Theory review on rare K decays: Standard Model and beyond

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The theoretical status of the rare $K \to \pi\nu\bar{\nu}$, $K_L \to \pi^0\ell^+\ell^-$ and $K_L \to \mu^+\mu^-$ decays in the Standard Model is reviewed. Their sensitivity to New Physics and their discriminating power is also illustrated.

1. Rare K decays in the Standard Model

One of the reasons why rare K decays are interesting is the good theoretical control reached over their predictions in the SM. In this section, the ingredients needed are shortly reviewed.

1.1. Electroweak structure

The electroweak processes driving the rare semi-leptonic K decays are the W box, Z and γ penguins[1], and lead to the amplitude

$$\mathcal{A}\left(K_L \to \pi^0 X\right) = \sum_{q=u,c,t} \left(\operatorname{Im}\lambda_q + \varepsilon \operatorname{Re}\lambda_q\right) y_q^X$$

with $X = \nu\bar{\nu}, \ell^+\ell^-$, $\lambda_q = V_{qs}^* V_{qd}$. Without the dependence of the loop functions y_q^X on the quark masses, CKM unitarity would imply vanishing FCNC (GIM mechanism). Now, looking at these dependences, combined with the scaling of the CKM elements, one can readily get a handle on the importance of each quark contribution.

For $X = \nu\bar{\nu}$, only the Z penguin and W box enter, $y_q^{\nu\bar{\nu}} \sim m_q^2$, and light quark contributions are suppressed. Since, in addition, $\varepsilon \sim 10^{-3}$ and $\operatorname{Re}\lambda_t \sim \operatorname{Im}\lambda_t$, indirect CP-violation is small. For K^+, the $\operatorname{Re}\lambda_{c,t}$ and $\operatorname{Im}\lambda_t$ parts contribute.

For $X = \ell^+\ell^-$, the photon penguin enters with its scaling $y_q^{\ell\ell} \sim \log(m_q)$ for $m_q \to 0$. Direct CP-violation is still short-distance dominated thanks to $\operatorname{Im}\lambda_u = 0$, but not indirect CP-violation, completely dominated by the long-distance u-quark photon penguin, $K_1 \to \pi^0\gamma^* \to \pi^0\ell^+\ell^-$.

For $K_L \to \ell^+\ell^-$, the structure is similar to $K \to \pi\nu\bar{\nu}$, up to the change $\operatorname{Im}\lambda_q \leftrightarrow \operatorname{Re}\lambda_q$ (no single-photon penguin).

*Work supported by the Schweizerischer Nationalfonds.

Finally, for charged leptons, there is also the double-photon penguin, which gives a CP-conserving contribution ($\sim \operatorname{Re}\lambda_q$) to $K_L \to \pi^0\ell^+\ell^-$ and $K_L \to \ell^+\ell^-$, and is completely dominated by long-distance (u-quark).

1.2. QCD corrections

Having identified the relevant electroweak structures, both perturbative and non-perturbative QCD effects have now to be included. This is done in three main steps:

Step 1: Integration of heavy degrees of freedom (top, W, Z), including perturbative QCD effects above M_W. This generates local FCNC operators, and Fermi-type four fermion local operators.

Step 2: Resummation of QCD corrections (running down). At the c threshold (similar for b, τ), four-fermion operators are combined to form closed c loops, which are then replaced by a tower of effective interactions in increasing powers of (external momentum)/(charm mass). The lowest order consists again of the dim.6 FCNC operators, while dim.8 operators are corrections scaling naively like $m_K^2/m_c^2 \sim 15\%$.

Step 3: To get the amplitude, it remains to compute the matrix elements of all the operators obtained. Those of dim.6 semi-leptonic operators can be extracted from $K_{\ell 2}, K_{\ell 3}$ decays (with Chiral Perturbation Theory (ChPT) corrections). Contributions from four-quark operators are represented directly in terms of meson fields in ChPT, such that non-local u-quark loops are represented as meson loops. The price to pay are some unknown low-energy constants, to be extracted from experiment. For dim.8 operators, an approximate matching is done with the ChPT representation of the u-quark contributions.

1.3. The $K \to \pi\nu\bar{\nu}$ decays in the SM

A high level of precision is attained for these modes. Dim.6 FCNC operators from the t-quark are known at NLO, while c-quark ones have recently been obtained at NNLO[2]. The matrix elements for these operators are known from $K_{\ell 3}$, including the leading isospin corrections [3]. For $K^+ \to \pi^+\nu\bar{\nu}$, residual c-quark effects from dim.8 operators, along with long-distance u-quark effects have also been computed[4]. For $K_L \to \pi^0\nu\bar{\nu}$, ICPV is of about 1%[5] while the CP-conserving contribution is less than 0.01%[6].

The SM predictions are $\mathcal{B}\left(K_L \to \pi^0\nu\bar{\nu}\right) = (2.81 \pm 0.56) \cdot 10^{-11}$ and $\mathcal{B}\left(K^+ \to \pi^+\nu\bar{\nu}\right) = (8.0 \pm 1.1) \cdot 10^{-11}$. The error on $K_L \to \pi^0\nu\bar{\nu}$ is dominated by $\operatorname{Im}\lambda_t$, while for $K^+ \to \pi^+\nu\bar{\nu}$, which receives a significant c-quark contribution, it breaks down to[2] scales (13%), m_c(22%), CKM, α_S, m_t (37%) and matrix-elements from $K_{\ell 3}$ and light-quark contributions (28%). Further improvements are thus possible through a better knowledge of m_c, of the isospin breaking in the $K \to \pi$ form-factors, or by a lattice study of higher-dimensional operators.

1.4. The $K_L \to \pi^0\ell^+\ell^-$ decays in the SM

Here the situation is more complicated. The t and c quark contributions, known to NLO, generate both the dimension-six vector $(\bar{s}d)_V(\ell\ell)_V$ and axial-vector $(\bar{s}d)_V(\ell\ell)_A$ operators. The former produces the $\ell^+\ell^-$ pair in a 1^{--} state, the latter in both 1^{++} and 0^{-+} states.

Indirect CP-violation is related to $K_S \to \pi^0\ell^+\ell^-$, dominated by the ChPT counterterm a_S[7]. NA48 measurements give $|a_S| = 1.2 \pm 0.2$[8]. Producing $\ell^+\ell^-$ in a 1^{--} state, it interferes with the contribution from the $(\bar{s}d)_V(\ell\ell)_V$ operator, arguably constructively[9,10].

The CP-conserving contribution from $Q_{1,\ldots,6}$ proceeds through two-photons, i.e. produces the lepton pair in either a helicity-suppressed 0^{++} or phase-space suppressed 2^{++} state. The LO corresponds to the finite two-loop process $K_L \to \pi^0 P^+P^- \to \pi^0\gamma\gamma \to \pi^0\ell^+\ell^-$, $P = \pi, K$, exactly predicted by ChPT, and produces only 0^{++} states. Higher order corrections are estimated using experimental data on $K_L \to \pi^0\gamma\gamma$ for both the 0^{++} and 2^{++} contributions[9,11].

Altogether, the predicted rates are $\mathcal{B}_{\rm SM}^{e^+e^-} = 3.54^{+0.98}_{-0.85}\ \left(1.56^{+0.62}_{-0.49}\right) \cdot 10^{-11}$ and $\mathcal{B}_{\rm SM}^{\mu^+\mu^-} = 1.41^{+0.28}_{-0.26}\ \left(0.95^{+0.22}_{-0.21}\right) \cdot 10^{-11}$ for constructive (destructive) interference. The errors are detailed in [9,11,12]. Overall, the error on a_S is currently the most limitative and better measurements of $K_S \to \pi^0\ell^+\ell^-$ would be welcomed.

Finally, the integrated forward-backward (or lepton-energy) asymmetry (see Refs in [12]) is generated by the interference between CP-conserving and CP-violating amplitudes. While for A_{FB}^e, no reliable prediction can be made because of the poor theoretical control on the 2^{++} contribution, the situation is better for A_{FB}^μ, for which the 0^{++} contribution is under control. Though the error is large, it can be used to fix the interference sign since $A_{FB}^\mu \approx -25\%$ or 15% depending on $\operatorname{sign}(a_S)$.

1.5. The $K_L \to \mu^+\mu^-$ decay in the SM

The short-distance (SD) piece from t and c-quarks is known to NLO and NNLO[13], resp., and is helicity-suppressed. Indirect CP-violation is negligible. The long-distance (LD) contribution, from the matrix elements of $Q_{1,\ldots,6}$, proceeds again through two-photons. Still, there are three differences with respect to $K_L \to \pi^0\ell^+\ell^-$.

First, the contribution from the imaginary part of the photon loop, estimated from $K_L \to \gamma\gamma$, is much larger than SD, and already accounts for the bulk of the experimental $K_L \to \mu^+\mu^-$ rate. Second, while the charged meson loop in $K_L \to \pi^0 P^+P^- \to \pi^0\gamma\gamma \to \pi^0\ell^+\ell^-$ is acting like a cut-off, and a finite result is found, now the two photons arise from the axial anomaly, and $K_L \to \pi^0, \eta, \eta' \to \gamma\gamma \to \mu^+\mu^-$ is divergent. Though still with a large theoretical error, the dispersive $\gamma\gamma$ part was estimated using experimental information on $K_L \to \gamma^*\gamma^*$ and the perturbative behavior of the $\bar{s}d \to \bar{u}u \to \gamma\gamma$ loop[14]. Finally, both SD and LD produce the lepton pair in the same 0^{-+} state and interfere with an unknown sign, which depends on that of $\mathcal{A}(K_L \to \gamma\gamma)$. In this respect, the progress made in [15] for treating $K_L \to \gamma\gamma$ points towards constructive interference between SD and LD. As shown there, confirmation of this sign could be obtained from better measurements of $K_S \to \pi^0\gamma\gamma$ or $K^+ \to \pi^+\gamma\gamma$.

2. New Physics in rare K decays

Being suppressed in the standard model, and in addition, the SM predictions being under theoretical control, makes the rare K decays ideal to get clear signals of New Physics (NP). Even if LHC finds NP signals before Kaon experiments, it will remain essential to probe the $\Delta S = 1$ sector. Indeed, in general, NP models involve many new flavor breaking parameters. Experimental information will be necessary to establish their structure, and thereby, give us some hints about a possible higher level of unification.

2.1. New Physics in $K \to \pi\nu\bar{\nu}$ decays

Model-independently, the present measurement of $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$ limits the possible effects in $\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})$, as expressed by the Grossman-Nir bound[16], $\mathcal{B}(K_L \to \pi^0\nu\bar{\nu}) \leq 4.4 \times \mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$, corresponding to $\leq 1.7 \cdot 10^{-9}$ (90%), about 50 times the SM prediction.

Many models have been considered along the years, like for example the enhanced EW penguins[17], Little Higgs[18], Extra dimensions[19],... which are encoded into $V \pm A$ FCNC operators, or leptoquark interactions[20], R-parity violating SUSY[16,21],... which can give rise also to new scalar/tensor FCNC interactions. We will here concentrate on the MSSM.

MSSM at large tanβ: When $\tan\beta = v_u/v_d \approx 50$ say, the Higgs couplings to quarks get significant higher-order loop effects. Of interest for $K \to \pi\nu\bar{\nu}$ is the charged Higgs contribution to the Z penguin[22], which exhibits a $\tan^4\beta$ behavior, is sensitive to δ_{RR}^D and is slowly decoupling when $m_{H^\pm} \to \infty$.

MSSM at moderate tanβ: In this case, the chargino penguins are dominant[23]. Also, the single mass insertion approximation is not sufficient, and these contributions probe the double $(\delta_{RL}^U)_{32}^*(\delta_{RL}^U)_{31}$ insertion. Many works have analysed the phenomenological consequences of these effects. Let us concentrate on two questions of relevance for future experiments.

First, how large the effect on $\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})$ can be, given the current measurement of $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$. This has been answered in [24], which showed that the GN bound can still be saturated in the MSSM. A full scan over the parameters was performed, using adaptive numerical algorithms (fig.1, top).

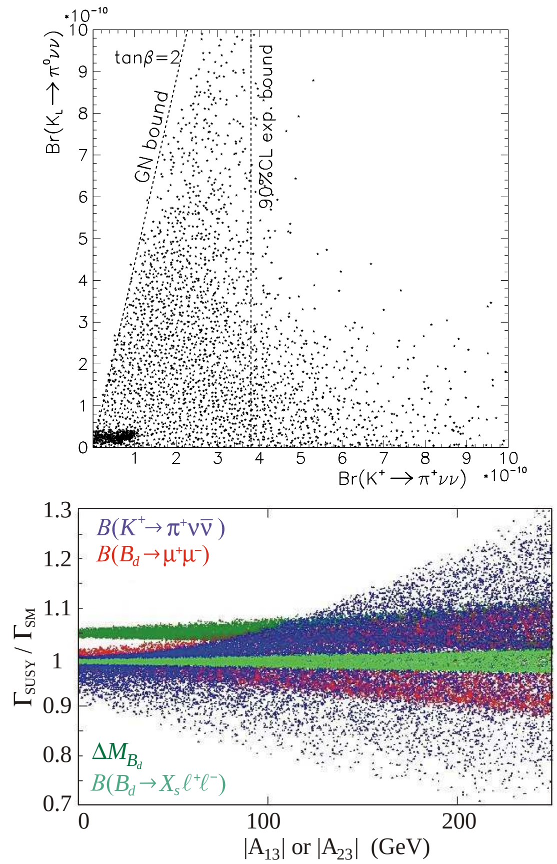

Figure 1. Scan results of [24](up) and [25] (down).

A second question, especially relevant after a SUSY discovery at LHC, is how does the constraint from $K \to \pi\nu\bar{\nu}$ on the trilinear terms A^U compare with those from other K and B physics observables. This has been answered in [25]. Fig.1 (bottom) shows that the $K \to \pi\nu\bar{\nu}$ decays are the most sensitive probe of that sector.

Minimal Flavor Violation: If the SM Yukawas remain the only source of flavor-symmetry breaking also beyond the SM, the FCNC remain tuned essentially by the CKM matrix, hence are suppressed[26]. This hypothesis can be enforced model-independently, or, e.g., within the MSSM. In this latter case, since the t-

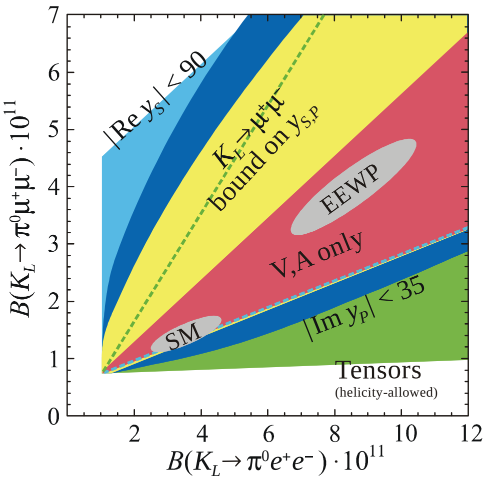

Figure 2. $\mathcal{B}\left(K_L \to \pi^0\mu^+\mu^-\right)$ against $\mathcal{B}\left(K_L \to \pi^0 e^+e^-\right)$ for various NP scenarios[12].

quark Yukawa is large, sizeable trilinear A^U terms are still allowed. As said previously, the $K \to \pi\nu\bar{\nu}$ modes are very sensitive to that sector.

Still, MFV does its job perfectly in killing any large deviation with respect to the SM. Though the MFV analyses in the literature differ in their parametrization, statistical treatment of errors, extraction of CKM elements and in the resulting correlations among observables, they all agree that the enhancement of $K \to \pi\nu\bar{\nu}$ never exceeds 25%[25,27].

2.2. New Physics in $K_L \to \pi^0\ell^+\ell^-$ decays

The $K_L \to \pi^0\ell^+\ell^-$ pair of decays is interesting at least for three reasons. First, compared to $K \to \pi\nu\bar{\nu}$, they can probe helicity-suppressed operators. Second, compared to $K_L \to \mu^+\mu^-$, the theoretical control on the SM part is better and further, $K_L \to \mu^+\mu^-$ is not sensitive to tensor operators. Finally, $K_L \to \pi^0 e^+e^-$ and $K_L \to \pi^0\mu^+\mu^-$ are two modes with very similar dynamics, but for the very different lepton masses. This makes them ideal to probe NP effects through their signatures in the pair[12].

Vector and axial-vector operators: The $(\bar{s}d)_V(\bar{\ell}\ell)_{V,A}$ operators, already present in the SM, arise for example from EEWP[17], MSSM,... In general, these models also affect $K \to \pi\nu\bar{\nu}$, and the sensitivity is slightly lower for $K_L \to \pi^0\ell^+\ell^-$ than for $K_L \to \pi^0\nu\bar{\nu}$. Anyway, they should not be disregarded because, contrary to the neutrinos, they offer the possibility to disentangle NP effects in $(\bar{s}d)_V(\bar{\ell}\ell)_V$ and $(\bar{s}d)_V(\bar{\ell}\ell)_A$. Indeed, $(\bar{s}d)_V(\bar{\ell}\ell)_A$ produces the final lepton pair also in a helicity-suppressed 0^{-+} state, hence contributes differently to $K_L \to \pi^0 e^+e^-$ and $K_L \to \pi^0\mu^+\mu^-$, while the $(\bar{s}d)_V(\bar{\ell}\ell)_V$ contribution is identical (up to phase-space corrections).

This is depicted by the red region in fig.2, which corresponds to the region in the $\mathcal{B}\left(K_L \to \pi^0 e^+e^-\right) - \mathcal{B}\left(K_L \to \pi^0\mu^+\mu^-\right)$ plane spanned leaving $(\bar{s}d)_V(\bar{\ell}\ell)_A$ and $(\bar{s}d)_V(\bar{\ell}\ell)_V$ operator coefficients arbitrary (but keeping lepton universality). Taking all the errors into account, this translates into the bounds $0.1 + 0.24\mathcal{B}^{ee} \le \mathcal{B}^{\mu\mu} \le 0.6 + 0.58\mathcal{B}^{ee}$ with $\mathcal{B}^{\ell\ell} = \mathcal{B}\left(K_L \to \pi^0\ell^+\ell^-\right)\cdot 10^{11}$.

Finally, the contribution from EMO operator $(\bar{s}\sigma^{\mu\nu}d)F_{\mu\nu}$ can always be absorbed into a redefinition of $(\bar{s}d)_V(\bar{\ell}\ell)_V$[28], and thus possible NP contributions to it cannot be disentangled.

Scalar and pseudoscalar operators, $(\bar{s}d)_S(\bar{\ell}\ell)_{S(P)}$, induce a CP-conserving (CP-violating) contribution, respectively. When these operators are helicity-suppressed, only the muon mode is significantly affected. Such a situation corresponds for example to the MSSM at large $\tan\beta$, where they arise from neutral Higgs penguins and are sensitive to down-squark mass insertions[29]. Combined with general V, A operators, the blue regions in fig.2 can be spanned.

Specific models like the MSSM can generate both $(\bar{s}d)_S(\bar{\ell}\ell)_{S,P}$ and $(\bar{s}d)_P(\bar{\ell}\ell)_{S,P}$ operators, contributing to $K_L \to \ell^+\ell^-$. Working out their relation, the current $\mathcal{B}\left(K_L \to \mu^+\mu^-\right)^{\text{exp}}$ corresponds to the yellow region in fig.2.

If the (pseudo-)scalar operators are helicity-allowed, the electron mode becomes more sensitive, simply because of the phase-space suppression. Such types of operators can arise from leptoquark tree-level exchanges[20] or sneutrino exchanges in SUSY without R-parity[21]. Still, operators contributing to $K_L \to e^+e^-$ will also be generated. Such contributions to an otherwise helicity-suppressed mode are very constrained by $\mathcal{B}\left(K_L \to e^+e^-\right)^{\text{exp}} = 9^{+6}_{-4}\cdot 10^{-12}$, and should not lead to a perceptible impact in fig.2.

Tensor and pseudotensor operators, $(\bar{s}\sigma_{\mu\nu}d)(\bar{\ell}\sigma^{\mu\nu}\ell)$ and $(\bar{s}\sigma_{\mu\nu}d)(\bar{\ell}\sigma^{\mu\nu}\gamma_5\ell)$ induce a CP-violating (CP-conserving) contribution, respectively. In case these operators are helicity-suppressed, being in addition phase-space suppressed, their impact on $K_L \to \pi^0\mu^+\mu^-$ is smaller than for scalar and pseudoscalar operators. In addition, in models like the MSSM, they are further suppressed by loop factors[30] and their impact can be expected to be small.

On the other hand, if helicity-allowed, there are at present no constraint on them, since they do not contribute to $K_L \to \ell^+\ell^-$. This is depicted by the green region in fig.2.

3. Conclusion

Rare K decays are very clean and sensitive probes of New Physics. They are promising not only to eventually get clear signals, but also to constrain the nature of the New Physics at play through the pattern of deviations they could exhibit with respect to the SM predictions.

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