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Theory Review on Rare K Decays in the Standard Model and Beyond

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- Outline

A- Rare K decays in the Standard Model

Anatomy of the decay processes

$$K \to \pi\nu\bar{\nu}, K_L \to \pi^0\ell^+\ell^-, K_L \to \ell^+\ell^-$$

B- Rare K decays beyond the Standard Model

Various models and possible signals

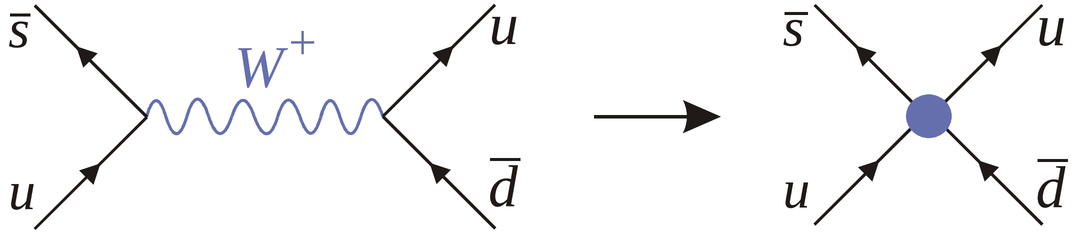
C- Conclusion

Rare K decays

In the Standard Model

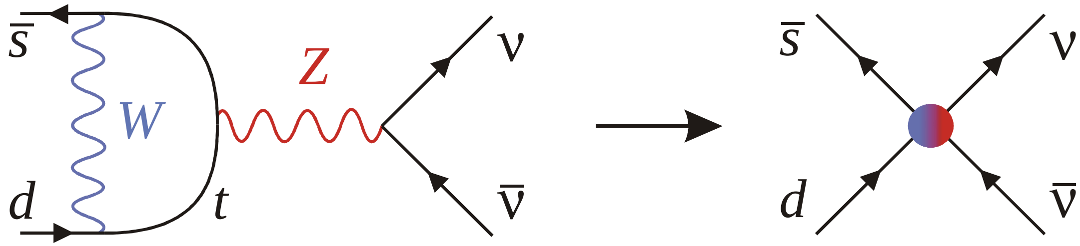
- # Electroweak FCNC

For the *charged current*, the Fermi interaction is obtained by integrating out the W:



$$H_{eff}(\bar{s}u \to \bar{d}u) = \frac{G_F}{\sqrt{2}} \lambda_u \; (\bar{s}u)_{V-A}(\bar{u}d)_{V-A} \qquad \lambda_q = V_{qd}V_{qs}^*$$

FCNC are generated at one-loop (penguin and box diagrams). Typically:

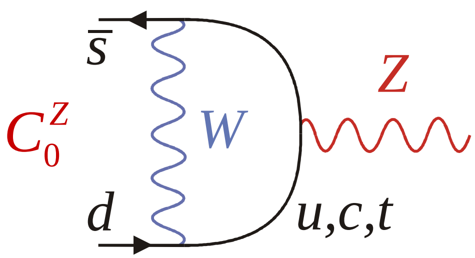


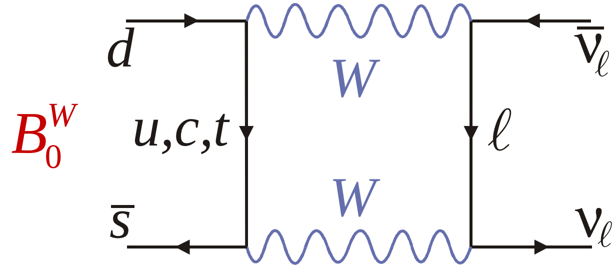
$$H_{eff}^{Z}(\bar{s}d \to \bar{\nu}\nu) = \frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2\theta_W} \lambda_t C_0^Z(x_t) \; (\bar{s}d)_{V-A}(\bar{\nu}\nu)_{V-A} \qquad x_q = \frac{m_q^2}{M_W^2}$$

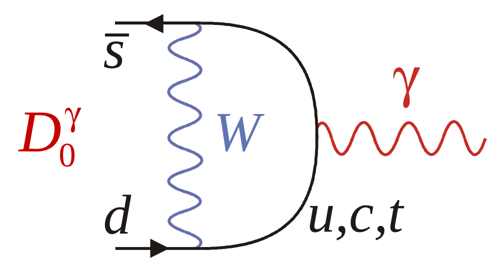
The Inami-Lim function C_0^Z generates a violation of the *GIM mechanism*:

$$\text{if } C_0^Z(x) = C^{st} \quad \Rightarrow \quad \lambda_u C_0^Z(x_u) + \lambda_c C_0^Z(x_c) + \lambda_t C_0^Z(x_t) = 0$$

FCNC







$$\sqrt{2}K_1 = K^0 - \bar{K}^0,\ \sqrt{2}K_2 = K^0 + \bar{K}^0,\ \langle \pi^0 | (\bar{s}d)_V | K^0 \rangle = -\langle \pi^0 | (\bar{d}s)_V | \bar{K}^0 \rangle$$

If only B_0^W and C_0^Z contribute, light quark effects are suppressed:

$$\langle \pi^0 \nu\bar{\nu} | H_{eff} | K_L \approx K_2 \rangle \sim \operatorname{Im}\lambda_u y_u^\nu + \operatorname{Im}\lambda_c y_c^\nu + \boxed{\operatorname{Im}\lambda_t y_t^\nu}$$

$$\langle \pi^0 \nu\bar{\nu} | H_{eff} | K_S \approx K_1 \rangle \sim \operatorname{Re}\lambda_u y_u^\nu + \boxed{\operatorname{Re}\lambda_c y_c^\nu + \operatorname{Re}\lambda_t y_t^\nu}$$

$$y_q^\nu \supset B_0^W, C_0^Z \sim \frac{m_q^2}{M_W^2}$$

When D_0^γ also contributes, long-distance effects may be significant:

$$\langle \pi^0 \ell^+\ell^- | H_{eff} | K_L \approx K_2 \rangle \sim \operatorname{Im}\lambda_u y_u^\ell + \boxed{\operatorname{Im}\lambda_c y_c^\ell + \operatorname{Im}\lambda_t y_t^\ell}$$

$$\langle \pi^0 \ell^+\ell^- | H_{eff} | K_S \approx K_1 \rangle \sim \boxed{\operatorname{Re}\lambda_u y_u^\ell} + \operatorname{Re}\lambda_c y_c^\ell + \operatorname{Re}\lambda_t y_t^\ell$$

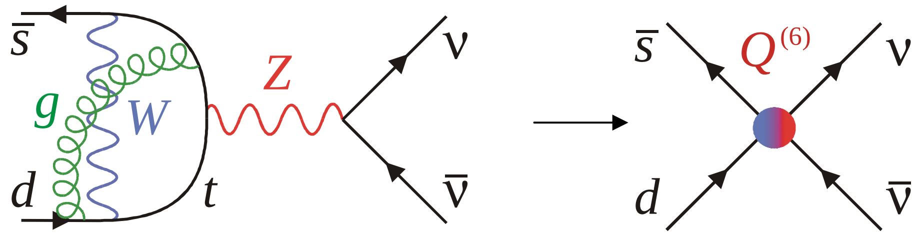
$$y_q^\ell \supset D_0^\gamma \sim \log\left(\frac{m_q}{M_W}\right)$$

Indirect CP-violation: $\langle \pi^0 \nu\bar{\nu}, \pi^0 \ell^+\ell^- | H_{eff} | K_{L(S)} \rangle = \varepsilon \langle \pi^0 \nu\bar{\nu}, \pi^0 \ell^+\ell^- | H_{eff} | K_{1(2)} \rangle$

- QCD corrections

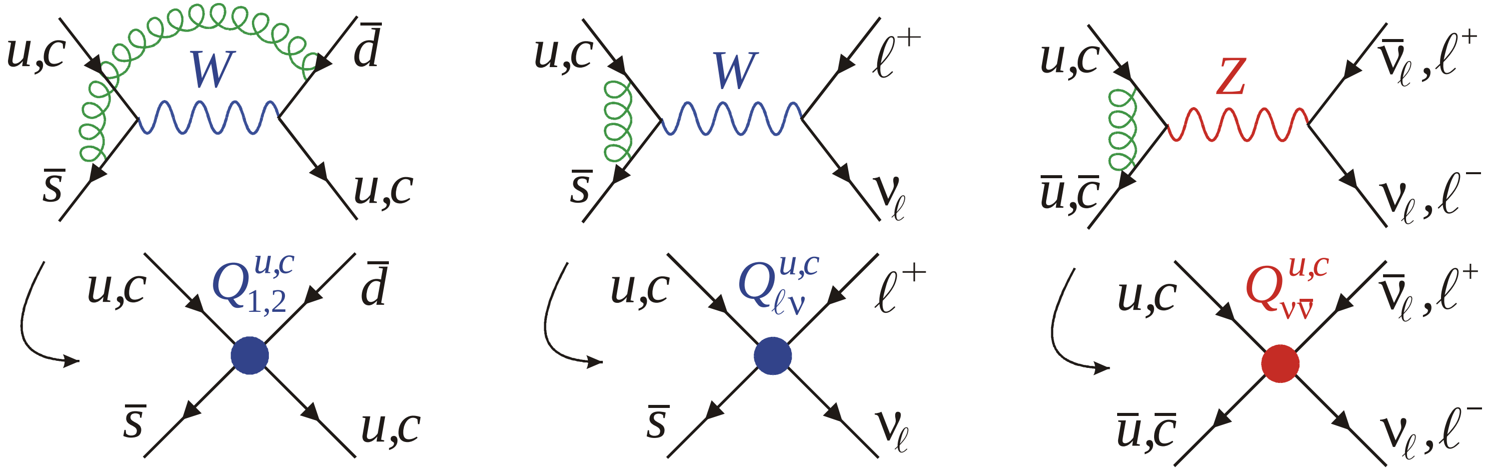
Step 1: integrating out the top, W, Z

Generates local FCNC operators, for example:



$$(\bar{s}d)_{V-A}(\bar{\nu}\nu)_{V-A}$$

Generates local Fermi four-fermion operators (all fermions except the top)



...

QCD corrections above M_W are computed perturbatively, and encoded into the Wilson coefficient initial values:

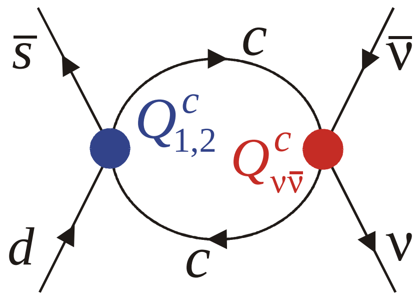
$$H_{eff}(M_W) \sim C_i(M_W)\, Q_i^{u,c} + y_{(6)}^{\nu}(M_W)\,(\bar{s}d)_{V-A}(\bar{\nu}\nu)_{V-A}$$

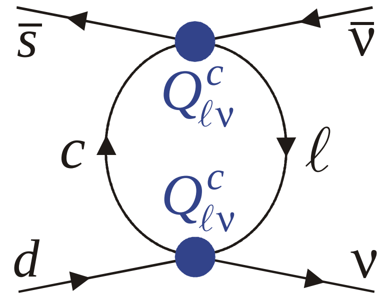
Step 2: crossing the charm quark threshold

QCD corrections are resummed (running down), leading to corrected values for the Wilson coefficients, at lower scales:

$$H_{eff}(m_c) \sim C_i(m_c)\,Q_i^{u,c} + y_{(6)}^{\nu}(m_c)\,(\bar{s}d)_{V-A}(\bar{\nu}\nu)_{V-A}$$

Four-fermion operators are combined to integrate out the c (similar for b and τ)





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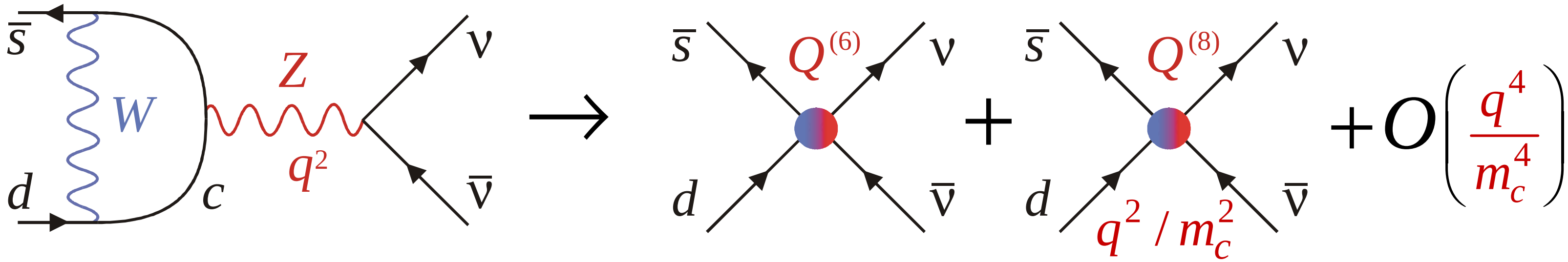
Momentum of external particles (with $q^2 \approx m_K^2$) → Dimension 8, 10,... operators:



$$H_{eff}(m_c) \sim C_i'(m_c)\,Q_i^{u} + y_{(6)}'^{\nu}(m_c)\,(\bar{s}d)_{V-A}(\bar{\nu}\nu)_{V-A} + y_{(8)}^{\nu}(m_c)\,(\bar{s}d)_{V-A}\,\partial^2(\bar{\nu}\nu)_{V-A} + \ldots$$

Step 3: computing matrix elements

$$H_{eff}(\mu) = C_i'(\mu)\, Q_i^u + y_{(6)}'^{\nu}(m_c)\,(\bar{s}d)_{V-A}(\bar{\nu}\nu)_{V-A} + y_{(8)}^{\nu}(m_c)\,(\bar{s}d)_{V-A}\partial^2(\bar{\nu}\nu)_{V-A} + \ldots$$

- *For dim. 6 semi-leptonic operators,* matrix elements extracted from experiment:

$$\langle \pi^0 | (\bar{s}d)_V | K^0 \rangle \approx \langle \pi^0 | (\bar{s}u)_V | K^+ \rangle, \qquad K^+ \to \pi^0 \ell^+ \nu_\ell \ (K_{\ell 3})$$

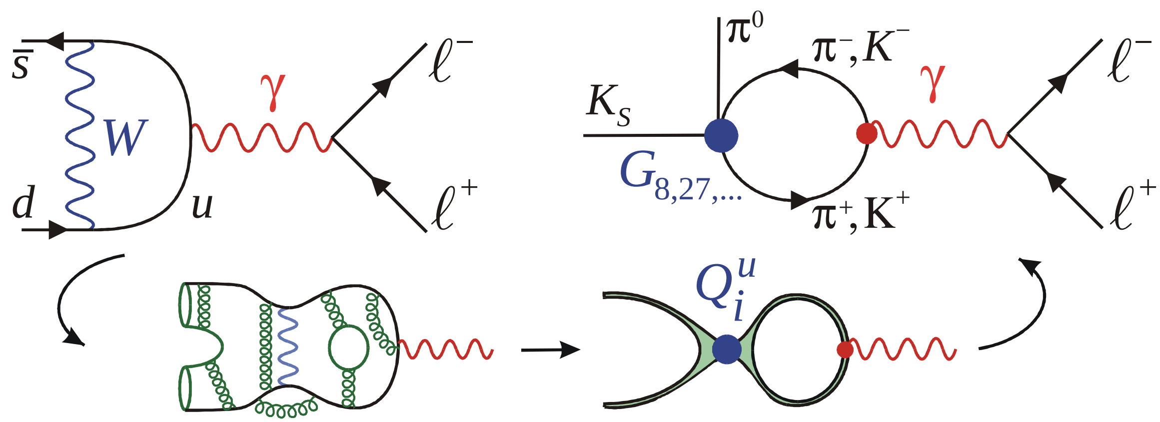
$$\langle 0 | (\bar{s}d)_A | K^0 \rangle \approx \langle 0 | (\bar{s}u)_A | K^+ \rangle, \qquad K^+ \to \ell^+ \nu_\ell \ (K_{\ell 2})$$

- *For dim. 6 four-quark operators*, matrix elements dealt with in ChPT:

$$\langle \pi^0 | \sum_{i=1}^{6} C_i'(\mu)\, Q_i^u \, | K^0 \rangle$$

$$\downarrow$$

$$G_8 (D_\mu U^\dagger D^\mu U)^{ds} + \ldots$$



Give CP-conserving contributions (ε' small), typically through photon penguins.
The Low-Energy Constants $G_{8,27,\ldots}$ are fixed from experiment.

- *For dim. 8 operators*, matrix elements from approximate matching with ChPT.

- The $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$ decays

$$Br(K_L \to \pi^0\nu\bar{\nu}) \approx \kappa^0 \left(\, |\mathrm{Im}\,\lambda_t X(x_t)|^2 \, \right)$$

$$Br(K^+ \to \pi^+\nu\bar{\nu}) \approx \kappa^+ \left(\, |\mathrm{Im}\,\lambda_t X(x_t)|^2 + \overbrace{|\underbrace{\mathrm{Re}\,\lambda_t X(x_t)}_{68\%} + \underbrace{\mathrm{Re}\,\lambda_c (P_c + \delta P_{u,c})}_{32\%}|^2}^{\sim Br(K_S \to \pi^0\nu\bar{\nu})} \right)$$

From $K_{\ell 3}$, with isospin corr.*

Marciano,Parsa ('96)

Precision Physics:

Dimension six *t-quark*: $X(x_t) \overset{NLO}{=} 1.464 \pm 0.041$ *Buchalla,Buras ('93)*

Dimension six *c-quark*: $P_c \overset{NNLO}{=} \lambda^4 (0.37 \pm 0.04)$ *Buras,Gorbahn,Haisch,Nierste ('05)*

Subleading *c-quark* dimension-eight operators
Residual *u-quark* long-distance contributions ($\mathrm{Re}\,\lambda_c \approx -\mathrm{Re}\,\lambda_u$) *Isidori,Mescia,C.S. ('05)*

$$\delta P_{u,c} = \lambda^4 (0.04 \pm 0.02)$$

$K_L \to \pi^0\nu\bar{\nu}$: - *Indirect CPV* $\approx 1\%$ *Buchalla,Buras ('96)*

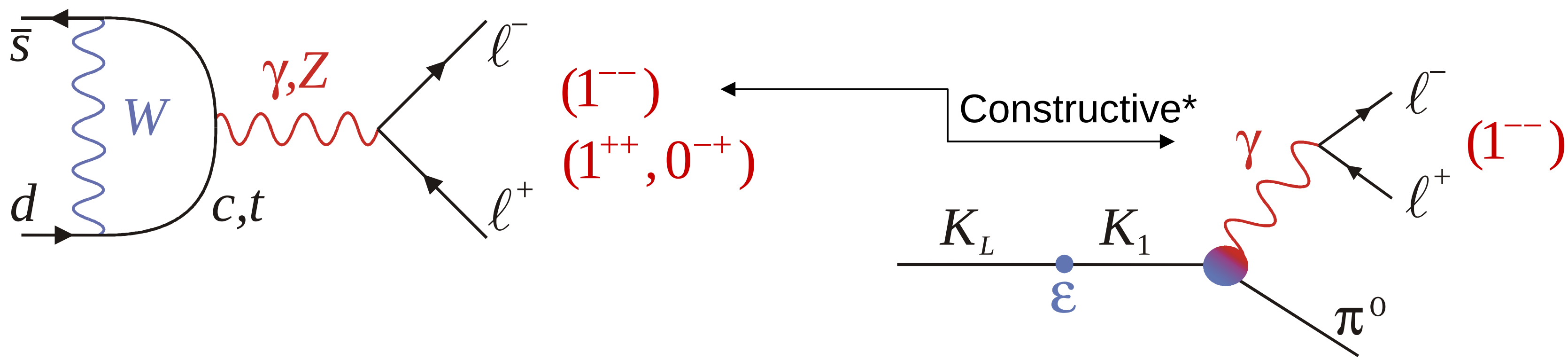
- *CPC* (dim. 8 from box with c,u) $\leq 0.01\%$ *Buchalla,Isidori ('98)*

• The $K_L \to \pi^0 \ell^+ \ell^-$ decay

1. Direct CPV: Two structures arise from top & charm integrations (known at NLO):

$$H_{eff}(\bar{s}d \to \ell^+\ell^-) = \frac{G_F \alpha}{\sqrt{2}} \left(y_{7V}\,(\bar{s}d)_{V-A}(\bar{\ell}\ell)_V + y_{7A}\,(\bar{s}d)_{V-A}(\bar{\ell}\ell)_A \right)$$

Vector: $C_0^Z, B_0^W, D_0^\gamma \to y_{7V}$, Axial-vector: $C_0^Z, B_0^W \to y_{7A}$



**Buchalla,D'Ambrosio,Isidori ('03)/de Rafael,Friot,Greynat ('04)*

2. Indirect CPV: $A(K_L \to \pi^0\ell^+\ell^-)_{ICPV} = \varepsilon\, A(K_S \approx K_1 \to \pi^0\ell^+\ell^-),\;\; \varepsilon \approx 10^{-3}$

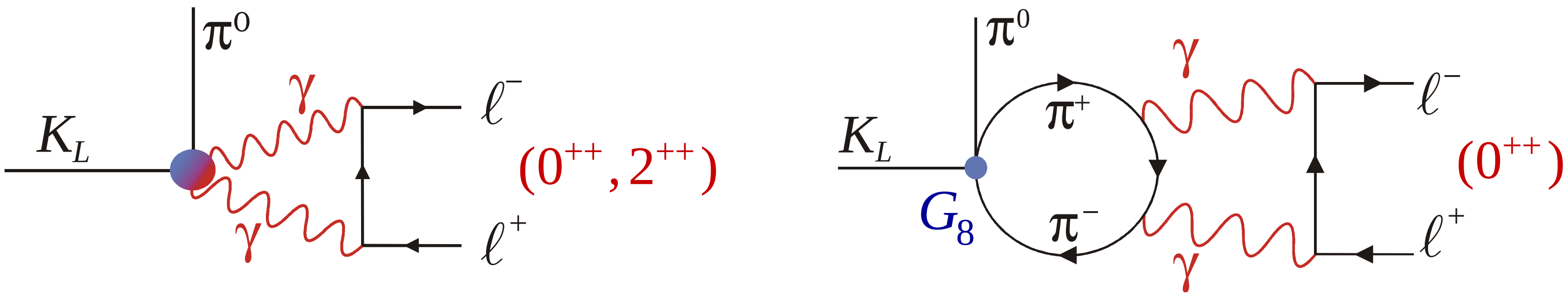
$\to$ Photon penguin, long-distance dominated: to be estimated using *ChPT*

- Meson loops are small; a single counterterm a_S dominates,
- From NA48 measurements of $B(K_S \to \pi^0\ell^+\ell^-)$: $|a_S| = 1.2 \pm 0.2$.

D'Ambrosio,Ecker,Isidori,Portolés ('98)

3. CP-conserving:

CP-conserving matrix elements of $Q_1, \ldots, Q_6$ give rise to pure long-distance contributions through $\gamma\gamma$ penguins:



ChPT $O(p^4)$ result is finite, and produces the lepton pair in a scalar state only.

Higher order effects estimated using the measurements of the $K_L \to \pi^0 \gamma\gamma$ rate and spectrum (KTeV & NA48):

- The ratio $R^{\ell}_{\gamma\gamma} = \dfrac{\Gamma(K_L \to \pi^0 \ell^+ \ell^-)_{J=0^{++}}}{\Gamma(K_L \to \pi^0 \gamma\gamma)}$ can be estimated theoretically within 30%.

  Isidori,Unterdorfer,C.S. ('04)

- Production of $(\gamma\gamma)_{J=2^{++}}$ is constrained by the low-energy end of the $\gamma\gamma$ spectrum, and is found negligible.

  Buchalla,D'Ambrosio,Isidori ('03)

4. Complete prediction

$$Br(K_L \to \pi^0 \ell^+ \ell^-) = \left(C_{\rm dir}^{\ell} \kappa^2 \pm C_{\rm int}^{\ell} |a_S| \kappa + C_{\rm mix}^{\ell} |a_S|^2 + C_{\gamma\gamma}^{\ell} \right) \cdot 10^{-12}$$

$$C_{\rm dir}^{e} \approx 2.3 (y_{7V}^2 + y_{7A}^2) \qquad C_{\rm dir}^{\mu} \approx 0.55 (y_{7V}^2 + 2.33 y_{7A}^2)$$

$$C_{\rm int}^{e} \approx 8.1 y_{7V} \qquad C_{\rm int}^{\mu} \approx 1.9 y_{7V}$$

$$C_{\rm ind}^{e} \approx 14.5, C_{\gamma\gamma}^{e} \approx 0 \qquad C_{\rm ind}^{\mu} \approx 3.4, C_{\gamma\gamma}^{\mu} \approx 5.2$$

¼ phase-space suppression

Helicity-suppressed

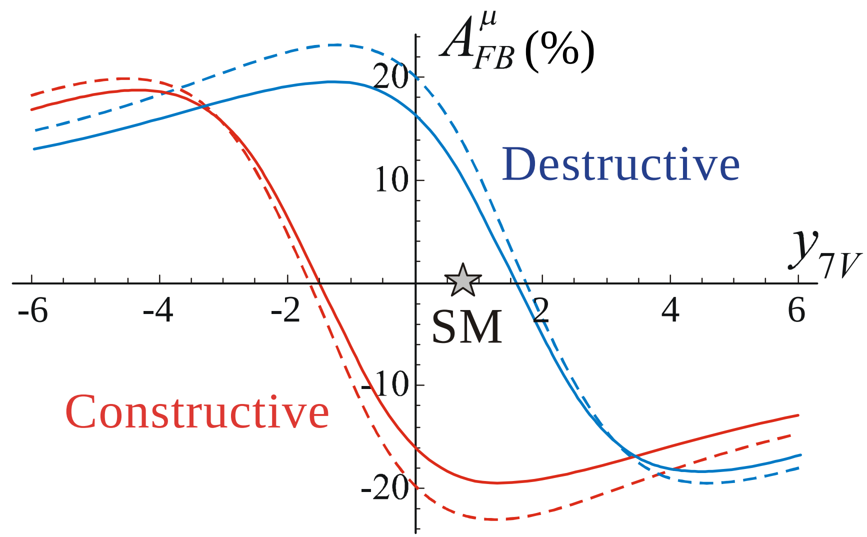
SM: $\kappa = \text{Im}\,\lambda_t 10^{-4} \approx 1.4, \; y_{7A} \approx -0.68, \; y_{7V} \approx 0.73$

5. Forward-Backward CP-asymmetry

$$A_{FB}^{\ell} = \frac{N(E_- > E_+) - N(E_- < E_+)}{N(E_- > E_+) + N(E_- < E_+)}$$

Helicity-suppressed, since proportional to the interference $CPC(0^{++}) \leftrightarrow CPV(1^{--})$

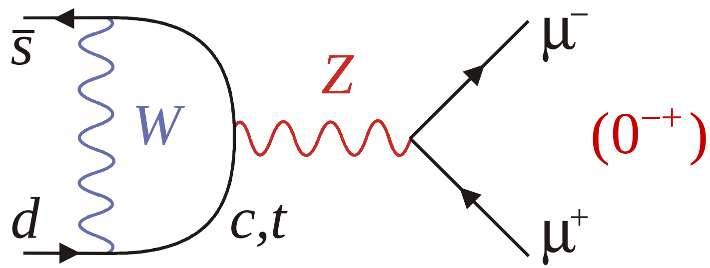
Can be used to fix the interference sign (i.e., sign of a_S)



Mescia, Trine, C.S. ('06)

- The $K_L \to \ell^+\ell^-$ decay

1. Short-distance (top & charm quark) is CP-conserving and helicity-suppressed:

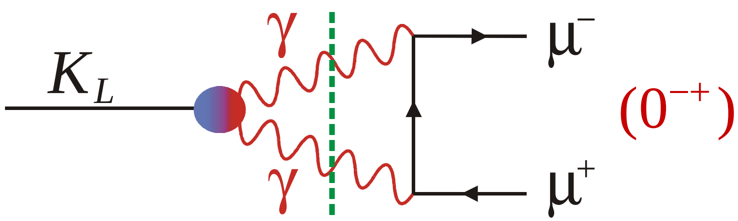


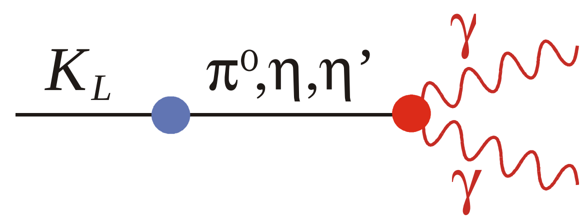
(known at NNLO)

Gorbahn & Haisch ('06)

Good theoretical control (no γ penguin), and indirect CPV very small.

2. Long-distance $\gamma\gamma$ penguin: the absorptive part is known precisely





Estimate for the (divergent) dispersive part, which interferes with SD, obtained from experimental data on $K_L \to \gamma^*\gamma^*$ + perturbative behavior of up-quark $\gamma\gamma$ penguin.

Isidori & Unterdorfer ('03)

3. Complete prediction:

$$y_{7A} \approx -0.68$$

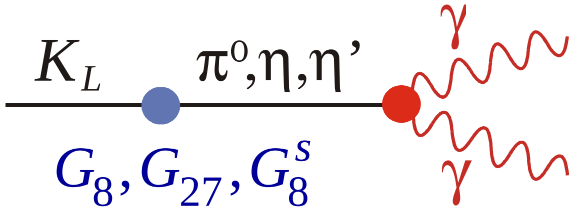
$$Br(K_L \to \mu^+\mu^-) \approx \big((1.1\,y_{7A} - 0.2 \pm 0.4^{+0.5}_{-0.5})^2 + 6.7\big) \cdot 10^{-9}$$

top, charm, Disp($\gamma\gamma$), Abs($\gamma\gamma$)

$$Br(e^+e^-) \approx 10^{-12}$$

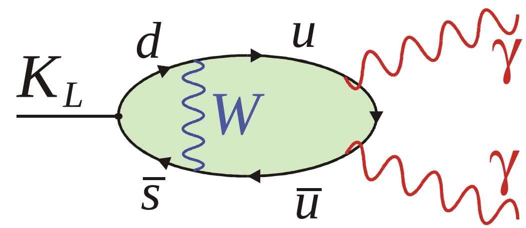
4. Interference sign? Requires the sign of $A(K_L \to \gamma\gamma)$: *Gerard, Trine, C.S ('05)*

Driven by Q_1 only $\to$ vanishes at LO in SU(3) ChPT.
U(3) ChPT needed to disentangle Q_1, Q_2 and Q_6
(partial use of Large N_C: *not* the factorization approx.!)



$$A_{\gamma\gamma} \approx \overbrace{(G_8^s + 2G_{27}/3)}^{\sim C_1(\mu_{hadr.})}\Big((0.46)_\pi - (1.83)_\eta - (0.12)_{\eta'}\Big)$$

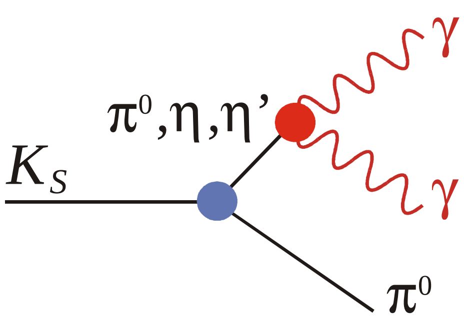
$$\to G_8^s / G_8 \approx \pm 1/3$$

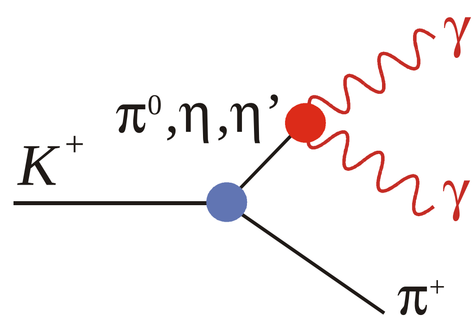


Theoretically, G_8^s can be estimated from the smooth Q_1, Q_2 non-perturbative evolution (with a reasonable penguin fraction in the $\Delta I = ½$ rule at the hadr. scale)

$$(C_1 + C_2)^2 (C_2 - C_1) = 1.0 \pm 0.3 \;\Rightarrow\; \begin{cases} G_8^s / G_8 = -0.38 \pm 0.12 \\ F_P \approx 65\%,\, F_{CC} \approx 35\% \end{cases}$$

Experimentally,
G_8^s can be fixed from:

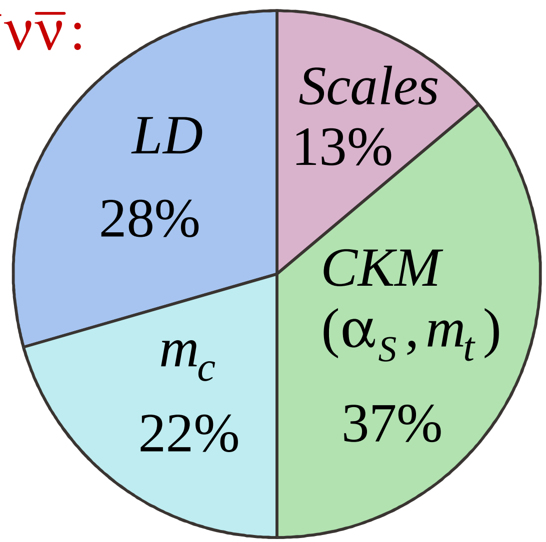




- Summary of current status in the SM:

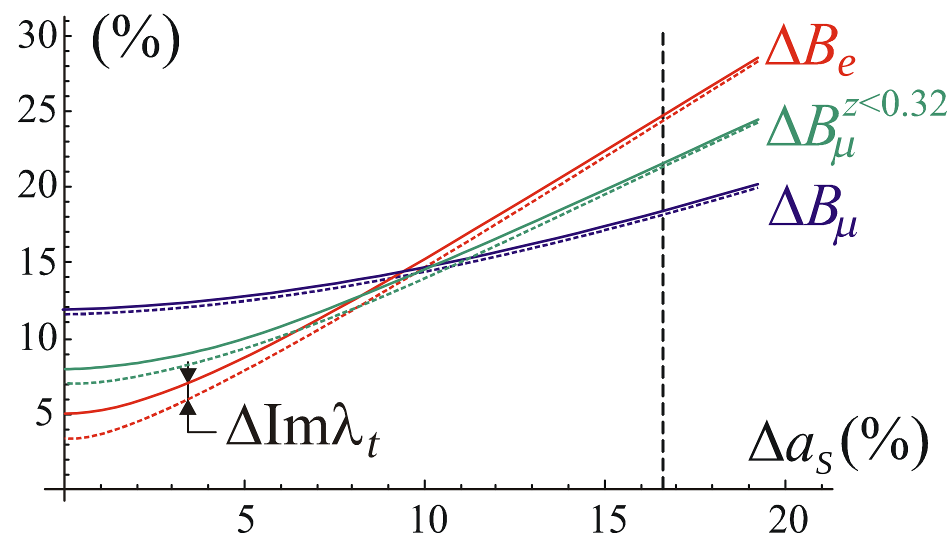
	Standard Model	Experiment	
$K_L \to \pi^0 \nu\bar{\nu}$	$2.81^{+0.56}_{-0.56} \cdot 10^{-11}$	$< 2.86 \cdot 10^{-7}$	E391a
$K_L \to \pi^0 e^+ e^-$	$3.54^{+0.98}_{-0.85} \cdot 10^{-11}$	$< 2.8 \cdot 10^{-10}$	KTeV
$K_L \to \pi^0 \mu^+ \mu^-$	$1.41^{+0.28}_{-0.26} \cdot 10^{-11}$	$< 3.8 \cdot 10^{-10}$	KTeV
$K^+ \to \pi^+ \nu\bar{\nu}$	$8.0^{+1.1}_{-1.1} \cdot 10^{-11}$	$14.7^{+13.0}_{-8.9} \cdot 10^{-11}$	E787 E949

Theory errors for $K^+ \to \pi^+ \nu\bar{\nu}$:



Buras,Gorbahn,Haisch,Nierste ('05,'06)

Theory errors for $K_L \to \pi^0 \ell^+ \ell^-$:



Rare K decays

Beyond the Standard Model

• Motivations

To get a clear signal of New Physics:

- FCNC are suppressed in the SM
- SM background under good theoretical control (both LD and SD).

New Physics in the $\Delta S = 1$ FCNC can be $O(10)$ with respect to the SM

To probe the nature of New Physics:

If NP effects are smaller, or if LHC finds NP signals before Kaon experiments:

It remains essential to probe the $\Delta S = 1$ sector.

Indeed, in general, NP models involve many new parameters, but this may be a necessary step towards understanding the flavor/family structure.

Information on $\Delta S = 1$ crucial to get hints about this higher level of unification.

• The $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$ decays

The GN model-independent bound still leaves room for large effects:

$$B(K_L \to \pi^0\nu\bar{\nu}) \le 4.4 \times B(K^+ \to \pi^+\nu\bar{\nu}) \approx 1.7 \cdot 10^{-9}$$

Grossman & Nir ('97)

$$(90\%\, C.L.)$$

1. Not within the MSSM

With general New Physics effects in the *Electroweak Penguins*,

$$H_{eff}(K \to \pi\nu\bar{\nu}) \sim y_L^\nu\, (\bar{s}d)_{V-A}(\bar{\nu}\nu)_{V-A} + y_R^\nu\, (\bar{s}d)_{V+A}(\bar{\nu}\nu)_{V-A}$$

Examples:	EEWP ← B physics	*Buras,Fleischer,Recksiegel,Schwab ('04)*
	Little Higgs	*Rai Choudhury,Gaur,Joshi,McKellar ('04)*
	Extra Dimensions	*Buras,Spranger,Weiler ('02)*
	...	

With general New physics effects in *New Operators*:

$$H_{eff}(K \to \pi\nu\bar{\nu}) \sim y_S^\nu\, (\bar{s}d)(\bar{\nu}\nu) + y_P^\nu\, (\bar{s}d)(\bar{\nu}\gamma_5\nu)$$

$$+ y_T^\nu\, (\bar{s}\sigma_{\mu\nu}d)(\bar{\nu}\sigma^{\mu\nu}\nu) + y_{\tilde{T}}^\nu\, (\bar{s}\sigma_{\mu\nu}d)(\bar{\nu}\sigma^{\mu\nu}\gamma_5\nu)$$

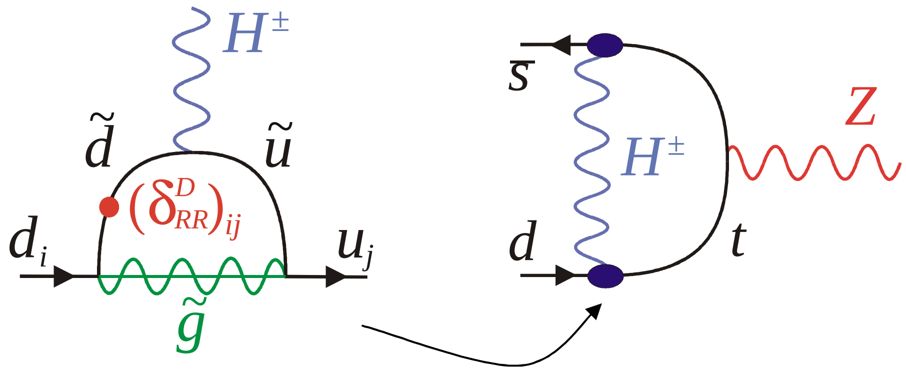
Examples: Leptoquarks, R-parity violation, LFV ($\bar{\nu}^i\Gamma\nu^j, i \neq j$),...

Grossman,Nir ('97)/Grossman,Isidori,Murayama ('03)/Deandrea,Welzel,Oertel ('04)/Deshpande,Ghosh,He ('04)

2. Within the MSSM

For large $\tan\beta = v_u / v_d \approx m_t / m_b \approx 50$, get sensitive to higher order effective vertices in the $H^{\pm}$ penguin:

Isidori & Paradisi ('06)

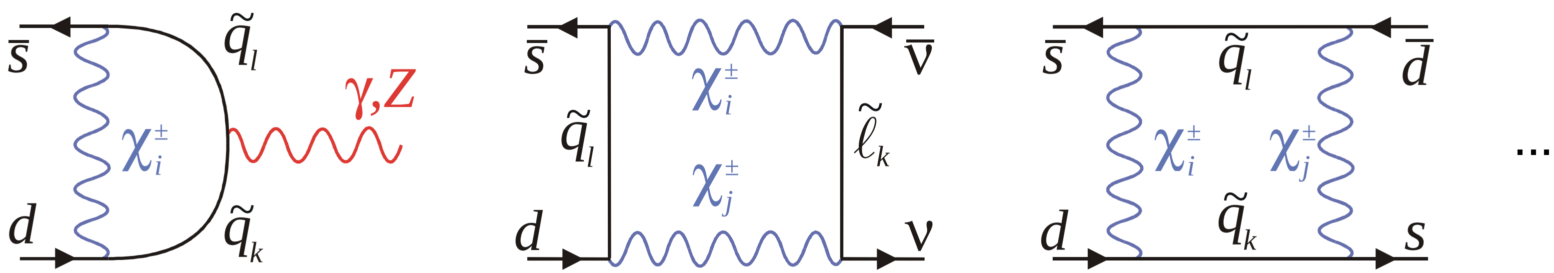


$(\bar{s}_R \gamma_\mu d_R)(\bar{\nu}_L \gamma^\mu \nu_L)$

$\sim (\tan\beta)^4$

Slow decoupling $\sim x_{tH} \log(x_{tH})$

For moderate $\tan\beta$, probe the up-squark sector through chargino penguins:



Beyond the single MIA: $\sim (\delta^{U}_{RL})^{*}_{32} (\delta^{U}_{RL})_{31}$, sensitive to up-squark $\boldsymbol{A}^{U}$ terms.

Nir,Worah ('98)/Buras,Romanino,Silvestrini ('98)/Colangelo,Isidori ('98)

Is it possible to saturate the GN bound in the MSSM?

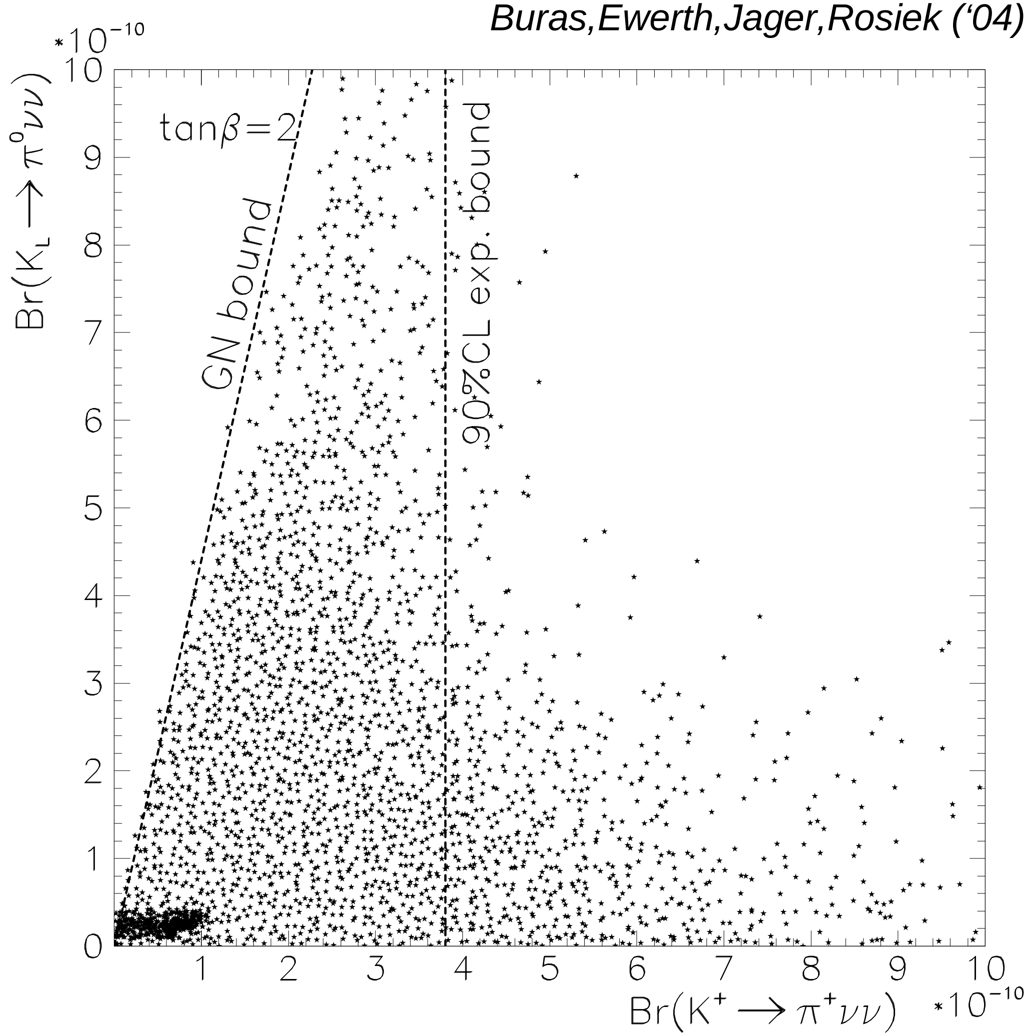
Full scan over MSSM parameters, checking compatibility with B, K and electroweak data, and vacuum stability bounds.

No Mass Insertion Approximation.

Adaptive scanning to search for maximal effects. *Brein ('04)*

Enhancement by a factor ~30 still allowed for the neutral mode.

Buras,Ewerth,Jager,Rosiek ('04)



Within K & B observables, the $K \to \pi\nu\bar{\nu}$ modes are the best probe of $\mathbf{A}^U$ terms

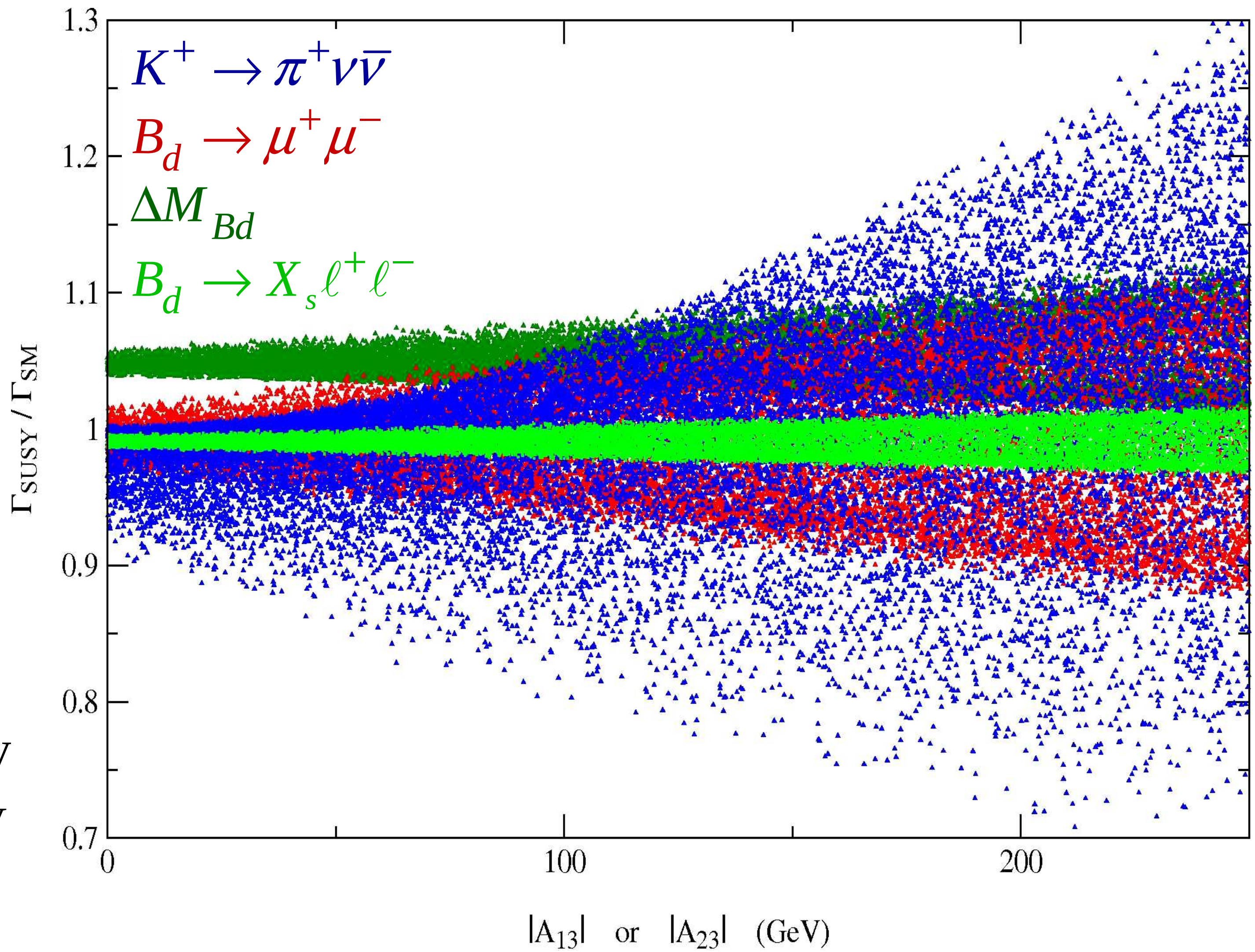
Scanning over trilinear terms:

$|\mathbf{A}_{13}^U|, |\mathbf{A}_{23}^U| \leq A_0 \lambda,$
$A_0 = 1 \text{ TeV}$
Phases left free.

Fixed sparticle masses:

$\tan\beta = 2-4$
$\mu = 500 \pm 10 \text{ GeV}$
$M_2 = 300 \pm 10 \text{ GeV}$
$m_{u_R} = 600 \pm 20 \text{ GeV}$
$m_{q_L} = 800 \pm 20 \text{ GeV}$
$others: 2 \text{ TeV}$

Isidori,Mescia,Paradisi,Trine,C.S. ('06)



3. Minimal Flavor Violation

To suppress FCNC, one invokes MFV defined in various ways:

Phenomenological:
No new operators, and CKM still rules all the FCNC (unique source for all CP-violation).

From symmetry principles:
SM Yukawas remain the only source of flavor-symmetry breaking.

- General: Parametrize the deviation of the penguin/box $B^W, C^Z, D^\gamma, \ldots$ still to be multiplied by CKM elements.

D'Ambrosio,Giudice,Isidori,Strumia ('02)
Bobeth,Bona,Buras,Ewerth,Pierini,Silvestrini,Weiler ('05)
…

- In the MSSM: Parametrize soft-breaking terms, and correspond to "minimal" departures with respect to mSUGRA (i.e. block-diagonal squark mass matrices in the super-CKM basis)

Buras,Gambino,Gorbahn,Jager,Silvestrini ('00)
D'Ambrosio,Giudice,Isidori,Strumia ('02)
Isidori,Mescia,Paradisi,Trine,C.S. ('06)
…

Large top-quark Yukawa → $\mathbf{A}^U$ → $K \to \pi\nu\bar{\nu}$

- *Maximal Effects*: Implementations differ in their *MFV parametrizations*, statistical *treatments of errors*, extraction of *CKM elements* and in the resulting *correlations among observables*. Still, *enhancement of $Br(K \to \pi\nu\bar{\nu})$ always less than 25%.*

- The $K_L \to \pi^0 \ell^+ \ell^-$ (and $K_L \to \ell^+ \ell^-$) decays

  - Can probe helicity-suppressed operators like those arising from Higgs FCNC.
  - Can probe tensor/pseudotensor interactions (no matrix elements for $K_L \to \ell^+ \ell^-$)

$$H_{eff}(K_L \to \pi^0 \ell^+ \ell^-) \sim$$
$$y_{7V}\,(\bar{s}\gamma_\mu d)(\bar{\ell}\gamma^\mu \ell) + y_{7A}\,(\bar{s}\gamma_\mu d)(\bar{\ell}\gamma^\mu \gamma_5 \ell)$$
$$+ y_S\,(\bar{s}d)(\bar{\ell}\ell) + y_P\,(\bar{s}d)(\bar{\ell}\gamma_5 \ell)$$
$$+ y_T\,(\bar{s}\sigma_{\mu\nu} d)(\bar{\ell}\sigma^{\mu\nu}\ell) + y_{\tilde{T}}\,(\bar{s}\sigma_{\mu\nu} d)(\bar{\ell}\sigma^{\mu\nu}\gamma_5 \ell)$$

(comprises all possible structures)

Two photons $K^0 - \bar{K}^0$		CPC CPV	$0^{++}(2^{++})$ 1^{--}
Vector Axial-Vector	y_{7V} y_{7A}	CPV CPV	1^{--} $1^{++}, 0^{-+}$
Pseudoscalar	y_P	CPV	0^{-+}
Scalar	y_S	CPC	0^{++}
Tensor	y_T	CPV	1^{--}
Pseudotensor	$y_{\tilde{T}}$	CPC	1^{+-}

Mescia,Trine,C.S ('06)

If helicity-suppressed: impact for muonic modes >> than for electronic ones.

If helicity-allowed: impact for muonic modes < than for electronic ones.
(phase-space suppression)

$$H_{eff}(K_L \to \ell^+ \ell^-) \sim -y'_{7A}\,(\bar{s}\gamma_\mu \gamma_5 d)(\bar{\ell}\gamma^\mu \gamma_5 \ell) + y'_S\,(\bar{s}\gamma_5 d)(\bar{\ell}\ell) + y'_P\,(\bar{s}\gamma_5 d)(\bar{\ell}\gamma_5 \ell)$$

1. Vector & Axial-vector operators

Arise from EEWP, extra Z, MSSM with moderate $\tan\beta\,(\chi^\pm, H^\pm$ penguins),...
In general, *less sensitive* than neutrino modes (~ 1/3).

- *Enhanced EW penguins*:

$$y_{7V}^{EEWP} \approx y_{7V}^{SM},\ y_{7A}^{EEWP} \approx 5 y_{7A}^{SM}$$

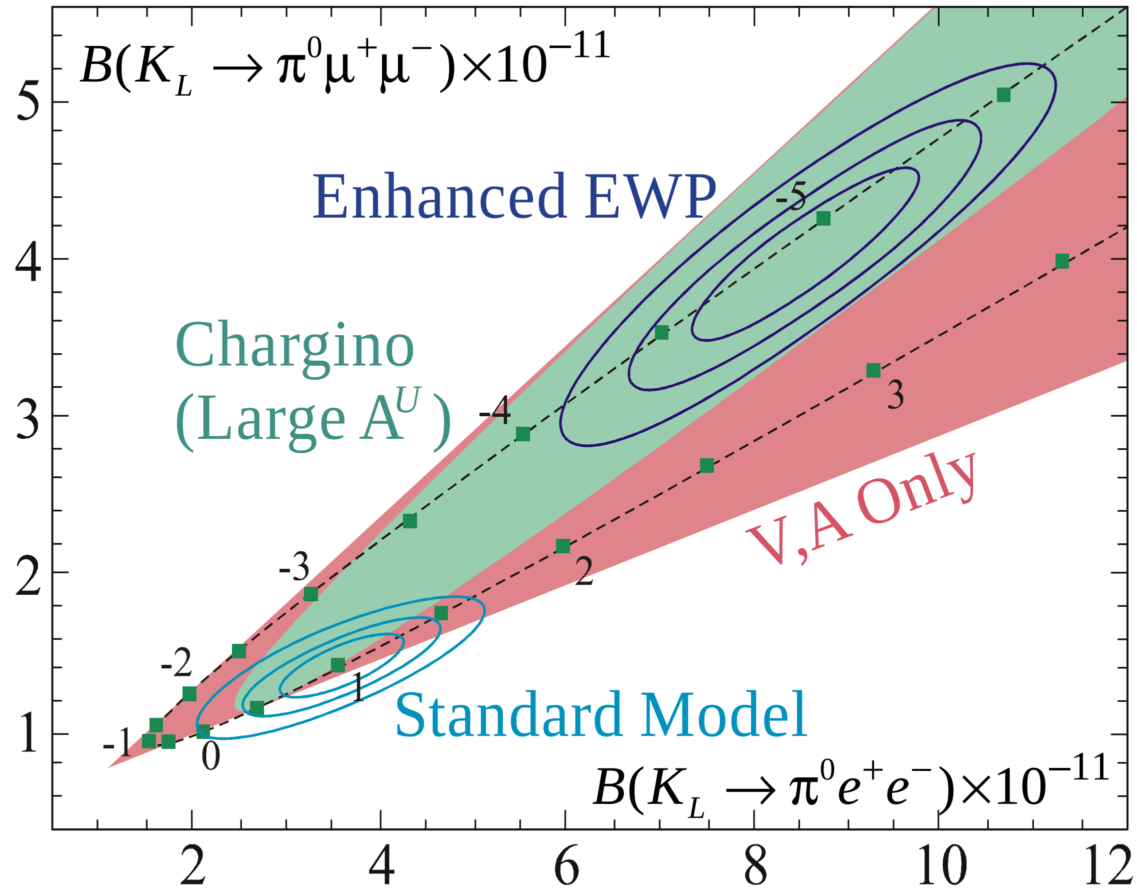
Buras,Fleischer,Recksiegel,Schwab ('04)

- *Chargino contributions*:
γ & Z penguins correlated
$\to$ restricted region even
for very large $\mathbf{A}^U$.

- *EMO* effectively absorbed as

$$y_\gamma\,(\bar{s}\sigma_{\mu\nu}d)F^{\mu\nu} \to y_{7V}$$

Buras,Colangelo,Isidori,Romanino,
Silvestrini ('00)



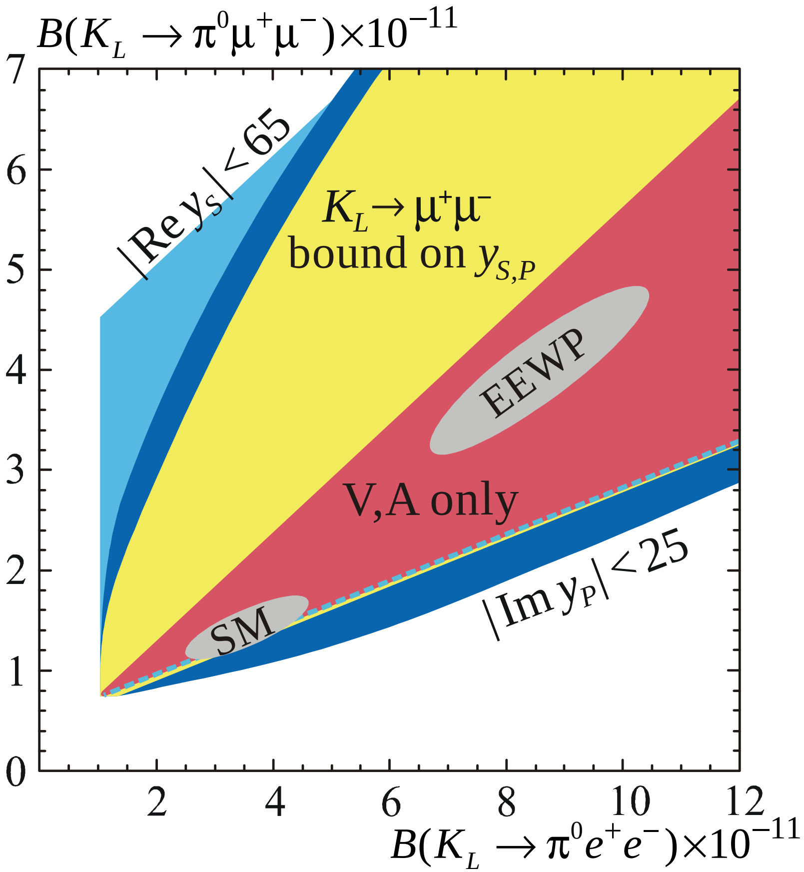
Bounds for general vector and axial vector FCNC operators (i.e. arbitrary y_{7A}, y_{7V}):

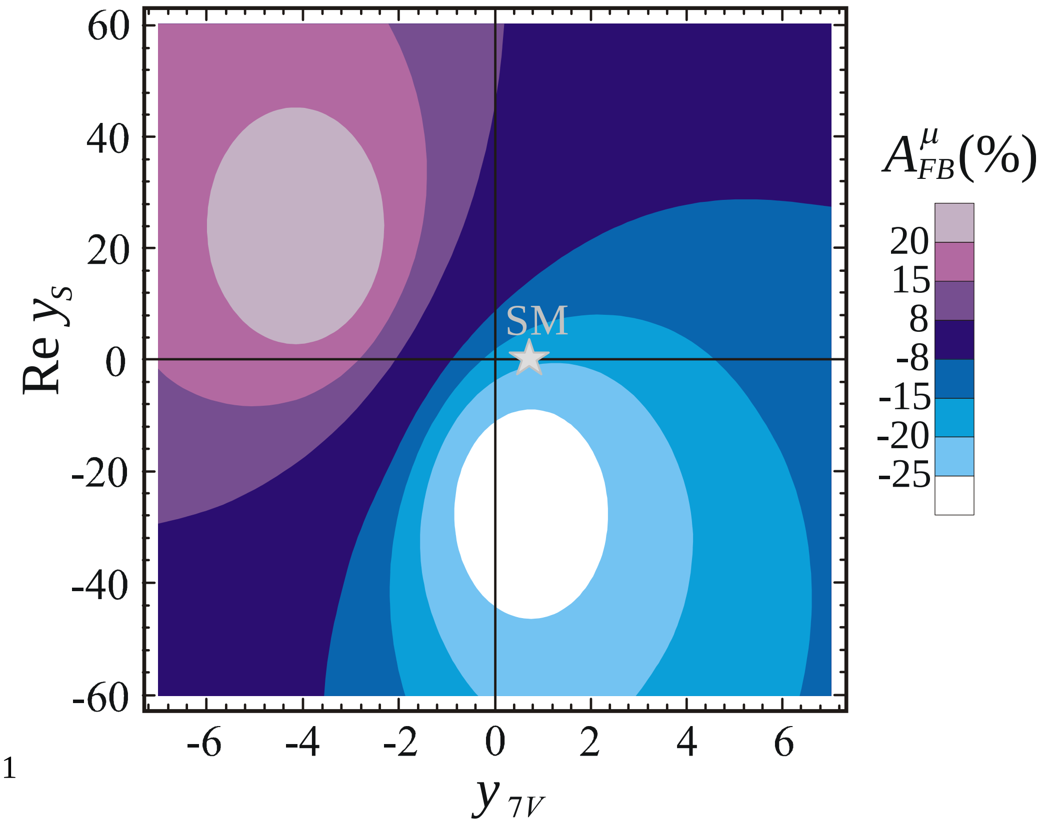
$$0.1\cdot 10^{-11} + 0.24 B(\pi^0 e^+ e^-) \le B(\pi^0\mu^+\mu^-) \le 0.6\cdot 10^{-11} + 0.58 B(\pi^0 e^+ e^-)$$

2. Scalar & Pseudoscalar operators

Helicity-suppressed: arise from neutral Higgs penguins at large $\tan\beta$ (similar to $B \to \mu^+\mu^-$, but sensitive to different mass insertions).

Isidori,Retico ('02)





Helicity-allowed: arise from tree-level leptoquark interactions (RPV,...). Impact completely negligible if these operators also contribute to $K_L \to e^+e^-$.

3. Tensor & Pseudotensor operators

Helicity-suppressed:

- In the MSSM, smaller than (pseudo-)scalar operators.
- Phase-space suppressed.

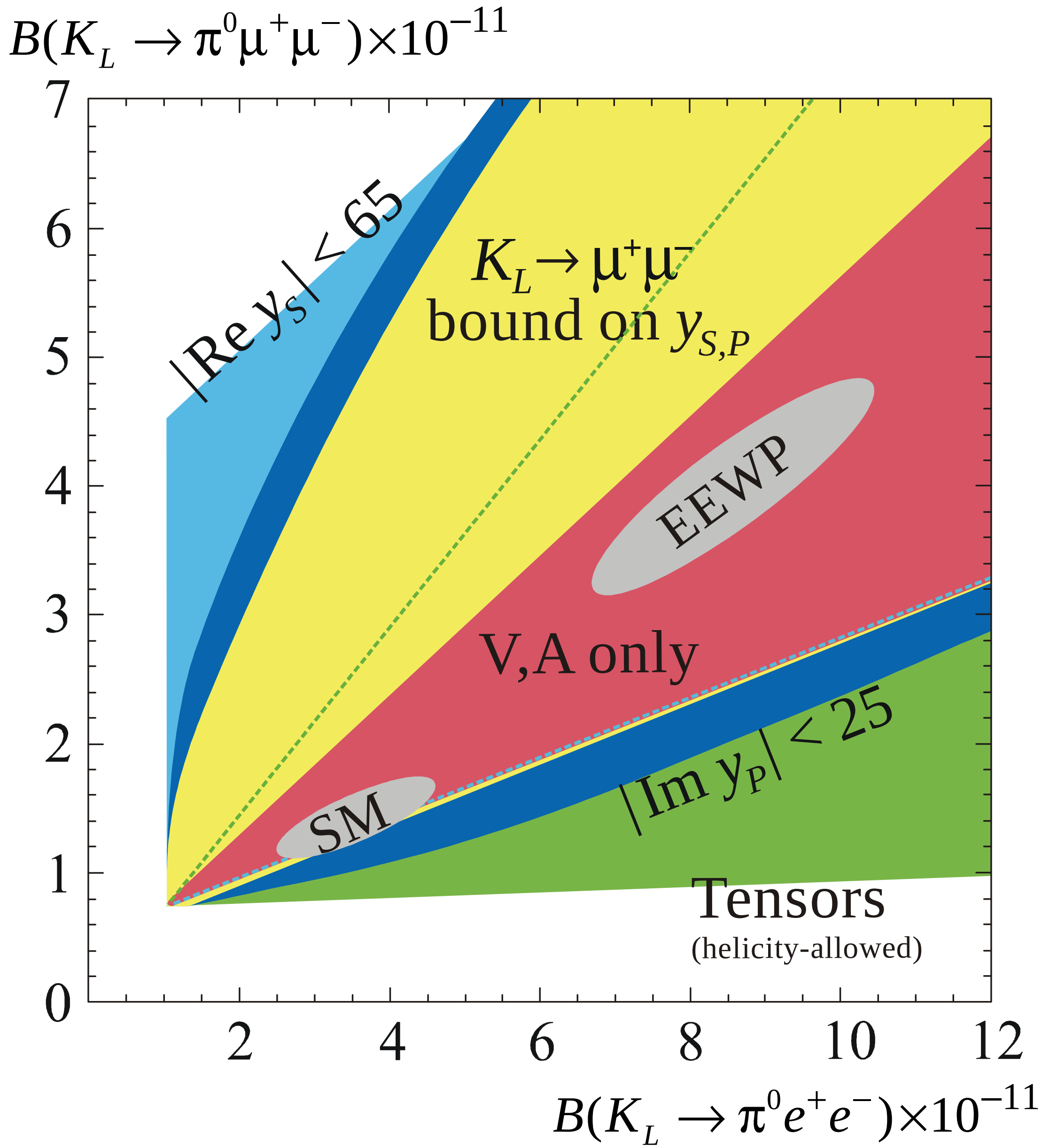
$\to$ *No visible impact*.

Helicity-allowed:

- Can arise from tree-level leptoquark interactions.
- No bound from $K_L \to \ell^+ \ell^-$.
- Even if similar interactions included for neutrino modes,

$$(\bar{s}\sigma_{\mu\nu} d)(\bar{\nu}\sigma^{\mu\nu}(1 \pm \gamma_5)\nu)$$

Still a large region allowed.



Conclusion

Theoretical control over the SM Contributions

– $K_L \to \pi^0 \nu\bar{\nu},\ K^+ \to \pi^+ \nu\bar{\nu}$ QCD effects are known to a high level of precision: NNLO for the dimension-six operators, with the smaller dimension-eight and LD contributions under control.

Possible improvements: *Isospin breaking in the vector/scalar form-factors*
Better estimate of charm-quark mass
Lattice study for higher-dimensional operators

– $K_L \to \pi^0 e^+ e^-,\ K_L \to \pi^0 \mu^+ \mu^-$ Long-distance effects under control, but could be improved. NLO effects for the running (sufficient).

Possible improvements: *Better measurements of* $K_S \to \pi^0 \ell^+ \ell^-$ *for* a_S
Better measurements of $K_L \to \pi^0 \gamma\gamma$ *for* $\gamma\gamma\,(0^{++}, 2^{++})$

– $K_L \to \mu^+ \mu^-$ QCD effects known to NNLO (dimension six), but large uncertainty for the long-distance, two-photon piece.

Possible improvements: *Better theoretical treatment of* Disp($\gamma\gamma$) (?)
Better measurements of $K_S \to \pi^0 \gamma\gamma, K^+ \to \pi^+ \gamma\gamma$
and $K_L \to \gamma^* \gamma^*$ *for* Sign(Disp($\gamma\gamma$))

Sensitivity to New Physics effects

Sensitive to *New Physics signals* and able to constrain the *nature of New Physics*.

- *MFV:* effects of ~ 20%-25% for the $\nu\bar{\nu}$ modes are possible, but MFV does its job perfectly in killing any large deviation from the SM.

  Very promising for reliably testing the MFV hypothesis.

- *Large trilinear up-squark couplings:* rare K decays are the most sensitive probe of this sector of the MSSM parameter space.

  Essential to investigate the nature of SUSY breaking mechanism

- *General New Physics:* $K_L \to \pi^0 \ell^+ \ell^-$ are sensitive to, and able to discriminate among, various New Physics effects not accessible from neutrino modes.

  The $K_L \to \pi^0 \ell^+ \ell^-$ *system important in the investigation of* $\Delta S = 1$ *FCNC*

If LHC finds New Physics, the four modes have to be measured!

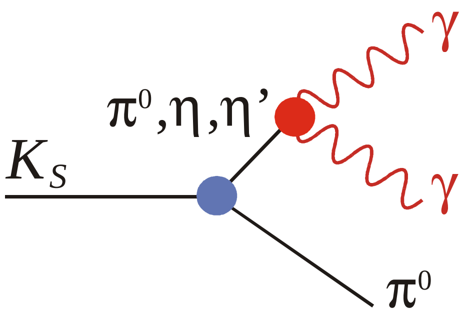
A clear signal of NP would no longer be the main goal,
but the *pattern of deviations with respect to the SM would become crucial*.

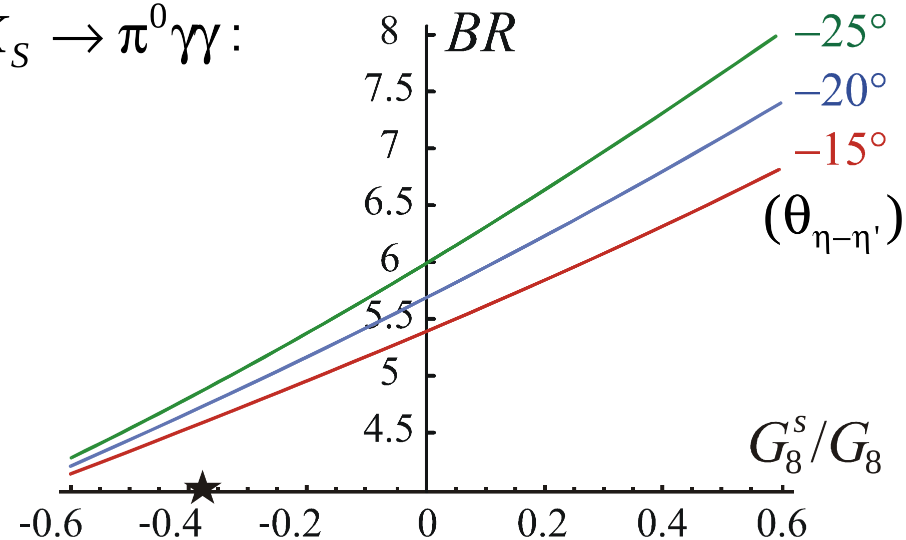
Back-up

Back-up

Backup 1: Sensitivity of radiative decays to the second octet LEC

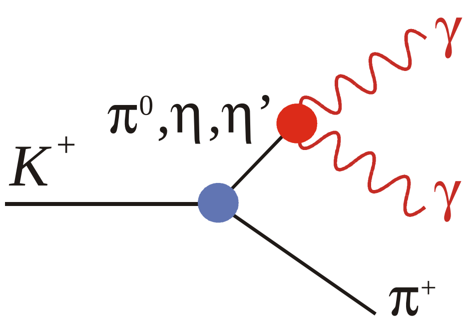
Experimentally, G_8^s could be fixed from $K_S \to \pi^0\gamma\gamma$:

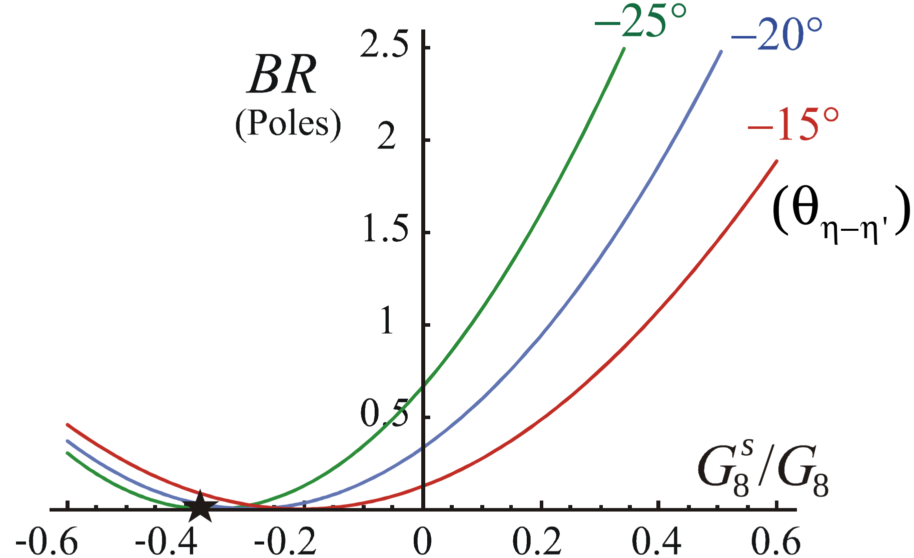




$$Br(K_S \to \pi^0\gamma\gamma)^{\exp}_{z>0.2} = (4.9 \pm 1.8)\cdot 10^{-8}$$

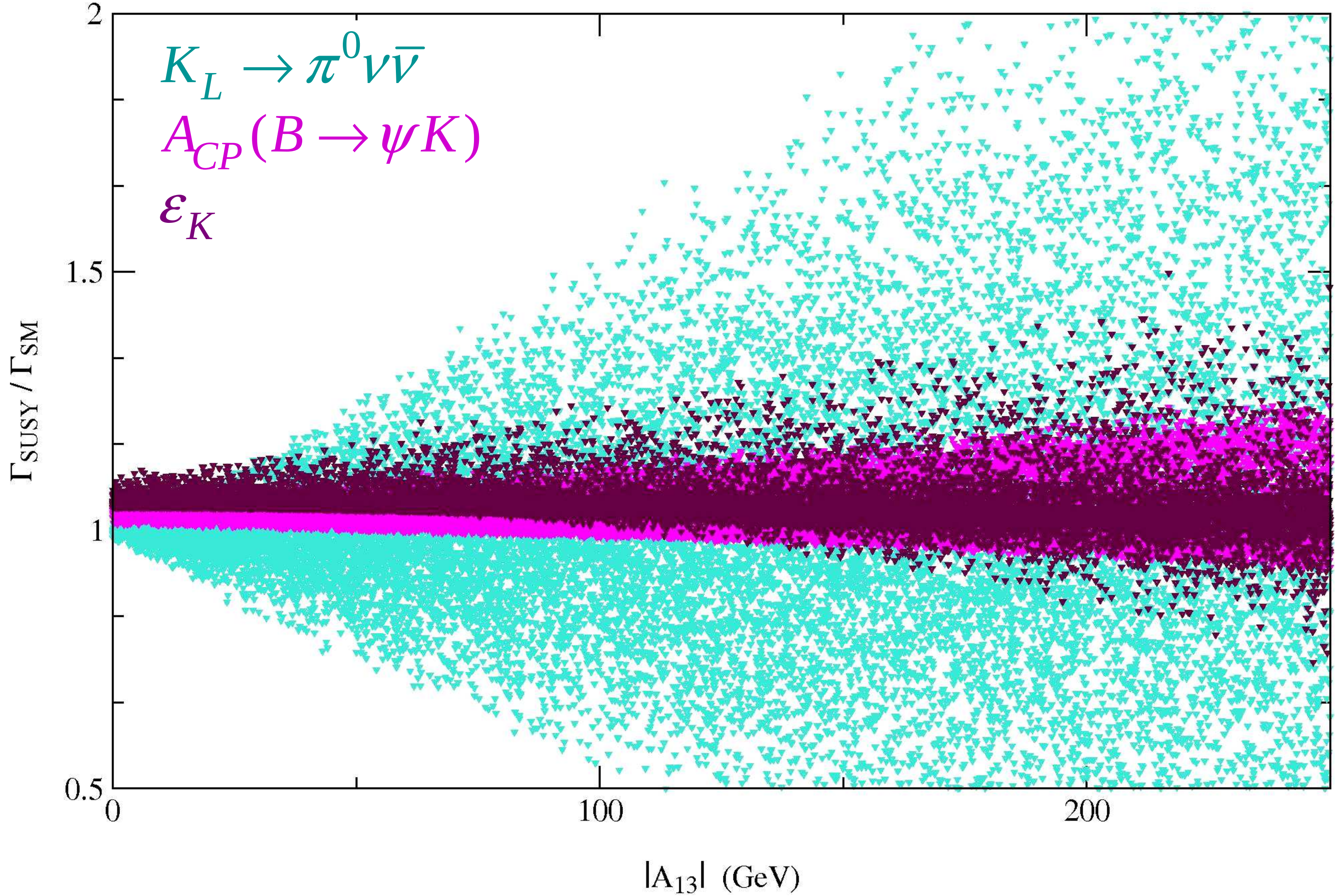
or from pole contributions to $K^+ \to \pi^+\gamma\gamma$





Backup 2: Sensitivities of CPV observables to A^U trilinear terms

The $K \to \pi\nu\bar{\nu}$ modes are the best probe of $\mathbf{A}^U$ terms

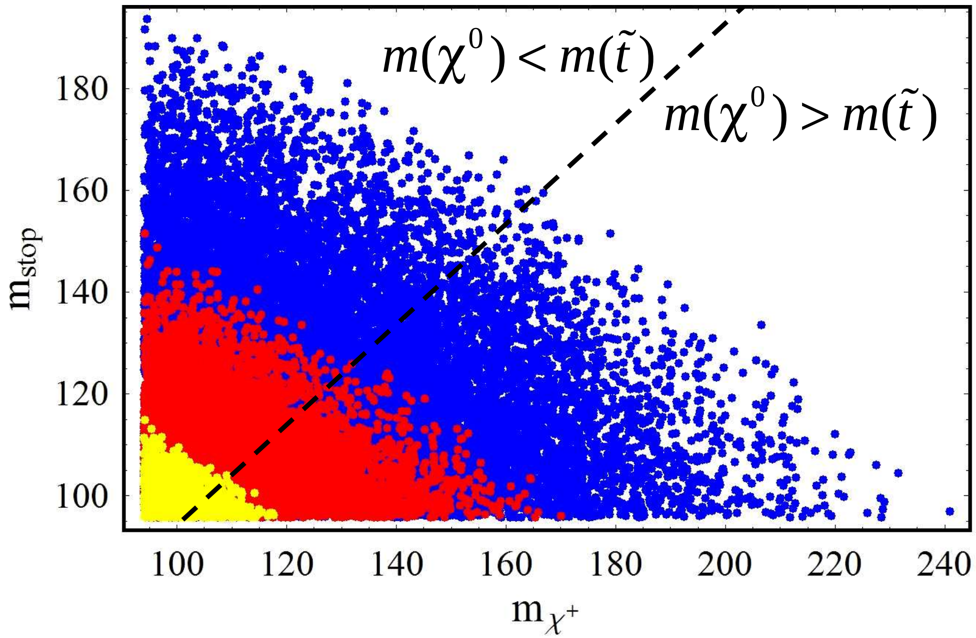


Backup 3: Anatomy of $K \to \pi\nu\bar{\nu}$ in MSSM with MFV

In the MSSM $\rightarrow$ Largest effect in the up-squark sector since enhanced by large top-quark Yukawa: $(\mathbf{m}_U)_{RL} = (a_4 - \cot\beta\mu^*)\mathbf{M}_u$

This makes $K \to \pi\nu\bar{\nu}$ an ideal test given its sensitivity to double MIA.

Isidori,Mescia,Paradisi,Trine,C.S. ('06)



- Colors $\Leftrightarrow$ enhancements of the $K_L \to \pi^0\nu\bar{\nu}$ mode by **10%, 12%, 15%.**
- Determining factors: lightest squark and chargino (~ higgsino) masses.
- Small correlation with $\Delta S = 2$
- Large correlation with $\Delta\rho$

Buras,Gambino,Gorbahn,Jager,Silvestrini ('00)

Adding the charged Higgs contribution, enhancements of ~ 20% for K^+, ~ 25% for K_L are possible with $\tan\beta = 2$, $m_{H+} > 300$ GeV (gets larger for smaller β).