Theory and Phenomenology of CP Violation

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In this talk I summarize a few peculiar features of CP violation in the Standard Model, focussing on the CP violation of quarks.

1. Introduction

CP Violation and more generally the flavour structure observed in elementary-partlcle interactions still remains one of the unexplained mysteries in high-energy physics. In the framework of the standard model flavour mixing and CP violation is encoded in the CKM matrix for the quarks and in the PMNS matrix for the leptons. However, this is only a parametrization, in which CP violation appears through irreducble phases in these matrices and which is up to now completely consistent with the experimental facts, at least with what is found at accelerator experiments.

On the other hand, the CP violation in the standard model is a small effect. In particular it is too small to create the observed matter-antimatter asymmetry of the universe, for which CP violation is an indispensable ingredient. The standard model is a local, Lorentz-covariant and causal quantum field theory, which means that we shall assume strict CPT conservation. Keeping this in mind, let us recall here the criteria established by Sakharov [1] for the appearance of a baryon-antibaryon asymmetry:

- There have to be baryon Number violating interactions:
$$\mathcal{L}(\Delta n_{\rm Bar} \neq 0) \neq 0\,.$$
- CP has to be violated in order to have different reaction rates for baryons and antibaryons
$$\Gamma(N \stackrel{\mathcal{L}(\Delta n_{\rm Bar}\neq 0)}{\longrightarrow} f) \neq \Gamma(\bar{N} \stackrel{\mathcal{L}(\Delta n_{\rm Bar}\neq 0)}{\longrightarrow} \bar{f})\,.$$
- The universe had to be out of thermal equilibrium, since in thermal equilibrium CPT invariance is equivalent to CP invariance.

Although the standard model has all these ingredients, it turns out that the observed matter-antimatter asymmetry cannot be accommodated by the standard model, and one reason is that CP violation turns out to be too small.

Furthermore, CP violation (and with it the complete flavour structure) of the Standard Model is quite peculiar and fully compatible with the data from particle accelerators. In the following I will focus on the CP violation of quarks and point out a few of these features which follow from the parametrization with the CKM matrix and which have a specific phenomenology and which are not easily reproduced by generic new physics models.

2. CP Violation in the Standard Model

In general, CP violation emerges in Lagrangian field theory through complex coupling constants, the phases of which are irreducible[1]. Schematically this means

$$\mathcal{L} = \sum_i a_i \mathcal{O}_i + h.c. \qquad (CP)\,\mathcal{O}_i (CP)^\dagger = \mathcal{O}_i^\dagger\,. \quad (1)$$

In the Standard Model there is only a single irreducible phase which is induced by the Yukawa couplings. It has become a general convention to define the phases of the fields such that the

[1]This means that the phases cannot be removed by phase redefinitions of the fields.

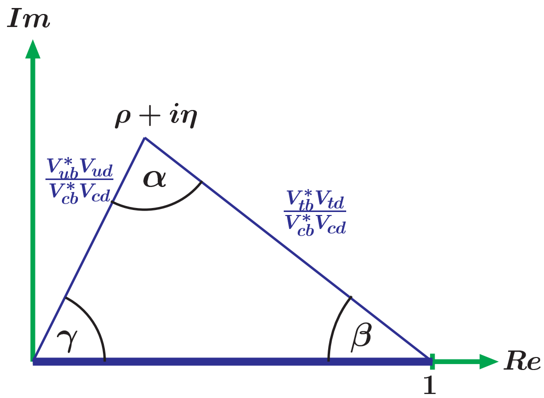

Figure 1. The standard unitarity triangle.

irreducible imaginary part appears in the CKM Matrix

$$V_{\rm CKM} \neq V^*_{\rm CKM} \tag{2}$$

In fact this is already one of the peculiarities of CP violation in the standard model, where the unique source of CP violation only appears in the charged current couplings.

There are infinitely many possible parametrizations of the CKM matrix in terms of three angles and a phase. One possibility, the PDG parametrization, may be written as a product of three rotations and a phase matrix in the form

$$U_{12} = \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}, \; U_{13} = \begin{bmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{bmatrix},$$
$$U_{23} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix}, \; U_\delta = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\delta_{13}} \end{bmatrix}.$$

where $s_{ij} = \sin\theta_{ij}$ etc. are the sines and cosines of the rotation angles. The standard parametrization is obtained by

$$V_{\rm CKM} = U_{23} U^\dagger_\delta U_{13} U_\delta U_{12} \tag{3}$$

In this parametrization large phases appear in V_{td} and V_{ub} which are small matrix elements as far as their absolute value is concerned.

The unitarity of $V_{\rm CKM}$ is usually depicted by the unitarity triangle. There are in total six unitarity relations (aside from the normalization relations for the rows and columns), which may be depicted as traingles in the complex plane. However, due to the hierarchy of the CKM matrix elements there are only two triangles with comparable sides which coincide to leading order in the Wolfenstein expansion. This triangle is shown in Fig. 1

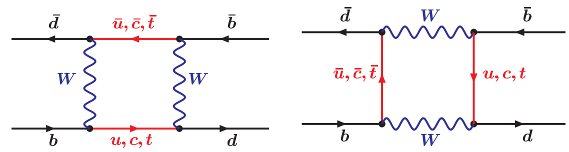

Figure 2. The box diagramms mediating $\Delta F = 2$ transitions in the standard model.

An invariant measure for the size of CP violation is the area of this triangle. In fact, on can show that all traingles have to have the same area due to unitarity. The area of all these triangles is proportional to the quantity

$$\begin{aligned} {\rm Im}\Delta &= {\rm Im} V_{ud} V^*_{td} V_{tb} V^*_{ub} \\ &= c_{12} s_{12} c^2_{13} s_{13} s_{23} c_{23} \sin\delta_{13} \end{aligned} \tag{4}$$

It is interesting to note that the maximal possible value for this quantity for some "optimized" values of the angles and the phase is

$$\delta_{\rm max} = \frac{1}{6\sqrt{3}} \sim 0.1 \tag{5}$$

while the value realized in nature is several orders smaller, $\delta_{\rm exp} \sim 0.0001$. This quantifies the statement that CP violation is a small effect.

Finally we note that CP violation vanishes in the case of degeneracies of up or down quark masses, in which case one may rotate away the CP violating phase. It has been noted by Jarlskog [2] that the following quantity is a clear indication for the presence of CP violation:

$$\begin{aligned} J &= {\rm Det}([M_u\,,\,M_d]) \\ &= 2i{\rm Im}\Delta(m_u - m_c)(m_u - m_t)(m_c - m_t) \\ &\quad \times (m_d - m_s)(m_d - m_b)(m_s - m_b) \end{aligned} \tag{6}$$

In second order in the weak interactions there is the possibility of flavour oszillations. The box-diagramms shown in fig. 2 can mediate transitions between $B_d \leftrightarrow \overline{B}_d$, $B_s \leftrightarrow \overline{B}_s$, $K^0 \leftrightarrow \overline{K}^0$ and also between $D^{)} \leftrightarrow \overline{B}^0$

For later use we note that the mixing amplitude for $B_d - \overline{B}_d$ mixing is proportional to the phase factor $\exp(2i\beta)$, while the phase in $B_s - \overline{B}_s$ mixing is negligibly small in the standard model. In the kaon system, the short distance contribution to

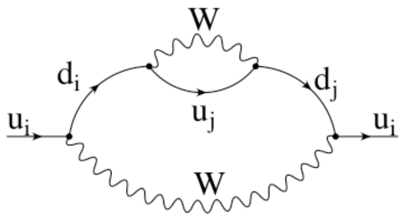

Figure 3. Feynman Diagram leading to an electric dipole moment for the up quark.

mixing is also proportional to $\exp(2i\beta)$, while the mixing in the up quark sector ($D-\overline{D}$ mixing) is heavily GIM suppressed.

The Flavour structure and the pattern of CP violation in the Standard Model is quite peculiar. Here I list a few of these peculiarities:

Strong CP violation:

The QCD sector of the standard model can also contain a CP violating interaction of the form

$$\mathcal{L}_{\text{strong CP}} = \theta \frac{\alpha_s}{\pi} \mathrm{Tr} \left\{ \vec{E} \cdot \vec{B} \right\} \tag{7}$$

where $\vec{E}$ and $\vec{B}$ are the chromo-electric and the chromo-magnetic field strenghts. The electric dipole moment of the neutron tells us that the coupling θ has to be extremely small. It has been suggested by Pecchei and Quinn [5] to introduce an additional symmetry to explain $\theta = 0$, but in these scenarios a new light pseudoscalar particle, the so-called axion appears for which we do not have any experimental evidence. It is fair to say that the strong CP problem is still unsolved.

Flavour Diagonal CP violation:

One of the most peculiar features of CP violation in the standard model is the enormous suppression of flavour diagonal CP violation. Assuming that the CKM matrix is the only source of CP violation, it is easy to see that an electric dipole moment of a quark can be induced only at the two loop level at least, evaluating a diagram as the one shown in fig. 3.

It has been shown by Shabalin [3] that the sum of all the two loop diagrams lead to a vanishing electric dipole moment for the quarks, and a non-vanishing contribution requires at least one more loop, which can be an additional gluon. Hence naive counting leads to the following estimate of the electric dipole moment of the neutron

$$d_e \sim e \frac{\alpha_s}{\pi} \frac{G_F^2}{(16\pi^2)^2} \frac{m_t^2}{M_W^2} \mathrm{Im}\Delta\, \mu^3 \sim 10^{-32} \mathrm{e\,cm} \tag{8}$$

where the factor m_t^2/M_W^2 originates from the GIM suppression, Δ encodes the necesary CKM factors and μ is a typical hadronic scale which we set to $\mu = 300$ MeV. This has to be compared to the current experimental limit which is [4]

$$d_{\text{exp}} \leq 3.0 \times 10^{-26} \mathrm{e\,cm}$$

Similar statements hold for other flavour diagonal and CP violating observables, and from the experimental side this strong suppression of these effects is supported.

Strong Suppression of CP violation in the up-quark sector:

The pattern of mixing and CP violation in the standard model is determined by the GIM mechanism. This means in particular, that the effects for the up, charm and the top quark are severely suppressed, since the mass differences between the down-type quarks are small compared to the weak boson masses. Furthermore, the mixing angles of the first and second generations into the third are small, which means that charm physics is basically a “two family” problem. Since CP violation with only two families is not possible through the CKM mechanism, this results in a strong suppression of CP violation in the charm sector. Again this fact seems to be supported by experiment.

3. Phenomenology of CP Violation

In the following we shall concentrate on CP violation in the decays of particles. In general, decays are mediated by an effective interaction consisting of a sum of local operators O_i

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{i=1}^{n} C_i O_i \tag{9}$$

where in the standard model C_i are coefficients containing the CKM factors (including possible phases) and the contributions from the QCD running from the weak scale down to the typical scale of process. Furthermore, in the standard model we have $n = 10$, where the operators $i = 1, 2$ are called tree operators, $i = 3, ..6$ are the QCD penguins and $i = 7, ...10$ are the electroweak penguins.

CP violation occurs through complex phases of the C_i in the following way. Consider an amplitude for a decay of a B meson into some final state f, with two contributions

$$A(B \to f) = \lambda_1 a_1 + \lambda_2 a_2 \tag{10}$$

where the λ_i are complex coupling constants and a_i are hadronic matrix elements $\langle B|O_i|f\rangle$. The amplitude of the CP conjugate process is obtained by conjugating the couplings λ_i, while the phases of the the hadronic matrix elements remain the same due to CP invariance of strong interactions. The CP asymmetry is given by

$$\begin{aligned} \mathcal{A}_{CP} &\propto \Gamma(B \to f) - \Gamma(\overline{B} \to \overline{f}) \\ &= 2\,\mathrm{Im}[\lambda_1 \lambda_2^*]\,\mathrm{Im}[a_1 a_2^*] \end{aligned} \tag{11}$$

Thus a CP asymmetry requires imaginary parts of the coupling constants, but also a *strong* phase difference between the hadronic matrix elements.

The neutral flavoured mesons can decay into a CP eigenstate. In this case we can have a strong phase difference from a different origin. Since the two mass eigenstates in the neutral flavoured meson systems (B_d, B_s, K^0 and D^0) have different mass eigenvalues, the time evolution creates a phase such that

$$\mathrm{Im}[a_1 a_2^*] \sim \sin(\Delta m\, t) \tag{12}$$

where Δm is the mass difference between the two eigenstates.Thus there will be a time dependent CP asymmetry which takes the form

$$\mathcal{A}_{CP}(t) = \frac{C\,\cos(\Delta m\, t) - S\,\sin(\Delta m\, t)}{\cosh(\Delta\Gamma\, t/2) + D\sinh(\Delta\Gamma\, t/2)} \tag{13}$$

where the coefficients satisfy $C^2 + S^2 + D^2 = 1$.

The problem in obtaining quantitative predictions for the CP asymmetries lies in our inability to perform a first principles calculation of the hadronic matrix elements. Thus we have to refer to approximate methods which fall into three classes which I shall briefly describe.

Flavour Symmetries:

Isospin or more genrerally flavour $SU(3)$ may be used to relate matrix elements for different processes [6]. While isospin is believed to be a good symmetry which is broken only by electromagnetism and the mass difference between the up and the down quark, the situation is worse for U- and V-spin which are the two other possible choices of the $SU(2)$ subgroup of $SU(3)$, since the mass of the strange quark is significantly higher than the one of the up and the down quark.

The breaking of flavour $SU(3)$ is not easily quantified. only for factorizable amplitudes $SU(3)$ breaking is included by a factor of f_π/f_K which also give the approximate size of the effects which have to be expected. However, the intrinsic uncertainties emerging from using flavour symmetries are hard to estimate.

Flavour symmetry arguments are often supplemented by arguments based on diagram topologies: A rearrangement of color indices will result in a color-suppression factor $1/N_C = 1/3$, an annihilation topology (i.e. a diagram where two quarks in a meson have to annihilate) results in a suppression by a factor f_M/m_M, where f_M and m_M are the decay constant and the mass of the meson M. Furthermore, a contraction of quark lines involving a quark loop (penguin contraction) involves a loop factor, the perturbative value of which is $1/(16\pi^2)$ but is generally expected to be between this value and unity.

This approach, which is completely model independent, allows us to have some semi-quantitative insight into rates and CP asymmetries, however, in many applications the uncertainties of the order of tens of percent are hard to estimate.

QCD Factorization and SCET:

It has recently been pointed out that in the limit of infinite heavy quark mass one may factorize the amplitudes of exclusive non-leptonic two body modes as sketched in fig. 4[7]. Thus in the infinite mass limit the amplitude can be expressed in terms of a non-perturbative (soft) form factor, the non-pertutbative wave functions of the hadrons and a perturbatively calculable hard scattering kernel T. This limit may be formulated

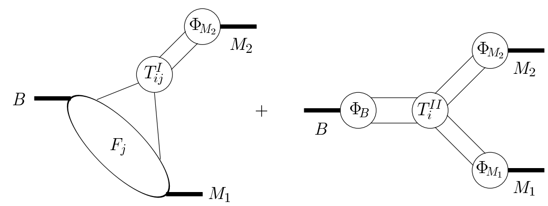

Figure 4. Sketch of QCD factorization

as an effective field theory, the soft collinear effective field theory (SCET) .

The most interesting observation from QCD factorization is that the strong phases in non-leptonic heavy hardon decays are perturbatively calculable to leading order, and thus are predicted to be small: Strong phases are either of order $\alpha_s(m)$ or suppressed by powers of $1/m$. However, the quantitative agreement with data is still not good, indicating sizable corrections of subleading orders in $1/m$. Although the number of subleading non-perturbative parameters is large, QCD factorization still provides a systematic approach to exclusive non-leptonic decays.

QCD (Light Cone) Sum Rules:

QCD sum rules are a well established method for the estimate of hadronic matrix elements. They rely on parton hadron duality and on the analyticity of the amplitudes. For the case of two-body non-leptonic B decays involving light mesons in the final state the so-called light cone sum rules are applied [8], where one of the light mesons is represented by its light cone wave function, while the decaying meson and the other light meson is interpolated by appropriate currents. This type of sum rules has been formulated recently directly in SCET.

It is interesting to note that the results from light-cone QCD sum rules quantitatively agree with the ones from QCD factorization. In particular, the statement about weak phases is the same in QCD sum rules.

In summary, the main obstacle to calculate CP violation in hadronic decays and thus to relate the measurements to the fundamental parameters is the evaluation of the hadronic matrix elements, in particular of their weak phases. Only in a few cases precise predictions are possible.

4. CP Violation and "New Physics"

Currently there is no convincing idea which explains flavour. Even in grand unified theories flavour is usually implemented by triplication of the spectrum, which will not give any explanation.

Most scenarios of "new physics" involve additional degrees of freedom, in particular, many models involve an extended Higgs sector. This in general implies additional couplings which may carry irreducible phases, i.e. to additional sources of CP violation. This additional CP violation generically also appears in flavour diagonal processes, in contradiction with the observed suppression of the e.g. electric dipole moments of particles. Furthermore, flavour changing neutral currents may appear, in some models even at tree level.

The pattern of CP violation and flavour parametrized by the CKM Ansatz is very special and in complete agreement with the observations in particle physics. For this reason, scenarios of "new physics" are thus formulated often with "minimal flavour violation" (MFV) which means that any flavour structure can be reduced to the Yukawa couplings and the CKM matrix. This makes these models consistent with present observations but does not explain the phenomenon of flavour.

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