implications for determination of $|V_{\rm ub}|$ from semileptonic decays — see below

Dressed Gluon Exponentiation

Renormalon resummation:

summation of PT to power accuracy, giving access to NP power corrections. Renormalons $\equiv n \to \infty$ asymptotics,

Sudakov resummation:

corrections that become large near the exclusive phase–space boundary, $m_{\rm jet} \to 0$

running coupling $\Longrightarrow$ non-summable series

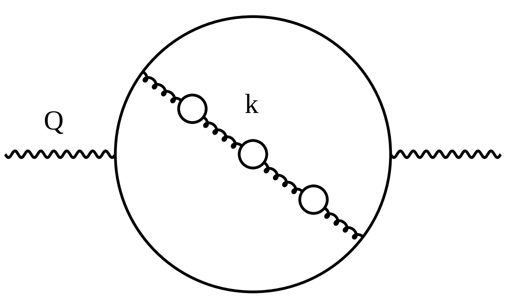
$$\int_0 dk^2 \left[\alpha_s \beta_0 \ln(k^2/Q^2)\right]^n \Longrightarrow \left[\alpha_s\beta_0\right]^n n!$$

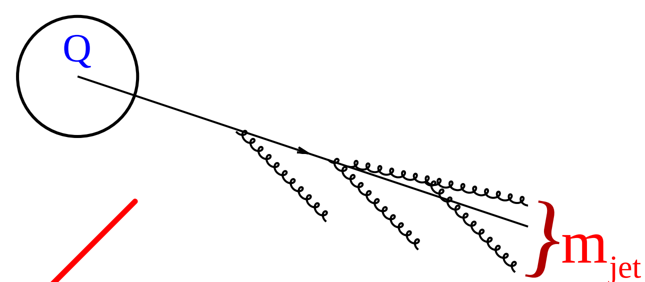
multiple soft and collinear emission

$$\left[C_F \alpha_s \ln^2(m_{\rm jet}^2/Q^2)\right]^n$$

dressing the gluon: $\alpha_s(\mu^2) \longrightarrow \alpha_s(k^2)$

exponentiation in moment space



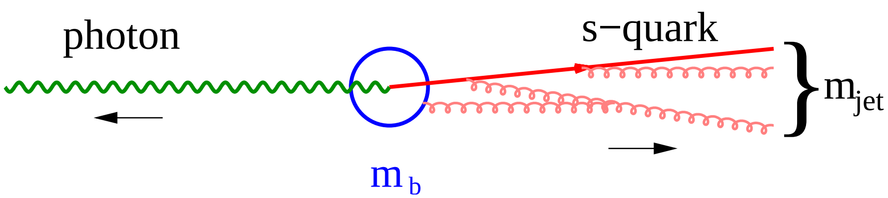


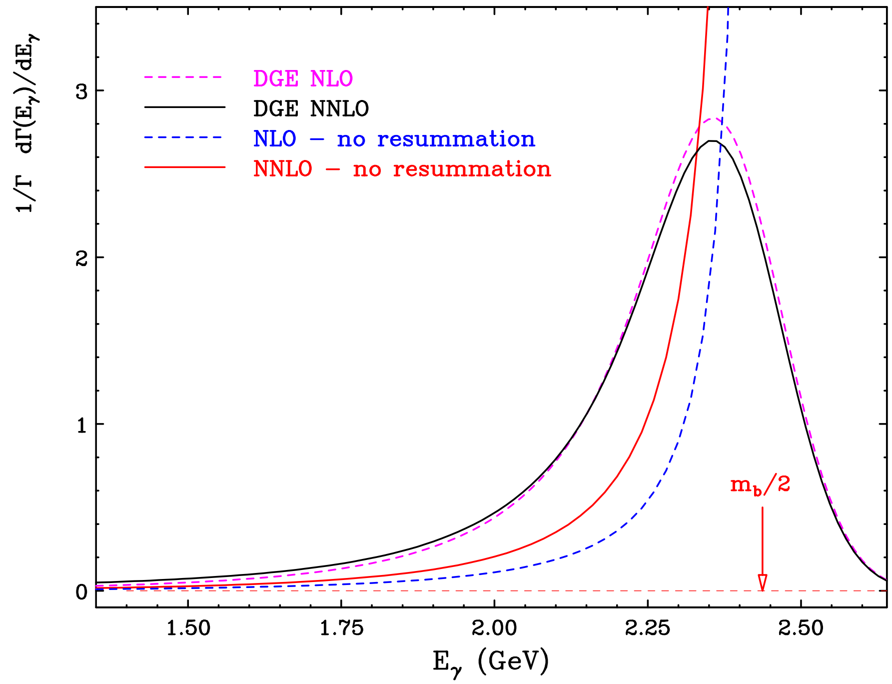
Dressed Gluon Exponentiation:

Sudakov resummation to power accuracy, giving access to non-perturbative power corrections that become large near the exclusive phase–space boundary.

The threshold region in PT and Sudakov logarithms

b decay into *light partons* is jet–like:





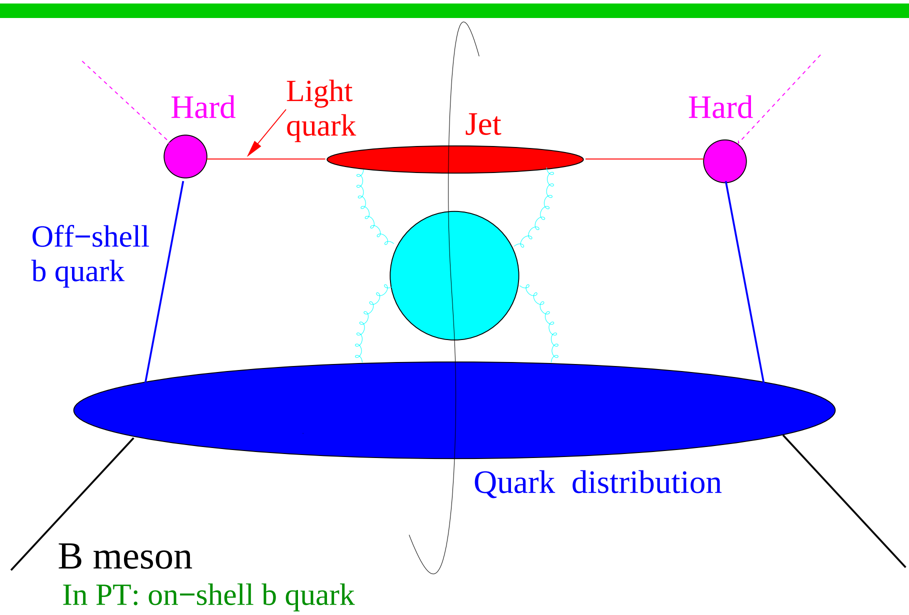
Dominated by soft & collinear radiation: **Sudakov logs**,

$$\ln(m_{\text{jet}}^2/m_b^2) = \ln(1-x) \longrightarrow -\infty$$

for $x \equiv 2E_\gamma/m_b \longrightarrow 1.$

- Real–emission contribution yield $\left(\frac{\alpha_s}{\pi}\right)^n \frac{\ln^k(1-x)}{1-x}$ with $0 \leq k \leq 2n-1$.
- All the moments are finite owing to **cancellation** with virtual corrections (IR safety).
- Resummation is necessary (even if α_s were small; but it's not small and it's running!).

Factorization in inclusive B decays



Korchemsky & Sterman '94

Hierarchy of scales $\Longrightarrow$ Factorization $\Longrightarrow$ Sudakov Resummation:

	Hard:		**Jet:**		**Quark Distribution — Soft:**
	m_b	$\gg$	$m_{\rm jet} = m_b\sqrt{1-x}$	$\gg$	$p^+_{\rm jet} \equiv E_{\rm jet} - \lvert\vec{p}_{\rm jet}\rvert = m_b(1-x)$
Moments	m_b	$\gg$	$m_b/\sqrt{N}$	$\gg$	m_b/N

$$\Gamma_N^b \quad \equiv \quad \int_0^1 dx \frac{1}{\Gamma^b_{\rm tot}}\frac{d\Gamma^b}{dx}\, x^{N-1} = H(m_b)J(m_b^2/N;\mu)S_b(m_b/N;\mu) + \ \mathcal{O}(1/N)$$

But power corrections, $(N\Lambda/m_b)^j$, distinguish the $\bar{B}$ meson $(S_{\bar{B}})$ from an on-shell b quark (S_b)!

Usually $S_b(m_b/N;\mu)$ is replaced by a "shape function", which is parametrized.

DGE approximates $S_{\bar{B}}(m_b/N;\mu)$ by $S_b(m_b/N;\mu)$ and parametrizes the ratio (pure powers).

DGE: the quark distribution function and power corrections

Evolution equation for the **quark distribution (soft) function** and the corresponding Borel sum:

$$\frac{d\ln S_N(m;\mu_F)}{d\ln m^2} = -\int_0^1 dx \frac{x^{N-1}-1}{1-x}\,\mathcal{S}\Big(\alpha_s\big((1-x)^2m^2\big)\Big) + \mathcal{O}(1/N)$$

$$= -\frac{C_F}{\beta_0}\int_0^\infty du\left(\frac{\Lambda^2}{m^2}\right)^u B_{\mathcal{S}}(u)\Gamma(-2u)\left(\frac{\Gamma(N)}{\Gamma(N-2u)}+\frac{1}{2u}\right)+\mathcal{O}(1/N)$$

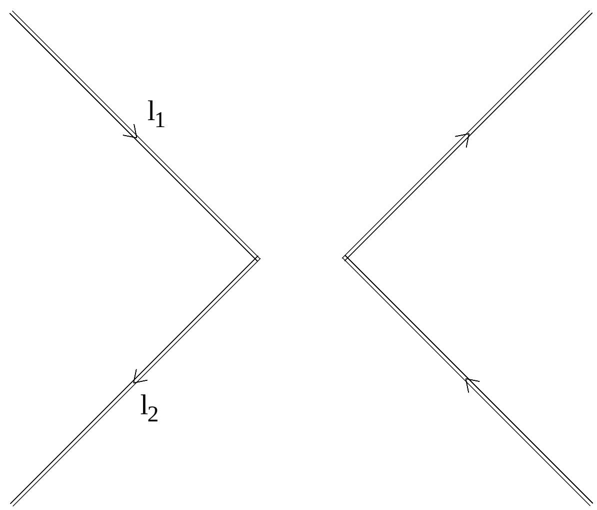
power ambiguities $\Rightarrow$ parametrically–enhanced power corrections $\mathcal{O}(N\Lambda/m)$ in the Sudakov exponent

$$S_N(m;\mu_F) \longrightarrow S_N^{\rm PV}(m;\mu_F)\times \underbrace{\exp\left\{-\sum_{k=1}^\infty \frac{\epsilon_k^{\rm PV}}{k!}\left(\frac{\Lambda}{m}\right)^k (N-1)(N-2)\ldots(N-k)\right\}}_{\text{power corrections on the scale } m/N}$$

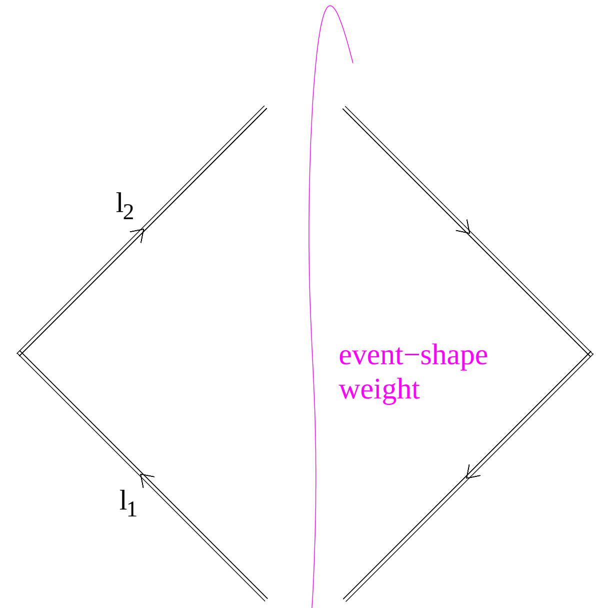
Upon choosing a prescription (e.g. PV) for the Borel integral, *the divergent sum is defined*.

- ❋ The pattern of power corrections can be deduced (large–β_0 limit):
  singularities in $\Gamma(-2u)$ at $u=\frac{k}{2} \Longrightarrow$ power corrections $(N\Lambda/m)^k$ in the exponent,
  except where $B_{\mathcal{S}}(u)=0$: **IR safety at power level**.
- ❋ NP parameters are defined in the same regularization: cancellation of renormalon ambiguities.

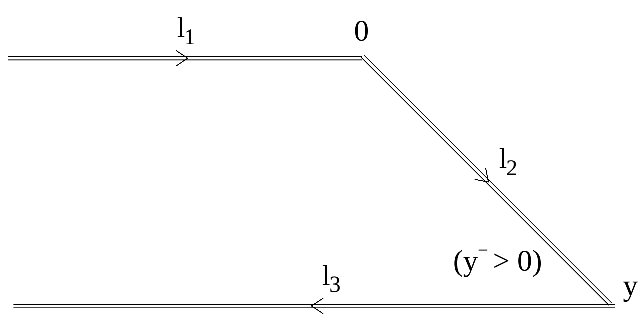
The soft function as a Wilson loop: examples



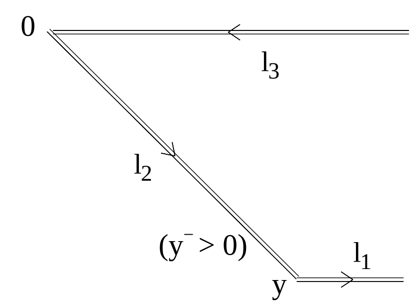
Drell−Yan or Higgs production



Event shapes in the two−jet limit



Heavy−quark distribution (b decay)



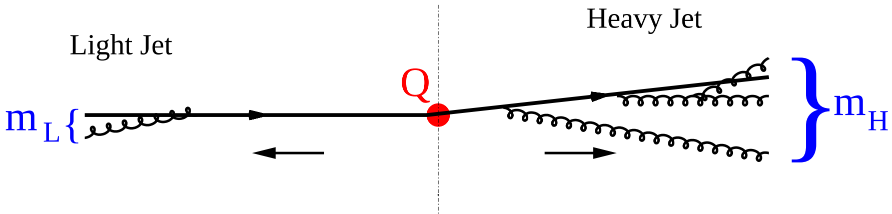
Heavy−quark fragmentaion (b production)

Soft anomalous dimensions in the large–β_0 limit: examples

$$B_{\mathcal{S}}(u) = \mathrm{e}^{\frac{5}{3}u}\frac{\sin \pi u}{\pi u}\, b_{\mathcal{S}}(u) \times \left(1 + \mathcal{O}(u/\beta_0)\right)$$

Observable	$b_{\mathcal{S}}(u)$	$B_{\mathcal{S}}(u) = 0$	power corrections
Drell-Yan e.g. in $p\bar{p} \to Z/\gamma^* \to l\bar{l}$	$2\frac{\Gamma^2(1-u)}{\Gamma(1-2u)}$	$u = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots$	$\left(\frac{\Lambda N}{Q}\right)^k$, k even Beneke & Braun '95
Event Shapes in $e^+e^- \to$ jets: Heavy Jet Mass Thrust c parameter	1 2 $2\frac{\Gamma^2(1+u)}{\Gamma(1+2u)}$	$u = 1, 2, 3, \ldots$	$\left(\frac{\Lambda N}{Q}\right)^k$, k odd
Heavy Quark Fragmentation Heavy Quark Distribution $(Q^2 = m^2)$	$(1-u)\frac{\pi u}{\sin \pi u}$	$u = 1$	$\left(\frac{\Lambda N}{m}\right)^k$, $k \neq 2$

Event–shape distributions in $e^+e^- \longrightarrow$ jets by DGE

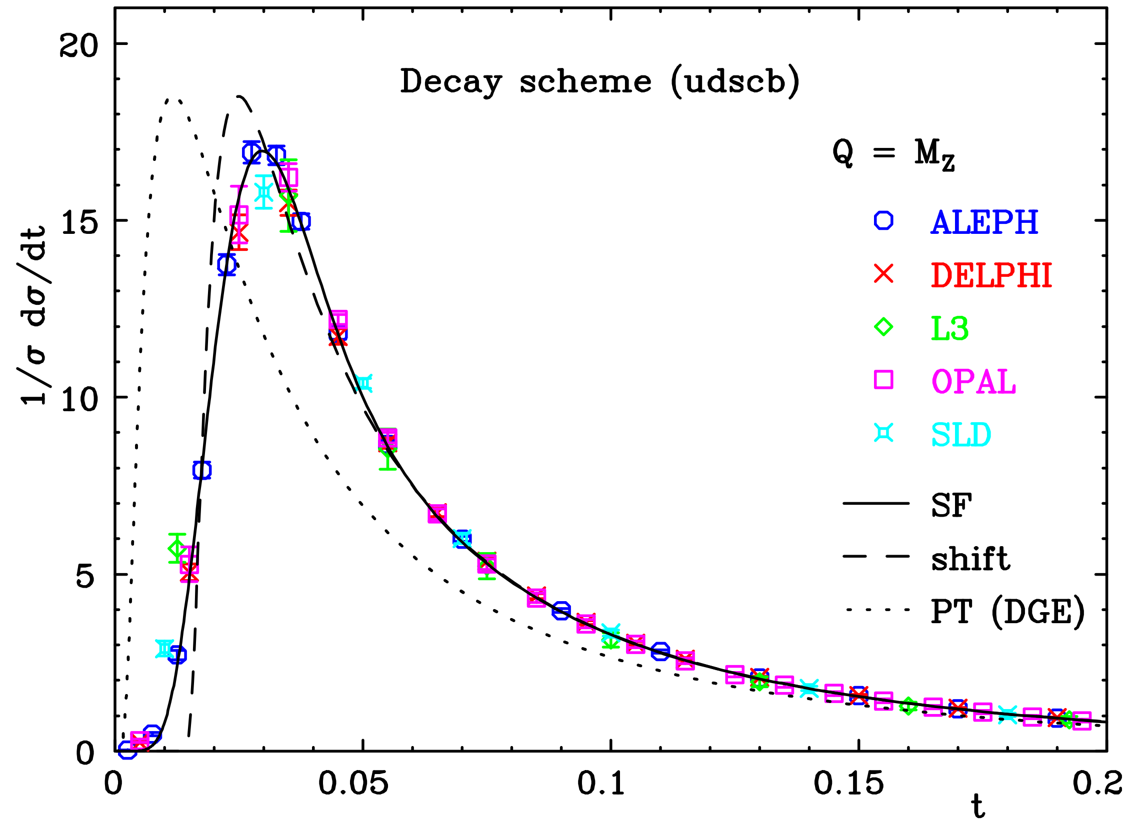


Quantifying hadronization effects in the two–jet limit by Power Corrections

Gardi & Rathsman '02

- DGE resummed result: dots
- Power–correction *predictions*:
  - ❋ shift in t
  - ❋ width unmodified

Thrust ($t = 1 - T = (m_H^2 + m_L^2)/Q^2$)



- Agreement of observed hadronization effects with renormalon prediction.
- Simple physical picture: in the two–jet limit hadronization is rapidity independent.
- DGE (with *a few* power corrections) captures the physics of the threshold region!

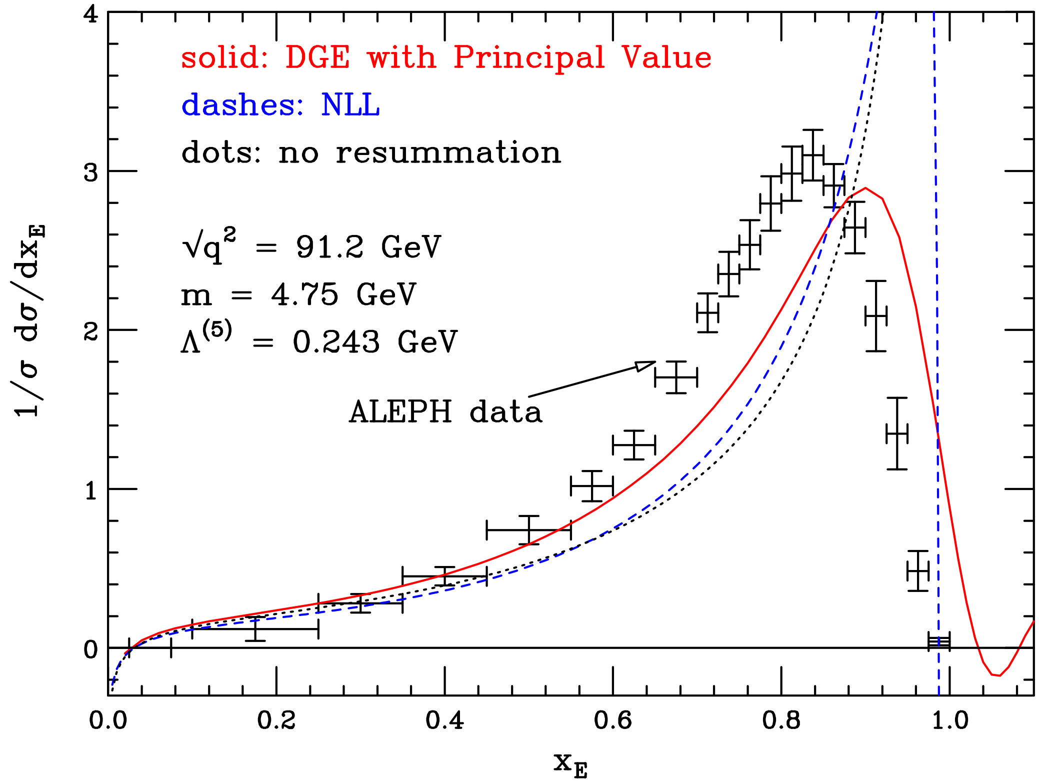
Heavy–Quark Fragmentation by DGE

Energy (E) distribution of B hadrons in inclusive $e^+e^-(Q) \to b\bar{b} \to B(E) + X_b$

$$x_E \equiv 2E/Q$$

- DGE result: red line
- Power–correction *predictions*:
  - ∗ shift in x_E
  - ∗ width unmodified

Cacciari & Gardi '03



Relation with B decay spectra:

- In perturbation theory heavy–quark fragmentation and on-shell decay spectra are controlled by *the same* Sudakov factor. (Gardi '04)
- The non-perturbative dynamics is different. In B decay the shift — the leading effect relating quark and meson decay spectra — is *determined by kinematics*: $\bar{\Lambda} \equiv M_B - m_b$

IR renormalons in inclusive B decay spectra

- The resummed E_γ spectrum is not influenced by the $u=\frac{1}{2}$, $\mathcal{O}(N\Lambda/m_b)$ ambiguity of the perturbative Sudakov exponent:

$$\frac{1}{\Gamma_{\rm tot}}\frac{d\Gamma}{dE_\gamma} = \frac{2}{m_b}\int_{c-i\infty}^{c+i\infty}\frac{dN}{2\pi i}\left(\frac{2E_\gamma}{m_b}\right)^{-N} H(m_b)\,\underbrace{J(m_b^2/N;\mu)S_b(m_b/N;\mu)}_{\text{Sud}(m_b,N)\;-\;\text{ambiguous}}$$

$$\simeq \frac{2}{M_B}\int_{c-i\infty}^{c+i\infty}\frac{dN}{2\pi i}\left(\frac{2E_\gamma}{M_B}\right)^{-N} H(m)J(m_b^2/N;\mu)\,\underbrace{S_b(m_b/N;\mu)\,\mathrm{e}^{-(N-1)\bar{\Lambda}/m_b}}_{u=\frac{1}{2}\text{ prescription independent}}$$

The cancellation is exact in all the moments, but it requires renormalon resummation in the Sudakov exponent and in $\bar{\Lambda} = M_B - m_b$

- $u=1$ renormalon is missing: no $\mathcal{O}((N\Lambda/m_b)^2)$ corrections. Width unmodified!

- $u=\frac{3}{2}$ renormalon is there: $\mathcal{O}((N\Lambda/m_b)^3)$ corrections are expected.

The resummed on-shell decay spectrum is a good approximation to the B meson one!

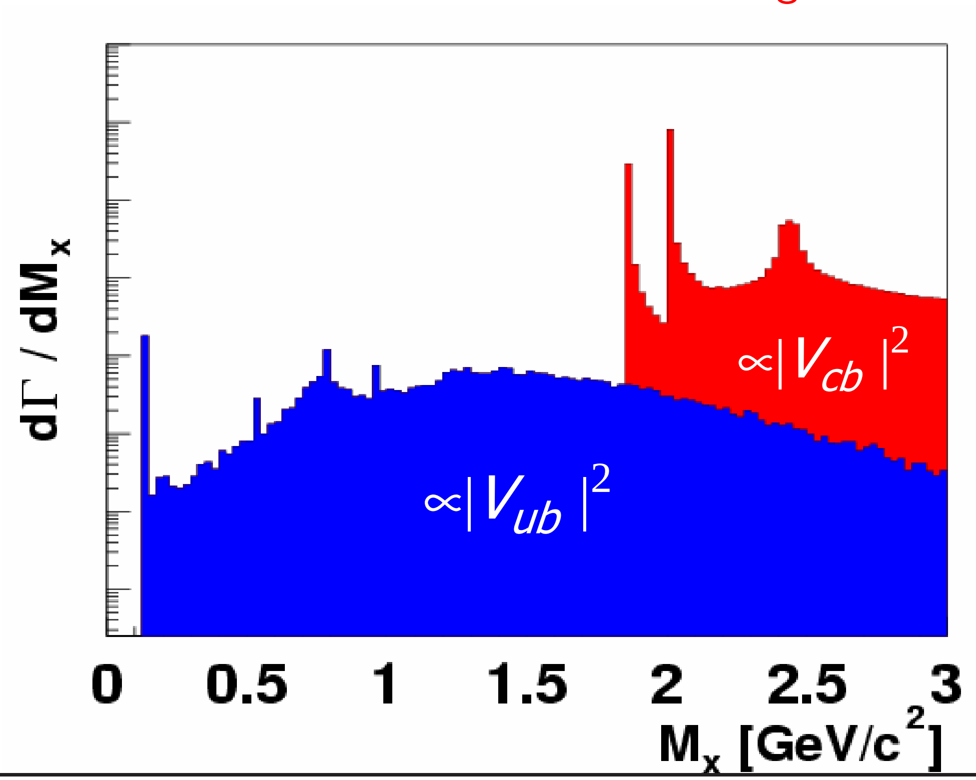
CKM and inclusive semileptonic decays

- Inclusive $b \to u$ has **an overwhelming** charm background:

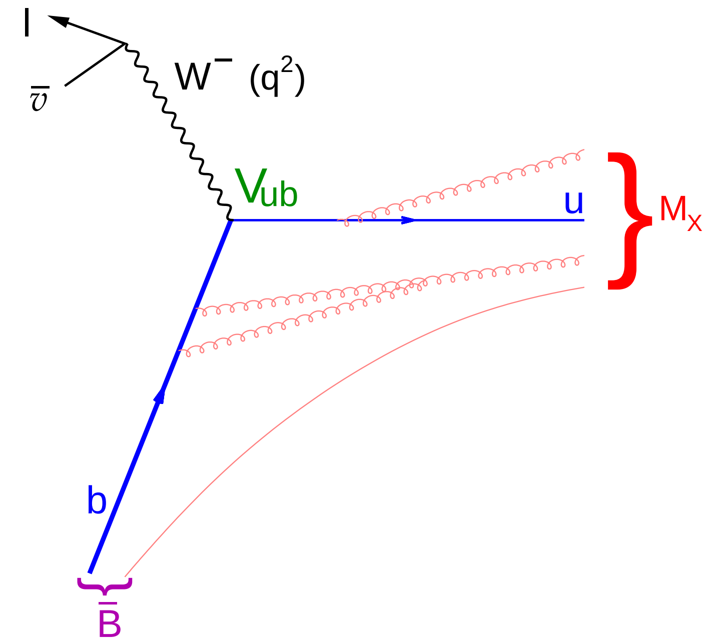
$$\frac{\Gamma(\bar{B} \to X_u l^- \bar{\nu})}{\Gamma(\bar{B} \to X_c l^- \bar{\nu})} = \frac{|V_{\rm ub}|^2}{|V_{\rm cb}|^2} \simeq \frac{1}{50}$$

- Fortunately $b \to c$ events always have $M_X > 1.7$ GeV.

Hadronic mass distribution on a log scale:



- Inclusive measurement of $b \to u$ can only be done for $M_X < 1.7$ GeV.
$\Longrightarrow$ To extract $|V_{\rm ub}|$ we need to compute the spectrum at small–M_X.



- Same dynamics as in $\bar{B} \to X_s \gamma$.
So far: "shape function" fit
Now: DGE calculation + power corrections

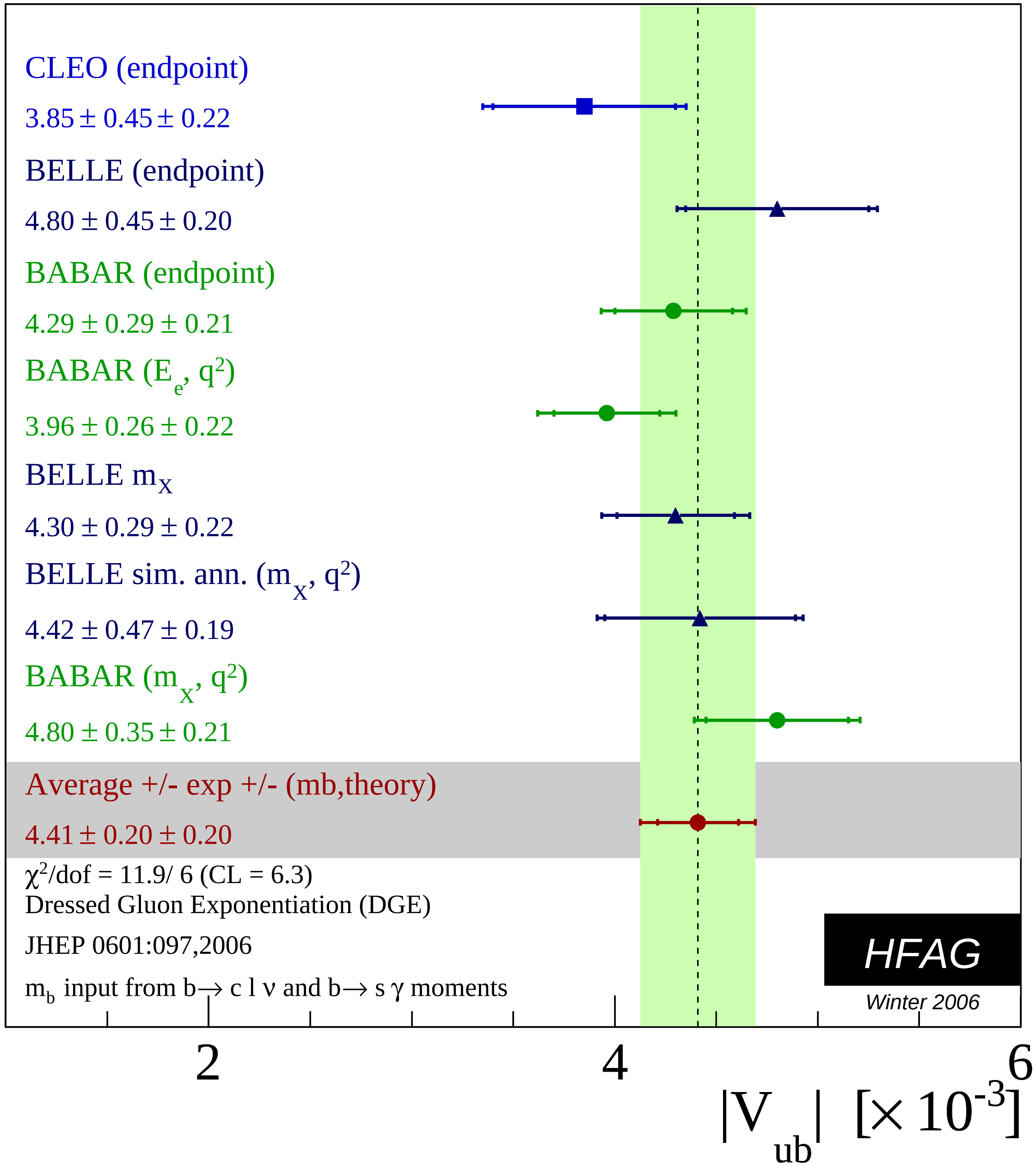
World Average $|V_{\rm ub}|$ using DGE

- CLEO, Belle & BaBar have performed several inclusive measurements of the partial $b \to u$ BF with *different kinematic cuts* on E_l, q^2, M_X, etc.

- March '06: each measurement was translated by the *Heavy Flavour Averaging Group* (*HFAG*) into a value for $|V_{\rm ub}|$ using DGE.

- The results are all consistent.

- The uncertainty is the smallest obtained so far.



Conclusions

- Progress towards NNLO calculation of the $\bar{B} \longrightarrow X_s\gamma$ branching fraction.
  - ☞ The theory uncertainty will decrease significantly ($\sim$ half) with full NNLO.
  - ☞ Important contributions (specifically G_{27}) are still missing.
  - ☞ Reduction of uncertainty with partial NNLO?

- Dressed Gluon Exponentiation: a new tool for inclusive decays spectra
  - ☞ DGE captures the physics of the threshold region:
    - ❋ resummed perturbation theory (with a first few power corrections in the exponent) can be used well into the threshold region — instead of a shape function
    - ❋ Observed pattern of non-perturbative corrections is consistent with the renormalon analysis; e.g. *width unmodified* in event-shapes, fragmentation and inclusive decays
  - ☞ Better determination of decay spectra by DGE allows for
    - ❋ precise extrapolation of $\mathcal{B}(\bar{B} \to X_s\gamma)$ below the region of measurement
    - ❋ precise determination of m_b from $\bar{B} \longrightarrow X_s\gamma$
    - ❋ precise determination of $|V_{\rm ub}|$ from inclusive charmless semileptonic decays.