b-Hadrons: Mixing and Lifetimes with a Lattice Perspective

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I review theoretical calculations of the nonperturbative parameters needed to describe B-meson mixing and b-hadron lifetime ratios and lifetime differences. I take a lattice QCD perspective and close with some comments on the current status of lattice calculations.

1. MIXING AND DECAY

The effective Hamiltonian and physical states for the $|B\rangle$, $|\overline{B}\rangle$ system are

$$\begin{pmatrix} M_{11} & M_{12} \\ M_{12}^* & M_{11} \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma_{11} \end{pmatrix}$$

$$|B_H\rangle = p|B\rangle + q|\overline{B}\rangle, \qquad |B_L\rangle = p|B\rangle - q|\overline{B}\rangle$$

where $|p|^2 + |q|^2 = 1$. The off-diagonal, $\Delta B = 2$, entries can be probed by measuring:

mass difference	$\Delta m = M_H - M_L \approx 2|M_{12}|$
width difference	$\Delta\Gamma = \Gamma_L - \Gamma_H \approx 2|\Gamma_{12}|\cos\phi$
CP asymmetry	$\left|\frac{q}{p}\right| - 1 \approx -\frac{1}{2}\mathrm{Im}\,\frac{\Gamma_{12}}{M_{12}}$

In the Standard Model (SM) the phase ϕ is expected to be small for $\Delta\Gamma_s$, while $\Delta\Gamma_d$ is negligible.

2. MIXING

In the SM, Δm_q for a neutral B_q-meson containing light quark q, with mass M_{B_q}, is governed by the matrix element,

$$\langle \overline{B}_q|(\overline{b}q)_{V-A}(\overline{b}q)_{V-A}|B_q\rangle = \frac{8}{3}B_{B_q} f_{B_q}^2 M_{B_q}$$

where f_{B_q} is the meson decay constant and B_{B_q} would be 1 if the vacuum insertion approximation were correct. Knowledge of $f_{B_q}\sqrt{B_{B_q}}$ for $q = d,s$ is needed to use experimental B_q mixing information to constrain the CKM unitarity triangle.

From the point of view of lattice calculations, the quantities with the least-correlated errors are

$$f_{B_s}\sqrt{B_{B_s}} \qquad \text{and} \qquad \xi \equiv \frac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}}.$$

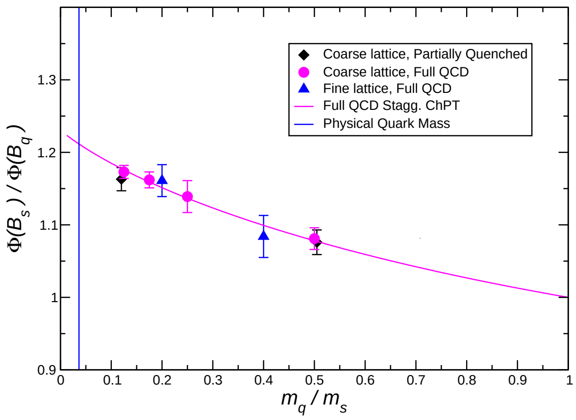

Figure 1. Chiral extrapolation of $\phi(B_s)/\phi(B_q) \equiv f_{B_s}\sqrt{m_{B_S}}/f_{B_s}\sqrt{m_{B_S}}$ using HPQCD lattice results [1].

The ratio ξ is most sensitive to chiral extrapolation errors, while $f_{B_s}\sqrt{B_{B_s}}$ is more sensitive to the remaining lattice systematics. Figure 1 shows the ratio $\phi(B_s)/\phi(B_q) \equiv f_{B_s}\sqrt{m_{B_S}}/f_{B_q}\sqrt{m_{B_q}}$ using HPQCD lattice results, taken from [1]. This ratio, and ξ to which it is closely related, depends quite strongly on the light quark mass. To get the physical result one has to reach the blue line on the left of the plot. Impressive progress has been made using staggered fermions to push down the mass of the light dynamical quarks in simulations. Figure 2 (updated from the one shown by Hashimoto at ICHEP04 [2]) shows a history of lattice results for f_{B_s}, culminating with the most recent 2+1-flavour dynamical simulations using staggered fermions. There is a discernible increase in the value of f_{B_s} in the unquenched results.

Results for B_{B_q} are not yet available from staggered

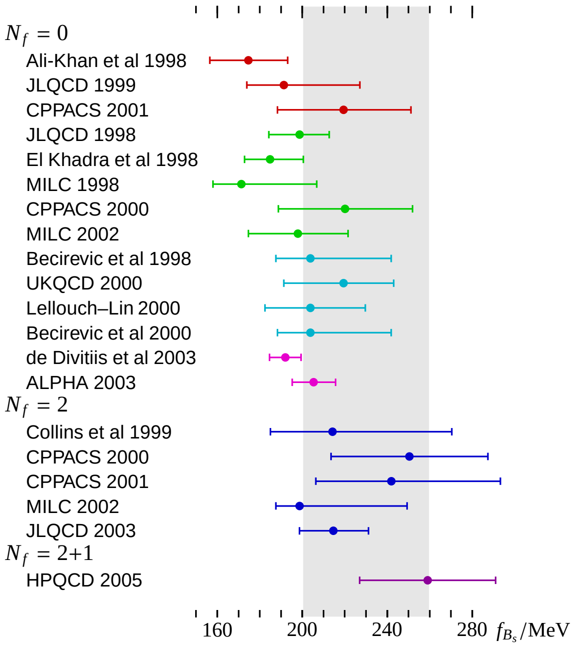

Figure 2. Lattice results for f_{B_s}, updated from [2] (see [2] for the $N_f = 0, 2$ references). The grey band is the average, $f_{B_s} = 230 \pm 30\,\mathrm{MeV}$ given in [2].

fermions, so rather than combine results from different formalisms, I prefer to quote the averages given by Hashimoto [2]:

$$f_{B_s} = 230(30)\,\mathrm{MeV},$$
$$f_{B_s}\sqrt{B_{B_s}} = 262(35)\,\mathrm{MeV},$$
$$\xi = 1.23(6).$$

However, the combination *is* done in [1,3,4] with the result that f_{B_s} and $f_{B_s}\sqrt{B_{B_s}}$ go up by about 30 MeV, while the central value of ξ is not much affected, but the quoted error is less: $\xi = 1.21(^{+5}_{-4})$.

3. LIFETIME RATIOS

The lifetime of a hadron H_b containing a b-quark is calculated from

$$\Gamma(H_b) = \frac{1}{m_{H_b}} \mathrm{Im}\langle H_b|\mathscr{T}|H_b\rangle$$

where

$$\mathscr{T} = i\int \mathrm{d}^4x \mathrm{T}\{H^{|\Delta B=1|}(x)H^{|\Delta B=1|}(0)\}$$

is a non-local product of two $|\Delta B{=}1|$ effective Hamiltonians. $H^{|\Delta B=1|}$ is known to NNLO [5]. The large energy release in a b decay allows an operator product expansion (OPE) of $\mathscr{T}$ as a series of local operators of increasing dimension and increasing inverse powers of m_b, with calculable coefficients (containing the CKM factors). This *heavy quark expansion* leads to an expression for the lifetime of the form

$$\Gamma(H_b) = \sum_k \frac{c_k(\mu)\langle H_b|O_k(\mu)|H_b\rangle}{m_b^k}.$$

The dependence of the coefficients c_k on the renormalisation scale μ cancels that of the $\Delta B = 0$ operators O_k to give a scale-independent physical result.

The operators occurring at the first few orders in $1/m_b$ (and leading order in QCD) are [6]:

$O(1)$	$\bar{b}b$
$O(1/m_b)$	no contribution
$O(1/m_b^2)$	$\bar{b}g_s\sigma\cdot Gb$, chromomagnetic operator
$O(1/m_b^3)$	$\bar{b}\Gamma q\bar{q}\Gamma b$, 4-quark operators, $\Delta B{=}0$

Their matrix elements are further expanded using heavy quark effective theory, leading to

$$\langle H_b|\bar{b}b|H_b\rangle = 1 - \frac{\mu_\pi^2 - \mu_G^2}{2m_b^2} + O(1/m_b^3)$$
$$\langle H_b|\bar{b}g_s\sigma{\cdot}Gb|H_b\rangle = 2\mu_G^2 + O(1/m_b)$$

The quantities μ_π and μ_G (which depend on the hadron, H_b) can be determined from masses and mass-splittings. The "1" appearing in the matrix element of $\bar{b}b$ is a universal term, corresponding to the decay of a free b quark.

Since the $O(1/m_b^2)$ terms are not large, any substantial deviations from equality of the hadron lifetimes should come from the $O(1/m_b^3)$ terms. In particular, there are *spectator effects* where the matrix elements first involve the light "spectator" quarks in H_b. These arise from 1-loop terms in the HQE, whereas the $O(1/m_b, 1/m_b^2)$ contributions come from 2-loop terms (see figure 3), and thus have a relative loop-factor enhancement of $16\pi^2$. QCD corrections to the Wilson coefficients of the HQE have been computed [7,8], which bring in penguin operators at $O(1/m_b^3)$.

The matrix elements of the $1/m_b^3$ operators have been calculated in quenched lattice QCD [9–11].

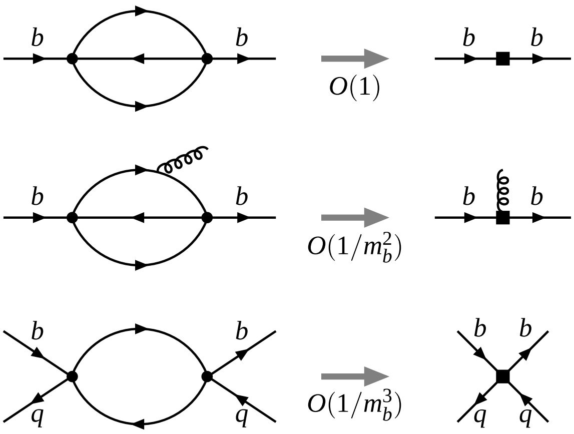

Figure 3. Generating local operators in the HQE.

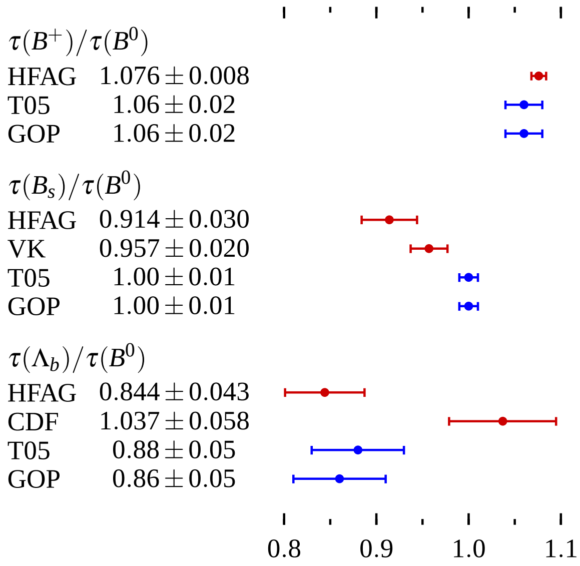

Figure 4. Lifetime ratios of b-flavoured hadrons (red: experiment, blue: theory). Experimental numbers are from HFAG [15], Van Kooten [14] and CDF [16]. T05 is the theoretical analysis of [17], updating [7], while GOP is from [13].

These calculations have not included the penguin contributions and so-called "eye" diagrams (for which a power-divergent subtraction has to be controlled). The missing parts are expected to be small SU(2) and SU(3) breaking effects for $\tau(B^+,B_s)/\tau(B^0)$, but could be important for $\tau(\Lambda_b)/\tau(B^0)$. The leading $O(1/m_b^4)$ spectator contributions have also been analysed [12,13], leading to eight new dimension-7 4-quark operators, whose matrix elements were evaluated from vacuum insertion for B-mesons, or using a quark-diquark model for b-baryons.

Analyses of the lifetime ratios incorporating all these pieces are compared to experiment in figure 4. The experimental picture for b-hadron lifetime ratios is in flux. At FPCP2006, Van Kooten [14] updated the average for $\tau(B_s)/\tau(B^0)$ compared to the HFAG [15] summary of early 2006, while at this meeting, a new measurement by CDF of $\tau(\Lambda_b)/\tau(B^0)$ was presented [16], with value 1 within errors. These numbers are shown in the figure.

The theoretical analyses, labelled T05 [17] and GOP [13] agree very well with each other. In the theoretical calculations, it is very hard to get a significant deviation from 1 for the ratio $\tau(B_s)/\tau(B^0)$, so that this could become a problem in comparison to experiment. On the other hand, the relevant matrix elements are large enough to accomodate a substantial deviation of $\tau(\Lambda_b)/\tau(B^0)$ from 1. If the recent CDF result is correct, it will be essential to revisit the lattice calculations.

4. WIDTH DIFFERENCES

The decay width difference between B_q and $\overline{B}_q$ depends on the off-diagonal $\Delta B=2$ matrix element

$$\Delta\Gamma_q = -\frac{1}{m_{B_q}}\langle\overline{B}_q|\mathscr{T}|B_q\rangle.$$

Once again the heavy quark expansion is used to organise this into a series of operator matrix elements with operators of increasing dimension, accompanying increasing inverse powers of m_b, and calculable coefficients:

$$\Delta\Gamma = \left(\frac{\Lambda}{m_b}\right)^3\left(\Gamma_3^0+\frac{\alpha_s}{4\pi}\Gamma_3^1+\cdots\right) + \left(\frac{\Lambda}{m_b}\right)^4(\Gamma_4^0+\cdots)+\left(\frac{\Lambda}{m_b}\right)^5(\Gamma_5^0+\cdots)+\cdots$$

The coefficient in the leading Γ_3^0 term was found long ago, while the QCD corrections to it, Γ_3^1 [18,19], and the Γ_4^0 piece [19–21] have been evaluated more recently. This year Γ_5^0 has also been considered [22].

The leading contribution, at $O(1/m_b^3)$, involves two dimension-6 operators, one of which is the same as

Table 1
Lifetime differences of neutral B mesons. Experimental numbers are from Van Kooten [14] for B_s and HFAG [15] for B_d.

	Expt	Theory [22]
$\Delta\Gamma_s/\Gamma_s$	0.14 ± 0.06	0.16 ± 0.05
$\Delta\Gamma_d/\Gamma_d$	0.009 ± 0.037	0.003 ± 0.001

appears in the expression for Δm. They are parameterised by $\langle \overline{B}_q|O_i|B_q\rangle = \text{const} \times f_{B_q}^2 B_i$. The parameters $B_{1,2}$ have been evaluated by several lattice simulations [23–29] and are not expected to be significantly different in unquenched calculations [30,31]. Four more dimension-7 operators appear at order $O(1/m_b^4)$: two of these are related to operators in the set of $\Delta B = 2$ operators whose matrix elements were calculated on the lattice in [28]; the others are estimated by vacuum insertion. Putting together all the ingredients [17,19] shows that the QCD corrections in Γ_3^1 and the $1/m_b$ corrections in Γ_4^0 are important. The size and same sign of these corrections led Lenz and Nierste [22] to consider $1/m_b^2$ corrections which turn out to be small; they also changed the operator basis to make the coefficient of the operator responsible for the mass difference dominant, which reduces the uncertainty from the QCD and $1/m_b$ corrections.

For $\Delta\Gamma_q/\Gamma_q$, two ways to quote a result are

- $\Delta\Gamma_q/\Gamma_q = \Delta\Gamma_{q,\text{theo}}\tau_{\text{expt}}$
  Pro: independent of new physics in mixing
  Con: depends on $f_{B_q}^2$
- $\Delta\Gamma_q/\Gamma_q = (\Delta\Gamma_q/\Delta m_q)_{\text{theo}}\Delta m_{q,\text{expt}}\tau_{\text{expt}}$
  Pro: theoretically clean
  Con: might depend on new physics in Δm_q

Using the first method, Lenz and Nierste find [22]

$$\Delta\Gamma_s/\Gamma_s = 0.158^{+0.046}_{-0.051},$$

where they have taken $f_{B_s} = 245\,\text{MeV}$. Both methods agree for $f_{B_s} = 221\,\text{MeV}$. Table 1 gives a comparison of theoretical and experimental results for both $\Delta\Gamma_s/\Gamma_s$ and $\Delta\Gamma_d/\Gamma_d$, showing good consistency.

5. CP VIOLATION PARAMETERS

For the CP-violation parameters $|q/p| - 1$, the QCD corrections at leading order in $1/m_b$ and the $1/m_b$ corrections have been calculated [19,32]. The results of two theoretical analyses, BBLN [32] and CFLMT [17,19] are given in table 2.

Table 2
CP violation parameters for neutral B mesons from two theoretical analyses.

	BBLN [32]	CFLMT [17,19]
$\left|\frac{q}{p}\right|_s - 1$	$-(1.1 \pm 0.2) \times 10^{-5}$	$-(1.28 \pm 0.28) \times 10^{-5}$
$\left|\frac{q}{p}\right|_d - 1$	$(2.5 \pm 0.6) \times 10^{-4}$	$(2.96 \pm 0.67) \times 10^{-4}$

6. PERSPECTIVE AND CONCLUSIONS

In today's lattice simulations, the *quenched* approximation, in which vacuum polarisation effects are neglected, has now largely been removed. Dynamical simulations have been done for meson decay constants, but for the majority of the four-quark operator matrix elements discussed here, we await updated unquenched calculations.

Much emphasis is now on reducing the masses of the light quarks in dynamical simulations, to control the chiral extrapolation. The challenge has been set by those collaborations using improved staggered quarks, where light quark masses down to around $1/8$ of the strange mass have been reached. Using staggered fermions requires using a "fourth root trick" to remove unphysical *tastes* (like extra unwanted flavours). There is no proof that this is correct, but there is growing circumstantial evidence and no counter example. We need simulations using alternative fermion formulations, including overlap or domain-wall quarks (with good chiral symmetry), improved Wilson and twisted mass quarks in order to gain full confidence.

For heavy quarks a variety of techniques are available, including QCD with heavy*ish* quarks plus extrapolation, non-relativistic QCD, discretised static quarks with $1/m_q$ corrections and relativisitic heavy (Fermilab/Tsukuba) quarks. The results from these different formulations have been consistent to date.

In conclusion, for neutral meson mixing we await fully unquenched calculations for the B_{B_q} parameters and confirmation of the results by more than one group. For lifetime ratios, the situation is very interesting given the evolution of the experimental results. Spectator effects can be large enough to explain a substantial difference between the Λ_b and B^0 lifetimes: a cancellation will have to be understood if these lifetimes are measured to be equal. Theoretical evaluations of $\tau(B_s)/\tau(B^0)$ hardly deviate from 1, so there could be a puzzle if the experimental ratio remains significantly different from 1. The hadronic uncertainty remains substantial for the lifetime ratios: updated lattice calculations would be welcome here.

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