Top Threshold Physics

André H. Hoang

Max-Planck-Institute for Physics

Munich

Outline

- Threshold Physics at the ILC
- Top Pair Threshold
  - measurements, experimental issues
  - theory issues
- Effective Theory (A): stable top quarks
- Effective Theory (B): finite lifetime effects
- Other Application:
  - $e^+e^- \to t\bar{t}H$
  - applications to LHC

    (anyone interested?)

Top Physics and the ILC

- e^+e^- collider: $E_{\rm cm} = 350$ GeV – 1 TeV
- Luminosity: $10^{34}\,{\rm cm}^{-2}{\rm s}^{-1} \longrightarrow 100\,{\rm fb}^{-1}$/year

  LC $\sim 10^5$ $t\bar{t}$ pairs [$\sigma_{\rm tot} \sim 1\,{\rm pb}$] ($e^+e^- \longrightarrow t\bar{t}$)

  LHC $\sim 10^8$ $t\bar{t}$ pairs [$\sigma_{\rm tot} \approx 850\,{\rm pb}$] ($gg \longrightarrow t\bar{t}$)

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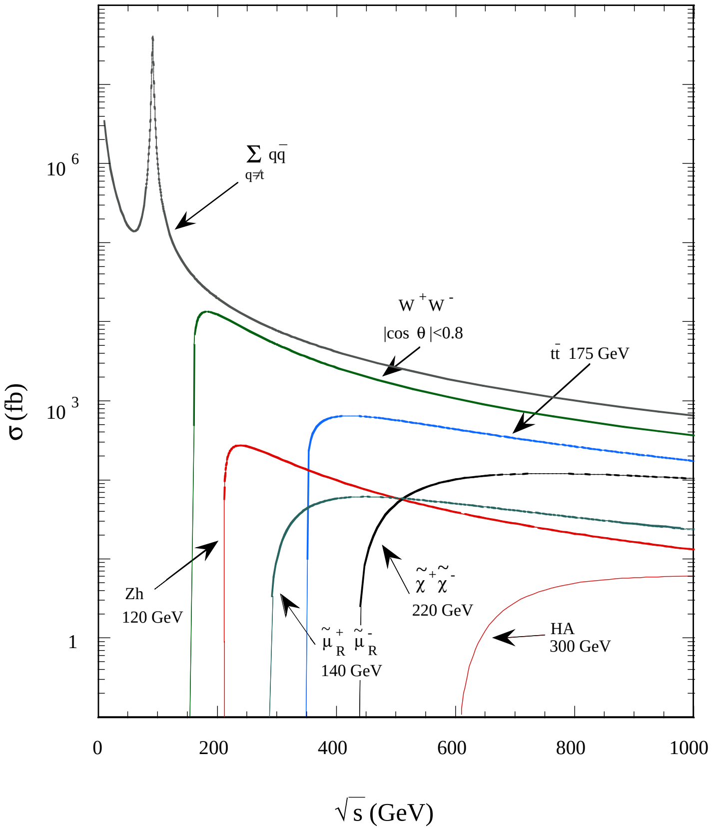
- Initial state tunable and very well known
  - Centre of mass energy variable

    $\longrightarrow$ threshold & continuum



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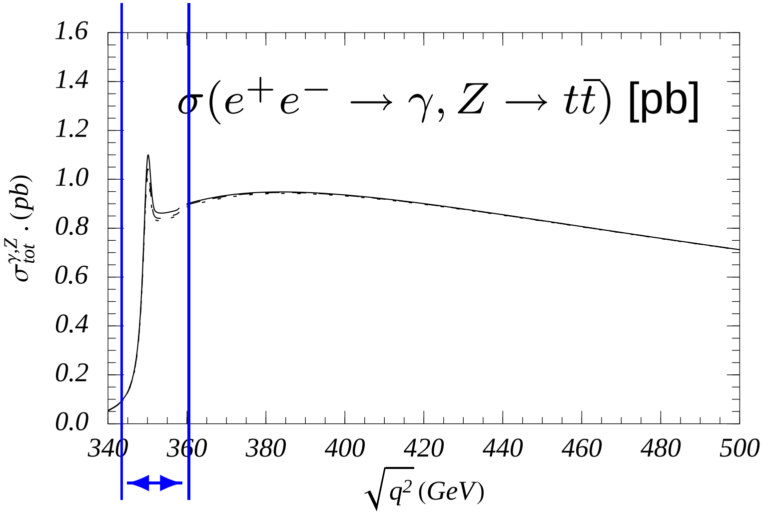

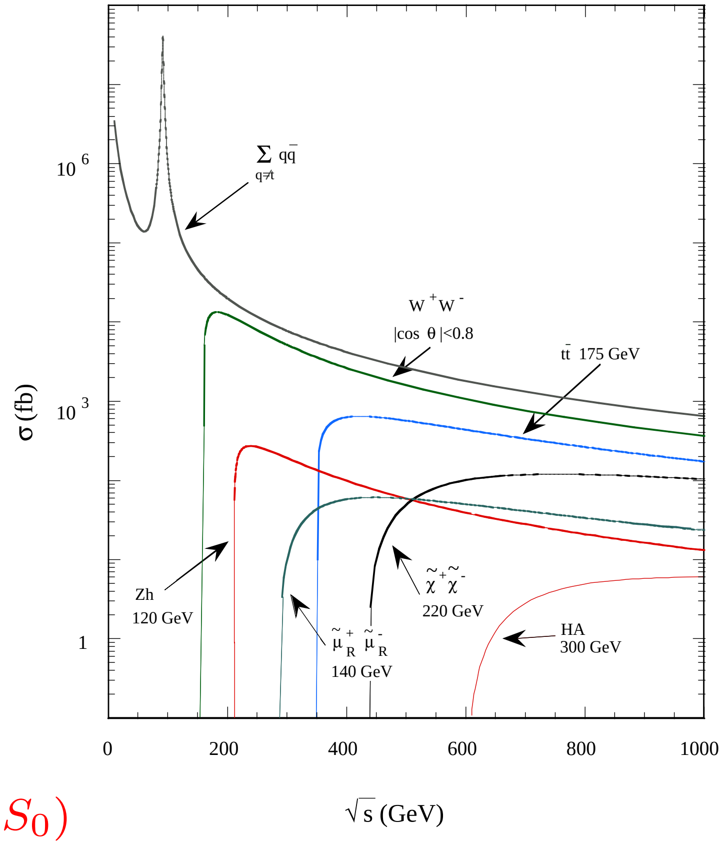

  - $e^\pm$ polarization: $P_- \sim 80\%$, $P_+ \sim 60\%$
  - $\gamma\gamma$ option: $e^+e^- \to t\bar{t}\,(^3S_1)$, $\gamma\gamma \to t\bar{t}\,(^1S_0)$

Threshold Measurements

Threshold Scan: $\sqrt{s} \simeq 350$ GeV (Phase I)

- ▷ count number of $t\bar{t}$ events
- ▷ color singlet state
- ▷ background is non-resonant
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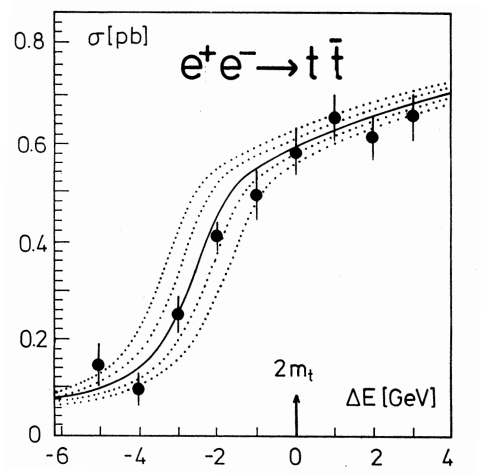

$\rightarrow \delta m_t^{\text{exp,stat}} \simeq 20$ MeV

$\mathcal{L} = 300\,\text{fb}^{-1}$
$9+1$ scan points
[Peralta, Martinez, Miquel]

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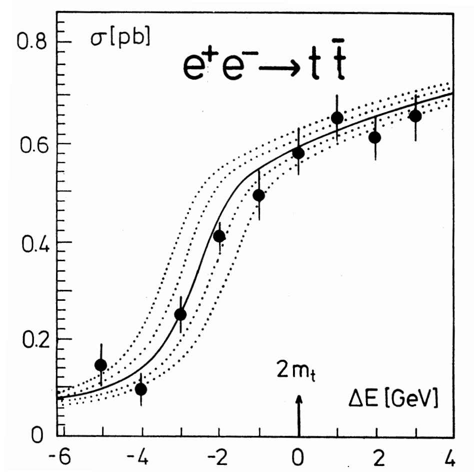

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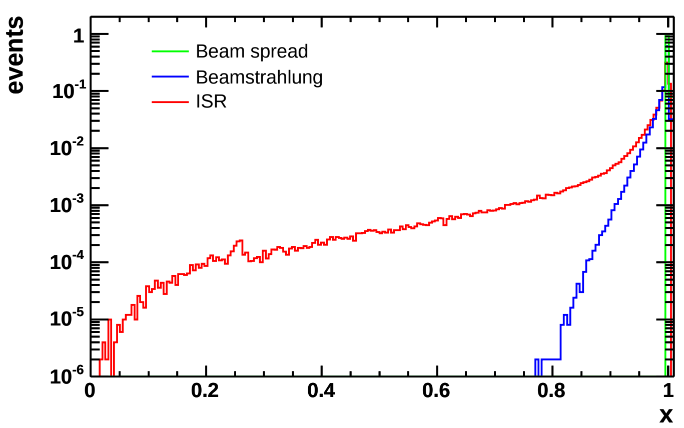
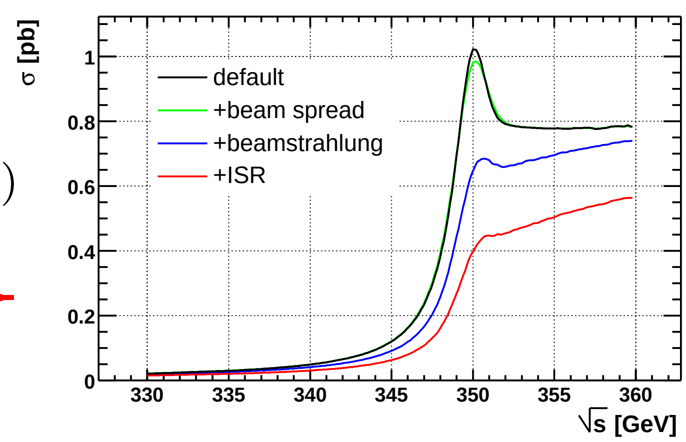
Simulations

Influence of Luminosity spectrum

Boogert 2004



$$\sigma(s) = \int_0^1 dx L(x) \sigma^0(x^2 s)$$



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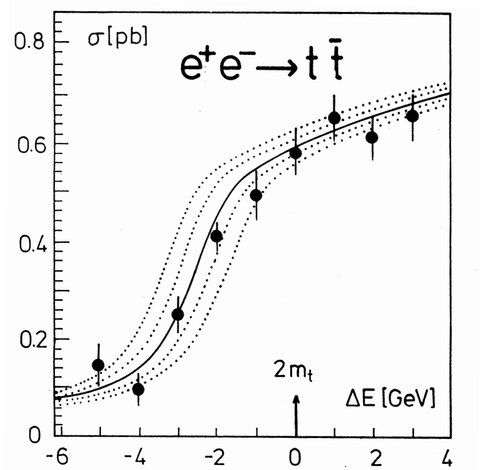

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$\rightarrow \delta m_t^{\mathrm{theory}} \simeq 100$ MeV

What mass?

$\sqrt{s}_{\mathrm{rise}} \sim 2m_t^{\mathrm{thr}} +$ conv.pert.series

(short distance mass: 1S $\leftrightarrow \overline{\mathrm{MS}}$)

For what ? $\rightarrow$ more examples: hep-ph/0306181

- elctroweak precision obs.'s: $\sin\theta_W \Big(1 + \delta(m_t, m_H, \ldots)\Big) = 1 - \frac{M_W^2}{M_Z^2}$
- Higgs precision obs.'s (e.g. MSSM):

$$m_h^2 \simeq M_Z^2 + \frac{G_F m_t^4}{\sin^2\beta} \ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_1}}{m_t^2}\right)$$

$\delta m_h^{\mathrm{exp}} = 200$ MeV (LHC)

$\delta m_h^{\mathrm{exp}} = 50$ MeV (ILC)

$\rightarrow \delta m_t^{\mathrm{required}} \approx \delta m_h^{\mathrm{exp}}$

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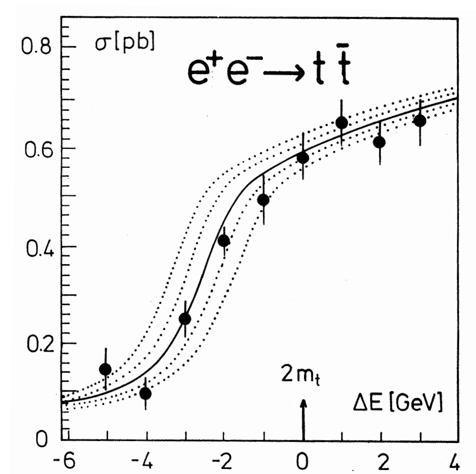

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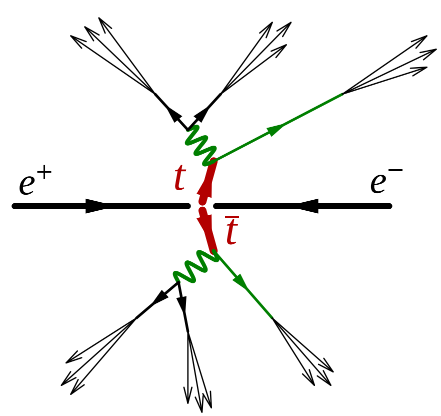
Reconstruction: any $\sqrt{s}$ (Phase I + II)

Chekanov,Morgunov:

- ▷ $e^+e^- \rightarrow 6$ jets (y^6_{cut})
- ▷ b-tagging
- ▷ $\vec{P}_1 + \vec{P}_2 < \Delta_p$
- ▷ $M_1 - M_2 < \Delta_M$



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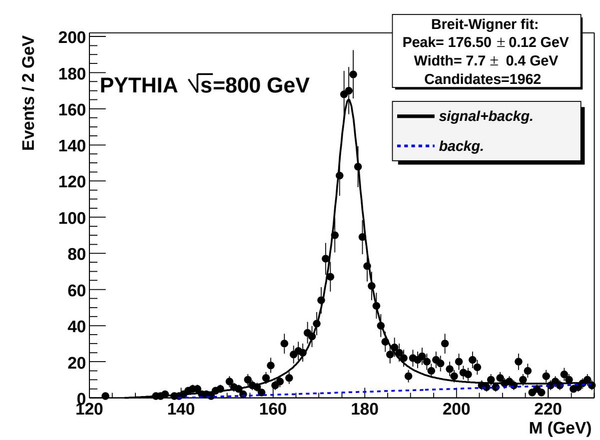
$(\mathcal{L} = 300\,\mathrm{fb}^{-1})$



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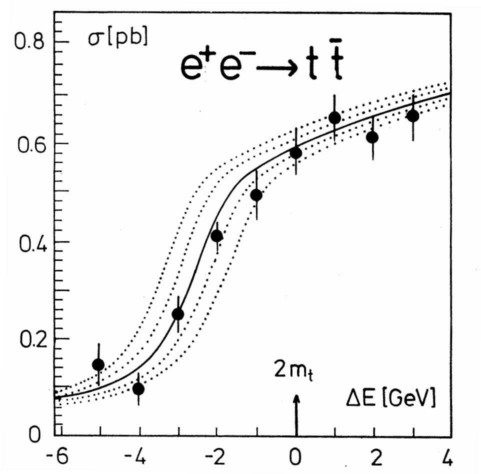

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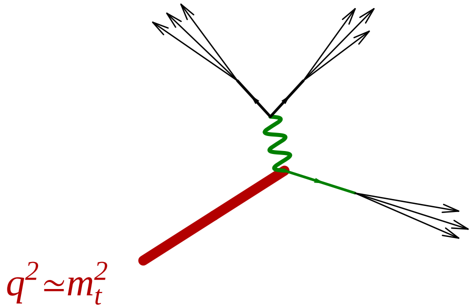

→ $\delta m_t^{\rm ex,stat} \simeq 100$ MeV ($\mathcal{L} = 300\,{\rm fb}^{-1}$)

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Pole Mass ?

ambiguity: $\Delta m_t \sim \Lambda_{\rm QCD}$

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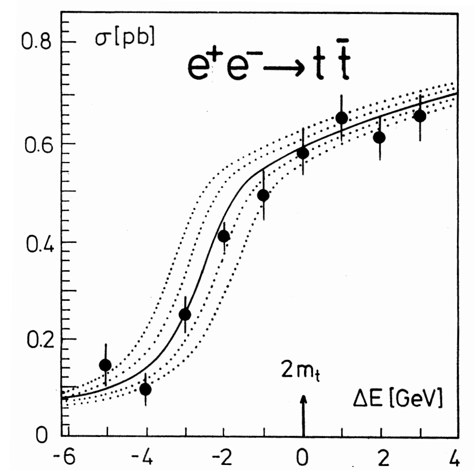

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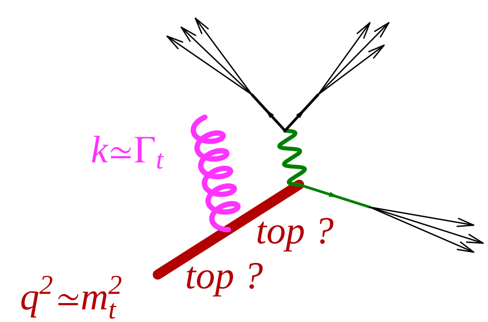

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$\Delta m_t \sim \pi\alpha_s(\Gamma_t)\,\Gamma_t$

There is s.th. to understand here !

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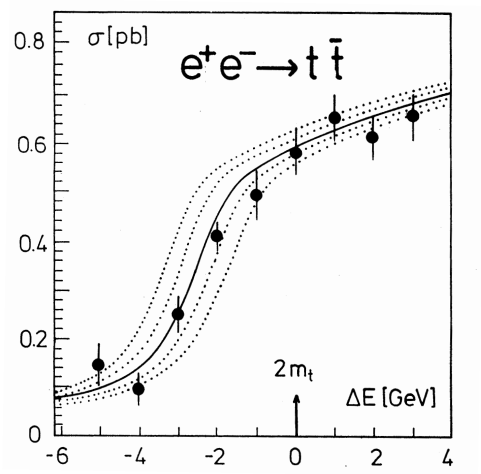

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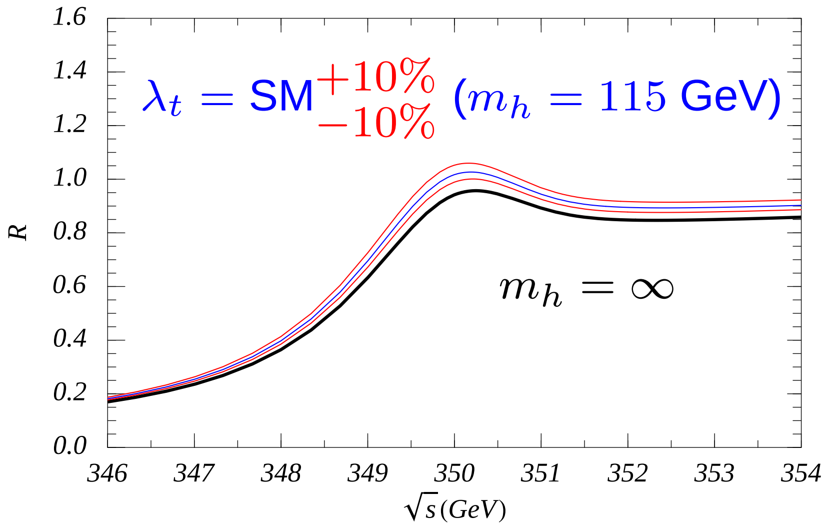

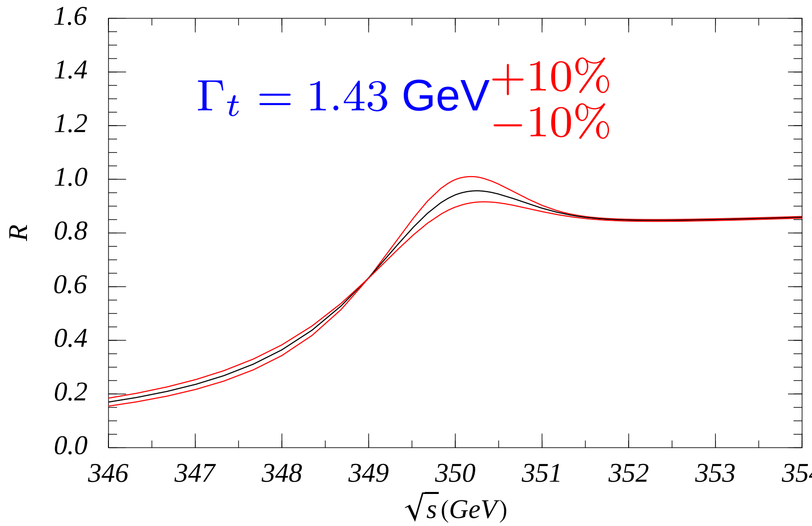

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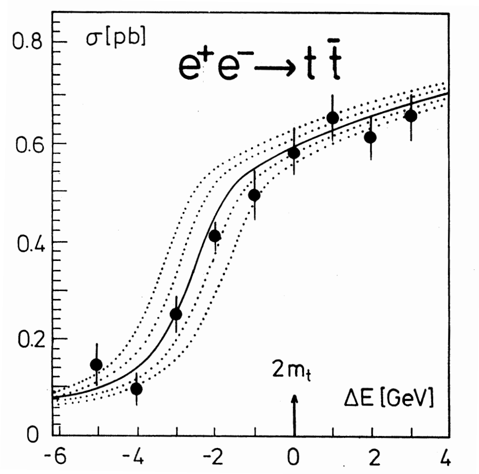

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Simulations δ(line-shape form) $\leftrightarrow$ δ(Lumi spectrum)

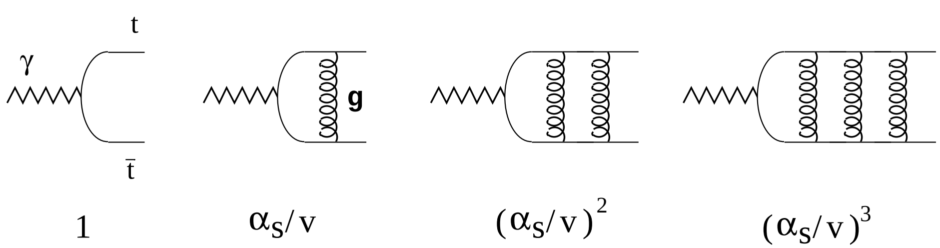
$(\delta\lambda_t/\lambda_t)^{\rm stat} = 15 - 50\%$	$(\delta\lambda_t/\lambda_t)^{\rm syst} = ?$	$(\delta\lambda_t/\lambda_t)^{\rm theo} \sim ?$
$(\delta\alpha_s(M_Z))^{\rm stat} = 0.001$	$(\delta\alpha_s(M_Z))^{\rm syst} = 0.002$	$(\delta\alpha_s(M_Z))^{\rm theo} \sim ?$
$(\delta\Gamma_t)^{\rm stat} = 50$ MeV	$(\delta\Gamma_t)^{\rm syst} = 15$ MeV	$(\delta\Gamma_t)^{\rm theo} \sim ?$

$\Rightarrow$ goal: $(\delta\sigma/\sigma)^{\rm theo} \leq 3\,\%$

Theory Issues

m_t (hard) $\gg$ $\mathbf{p} \sim m_t v$ (soft) $\gg$ $E \sim m_t v^2$ (ultrasoft)

- perturbation theory in α_s breaks down $(\alpha_s/v)^n$



"Coulombsingularities"

$\longrightarrow$ Schrödinger Equation

$v \sim \alpha_s$

- perturbation theory in α_s breaks down $\longrightarrow$ large logs $(\alpha_s \ln v)^n$

$m_t = 175$ GeV, $\mathbf{p} \sim 25$ GeV, $E \sim 4$ GeV $\Rightarrow \ln\left(\frac{m_t^2}{E^2}\right) = 8$ $\longrightarrow$ RGE's

"multi-scale problem"

Theory Issues

- $\boxed{\Gamma_t \approx 1.5\,\text{GeV} \gg \Lambda_{\rm QCD}}$ $\Rightarrow$ $v = \sqrt{\frac{E}{m}} \longrightarrow v_{\rm eff} = \sqrt{\frac{E+i\Gamma_t}{m}}$

  (Fadin,Khoze)

  $\Rightarrow m_t \gg p = m v_{\rm eff} \gg E = m v_{\rm eff}^2 \gtrsim \Lambda_{\rm QCD}$ always true !

  $\Rightarrow$ top threshold entirely perturbative ! $\longrightarrow$ "Schrödinger theory"

- $\boxed{E \sim \Gamma_t:}$ top quarks are always produced off-shell !

  - non-factorizable corrections
  - final-state interferences (signal $\leftrightarrow$ non-signal/background)
  - gauge invariance

"theory for unstable particles"

Degrees of Freedom

- fields for degrees of freedom that can resonate for the quark-antiquark system

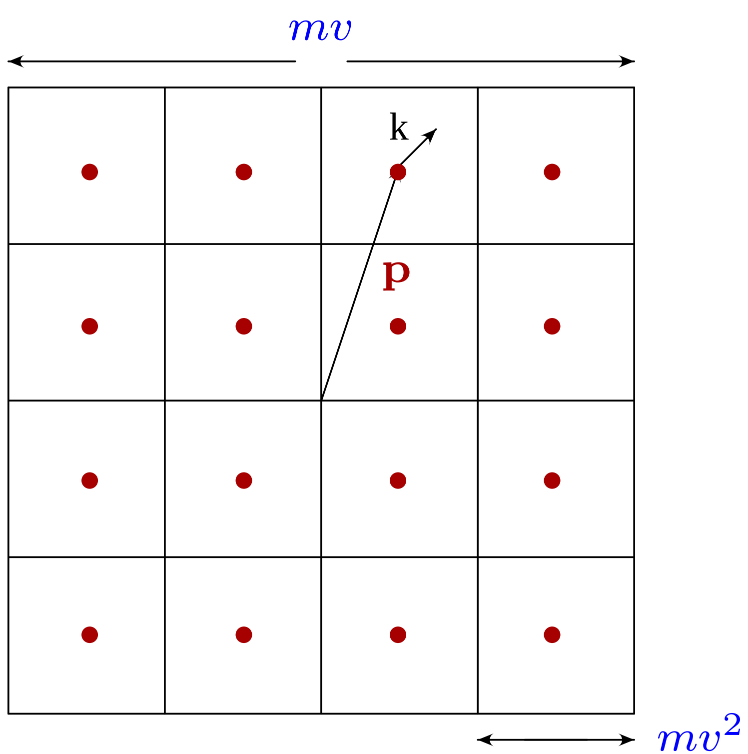
potential quarks	ψ_p, χ_p:	$(k_0, \mathbf{k}) \sim (mv^2, mv)$	$m_t \gg \mathbf{p} \gg E$
soft gluons	$A_q{}^\mu$:	$(k_0, \mathbf{k}) \sim (mv, mv)$	&
ultrasoft gluons	A^μ:	$(k_0, \mathbf{k}) \sim (mv^2, mv^2)$	$E = \mathbf{p}^2/m$

- vNRQCD label formalism:

$$(P^0, \mathbf{P}) = (0, \mathbf{p}) + (k^0, \mathbf{k})$$

soft component label — ultrasoft component dynamic variable

$$\psi_{\rm QCD}(x) \to \sum_{\mathbf{p}} e^{i\mathbf{p}.\mathbf{x}}\, \psi_{\mathbf{p}}(x)$$



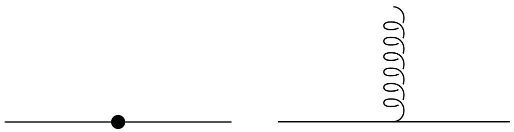
vNRQCD (stable quarks)

$\underline{\mathcal{L} = \mathcal{L}_{\rm usoft} + \mathcal{L}_{\rm potential} + \mathcal{L}_{\rm soft}}$

Luke, Manohar, Rothstein, Stewart, AH

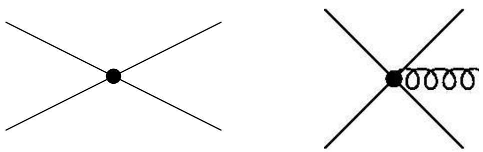
$$D^\mu = \partial^\mu + i\mu_U^\epsilon g_s(m\nu^2)A^\mu$$

$\mathcal{L}_{\rm usoft}$:



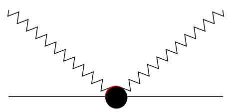
$$\psi_{\mathbf{p}}^\dagger(x)\left\{iD^0 - \frac{(\mathbf{p}-i\mathbf{D})^2}{2m_t} - \delta m_t(\nu)\right\}\psi_{\mathbf{p}}(x)$$

$\mathcal{L}_{\rm potential}$:



$$\mu_S^{2\epsilon}\, V(\nu)\, \psi_{\mathbf{p}'}^\dagger \psi_{\mathbf{p}} \chi_{-\mathbf{p}'}^\dagger \chi_{-\mathbf{p}}$$

$\mathcal{L}_{\rm soft}$:



$$\mu_S^{2\epsilon}\, U_{\mu\nu}(\nu)\ \psi_{\mathbf{p}'}^\dagger A_q^\mu A_{q'}^\nu \psi_{\mathbf{p}}$$

$$V = \left[\frac{\mathcal{V}_c(\nu)}{\mathbf{k}^2} + \frac{\mathcal{V}_k(\nu)\pi^2}{m|\mathbf{k}|} + \frac{\mathcal{V}_r(\nu)(\mathbf{p}^2+\mathbf{p}'^2)}{2m^2\mathbf{k}^2} + \frac{\mathcal{V}_2(\nu)}{m^2} + \frac{\mathcal{V}_s(\nu)}{m^2}\mathbf{S}^2 + \frac{\mathcal{V}_\Lambda(\nu)}{m^2}\Lambda + \frac{\mathcal{V}_t(\nu)}{m^2}T\right]$$

$$\mathbf{k} \equiv \mathbf{p} - \mathbf{p}'$$

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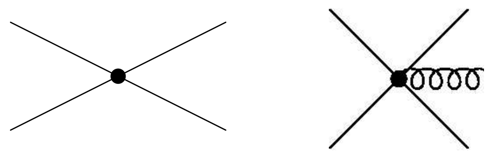

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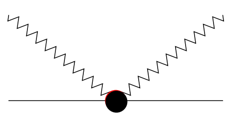
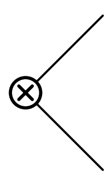


$$\mu_S^{2\epsilon}\, U_{\mu\nu}(\nu)\;\psi_{\mathbf{p}'}^\dagger A_q^\mu A_{q'}^\nu \psi_{\mathbf{p}}$$

<u>external currents:</u> (production & annihilation)



$$\mathbf{O}_{\mathbf{p}} = C(\nu)\cdot(\psi_{\mathbf{p}}^\dagger\,\boldsymbol{\sigma}\,\tilde{\chi}_{-\mathbf{p}}^*) + \cdots \qquad t\bar{t}\,(^3S_1)$$

Cross Section at NNLL Order

Schematic:

$$\sigma_{\rm tot} \;\propto\; {\rm Im}\left[\int d^4x\, e^{-i\hat{q}x} \left\langle 0 \left| T\, \mathbf{O}_{\mathbf{p}}^{\dagger}(0)\, \mathbf{O}_{\mathbf{p}'}(x) \right| 0 \right\rangle\right]$$

$$\propto\; {\rm Im}\left[C(\nu)^2\, G(0,0,\sqrt{s})\right]$$

$$\left(-\frac{\nabla^2}{m} - \frac{\nabla^4}{4m^3} + V(\mathbf{r}) - (\sqrt{s} - 2m - 2\delta m) - i\Gamma_t\right) G(\mathbf{r},\mathbf{r}') = \delta^{(3)}(\mathbf{r}-\mathbf{r}')$$

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fully known
at NNLL order ✓

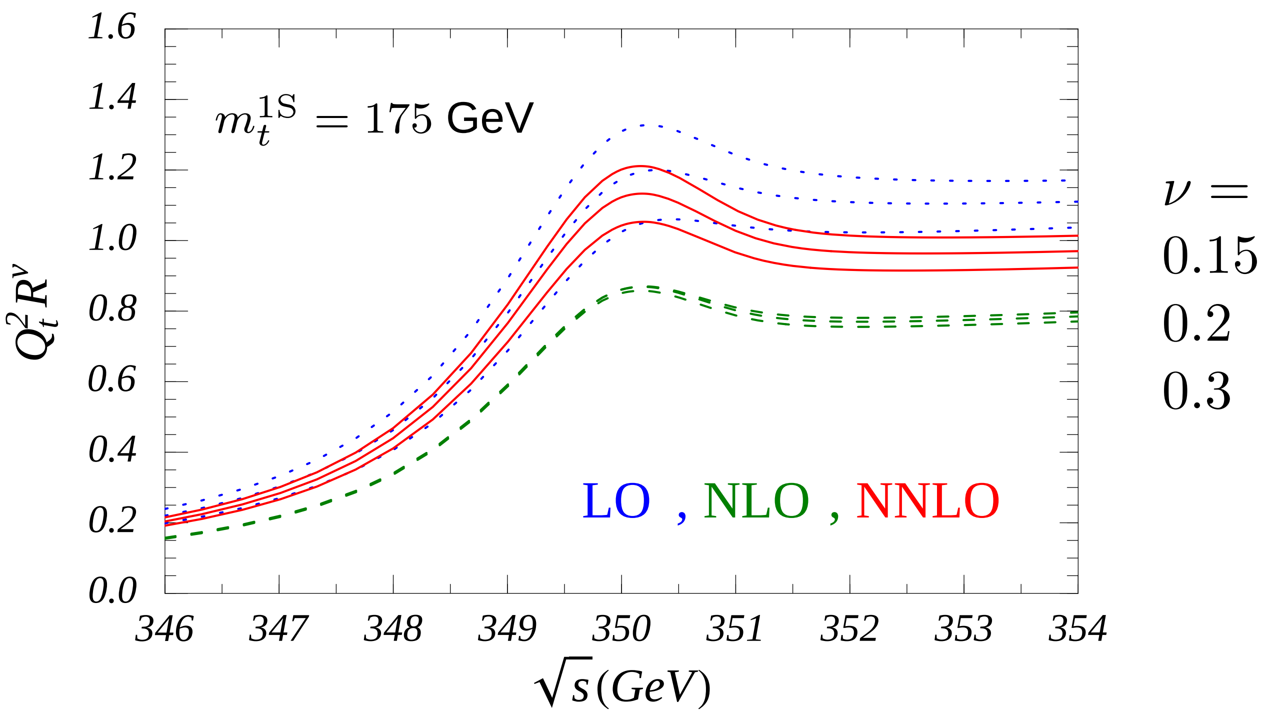
NLL ✓
NNLL (non-mixing) ✓
NNLL (mixing) unknown

Manohar,Stewart; AHH ’99-’03
Pineda,Soto ’00-’01

Cross Section at NNLL Order

AH

1S mass - fixed order approach

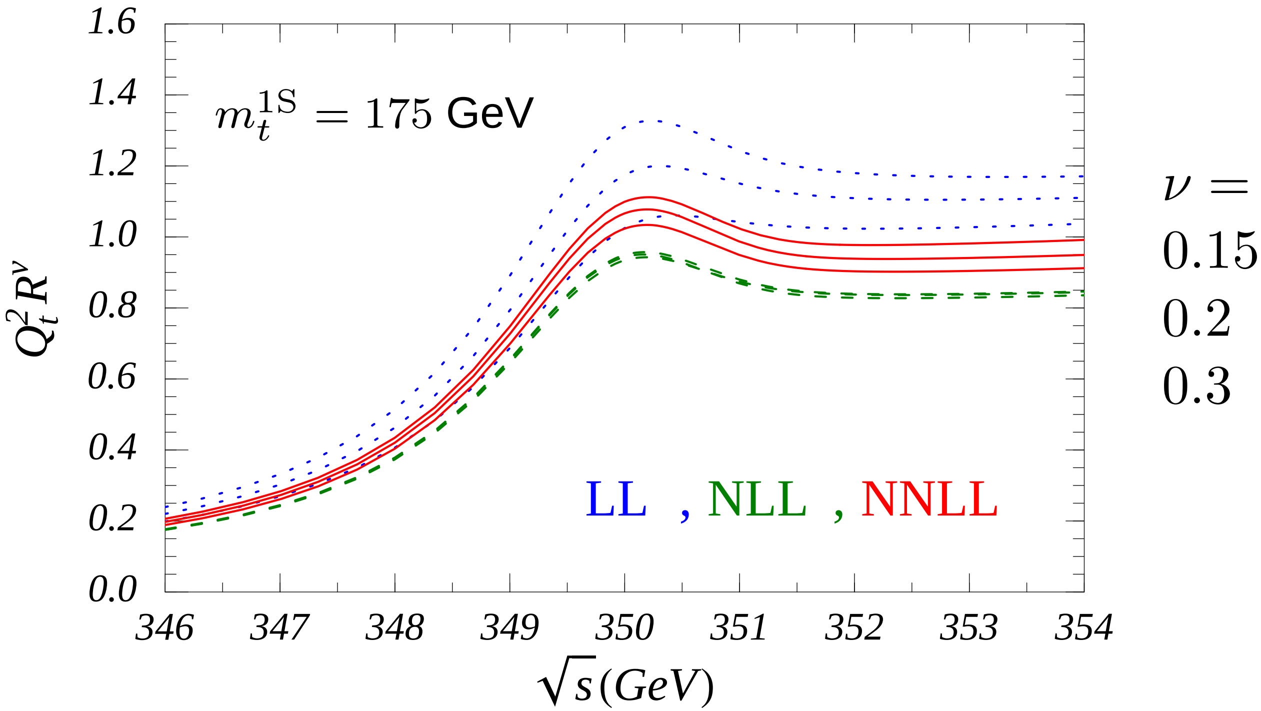


- peak position stable (threshold masses: 1S, PS, . . .)
- sensitivity to prescriptions of factorization/renormalization scale setting
- progress in NNNLO computations: $\longrightarrow$ A. Signer's talk

Cross Section at NNLL Order

AH

1S mass - RG-improved, with NNLL non-mixing terms



- RGI expansion shows better convergence
- theory error: $\delta\sigma_{t\bar{t}}/\sigma_{t\bar{t}} \sim \pm 6\%$ goal: 3%

$\longrightarrow$ full NNLL (mixing) running of $C(\nu)$ required $\rightarrow$ w.i.p.

vNRQCD (unstable quarks)

"inclusive treatment"

⇒ Optical Theory: effective complex indices of refraction for absorptive processes

⇒ vNRQCD: contributions from real Wb final states included in EFT matching conditions to QCD+ew. theory (=SM)

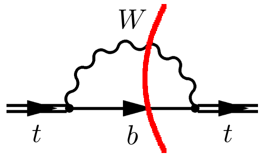
- complex matching conditions
- effective Lagrangian non-hermitian
- total rates through the optical theorem

Christoph Reisser, AH; Phys. Rev. D 71, 074022 (2005)

vNRQCD (unstable quarks)

time dilatation correction

quark bilinears:



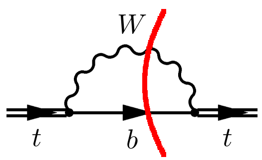
$$iD^0 - \frac{\mathbf{p}^2}{2m_t} + \delta m_t \quad \Longrightarrow \quad iD^0 - \frac{\mathbf{p}^2}{2m_t} + \delta m_t + i\frac{\Gamma_t}{2}\left(1 - \frac{\mathbf{p}^2}{2m_t^2}\right)$$

- power counting: $\Gamma_t \propto m_t g^2 \sim m_t v^2 \quad \Rightarrow \quad \boxed{g \sim g' \sim v \sim \alpha_s}$ $\longrightarrow$ gauge invariance
- finite lifetime is LL effect, LO: $E \rightarrow E + i\Gamma_t$ Fadin,Khoze

vNRQCD (unstable quarks)

quark bilinears:

time dilatation correction



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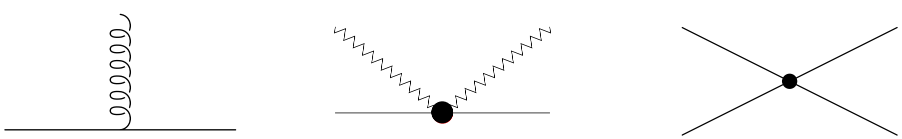
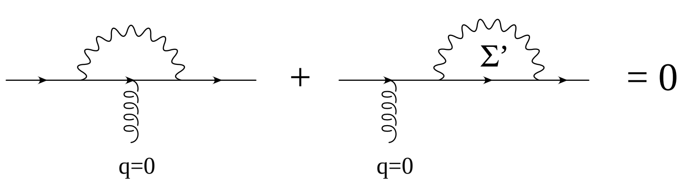
gluon interactions & potentials:



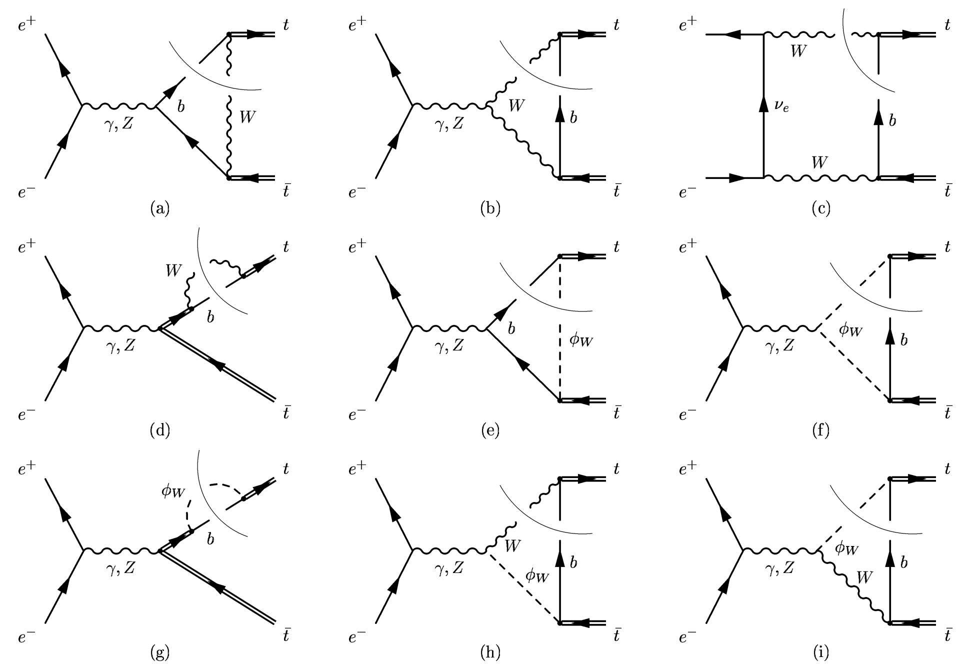


- electroweak corrections either beyond NNLL order or vanish due to gauge cancellations

$\longrightarrow$ ultrasoft gluon interference effects vanish at NLL [Khoze etal., Melnikov etal.] and NNLL order (new !)

vNRQCD (unstable quarks)

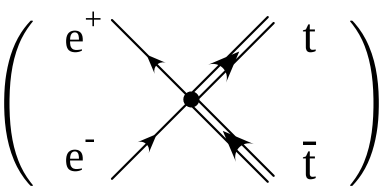
Currents:



real virtual electroweak
& and QCD corrections

bW^+ and $\bar{b}W^-$ cuts

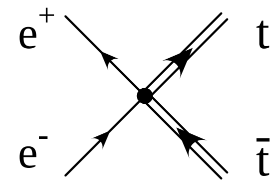
$$\mathbf{O_p} = \left[C^{\rm LL} + C^{\rm NLL} + C^{\rm NNLL} + iC^{\rm NNLL}_{\rm abs} \cdots \right] \cdot \left(\quad \right) + \cdots$$



vNRQCD (unstable quarks)

Currents:

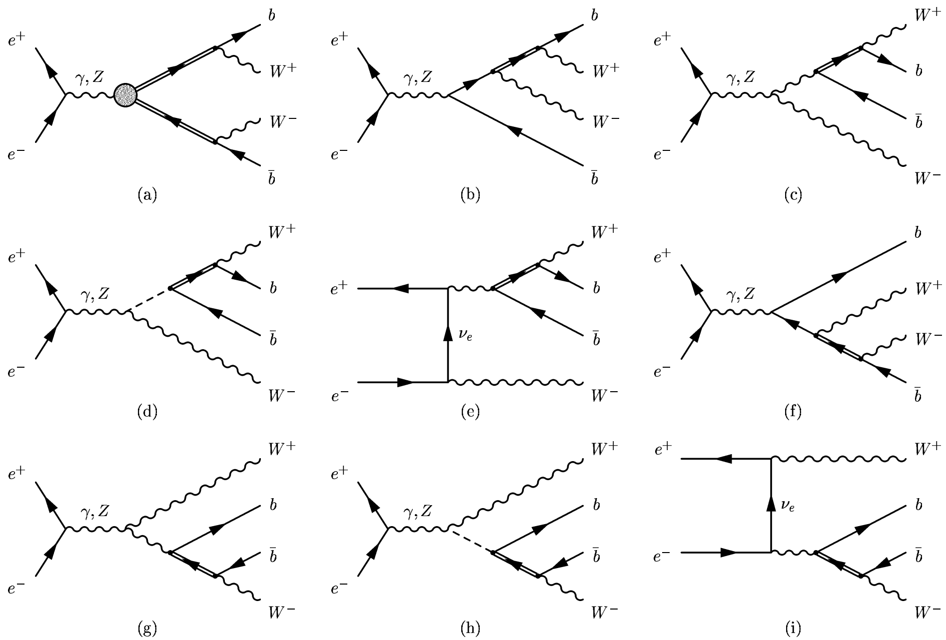
$$\mathbf{O_p} = \left[C^{\rm LL} + C^{\rm NLL} + C^{\rm NNLL} + iC^{\rm NNLL}_{\rm abs} \ldots \right] \cdot \left(\right) + \cdots$$



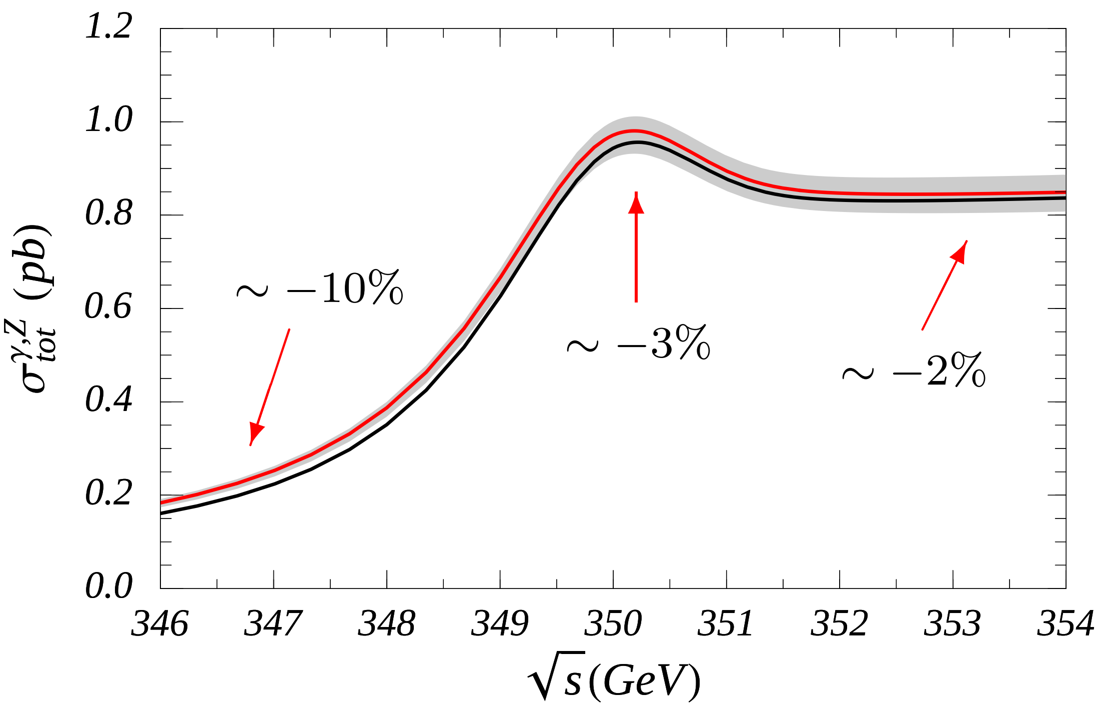
$$\sigma_{\rm tot} \propto \mathrm{Im}\left[\mathrm{C}(\nu)^2\, \mathrm{G}(0,0,\sqrt{\mathrm{s}} + \mathrm{i}\Gamma_{\rm t}) \right]$$

- accounts for irreducible interference contributions:

  resonant $\leftrightarrow$ non-resonant $W^+W^-b\bar{b}$ final states



Total Cross Section

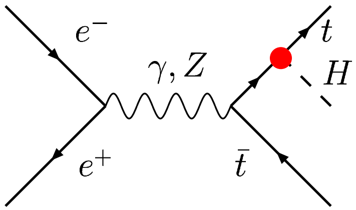


C. Reisser, AH

- also included summation of phase space logs $\sim (\alpha_s \ln v)^n$
- corrections comparable to NNLL QCD corrections
- shift in the peak position: $30 - 50$ MeV $(\delta m_t^{\text{ex}} \approx 50$ MeV$)$
- missing: phase space matching, QED effects (SCET+NRQCD), full NNLL ew

Other Application

$e^+e^- \rightarrow t\bar{t}H$ $\longrightarrow$ top-Yukawa coupling



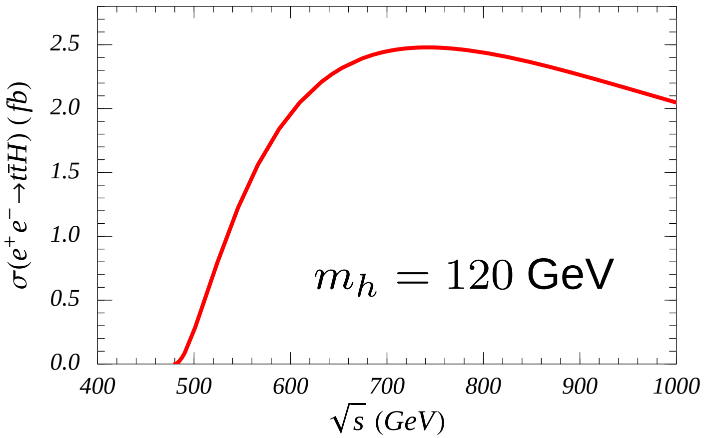
- Theory Status: $\sigma(e^+e^- \rightarrow t\bar{t}H)$

Born ✓ [Gaemers et al., Djouadi et al.]

1-loop ew. ✓ [Denner et al., Belanger et al., You et al.]

$\mathcal{O}(\alpha_s)$ fixed-order ✓ [Dittmaier et al., Dawson et al.]

NLL large-E_H QCD endpoint corrections ✓ [Cailin Farrell, AHH]



800 GeV: $\delta\lambda_t/\lambda_t = 5\%$ $(\mathcal{L} = 1000\text{fb}^{-1})$

[Gay;Besson;Winter]

compare LHC: $\delta\lambda_t/\lambda_t = 12\%$ $(\mathcal{L} = 300\text{fb}^{-1})$

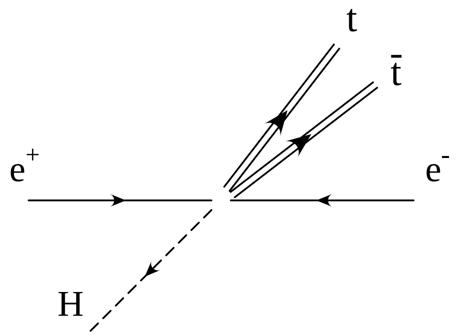
500 GeV: $(\delta\lambda_t/\lambda_t)^{\text{stat}} = 30\%$ $(\mathcal{L} = 1000\text{fb}^{-1})$

[Juste] $\rightarrow$ Born theory predictions

Other Application

$e^+e^- \to t\bar{t}H$

$\to$ region of large Higgs energy



$\to$ $t\bar{t}$ collinear

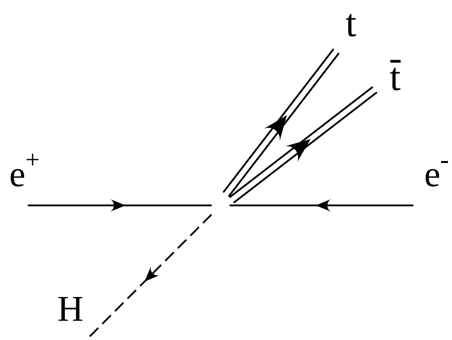
$\to$ QCD effects localized in $t\bar{t}$ system

$\Rightarrow$ $t\bar{t}$ dynamics non-relativistic

Other Application

$e^+e^- \longrightarrow t\bar{t}H$

$\longrightarrow$ region of large Higgs energy



$\longrightarrow t\bar{t}$ collinear

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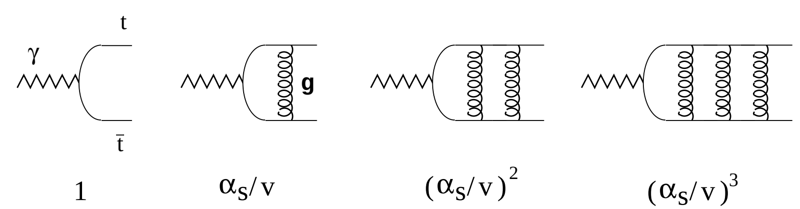
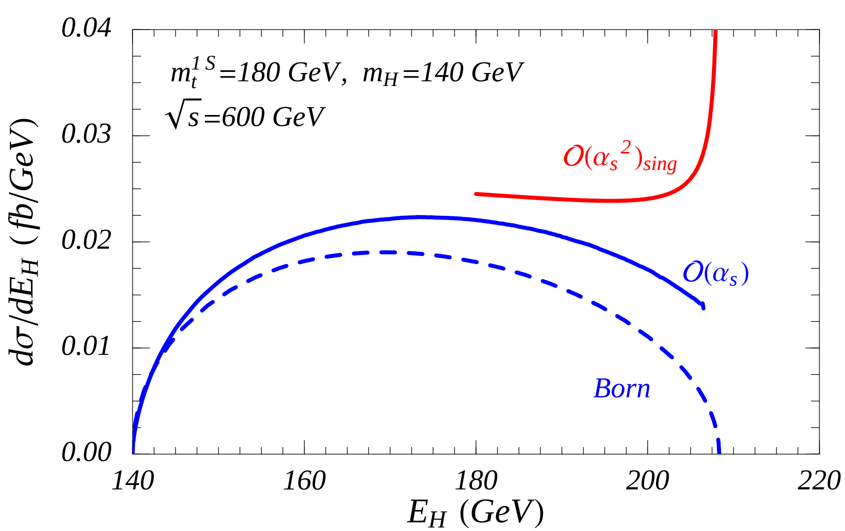




$\longrightarrow$ singularities: $\sim (\alpha_s/v)^n$,

$\sim (\alpha_s \ln v)^n$

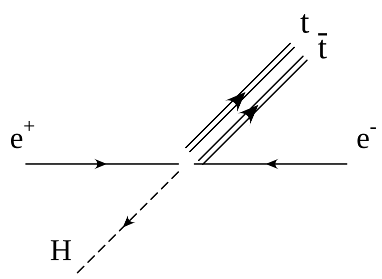
$\longrightarrow$ fixed order expansion breaks down

$\Rightarrow$ summation of singular terms

Other Application

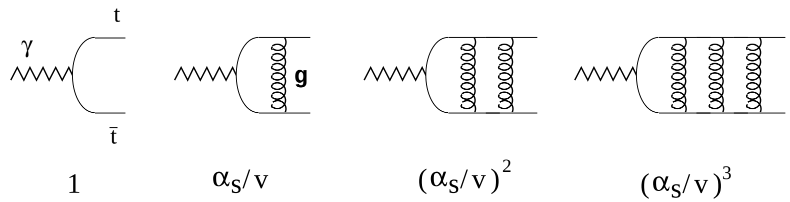
$e^+e^- \to t\bar{t}H$ $\qquad \to$ factorization formula

$$\left(\frac{d\sigma}{dE_H}\right)_{\mathrm{E_H \approx E_H^{max}}} \sim C^2(\mu, \sqrt{s}, m_t, m_H) \times \mathrm{Im}[G(0,0,v,\mu)]$$



$+$ hard QCD corrections

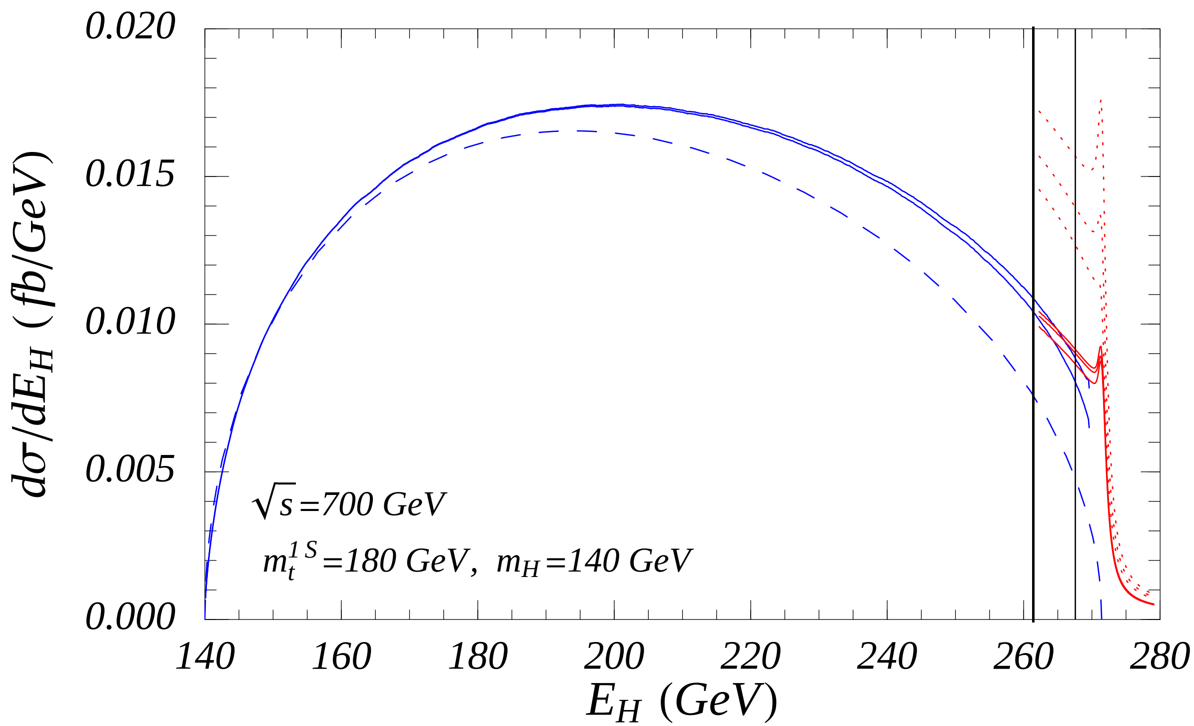
soft & ultrasoft QCD corrections



NLL formalism: Cailin Farrell, AHH; Phys.Rev.D72,014007 (2005)

Other Application

$e^+e^- \to t\bar{t}H$ $\longrightarrow$ NLL Higgs energy spectrum

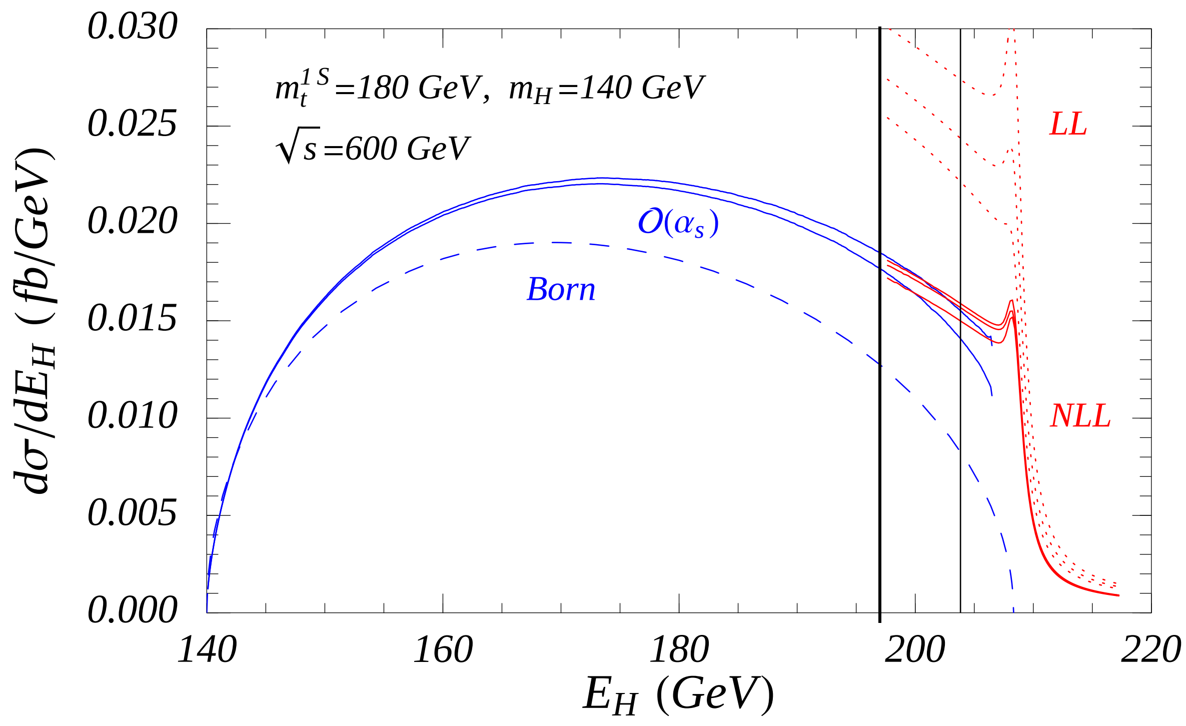


Farrell, AHH

- large E_H endpoint regions increases for smaller $\sqrt{s}$ / larger m_H

Other Application

$e^+e^- \to t\bar{t}H$ $\to$ NLL Higgs energy spectrum



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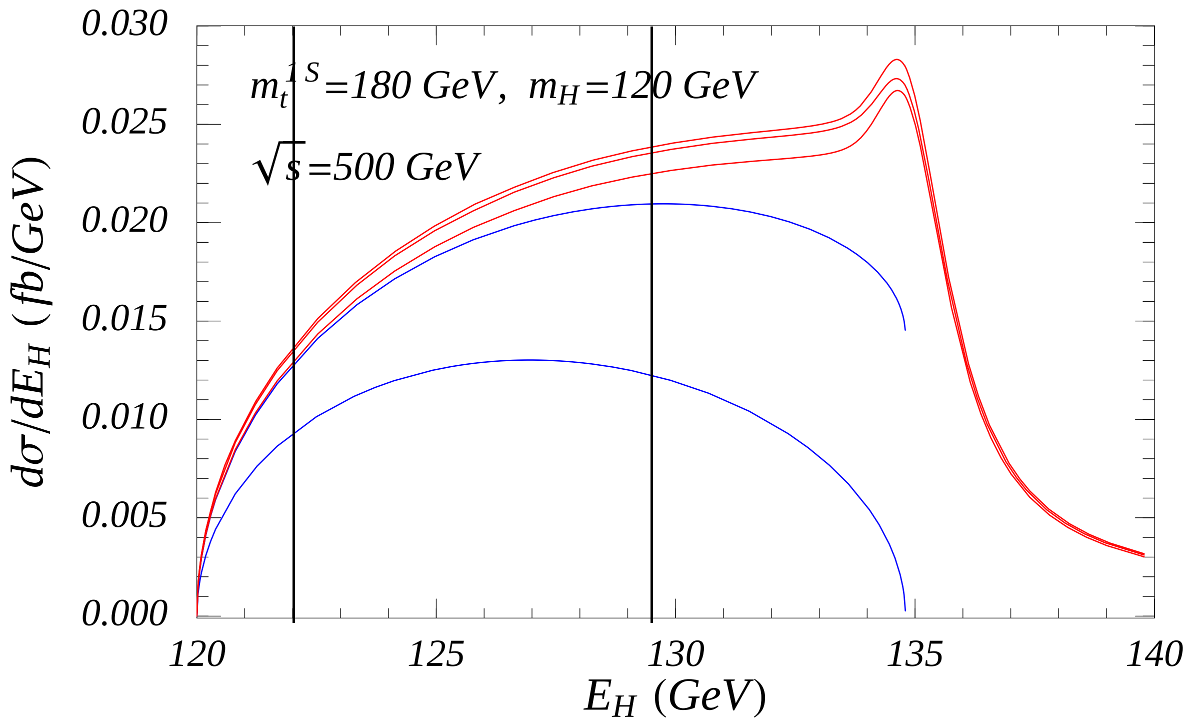
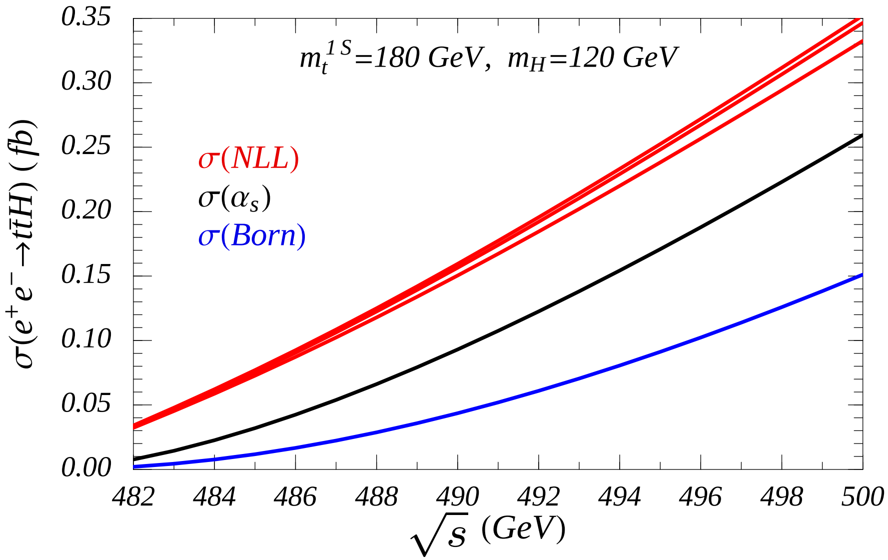


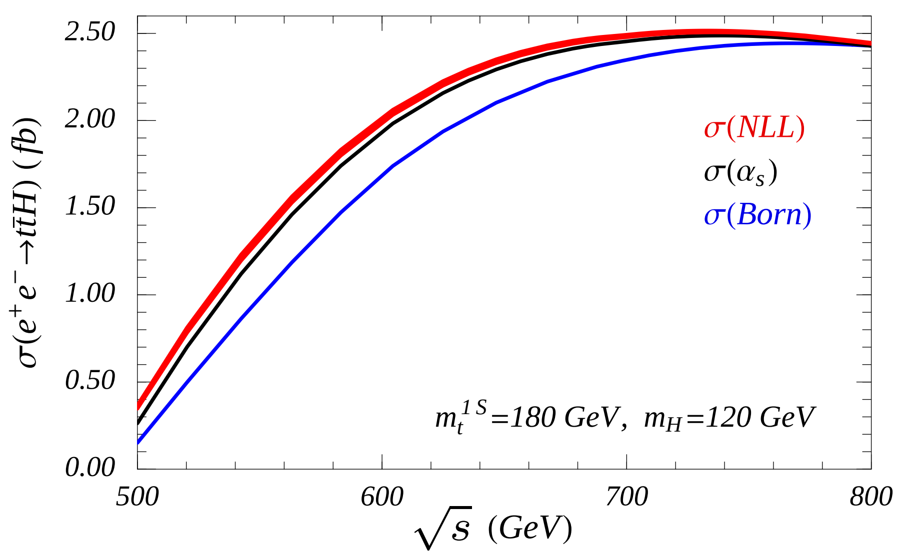
Farrell, AHH

- large E_H endpoint regions increases for smaller $\sqrt{s}$ / larger m_H

Other Application

$e^+e^- \to t\bar{t}H$ $\quad\to$ total cross section





500 GeV:

- factor 2 enhancement over tree level from summation of $(\alpha_s/v)^n$, $(\alpha_s \ln v)^n$ terms
- essential for realistic studies for ILC (phase I)
- another factor of 2 enhancement for $P_- = -80\%$, $P_+ = +60\%$

$\Rightarrow \quad (\delta\lambda_t/\lambda_t)^{\rm ILC}_{\rm 500\ GeV} \sim 30\% \quad \overset{\sim}{\longrightarrow} \quad 10 - 15\%$

Conclusion

- Top threshold physics ($m_{t\bar{t}} \approx 2m_t$) unique QCD $\leftrightarrow$ electroweak laboratory
  - hard & nonrelativistic & soft-collinear dynamics
  - unstable particles ($\Gamma_t \sim E_{\rm kinetic}$)
- Master application: $e^+e^- \to t\bar{t}$ @ ILC
  - QCD dynamics understood (NNLL RGE-improved, N^{3}LO fixed-order)
  - effects of ew. corr.'s and finite top lifetime $\to$ w.i.p.
  - top mass: $\delta m_t^{1S} \simeq (\pm 100~\text{GeV})^{\rm theo} + (\pm 50~\text{GeV})^{\rm exp}$ ✓
  - λ_t, Γ_t: $\to d\sigma/\sigma \approx \pm 6\%$ goal: 3%
- top threshold essential for $\sigma(e^+e^- \to t\bar{t}H)$ @ 500 GeV $\to \lambda_t$
- Other applications:
  - ⋆ $t\bar{t}$ threshold at Tevtraon/LHC (presumably small, but maybe important)
  - ⋆ squark pair production ILC+LHC (particularly for heavy squarks)
  - ⋆ particle decay rates: $H \to gg$ ($m_h \approx 2m_t$)

    $X \to t\bar{t} + Y$

Colors

This is blue

This is red

This is brown

This is magenta

This is Dark Green

This is Dark Blue

This is Green

This is Cyan

Test how this color looks

Test how this color looks

Test how this color looks

Test how this color looks

Test how this color looks