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A theoretical update on $t\bar{t}$ pair production near threshold

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BASED ON WORK DONE IN COLLABORATION WITH

M. BENEKE, S. CHAPOVSKY, A. PINEDA, V. SMIRNOV, G. ZANDERIGHI

outline

Current Status

- “fixed order” top threshold scan
- resummation of $\ln v$

A “NNLL” Computation

with A.Pineda

- outline of calculation
- QED effects
- theoretical error
- future improvements

Unstable Particles

with M.Beneke, S.Chapovsky, G.Zanderighi

- another effective theory
- toy model
- towards pair production near threshold

Conclusions

threshold scan

$t\bar{t}$ near threshold: $E = \sqrt{s} - 2m \sim mv^2 \sim m\alpha_s^2$

Problem with three scales: $m; \ \vec{p} \sim mv \sim m\alpha_s; \ E \sim mv^2 \sim m\alpha_s^2$

Hierarchy of scales: $m \gg mv \gg mv^2 \gg \Lambda_{\rm QCD}$

fixed order:

$$\sigma(= R) = v \sum_n \left(\frac{\alpha_s}{v}\right)^n \times \left\{1 \ (LO); \alpha_s, v \ (NLO); \alpha_s^2, v^2, \alpha_s v \ (NNLO)\right\}$$

resummed:

$$\sigma = v \sum_n \left(\frac{\alpha_s}{v}\right)^n \sum_l (\alpha_s \ln v)^l \times \left\{1 \ (LL); \alpha_s, v \ (NLL); \alpha_s^2, v^2, \alpha_s v \ (NNLL)\right\}$$

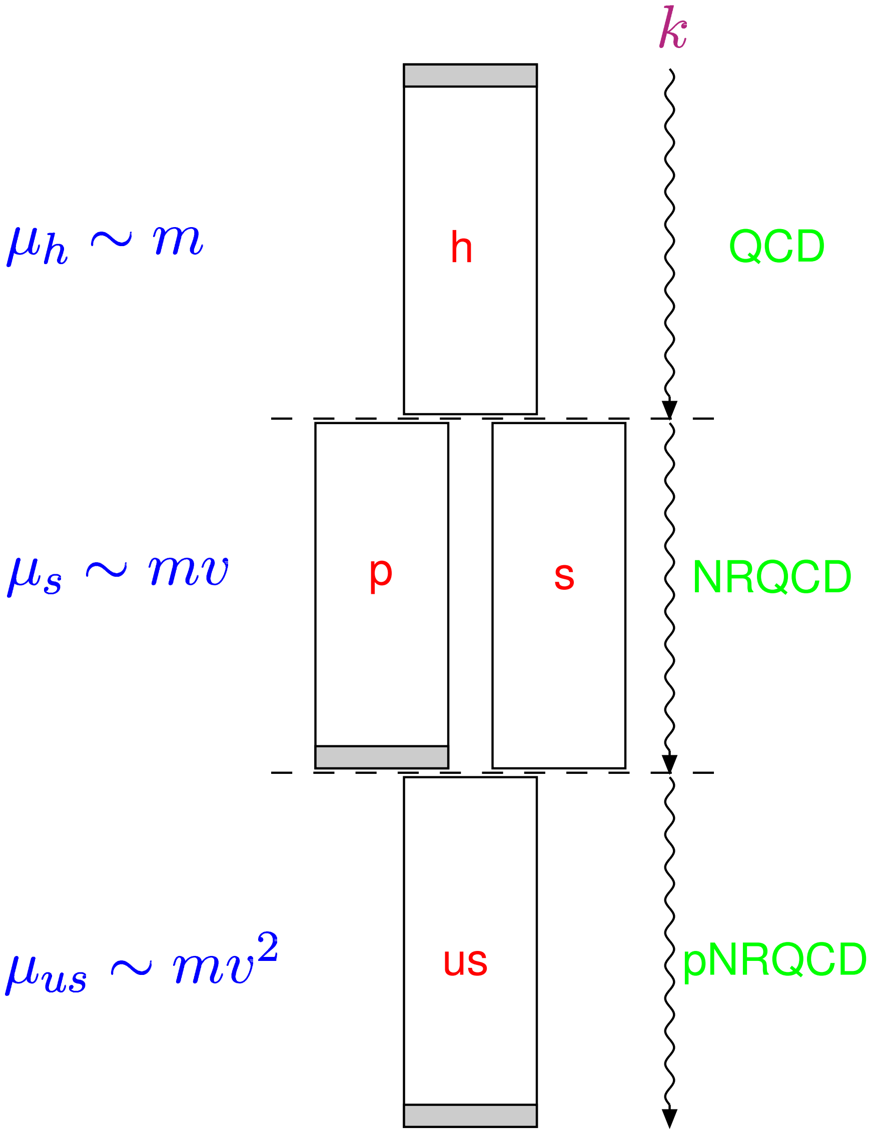
fixed order

- exploit $\alpha_s \ll 1$ and $v \ll 1$ $\to$ double expansion
- identify modes [Beneke, Smirnov] $\Rightarrow$ asymptotic expansion (method of regions)

hard	$p^\mu \sim m$
soft	$p^\mu \sim mv$
potential	$p^0 \sim mv^2$; $\vec{p} \sim mv$
ultrasoft	$p^\mu \sim mv^2$

- integrate out 'unwanted' modes (final state described by potential quarks and ultrasoft gluons):
  QCD (h,s,p,u) $\longrightarrow$ NRQCD (s,p,u)$\longrightarrow$ pNRQCD (p$|_q$,u)
- matching of currents
- done to NNLO [Beneke et.al; Hoang et.al; Melnikov et.al; Yakovlev; . . .]
- use threshold mass, not pole mass [Bigi et.al; Beneke; Hoang et.al; Pineda]

fixed order



In pNRQCD: Schrödinger equation:

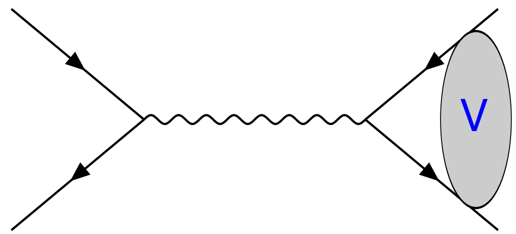
$$\delta(\vec{r}-\vec{r}\,') =$$

$$\left(-\frac{\vec{\partial}^2}{m} + V(\vec{r}-\vec{r}\,') - \bar{E}\right) G(\vec{r},\vec{r}\,',\bar{E})$$

where

$$V(r) = -\frac{C_F \alpha_s}{r} + \ldots$$

For $t\bar{t}$ production: $\bar{E} = E + i\Gamma_t$
not correct at NNLO [Fadin, Khoze]



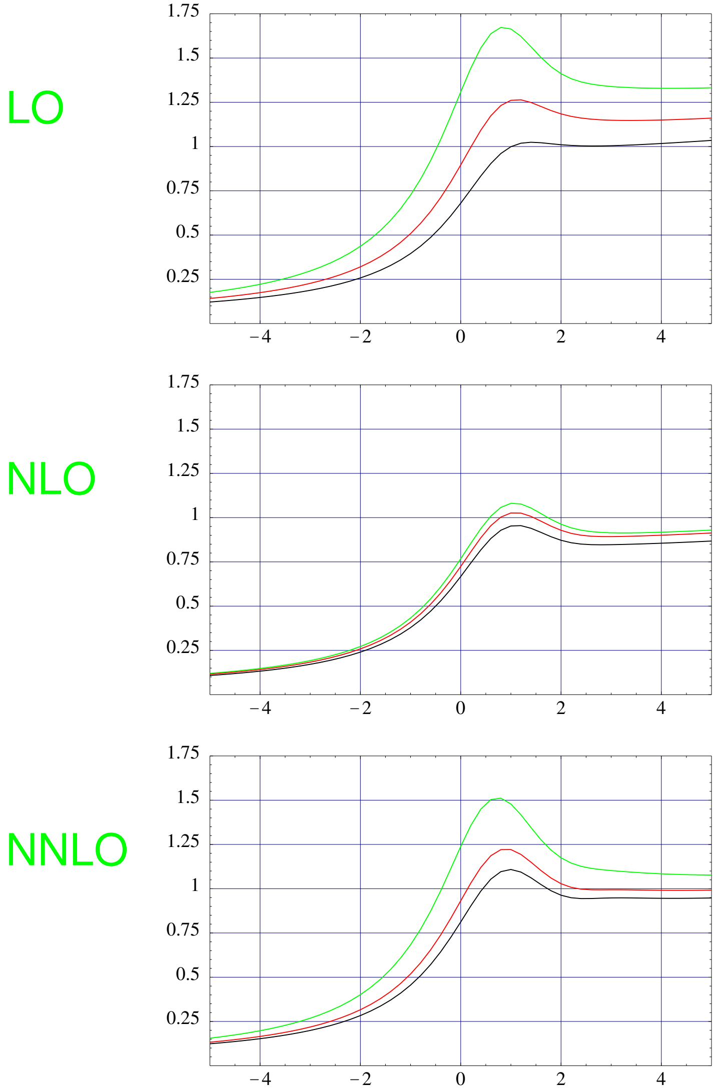
fixed order

→ stabilization of peak position
→ no improvement in normalization

here: PS-mass

$$m_{PS}(\mu_F) \equiv m + \frac{1}{2}\int^{\mu_F} \frac{d^3\vec{q}}{(2\pi)^3} V_{\text{Coul}}$$

parameters:
$m_t = 175$ GeV,
$\Gamma_t = 1.40$ GeV,
$\alpha_s(M_Z) = 0.118$,
scale of α_s: 15, 30, 60 GeV
$\mu_F = 20$ GeV

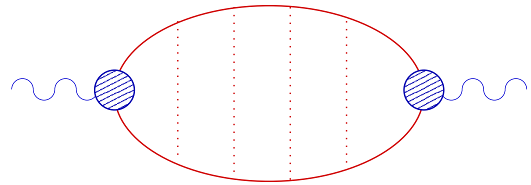


$E \equiv \sqrt{s} - 2m_{\text{PS}}$ in GeV

resummation of $\ln v$

- resummation of $\ln v$ is mandatory for satisfactory description of normalization of cross section
- done at “NNLL” using vNRQCD [Hoang, Manohar, Stewart, Teubner]
- vNRQCD fixes the correlation between the scales from the start $\mu_{us} = \mu_s^2/m$
- here I present an alternative evaluation of R_{NNLL}, keeping the two-stage matching (work with A.Pineda):
  - QCD $\to$ NRQCD, RGI coefficients for R_{NNLL} known [Bauer, Manohar; Pineda]
  - NRQCD $\to$ pNRQCD, RGI coefficients for R_{NNLL} known [Pineda]
  - the correlation between the scales is taken into account in the RG solutions
  - matching of current: known at NLL, but not yet fully at NNLL [Manohar, Stewart, Hoang; Pineda]
  - potential loops: higher dim operators mix back into current matching coeff. $\Rightarrow$ have to take into account the NLL running of NRQCD operators in RGE.
  - done (so far) only for the spin dependent term [Penin, Pineda, Steinhauser, Smirnov], thus NNLL $\Rightarrow$ “NNLL”

We use dimensional regularization throughout, perform all calculations in momentum space and always use $\overline{\mathrm{MS}}$-subtraction [Beneke, AS, Smirnov]



$$G_c(\vec{r},\vec{r}',E)\Big|_{\vec{r}=\vec{r}'=0} \equiv \int \frac{d^d\vec{p}}{(2\pi)^d}\frac{d^d\vec{p}'}{(2\pi)^d}\,\tilde{G}_c(\vec{p},\vec{p}',E)$$

$$\tilde{G}_c(\vec{p},\vec{p}',E) = (2\pi)^d\delta^{(d)}(\vec{p}-\vec{p}')\,\frac{-1}{E-\vec{p}^2/m} + \frac{4\pi C_F\alpha_s}{(E-\vec{p}^2/m)\,(\vec{p}-\vec{p}')^2\,(E-\vec{p}'^{\,2}/m)} + \text{finite}$$

$$G_c(0,0,E) = -\frac{\alpha_s\,C_F\,m^2}{4\pi}\left(\frac{1}{2\lambda}+\frac{1}{2}\ln\frac{-4mE}{\mu^2}-\frac{1}{2}+\gamma_E+\psi(1-\lambda)\right)$$

where $\lambda \equiv C_F\,\alpha_s/(2\sqrt{-E/m})$; This sums all potential gluon (ladder) diagrams

require "D-dim" operators e.g. $\frac{\mathbf{L}^2}{2\pi r^3} \to \left(\frac{\vec{p}^2-\vec{p}'^{\,2}}{\vec{q}^2}\right)^2 - 1$ and $\sigma^i B^i \equiv \frac{i}{4}[\sigma^i,\sigma^j]G^{ij}$

The renormalization group improved pNRQCD potential: [Pineda, AS]

$$
\begin{aligned}
V_{NNLL} \quad = \quad & -4\pi C_F \frac{\alpha_{\tilde{V}_s}}{\vec{q}^2} \\
& - C_F C_A D_s^{(1)} \frac{\pi^2}{m q^{1+2\epsilon}} (1-\epsilon) \frac{(4\pi)^\epsilon \Gamma^2(\frac{1}{2}-\epsilon)\Gamma(\frac{1}{2}+\epsilon)}{\pi^{3/2}\Gamma^2(1-2\epsilon)} \\
& - \frac{2\pi C_F D_{1,s}^{(2)}}{m^2} \frac{\vec{p}^2 + \vec{p}'^{\,2}}{\vec{q}^2} + \frac{\pi C_F D_{2,s}^{(2)}}{m^2} \left(\left(\frac{\vec{p}^2 - \vec{p}'^{\,2}}{\vec{q}^2} \right)^2 - 1 \right) \\
& + \frac{3\pi C_F D_{d,s}^{(2)}}{m^2} - \frac{4\pi C_F D_{S^2,s}^{(2)}}{d\, m^2} [\mathbf{S}_1^i, \mathbf{S}_1^j][\mathbf{S}_2^i, \mathbf{S}_2^j] \\
& + \frac{4\pi C_F D_{S_{12},s}^{(2)}}{d\, m^2} [\mathbf{S}_1^i, \mathbf{S}_1^r][\mathbf{S}_2^i, \mathbf{S}_2^j] \left(\delta^{rj} - d\, \frac{\mathbf{q}^r \mathbf{q}^j}{\mathbf{q}^2} \right) \\
& - \frac{6\pi C_F D_{LS,s}^{(2)}}{m^2} \frac{\mathbf{p}^i \mathbf{q}^j}{\mathbf{k}^2} \left([\mathbf{S}_1^i, \mathbf{S}_1^j] + [\mathbf{S}_2^i, \mathbf{S}_2^j] \right)
\end{aligned}
$$

and the current: (Z exchange not included)

$$\bar{Q}\gamma^{\mu}Q(0) = c_1\chi^{\dagger}\sigma^{i}\psi - \frac{c_2}{6m^2}\chi^{\dagger}\sigma^{i}\left(i\mathbf{D}\right)^2\psi + \ldots$$

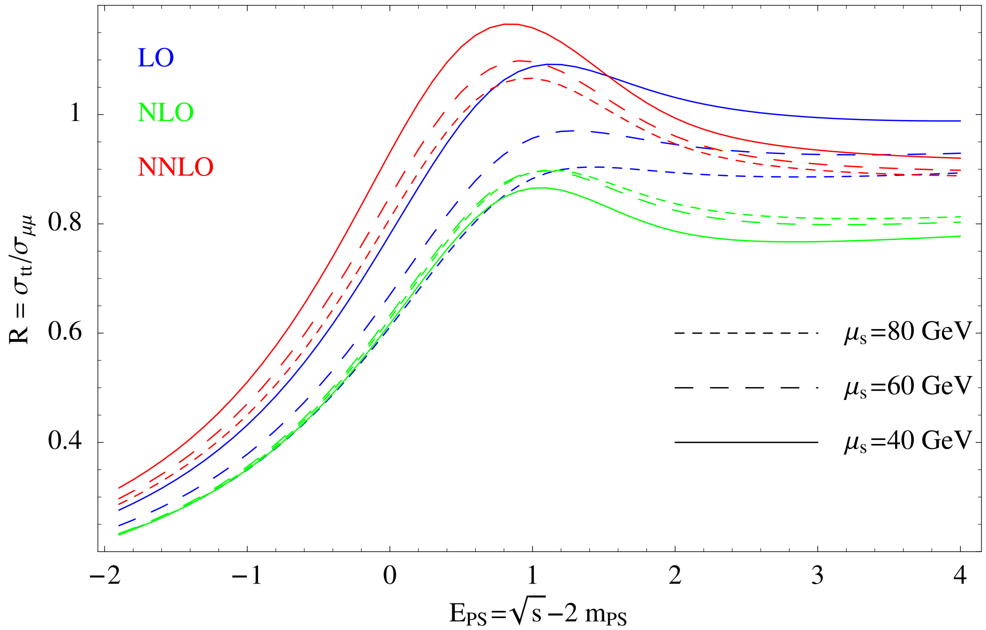
evaluate insertions [Beneke, AS, Smirnov]

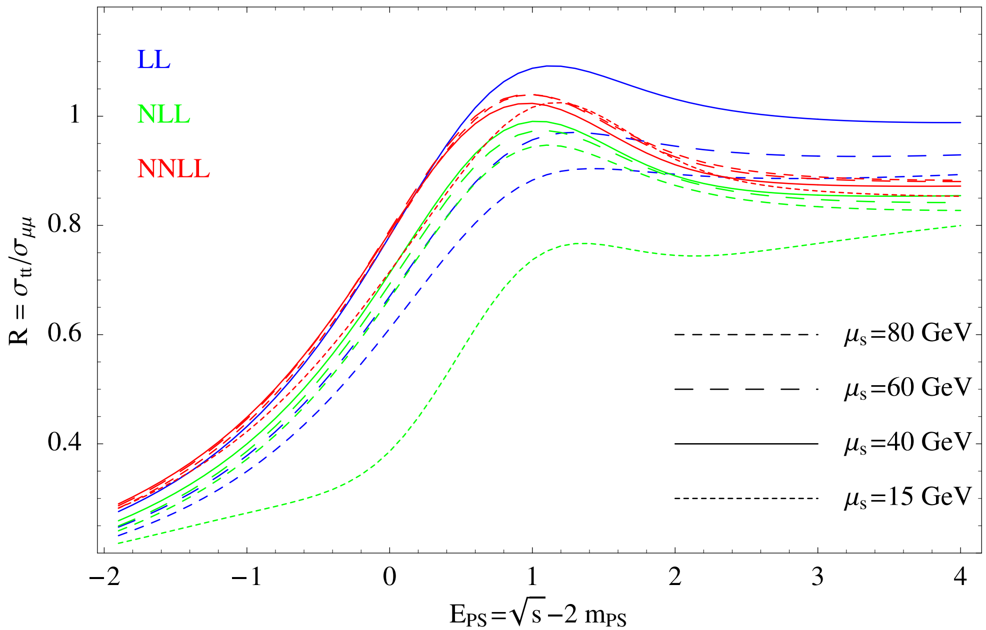
$$\delta G_c(0,0,E) = \int \prod \frac{d^d\vec{p}_i}{(2\pi)^d}\,\tilde{G}_c(\vec{p}_1,\vec{p}_2,E)\delta V(\vec{p}_2,\vec{p}_3)\tilde{G}_c(\vec{p}_3,\vec{p}_4,E)$$

include (trivial) QED corrections $\alpha \sim \alpha_s^2$

- NLO: $V \to V - 4\pi\alpha e_q^2/q^2$ single potential photon exchange suppressed by $\alpha/v \sim \alpha_s^2/v \sim v$
- NNLO: $c_1 \to c_1 - 2e_q^2\alpha/\pi$
- NNLO: double potential photon exchange $(\alpha/v)^2 \sim v^2$

“nnll”



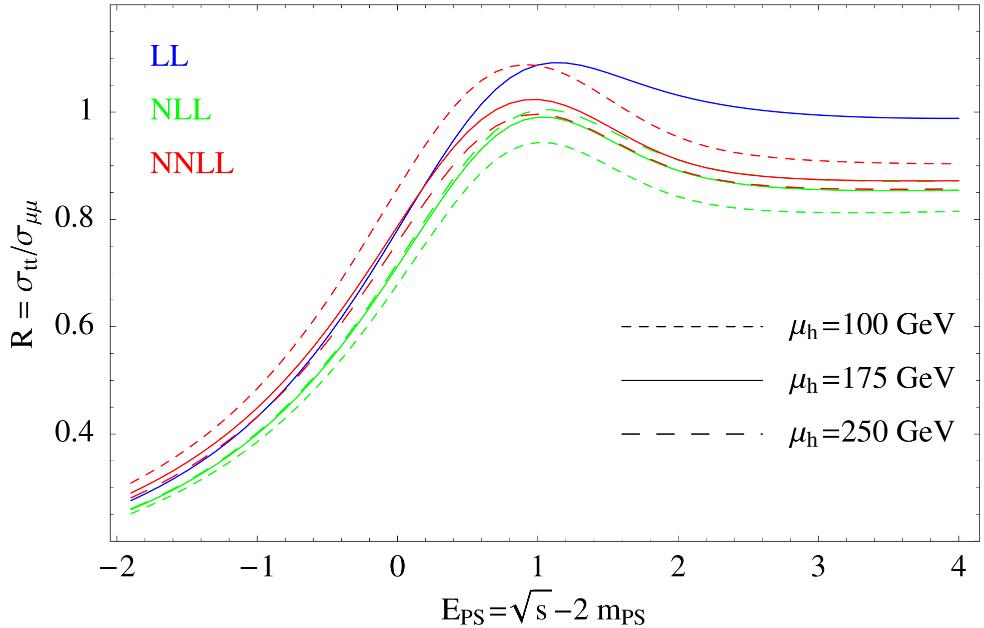


variation of soft scale:

$$\mu_s \sim 2(m\sqrt{E^2+\Gamma_t^2})^{1/2}$$
$$m_{PS} = 175\ \text{GeV}$$
$$\Gamma_t = 1.4\ \text{GeV}$$
$$\mu_F = 20\ \text{GeV}$$

- big improvement, as seen previously [Hoang, Manohar, Stewart, Teubner]
- problem for small scales only due to missing out multiple insertions → can be recified
- NLL and NNLL bands do not overlap
- this scale dependence is not a reliable estimate for the theoretical error

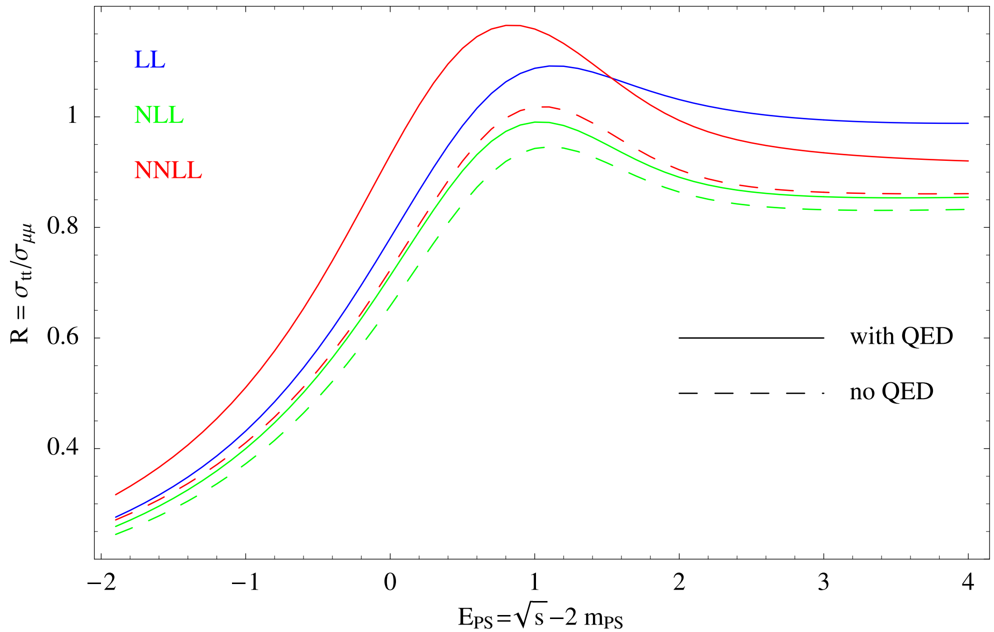
“nnll”



variation of hard scale:

$$\mu_h \quad \sim \quad m$$

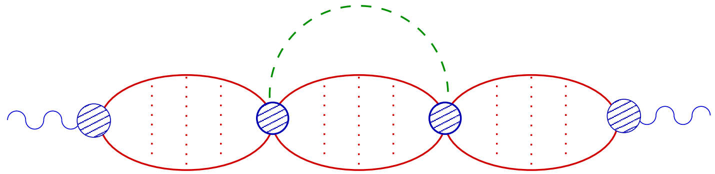
- larger than soft scale dependence
- NLL and NNLL bands now do overlap



QED corrections:

- shift $\delta m_t \sim 300$ MeV
- at this level even small corrections are important

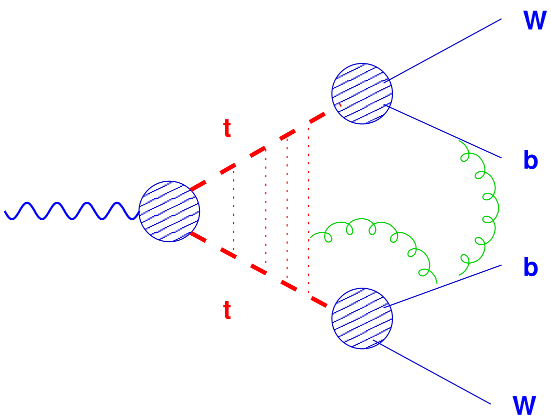
future improvements



- needed: c_1 fully NNLL
- ultrasoft effects (retardation effects)
  - due to chromoelectric dipole operator $\vec{x} \cdot \vec{E}$
  - NNNLO effects α_s^3 (NNLL part $\alpha_s^3 \ln \alpha_s$ already included)
  - potentially particularly important: $\alpha_s^3 \sim \alpha_s^2 \alpha_s(\mu_{us})$
- full NNNLO ......
- effects due to the instability of top
  - electroweak effects are important at this level $\alpha_{ew} \sim \alpha_s^2$
  - $E \to E + i\Gamma_t$ is not correct at NNLO
  - the whole concept of $\sigma(e^+e^- \to t\bar{t})$ breaks down
- more exclusive quantities

unstable particles

- Strictly speaking, it does not make sense to talk about $\sigma(e^+e^- \to t\bar{t})$ (or any cross section with an unstable particle in the final state).
- for threshold scan, $\delta m_t \ll \Gamma_t$, thus $\sigma(e^+e^- \to t\bar{t}) \to \sigma(e^+e^- \to W^+W^-b\bar{b}) \to \sigma(e^+e^- \to f_1 f_2 f_3 f_4 b\bar{b})$
- QCD and electroweak effects



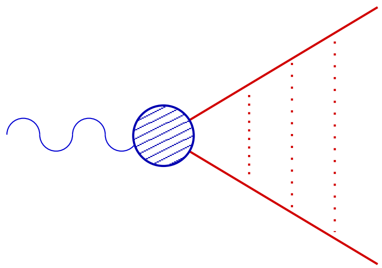
plus QED radiation from all charged particles (also incoming e^+e^-)

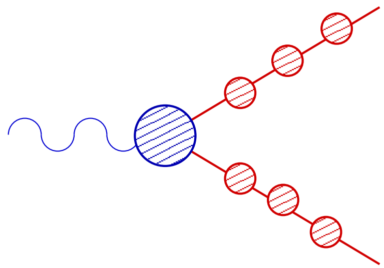
- electroweak effects are important! partially computed [Hoang, Reisser]

basic problem

- top quark propagator $(E - \frac{\vec{p}^2}{2m_t})^{-1}$ scales as $\frac{1}{mv^2} \sim \frac{1}{m\alpha_s^2} \sim \frac{1}{m\alpha_{ew}}$
- the width $\Gamma_t \sim m\alpha_{ew}$ is a LO effect [Fadin, Khoze]

$$\frac{1}{E - \frac{\vec{p}^2}{2m_t}} \rightarrow \frac{1}{E - \frac{\vec{p}^2}{2m_t} + i\Gamma_t}$$





Coulomb singularity $v \to 0$

resum $(\alpha_s/v)^n$ (potential gluon exchange)

systematic expansion in α and v

propagator pole $\Gamma \to 0$

resum $(\Gamma/m)^n$ (self-energy insertions)

systematic expansion in α and Γ

basic solution

- use effective theory methods (again!) to systematically expand in Γ/m [Chapovsky, Khoze, AS, Stirling]
- identify relevant modes (depends on details of observable) $\rightarrow$ asymptotic expansion [Beneke, Chapovsky, AS, Zanderighi]
- integrate out ‘unwanted’ modes $\rightarrow$ tower of effective theories (Unstable Particle Effective Theory)
- hard effects correspond to factorizable corrections
- non-factorizable corrections due to still dynamical modes
- this is neither a “quick-fix” nor a “free lunch”, it is a method to identify the minimal amount of calculation to be done for a systematic expansion in the small parameters (as for NRQCD)
- gauge invariance is automatic since the split into the various contributions respects gauge invariance

toy model

- Lagrangian:

$$\mathcal{L} = (D_\mu \phi)^\dagger D^\mu \phi - \hat{M}^2 \phi^\dagger \phi + \overline{\psi} i \not{D} \psi + \overline{\chi} i \not{\partial} \chi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$
$$- \frac{1}{2\xi} (\partial_\mu A^\mu)^2 + y \phi \overline{\psi} \chi + y^* \phi^\dagger \overline{\chi} \psi - \frac{\lambda}{4} \left(\phi^\dagger \phi \right)^2 - \mathcal{L}_{\rm ct}$$

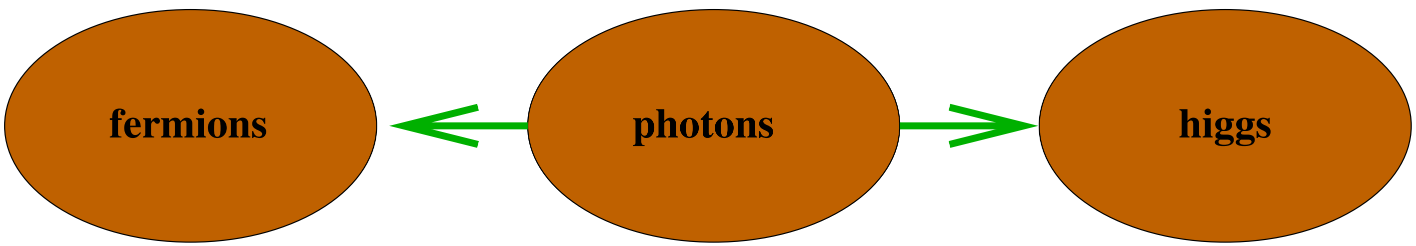
- Process:

$$\bar{\nu}(p_1) e^-(p_2) \to \phi \to X$$

  with $s - \hat{M}^2 \sim M\Gamma$. Use optical theorem and compute $\mathrm{Im}\, \mathcal{T}$
- scales: decay time $1/M$, lifetime $1/\Gamma \gg 1/M$
- expand in α and $\delta \equiv (s - \hat{M}^2)/\hat{M}^2 \sim \Gamma/M \sim \alpha$
- fermions: SCET; scalar (higgs): H"Q"ET

modes

Soft-Collinear Effective Theory	+	Heavy "Quark" Effective Theory
fermions		higgs
$p^\mu = (n_+p)\frac{n_-}{2} + (n_-p)\frac{n_+}{2} + p_\perp$		$q^\mu = Mv^\mu + k^\mu; \quad q_\top = q^\mu - v^\mu(qv)$
$n_\pm^2 = 0, \quad n_+n_- = 2$		$v^\mu \equiv (q_1^\mu + q_2^\mu)/\sqrt{s}, \quad v^2 = 1$
hard: $p \sim M$		hard: $k^\mu \sim M$
(u)soft: $p \sim M\delta$		soft: $k^\mu \sim \delta$
collinear: $p_\perp \sim M\delta^{1/2};\ n_+p \sim M;\ n_-p \sim M\delta$		



effective theory

underlying theory

$\mathcal{L}(\phi_h, \phi_c, \phi_s)$

dynamical modes:

hard, collinear, soft

integrate out hard modes

factorizable corrections

non-factorizable corrections

effective theory

$\mathcal{L} = \sum_n c_n(\mathrm{h})\, O_n(\phi_c, \phi_s)$

dynamical modes:

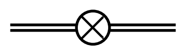
collinear, soft

The effective Lagrangian for the NLO line shape:

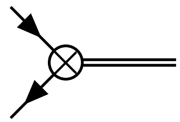
$$
\begin{aligned}
\mathcal{L}_{\rm eff} \;=\; & -\frac{1}{4} F_{\rm s}^{\mu\nu} F_{{\rm s}\mu\nu} + 2\hat{M}\,\phi_v^\dagger \left(i(vD_{\rm s}) - \frac{\Delta}{2} \right) \phi_v + 2\hat{M}\,\phi_v^\dagger \left(\frac{iD^2_{{\rm s}\top}}{2\hat{M}} + \frac{\Delta^2}{8\hat{M}} \right) \phi_v \\
& + \bar{\psi}_{\rm s} i \not{D}_{\rm s} \psi_{\rm s} + \bar{\chi}_{\rm s} i \not{\partial} \chi_{\rm s} + \bar{\psi}_{n-} \left(in_- D + \not{D}_{c\top} \frac{i}{n_+ D_c} \not{D}_{c\top} \right) \psi_{n-} \\
& + C \left(y\,\phi_v \bar{\psi}_{n-} \chi_{n+} + y^*\,\phi_v^\dagger \bar{\chi}_{n+} \psi_{n-} \right) + \frac{yy^* B}{4\hat{M}^2} \left(\bar{\psi}_{n-} \chi_{n+} \right) \left(\bar{\chi}_{n+} \psi_{n-} \right) + \ldots
\end{aligned}
$$

Matching coefficients (contain hard effects)

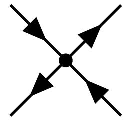
- $\Delta \equiv (\bar{s} - \hat{M}^2)/\hat{M} = \alpha\,\Delta^{(1)} + \alpha^2 \Delta^{(2)} + \ldots$
  In the pole scheme: $\Delta = -i\Gamma$

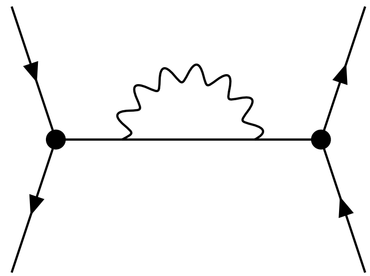


- $C = 1 + \alpha\, C^{(1)} + \ldots$



- $B = 1 + \alpha B^{(1)} + \ldots$



Consider self-energy diagrams  + higher orders

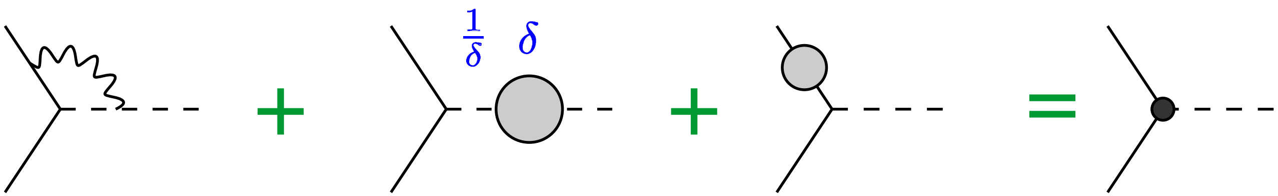
split self-energy into hard and soft part $\Pi(s) = \Pi_h(s) + \Pi_s(s)$ and expand the hard part of the self energy $\Pi_h(s) = \hat{M}^2 \sum \alpha^k \delta^l \Pi^{(k,l)}$

- $\Pi^{(1,0)}$ (gauge independent) $\rightarrow$ $\Delta^{(1)}$ (LO, Propagator)
- $\Pi^{(1,1)}$ (gauge dependent) $\rightarrow$ $C^{(1)}$ (NLO)
- $\Pi^{(1,2)}$ (gauge dependent) $\rightarrow$ $B^{(1)}$ (NNLO)
- $\Pi^{(2,0)}$ and $\Pi^{(1,0)}\Pi^{(1,1)}$ (separately gauge dependent) $\rightarrow$ $\Delta^{(2)}$ (NLO, gauge independent)
- Π_s (gauge dependent) $\rightarrow$ diagram in effective theory (NLO)

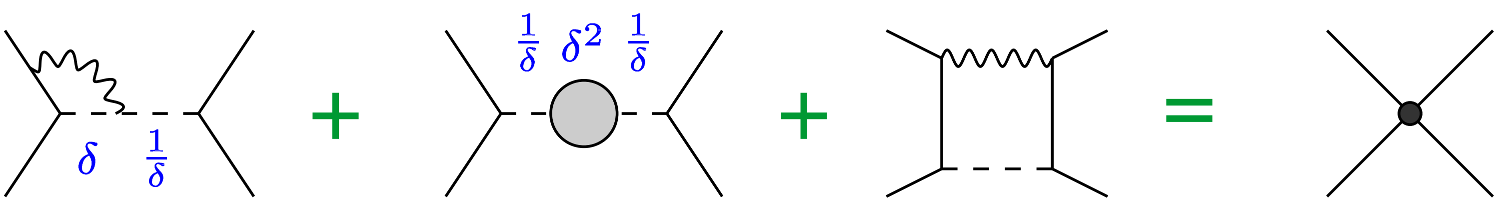
matching

Matching of C (in $\overline{\rm MS}$ scheme)

$$C = 1 + \frac{\alpha_y}{4\pi}\left[\ln\frac{M^2}{\mu^2} - \frac{1}{4} - \frac{i\pi}{2}\right] +$$
$$\frac{\alpha_g}{4\pi}\left[-\frac{1}{\bar{\epsilon}^2} + \frac{1}{\bar{\epsilon}}\left(\ln\frac{M^2}{\mu^2} - \frac{5}{2}\right) - \frac{1}{2}\ln^2\frac{M^2}{\mu^2} + \frac{7}{4}\ln\frac{M^2}{\mu^2} - \frac{15}{4} - \frac{\pi^2}{12}\right]$$



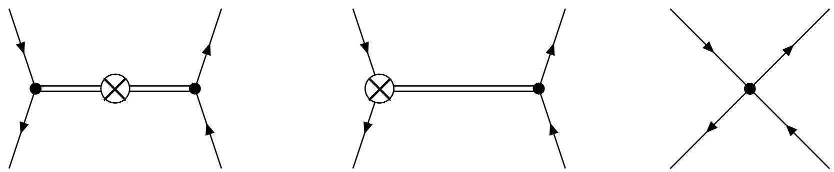
Matching of B at order α (contributes at NNLO)



gauge dependence cancels

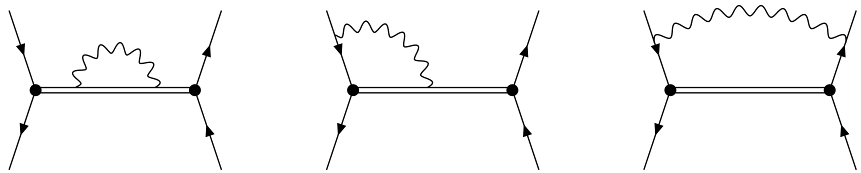
results

Forward scattering amplitude at NLO:



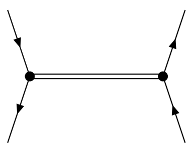
$$i\,\mathcal{T}_h^{(1)} = i\,\mathcal{T}^{(0)} \times$$

$$\left(2\,C^{(1)} - \frac{[\Delta^{(1)}]^2}{8\mathcal{D}\hat{M}} + \frac{\Delta^{(2)}}{2\mathcal{D}} - \frac{\mathcal{D}}{2\hat{M}}\right)$$



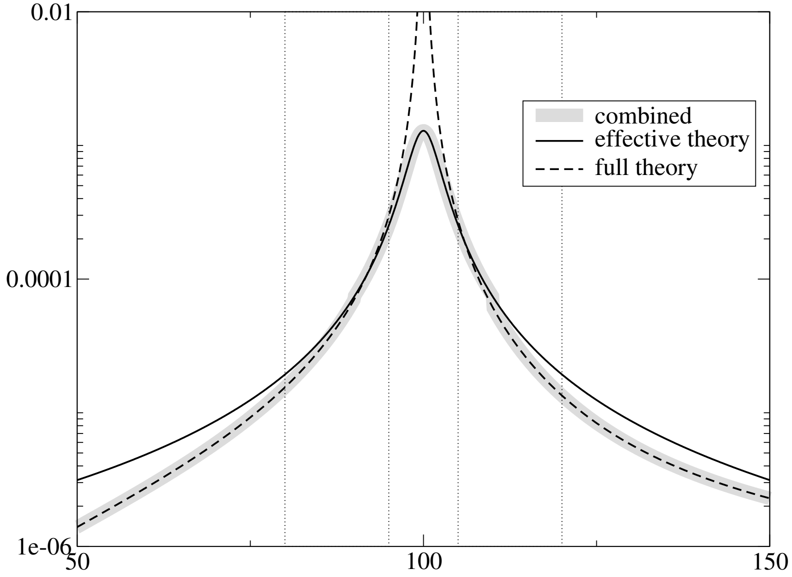
$$i\,\mathcal{T}_s^{(1)} = i\,\mathcal{T}^{(0)}\,\frac{\alpha_g}{4\pi}\left(\frac{-2\mathcal{D}}{\mu}\right)^{-2\epsilon} \times$$

$$\left(\frac{2}{\epsilon^2} + \frac{2}{\epsilon} + 4 + \frac{5\pi^2}{6}\right)$$

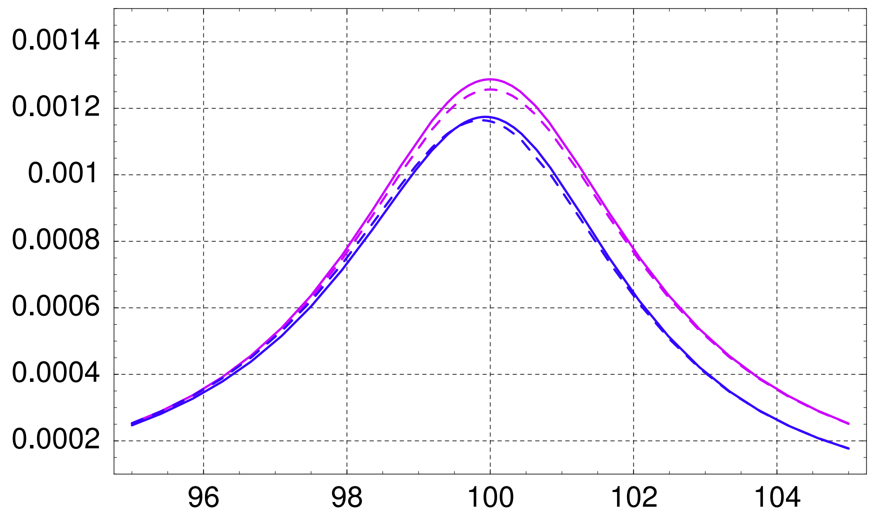
where  $= i\,\mathcal{T}^{(0)} = \frac{-yy^* s}{4\hat{M}\mathcal{D}}$ with $\mathcal{D} \equiv \sqrt{s} - \hat{M} - \frac{\Delta^{(1)}}{2}$

poles $1/\epsilon$ cancel when adding soft and hard contributions (up to initial state collinear singularity)

Partonic cross section for $M = 100$ GeV as a function of $\sqrt{s}$.



full range of $\sqrt{s}$: matching of resonant to off-resonant cross section



resonant region: LO vs. NLO for pole and $\overline{\mathrm{MS}}$ scheme

towards $x\bar{x}$ near threshold

- more realistic processes:
  - higgs $\to$ fermion (t) : H"Q"ET $\to$ HQET
  - higgs $\to$ gauge boson (W, Z) . With $p^\mu = Mv^\mu + k^\mu$ we get $p^2 - \xi M^2 = (1-\xi)M^2 + 2M(vk) + k^2$ and the propagator:

$$\frac{i}{p^2 - M^2}\left(-g^{\mu\nu} + (1-\xi)\frac{p^\mu p^\nu}{p^2 - \xi M^2}\right) \to \frac{i}{2M(vk)}\left(-g^{\mu\nu} + v^\mu v^\nu\right)$$

  massive field, 3 polarizations, gauge invariant [Beneke, Kauer, AS, Zanderighi]
- pair production near threshold $t\bar{t};\ W^+W^-$: HQET $\to$ NRQ(C/E)D. Due to potential gluons/photons $(vk) \sim k^2$ since $k^\mu_{\rm pot} \sim (M\nu^2, m\vec{\nu})$
- additional operators, for $t\bar{t}$ e.g. $\bar{e}_L e_L\, \chi^\dagger \psi$
- more exclusive final states: expand also phase-space integrals
- resummation of $\log(\Gamma/M)$ via standard RGE techniques

conclusions

- the theory for $t\bar{t}$ production near threshold is in good shape
- but the usual statement $\delta m_t \sim 100\mathrm{MeV}$ for ILC relies on further theoretical progress (and the patience to actually do a threshold scan!!)
  - full NNLL !!
  - at least ultrasoft (if not full) NNNLO
  - fully take into account instability of top quark
- more exclusive final states ?
- tools are set up, but a lot of (tedious) additional work required
- this is one of the rare problems that is very fascinating from a theoretical point of view and extremely relevant from an experimental point of view