B Mixing and Lifetimes

. . . with a Lattice Perspective

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BEACH 2006

Lancaster University, 2–8 July 2006

Contents

Mixing and Decay

Effective Hamiltonian matrix for $|B\rangle$, $|\bar{B}\rangle$ system:

$$\begin{pmatrix} M_{11} & M_{12} \\ M_{12}^* & M_{11} \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma_{11} \end{pmatrix}$$

Physical eigenstates:

$$|B_H\rangle = p|B\rangle + q|\bar{B}\rangle \qquad \text{with } |p|^2 + |q|^2 = 1$$
$$|B_L\rangle = p|B\rangle - q|\bar{B}\rangle$$

Probe off-diagonal entries ($\Delta B = 2$) with

Mass difference $\qquad \Delta m = M_H - M_L \approx 2|M_{12}|$

Lifetime difference $\qquad \Delta\Gamma = \Gamma_L - \Gamma_H \approx 2|\Gamma_{12}|\cos\phi$

CP asymmetry $\qquad \left|\frac{q}{p}\right| - 1 \approx \frac{1}{2}\mathrm{Im}\,\frac{\Gamma_{21}}{M_{21}}$

Mixing

$$M_{12}^{q} = \frac{G_F^2}{12\pi^2}(V_{tq}^{*}V_{tb})^2 M_W^2 S_0(x_t)\eta_B\, B_{B_q} f_{B_q}^2 M_{B_q}$$

with $\Delta B = 2$ matrix element parametrised by

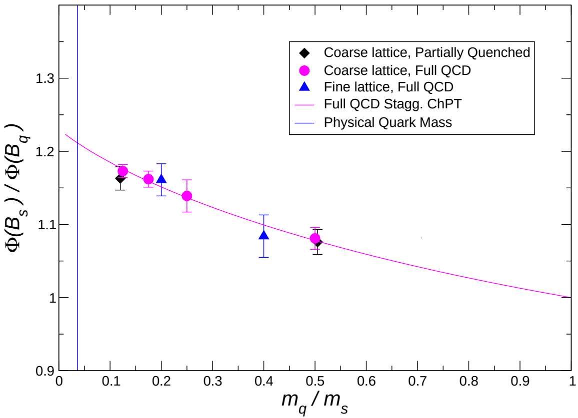
$$\langle \bar{B}_q | (\bar{b}\gamma^{\mu} L q)(\bar{b}\gamma_{\mu} L q) | B_q \rangle = \frac{8}{3} B_{B_q} f_{B_q}^2 M_{B_q}$$

- $f_B\sqrt{B}$ relevant quantity for mixing
- Quantities with least-correlated errors in LQCD are

$$f_{B_s}\sqrt{B_{B_s}} \qquad \text{and} \qquad \xi \equiv \frac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}}$$

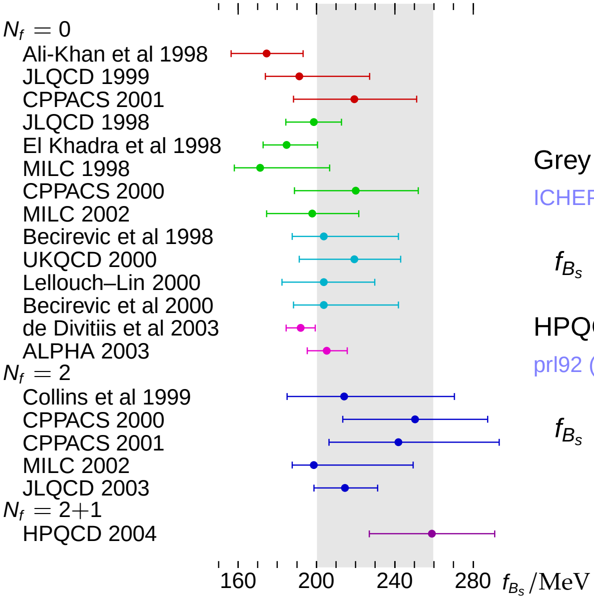
- ξ most sensitive to chiral extrapolation (Kronfeld–Ryan 2002), other errors tend to cancel in ratio

Mixing: Chiral Extrapolation



Plot of $\Phi(B_s)/\Phi(B_q) = f_{B_s}\sqrt{m_{B_s}}/f_{B_q}\sqrt{m_{B_q}}$ (HPQCD results, shown in Wingate hep-ph/0604254)

f_{B_s} History



Grey band, Hashimoto, ICHEP 2004:

$$f_{B_s} = 230 \pm 30\,\mathrm{MeV}$$

HPQCD (Wingate et al, prl92 (2004) 162001):

$$f_{B_s} = 260 \pm 29\,\mathrm{MeV}$$

Mixing

- B_q results not yet available from staggered fermions
- Rather than combine f_{B_s} and B_{B_s} from different formalisms, I would stick with the averages:

$$f_{B_s} = 230(30)\,\mathrm{MeV}$$

$$f_{B_s}\sqrt{\hat{B}_{B_s}} = 262(35)\,\mathrm{MeV}$$

Hashimoto, ICHEP2004

$$\xi = 1.23(6)$$

- The combination *is* done in Okamoto Lattice2005, Mackenzie FPCP2006, Wingate hep-ph/0604254:
  - f_{B_s} and $f_{B_s}\sqrt{\hat{B}_{B_s}}$ go up by $\approx 30\,\mathrm{MeV}$
  - ξ not much affected but quoted error less [$1.21(^{+5}_{-4})$]

Lifetime Ratios: Experiment

HFAG hep-ex/0603003

$$\frac{\tau(B^+)}{\tau(B^0)} = 1.076 \pm 0.008\,\mathrm{ps}$$

$$\frac{\tau(B_s)}{\tau(B^0)} = 0.914 \pm 0.030\,\mathrm{ps}$$

$$\frac{\tau(\Lambda_b)}{\tau(B^0)} = 0.844 \pm 0.043\,\mathrm{ps}$$

But using van Kooten, FPCP2006, hep-ex/0606005 for $\tau(B_s)$:

$$\frac{\tau(B_s)}{\tau(B^0)} = 0.957 \pm 0.020\,\mathrm{ps}$$

Lifetime of hadron H_b containing a b-quark

$$\Gamma(H_b) = \frac{1}{m_{H_b}} \mathrm{Im}\langle H_b|\mathcal{T}|H_b\rangle$$

where

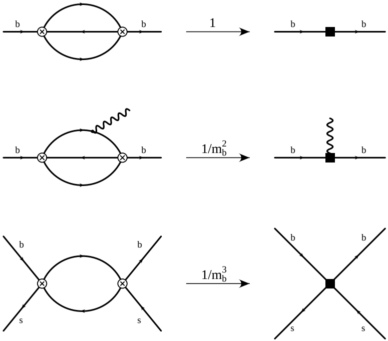
$$\mathcal{T} = i \int d^4x \, \mathrm{T}\left[\mathcal{H}^{|\Delta B|=1}(x)\mathcal{H}^{|\Delta B|=1}(0)\right]$$

- Effective Hamiltonian $\mathcal{H}^{|\Delta B|=1}$ known to NLO Buchalla et al, Ciuchini et al and NNLO Gorbahn–Haisch
- Heavy Quark Expansion: large energy release in b decay allows OPE of $\mathcal{T}$ as series of local operators of increasing dimension with increasing inverse powers of m_b and calculable coefficients (containing CKM factors)

$$\Gamma(H_b) = \sum_k \frac{c_k(\mu)\langle H_b|O_k(\mu)|H_b\rangle}{m_b^k}$$

Leading Order Analysis

Neubert–Sachrajda, 1996



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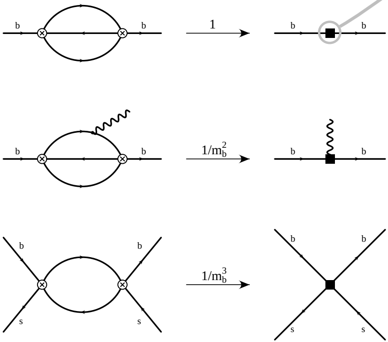
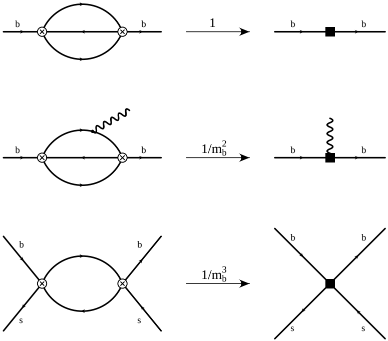


- $O(1)$: $\bar{b}b$ free quark decay

$$\langle \bar{b}b \rangle \stackrel{\text{HQET}}{=} 1 - \frac{\mu_\pi^2 - \mu_G^2}{2m_b^2} + O(m_b^{-3})$$

Leading Order Analysis

Neubert–Sachrajda, 1996



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Leading Order Analysis

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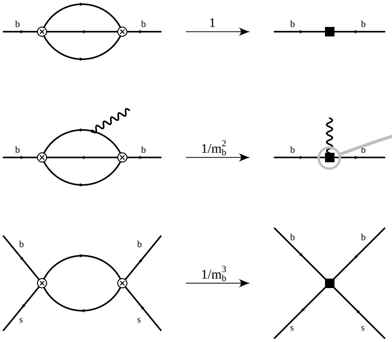


- $O(1)$: $\bar{b}b$ free quark decay

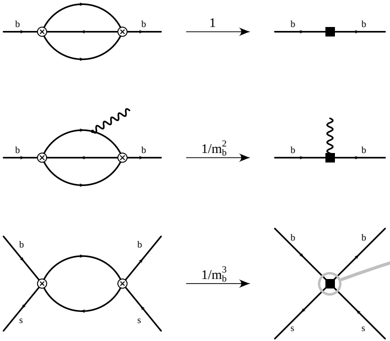
  $\langle \bar{b}b \rangle \stackrel{\text{HQET}}{=} 1 - \frac{\mu_\pi^2 - \mu_G^2}{2m_b^2} + O(m_b^{-3})$
- $O(1/m_b)$: no contribution
- $O(1/m_b^2)$: $\bar{b}g_s\sigma{\cdot}G\,b$

  chromomagnetic operator

  $\langle \bar{b}g_s\sigma{\cdot}G\,b \rangle \stackrel{\text{HQET}}{=} 2\mu_G^2 + O(m_b^{-1})$

Leading Order Analysis

Neubert–Sachrajda, 1996



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- $O(1/m_b^3)$: $\bar{b}\Gamma q\, \bar{q}\Gamma b$ spectator effects (1-loop $\rightarrow$ $16\pi^2$ factor): four $\Delta B = 0$ 4-quark operators

Leading Order Analysis

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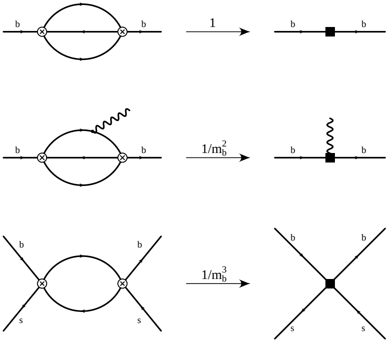

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- $O(1/m_b)$: no contribution
- $O(1/m_b^2)$: $\bar{b}g_s\sigma{\cdot}G\,b$ chromomagnetic operator
  $$\langle \bar{b}g_s\sigma{\cdot}G\,b \rangle \overset{\text{HQET}}{=} 2\mu_G^2 + O(m_b^{-1})$$
- $O(1/m_b^3)$: $\bar{b}\Gamma q\,\bar{q}\Gamma b$ spectator effects (1-loop $\to$ $16\pi^2$ factor): four $\Delta B{=}0$ 4-quark operators

- $O(1/m_b^3)$ spectator effects dominate lifetime differences
- …if matrix elements of 4-quark operators not too small

Lifetime Ratios

- NLO QCD corrections to Wilson coefficients

  Beneke–Buchalla–Greub–Lenz–Nierste, 2002

  Franco–Lubicz–Mescia–Tarantino, 2002

  $\longrightarrow$ brings in penguin operators at $O(1/m_b^3)$
- Matrix elements from LQCD

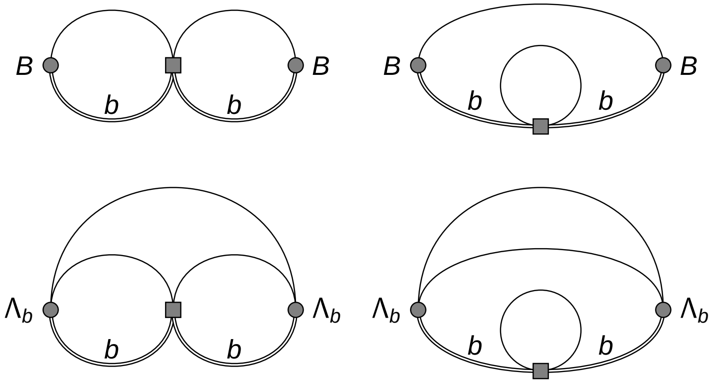
  Di Pierro–Sachrajda, Di Pierro–Michael–Sachrajda, 1998/99

  APE, Becirevic et al, 2001
- Leading $O(1/m_b^4)$ spectator contributions from eight dimension-7 4-quark operators

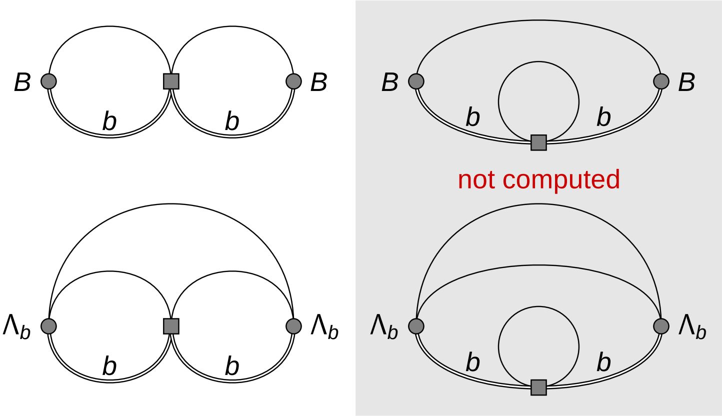
  Gabbiani–Onishchenko–Petrov, 2004

  $\longrightarrow$ yet more matrix elements ... estimated from vacuum insertion (B-mesons) or quark-diquark model (baryon)

Lattice Matrix Elements



Lattice Matrix Elements



Penguin contributions and “eye” diagrams not computed. Tend to cancel for $\tau(B^+)/\tau(B_d)$ and (less so) for $\tau(B_s)/\tau(B_d)$ but not for $\tau(\Lambda_b)/\tau(B_d)$

Matrix elements: Leading operators

$$O_1^q = (\bar{b}\gamma^\mu L q)(\bar{q}\gamma_\mu L b) \qquad O_3^q = (\bar{b}\gamma^\mu t^a L q)(\bar{q}\gamma_\mu L t^a b)$$
$$O_2^q = (\bar{b} L q)(\bar{q} L b) \qquad O_4^q = (\bar{b} L t^a q)(\bar{q} L t^a b)$$

Parametrise:

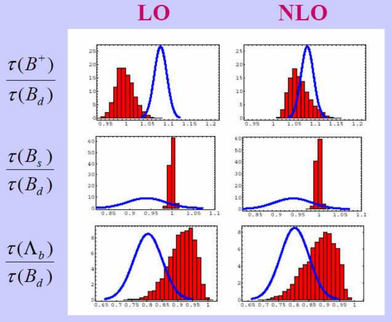
$$\frac{\langle B_q | O_{1,2}^q | B_q \rangle}{2M_{B_q}} = \frac{f_{B_q}^2 M_{B_q}}{2}(B_{1,2}^q + \delta_{1,2}^{qq})$$
$$\frac{\langle B_q | O_{3,4}^q | B_q \rangle}{2M_{B_q}} = \frac{f_{B_q}^2 M_{B_q}}{2}(\epsilon_{3,4}^q + \delta_{3,4}^{qq})$$
$$\frac{\langle B_q | O_i^{q'} | B_q \rangle}{2M_{B_q}} = \frac{f_{B_q}^2 M_{B_q}}{2}\delta_i^{qq'} \quad (q \neq q')$$

$$\frac{\langle \Lambda_b | O_1^q | \Lambda_b \rangle}{2M_{\Lambda_b}} = \frac{f_B^2 M_B}{2}(L_1 + \delta_1^{\Lambda,q})$$
$$\frac{\langle \Lambda_b | O_3^q | \Lambda_b \rangle}{2M_{\Lambda_b}} = \frac{f_B^2 M_B}{2}(L_3 + \delta_3^{\Lambda,q})$$

VIA $\rightarrow B \approx 1$, $\epsilon \approx 0$. LQCD confirms
di Pierro–Sachrajda (1998), Becirevic et al (2001)
δ's not calculated

$L_{1,3}$ from exploratory calculation
di Pierro et al (1999)
δ's not calculated

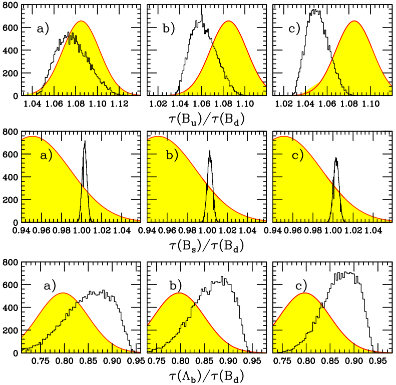
Lifetime Ratios: Theory



Tarantino CKM2005, Beauty2005,

Franco-Lubicz-Mescia-Tarantino,

npb633 (2002) 212



Gabbiani-Onishchenko-Petrov

prd70 (2004) 094031

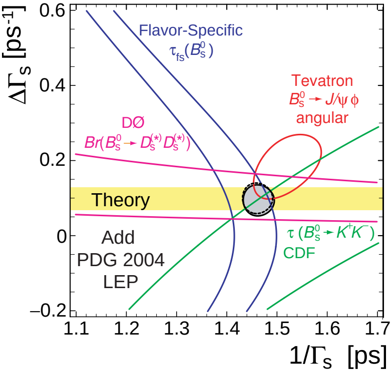
Lifetime Ratios: Comparison

	Expt	T05	GOP
$\dfrac{\tau(B^+)}{\tau(B^0)}$	1.076 ± 0.008	1.06 ± 0.02	1.06 ± 0.02
$\dfrac{\tau(B_s)}{\tau(B^0)}$	0.957 ± 0.020	1.00 ± 0.01	1.00 ± 0.01
$\dfrac{\tau(\Lambda_b)}{\tau(B^0)}$	0.844 ± 0.043	0.88 ± 0.05	0.86 ± 0.05

- Expt is HFAG with van Kooten, FPCP2006 for $\tau(B_s)$
- T05 is Tarantino CKM2005, updating analysis of Franco et al, 2002
- GOP is Gabbiani–Onishchenko–Petrov, 2004

$\Delta\Gamma_d$ and $\Delta\Gamma_s$ Experiment

van Kooten, FPCP2006



HFAG 2006 (prelim)

$$\frac{\Delta\Gamma_d}{\Gamma_d} = 0.009 \pm 0.037$$

$$\Delta\Gamma_s = 0.097 \pm 0.042\,\mathrm{ps}^{-1}$$

$$\bar{\tau} = 1/\Gamma_s = 1.461 \pm 0.030\,\mathrm{ps}$$

$$\frac{\Delta\Gamma_s}{\Gamma_s} \approx 0.14$$

Lifetime Differences

- Lifetime difference for hadron H_b depends on off-diagonal decay matrix element ($\Delta B = 2$):

$$\Delta\Gamma_{H_b} = -\frac{1}{m_{H_b}}\langle \bar{H}_b|\mathcal{T}|H_b\rangle$$

- Use HQE to organise as series of operators of increasing dimension with calculable coefficients containing inverse powers of m_b
  - leading contribution at $O(1/m_b^3)$: two dim-6 operators
  - at $O(1/m_b^4)$, four more dim-7 operators
- To complete the calculation need matrix elements of the operators

$\Delta\Gamma_s$, $\Delta\Gamma_d$: Theoretical Status

$$\Delta\Gamma = \left(\frac{\Lambda}{m_b}\right)^3\left(\Gamma_3^{(0)} + \frac{\alpha_s}{4\pi}\Gamma_3^{(1)} + \cdots\right) + \left(\frac{\Lambda}{m_b}\right)^4\left(\Gamma_4^{(0)} + \cdots\right) + \left(\frac{\Lambda}{m_b}\right)^5\left(\Gamma_5^{(0)} + \cdots\right) + \cdots$$

- $\Gamma_3^{(0)}$: Hagelin, Buras et al, Datta et al, Voloshin et al, Chau, Franco et al (from 1981)
- $\Gamma_3^{(1)}$: Beneke–Buchalla–Greub–Lenz–Nierste (1998), Ciuchini–Franco–Lubicz–Mescia–Tarantino (2003)
- $\Gamma_4^{(0)}$: Beneke–Buchalla–Dunietz (1996), Dighe et al (2001), Ciuchini–Franco–Lubicz–Mescia–Tarantino (2003)
- $\Gamma_5^{(0)}$: Lenz–Nierste (2006) prelim

Lifetime Differences: Matrix Elements

- Leading contribution in $1/m_b$: two operators

$$O_1 = (\bar{b}\gamma^\mu Lq)(\bar{b}\gamma_\mu Lq), \qquad O_2 = (\bar{b}Lq)(\bar{b}Lq)$$

  - O_1 also determines Δm
  - parameterize as $\langle \bar{B}_q | O_i^q | B_q \rangle = \text{const} \times f_{B_q}^2 B_i$
  - $B_{1,2}$ from lattice: Gimènez–Reyes (2000), Hashimoto et al (2000), JLQCD (2001–2003), Becirevic et al (2000, 2001)
- Subleading contribution: four dimension-7 operators
  - two operators related to complete set of $\Delta B = 2$ operators calculated by lattice (Becirevic et al (2001)); others by vacuum insertion

B_1, B_2 from the Lattice

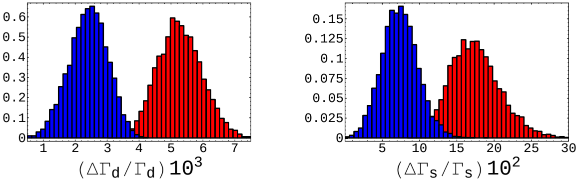
Matrix elements for leading contribution in $1/m_b$. Two operators:

$$O_1 = (\bar{b}\gamma^\mu Lq)(\bar{b}\gamma_\mu Lq), \qquad O_2 = (\bar{b}Lq)(\bar{b}Lq)$$

	B_1^s	B_2^s	
HQET, static	$0.83(5)(6)$	$0.81(2)(10)$	Gimènez–Reyes, 2000
NRQCD, $O(1/m_b)$	$0.85(3)(11)$	$0.82(2)(11)$	Hashimoto et al, 2000
NRQCD, $N_f = 2$	$0.85(2)(6)$	$0.84((6)(8)$	JLQCD, 2001–2003
QCD, $m_Q \to m_b$	$0.91(3)\binom{0}{6}$	$0.86(2)\binom{2}{3}$	APE, Becirevic et al, 2000
QCD & HQET	$0.87(2)(5)$	$0.84(2)(4)$	APE, Becirevic et al, 2001

Lifetime Differences: Theory Output

- QCD NLO corrections important



LO red, NLO blue (Ciuchini et al (2003), Tarantino Beauty2005)

- $1/m_b$ corrections important
- Size and same sign of corrections above led Lenz–Nierste to consider $1/m_b^2$ corrections: turn out to be small (Lenz–Nierste (2006) prelim)
- Change operator basis to make coefficient of Δm_s operator dominant: reduces uncertainty from QCD and $1/m_b$ corrections (Lenz–Nierste (2006) prelim)

$\Delta\Gamma_s/\Gamma_s$: Theory Output

Two ways to quote a number:

- $\Delta\Gamma/\Gamma = \Delta\Gamma_{\text{theo}}\,\tau_{\text{expt}}$
  Pro: independent of new physics in mixing
  Con: depends on $f_{B_s}^2$
- $\Delta\Gamma/\Gamma = (\Delta\Gamma/\Delta m)_{\text{theo}}\,\Delta m_{s,\text{expt}}\,\tau_{\text{expt}}$
  Pro: theoretically clean
  Con: might depend on new physics in Δm_s

Lenz–Nierste favour first method and find

$$(\Delta\Gamma/\Gamma)_s = 0.158^{+0.046}_{-0.051}$$ Lenz–Nierste (2006) prelim

They use $f_{B_s} = 245\,\mathrm{MeV}$ (both methods agree for $f_{B_s} = 221\,\mathrm{MeV}$)

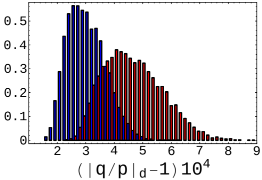
Lifetime Differences: Comparison

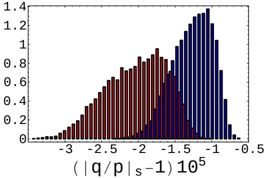
	Expt	Theory
$\frac{\Delta\Gamma_s}{\Gamma_s}$	0.14 ± 0.06	0.16 ± 0.05
$\frac{\Delta\Gamma_d}{\Gamma_d}$	0.009 ± 0.037	0.003 ± 0.001

- Expt is van Kooten, FPCP2006 for B_s, HFAG, hep-ex/0603003 for B_d
- Theory is Lenz–Nierste (2006) prelim

CP-violation Parameters

- Determination of NLO QCD and $1/m_b$ corrections by Beneke–Buchalla–Lenz–Nierste (2003) and Ciuchini–Franco–Lubicz–Mescia–Tarantino (2003)
- Comparison of LO (red) and NLO (blue) (Ciuchini et al, updated Tarantino Beauty2005)





	BBLN	CFLMT
$\left\|\frac{q}{p}\right\|_d - 1$	$(2.5 \pm 0.6) \times 10^{-4}$	$(2.96 \pm 0.67) \times 10^{-4}$
$\left\|\frac{q}{p}\right\|_s - 1$	$-(1.1 \pm 0.2) \times 10^{-5}$	$-(1.28 \pm 0.28) \times 10^{-5}$

LQCD

- Lattice computations put QCD on a finite discrete Euclidean space-time lattice and do functional integrals by Monte-Carlo
- Quantities which can be calculated include:
  - hadronic masses (and hence quark masses)
  - matrix elements of form

$$\begin{array}{rl} \langle 0|\mathcal{O}|H\rangle & \text{eg leptonic decay constants} \\ \langle H_2|\mathcal{O}|H_1\rangle & \text{eg semileptonic form-factors} \end{array}$$

    where $\mathcal{O}$'s are local composite operators and H, H_1 and H_2 are hadrons.
- Recently have learned how to evaluate matrix elements with two-hadron states below inelastic threshold.
- The *quenched* approximation, in which vacuum polarization effects are neglected is now largely removed.

Improving Precision: Light Quarks

- Emphasis now on reducing masses of the *u* and *d* quarks in the simulations, to control *chiral extrapolation*.

	m_q/m_s	m_π MeV	m_π/m_ρ
SU(3) limit	1	690	0.68
Currently typical	1/2	490	0.55
Impressive	1/4	340	0.42
MILC	1/8	240	0.31
Physical	1/25	140	0.18

- Want to use lattice actions with $\sim O(a^2)$ discretization errors ($a =$ lattice spacing) which give good control of chiral behaviour at reasonable computational cost.
- Challenge set by MILC (and collaborating groups using their data) using Improved Staggered Fermions, who have calculated many quantities with small quoted errors.

Staggered Fermions

- Unphysical *tastes* removed by taking 4th root of fermionic determinant
- No proof that this is correct, but growing circumstantial evidence and no counter-example
- Staggered chiral perturbation theory has to include the a-dependence and has many parameters (e.g. over 50 for f_π). The massless limit ($m \longrightarrow 0$) cannot be taken before the continuum limit ($a \longrightarrow 0$)
- matching lattice $\leftrightarrow$ continuum is done perturbatively

Staggered Fermions and Others

- Unphysical *tastes* removed by taking 4th root of fermionic determinant
- No proof that this is correct, but growing circumstantial evidence and no counter-example
- Staggered chiral perturbation theory has to include the a-dependence and has many parameters (e.g. over 50 for f_π). The massless limit ($m \longrightarrow 0$) cannot be taken before the continuum limit ($a \longrightarrow 0$)
- matching lattice $\leftrightarrow$ continuum is done perturbatively
- Confirmation of extrapolations and procedures by other groups would be welcome
- Other light quark actions are also being used: improved Wilson, twisted mass, domain wall/overlap. Results from these with comparable parameters to those now available with staggered quarks could lead to full confidence in the results.

Improving Precision: Heavy Quarks

Typical lattice spacings $1.5\,\mathrm{GeV} < a^{-1} < 3\,\mathrm{GeV}$ preclude *direct* simulation of *b*-quarks (and questionable even for *c*-quarks)
Actions used for heavy quarks include

- QCD with heavy*ish* quarks and extrapolation in mass
- HQET
- NRQCD
- Fermilab/Tsukuba action

Results from different formulations have been consistent

Heavy Quarks

HQET

Challenge is to go beyond static limit to $O(1/m_b)$ and to perform nonperturbative renormalisation.

ALPHA collaboration

NRQCD

Expansion in velocity of heavy quarks. Particularly applicable to quarkonium but also used in heavy-light physics. No continuum limit [errors of order $1/(m_b a)^n$]

Fermilab/Tsukuba action

Nonrelativistic interpretation: breaks hypercubic to cubic symmetry $\longrightarrow$ more terms in action and operators for matrix elements

El Khadra–Kronfeld–Mackenzie (1996)

Ideas being developed for nonperturbative determination of parameters in action Lin–Christ (2005,06)

Conclusions

- Mixings: awaiting fully-unquenched results for B parameter and confirmation by more than one group
- Lifetime ratios and differences:
  - spectator effects can be large enough to explain Λ_b lifetime puzzle
  - B_s could get interesting?
  - hadronic input uncertainty: need to update LQCD matrix elements
- LQCD: unquenched simulations now standard, with much attention focused on light quarks (choice of action and chiral extrapolation).