Bottom quark fragmentation in top quark decay

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1. b-quark spectrum in top quark decay at NLO
2. Collinear and soft resummations
3. Monte Carlo event generators (HERWIG and PYTHIA)
4. Fits of hadronization models to LEP and SLD data
5. B-hadron spectrum in top decay
6. Conclusions

G.C. and A. Mitov, NPB 623 (2002) 247; 676 (2004) 346; M. Cacciari, G.C. and A. Mitov, JHEP 0212 (2002) 015; G.C. and V. Drollinger, NPB 730 (2005) 82

Reliable description of multiple radiation in top production and decay and of b-quark fragmentation is fundamental to perform trustworthy measurements of top properties, e.g. m_t

Monte Carlo event generators (HERWIG/PYTHIA) widely used to simulate top production and decay and b hadronization

Tevatron:

b-fragmentation contributes to Monte Carlo systematics in m_t measurement

LHC:

$J/\psi +$ lepton final states (10^3/year of high luminosity)

$t \to bW$ **;** $b \to B \to J/\psi\ X$ **;** $J/\psi \to \mu^+\mu^-$ **;** $W \to \ell\nu_\ell$

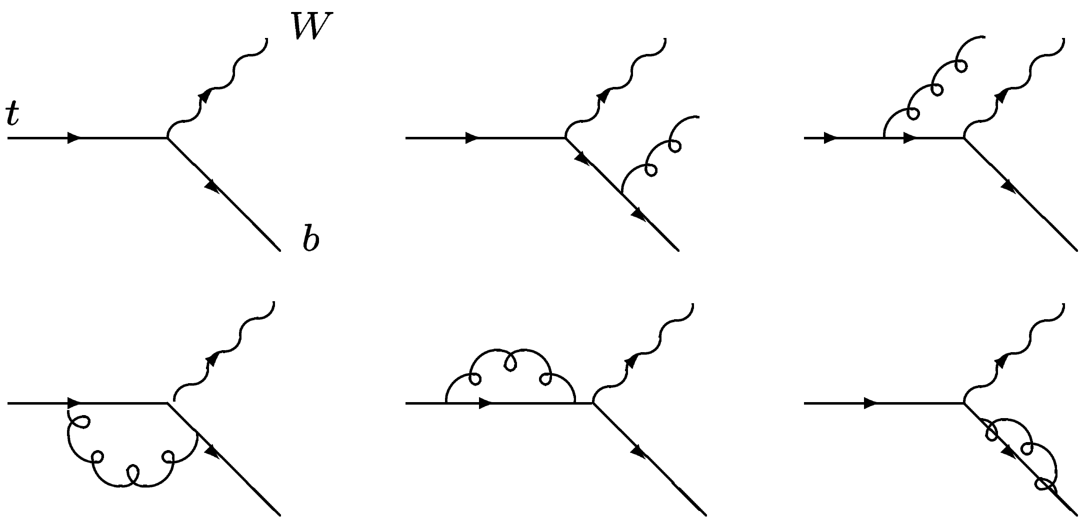
A. Kharchilava, PLB 476 (2000) 73 **(PYTHIA)**

$m_{\ell J/\psi} = 0.51\ m_t - 23$ GeV

$\Delta m_{\ell J/\psi} \simeq 0.5$ GeV $\Rightarrow \Delta m_t \simeq 1$ GeV

b-fragmentation: $\Delta m_t(\mathrm{frag}) \simeq 0.6$ **GeV**

Top decay at NLO



$$t(q) \rightarrow b(p_b)W(p_W)\,\left(g(p_g)\right)$$

$$x_b = \frac{1}{1-m_W^2/m_t^2}\,\frac{2p_b\cdot p_t}{m_t^2}$$

$$\frac{1}{\Gamma_0}\frac{d\Gamma}{dx_b} = \delta(1-x_b) + \frac{\alpha_S(\mu)}{2\pi}\left[P_{qq}(x_b)\ln\frac{m_t^2}{m_b^2} + A(x_b)\right] + \mathcal{O}\left[\left(\frac{m_b}{m_t}\right)^p\right]$$

$$P_{qq}(x_b) = C_F\left(\frac{1+x_b^2}{1-x_b}\right)_+ \qquad \alpha_S(\mu)\ln\frac{m_t^2}{m_b^2} \simeq \mathcal{O}(1)$$

Perturbative fragmentation functions

B. Mele and P. Nason, NPB 361 (1991) 626

$$\frac{1}{\Gamma_0}\frac{d\Gamma}{dx_b}(x_b, m_b \neq 0) = \frac{1}{\Gamma_0}\sum_i \int_{x_b}^1 \frac{dz}{z}\frac{d\hat{\Gamma}_i}{dz}(z, m_i = 0, \mu_F) D_i\left(\frac{x_b}{z}, \mu_F, m_b\right) + \mathcal{O}\left[\left(\frac{m_b}{m_t}\right)^p\right]$$

$D_i(x_b, \mu_F, m_b)$: **perturbative fragmentation function (PFF)**

$$\frac{1}{\Gamma_0}\frac{d\hat{\Gamma}_b}{dz} = \delta(1-z) + \frac{\alpha_S(\mu)}{2\pi}\left[P_{qq}(z)\left(-\frac{1}{\epsilon} + \gamma_E - \ln 4\pi\right) + \hat{A}(z)\right]$$

$\overline{\mathrm{MS}}$ **coefficient function:**

$$\left(\frac{1}{\Gamma_0}\frac{d\hat{\Gamma}_b}{dz}\right)^{\overline{\mathrm{MS}}} = \delta(1-z) + \frac{\alpha_S(\mu)}{2\pi}\hat{A}(z)$$

$$\frac{1}{\Gamma_0}\frac{d\Gamma}{dx_b}(m_b) = \left(\frac{1}{\Gamma_0}\frac{d\hat{\Gamma}_b}{dx_b}(m_b = 0)\right)^{\overline{\mathrm{MS}}} \otimes D_b^{\overline{\mathrm{MS}}}(m_b)$$

DGLAP equations for PFF's:

$$\frac{d}{d\ln\mu_F^2}D_i(x_b,\mu_F,m_b)=\sum_j\int_{x_b}^1\frac{dz}{z}P_{ij}\left(\frac{x_b}{z},\alpha_S(\mu_F)\right)D_j(z,\mu_F,m_b)$$

Initial condition $D(x_b,\mu_{0F})$ is process-independent:

$$D_b(x_b,\mu_{0F},m_b)=\delta(1-x_b)+\frac{\alpha_S(\mu_0)C_F}{2\pi}\left[\frac{1+x_b^2}{1-x_b}\left(\ln\frac{\mu_{0F}^2}{m_b^2}-2\ln(1-x_b)-1\right)\right]_+$$

$$\frac{dD_N(\mu_F,m_b)}{d\ln\mu_F^2}=\frac{\alpha_S(\mu_F)}{2\pi}\left[P_N^{(0)}+\frac{\alpha_S(\mu_F)}{2\pi}P_N^{(1)}\right]D_N(\mu_F,m_b)$$

$$D_N(\mu_F,m_b)=D_N(\mu_{0F},m_b)\exp\left\{\frac{P_N^{(0)}}{2\pi b_0}\ln\frac{\alpha_S(\mu_{0F})}{\alpha_S(\mu_F)}+\frac{\alpha_S(\mu_{0F})-\alpha_S(\mu_F)}{4\pi^2 b_0}\left[P_N^{(1)}-\frac{2\pi b_1}{b_0}P_N^{(0)}\right]\right\}$$

$$\begin{aligned}D_N(\mu_F,m_b) &= D_N(\mu_{0F},m_b)\exp\Big\{C_{1,0}\alpha_S(\mu_F)+C_{1,1}\alpha_S(\mu_F)\ln(\mu_F^2/\mu_{0F}^2)\dots\\ &+ C_{n,n-1}\alpha_S^n(\mu_F)\ln^{n-1}(\mu_F^2/\mu_{0F}^2)+C_{n,n}\alpha_S^n(\mu_F)\ln^n(\mu_F^2/\mu_{0F}^2)+\dots\Big\}\end{aligned}$$

Resummation of LLs $\alpha_S^n\ln^n(\mu_F^2/\mu_{0F}^2)$ and NLLs $\alpha_S^n\ln^{n-1}(\mu_F^2/\mu_{0F}^2)$

$\mu_{0F}\simeq m_b$ **and** $\mu_F\simeq m_t$: **resummation of NLL** $\ln(m_t^2/m_b^2)$ **(collinear resummation)**

Soft-gluon radiation

Region $x_b \to 1$ corresponds to soft-gluon radiation

Bottom quark spectrum presents terms $1/(1-x_b)_+$ **and** $[\ln(1-x_b)/(1-x_b)]_+$

$$\frac{1}{(1-x_b)_+} \to \ln N \qquad \left[\frac{1}{1-x_b}\ln(1-x_b)\right]_+ \to \ln^2 N$$

$$\hat{\Gamma}_N(m_t,\mu,\mu_F) = 1 + \frac{\alpha_S(\mu)C_F}{2\pi}\left\{2\ln^2 N + \left[4\gamma + 2 - 4\ln(1-w) - 2\ln\frac{m_t^2}{\mu_F^2}\right]\ln N + K(m_t,\mu_F) + \mathcal{O}\left(\frac{1}{N}\right)\right\}$$

$z = 1 - x_b, \quad k^2 = (p_b+p_g)^2(1-z) = 2E_g^2(1-\cos\theta_{bg}) \simeq E_g^2\sin^2\theta_{bg}$

$$\Delta_N = \exp\left\{\int_0^1 dz \frac{z^{N-1}-1}{1-z}\int_{\mu_F^2}^{m_t^2(1-w)^2(1-z)^2}\left[\frac{dk^2}{k^2}A\left[\alpha_S(k^2)\right] + S\left[\alpha_S\left(m_t^2(1-w)^2(1-z)^2\right)\right]\right]\right\}$$

$$= \exp\left[\ln N g_1 + g_2\right]$$

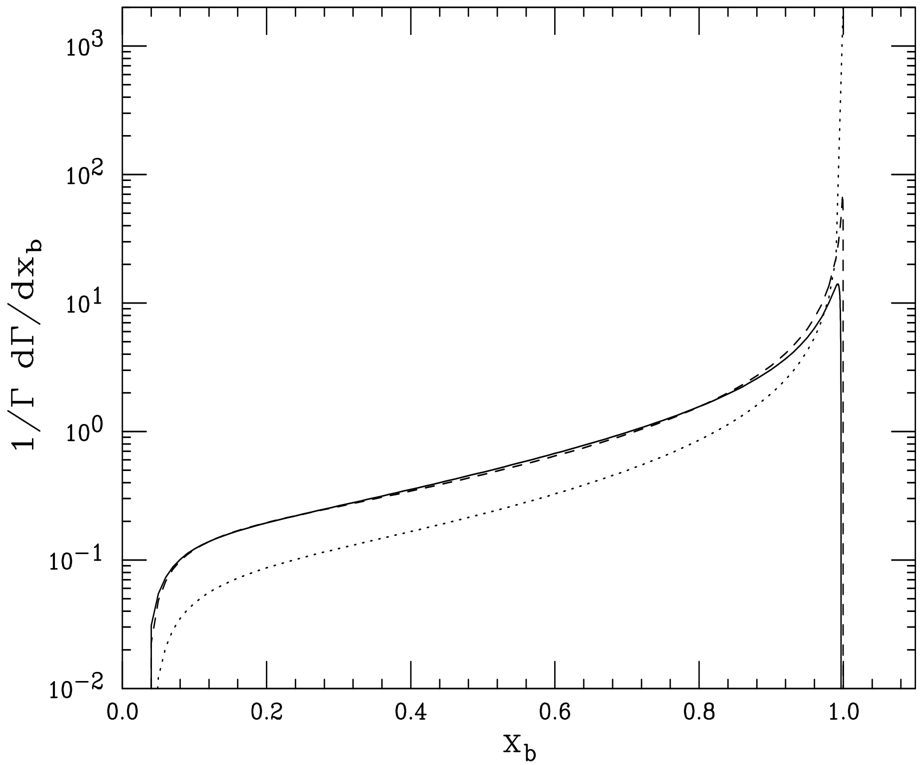
$$A(\alpha_S) = \sum_{n=1}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^n A^{(n)} \quad ; \quad S(\alpha_S) = \sum_{n=1}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^n S^{(n)}$$

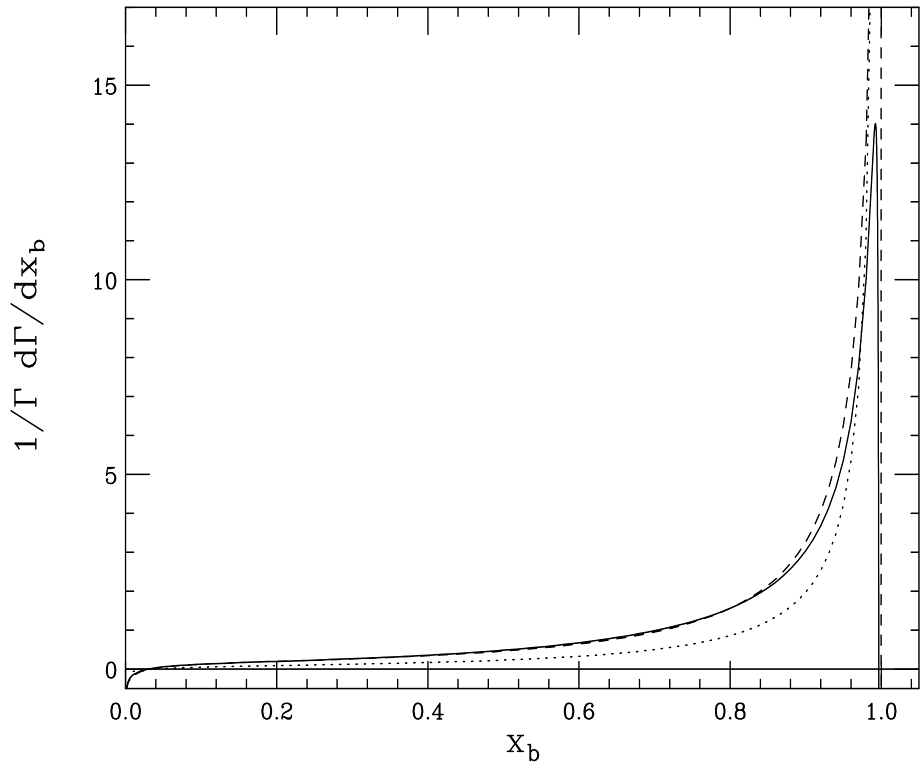
$g_1 \ln N$ **resums LL** $A^{(1)}$ **:** $\alpha_S \ln^2 N, \alpha_S^2 \ln^4 N \ldots \alpha_S^n \ln^{n+1} N$

g_2 **resums NLL** $A^{(2)}, S^{(1)}$ **:** $\alpha_S \ln N, \alpha_S^2 \ln^2 N \ldots \alpha_S^n \ln^n N$

b-quark energy spectrum in top decay

m_t=175 GeV, m_b=5 GeV, m_W=80.425 GeV, $\mu_F = \mu = m_t$, $\mu_0 = \mu_{0F} = m_b$, $\Lambda_{\overline{MS}}$=200 GeV



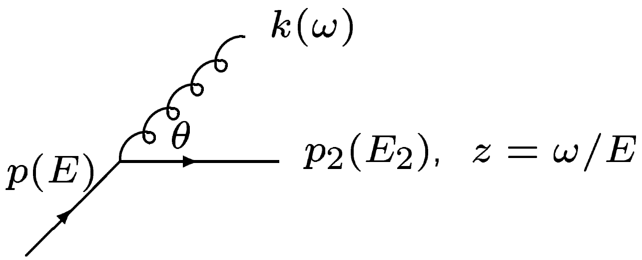


Solid: soft and collinear resummation Dashes: only collinear resummation

Dots: massive NLO without resummation

Alternative approach: Monte Carlo event generators

Multiple radiation in the soft or collinear approximation



$$dP = \frac{\alpha_S}{2\pi}\, \hat{P}(z) dz\, \frac{dQ^2}{Q^2}\, \Delta_S(Q^2_{\rm max}, Q^2)$$

Q^2: ordering variable

$\Delta_S(Q^2_{\rm max}, Q^2)$ **Sudakov form factor: no radiation in** $[Q^2, Q^2_{\rm max}]$

$$\Delta_S(Q^2_{\rm max}, Q^2) = \exp\left[-\frac{\alpha_S}{2\pi}\int_{Q^2}^{Q^2_{\rm max}} \frac{dQ'^2}{Q'^2}\int_{z_{\rm min}}^{z_{\rm max}} dz\, \hat{P}(z)\right]$$

HERWIG : $Q^2 = E^2(1-\cos\theta) \simeq E^2\theta^2/2$ **Soft approximation: angular ordering**

PYTHIA (up to 6.2 version): $Q^2 = p^2$

It includes angular ordering only by an additional veto

PYTHIA 6.3: $Q^2 = k_T^2$

Hard and large-angle radiation: matrix-element corrections

Implemented for top decay, not for top production

Parton showers and resummation: iterating dP for multiple radiation:

$$
\begin{aligned}
dP_1 &= \frac{\alpha_S}{2\pi} dz_1 \hat{P}(z_1) \frac{dQ_1^2}{Q_1^2},\ dP_2 = \frac{\alpha_S}{2\pi} dz_2 \hat{P}(z_2) \frac{dQ_2^2}{Q_2^2} dP_1, \ldots, \\
dP_n &= dP_{n-1} \frac{\alpha_S}{2\pi} dz_n \hat{P}(z_n) \frac{dQ_n^2}{Q_n^2}
\end{aligned}
$$

$$
P_n \sim \alpha_S^n \int_{Q_0^2}^{Q^2} \frac{dQ_1^2}{Q_1^2} \int_{Q_0^2}^{Q_1^2} \frac{dQ_2^2}{Q_2^2} \ldots \int_{\epsilon_1}^{1-\epsilon_1} dz_1 \hat{P}(z_1) \int_{\epsilon_2}^{1-\epsilon_2} dz_2 \hat{P}(z_2) \ldots
$$

Example: all $q \to qg$, $z \to 0$, $Q^2 \propto \theta^2 \to 0$

$$
\hat{P}(z) = C_F \frac{1+(1-z)^2}{z} \simeq \frac{2C_F}{z}
$$

Soft and collinear limit: resummation of double logarithms $\sim \alpha_S^n L^{2n}$

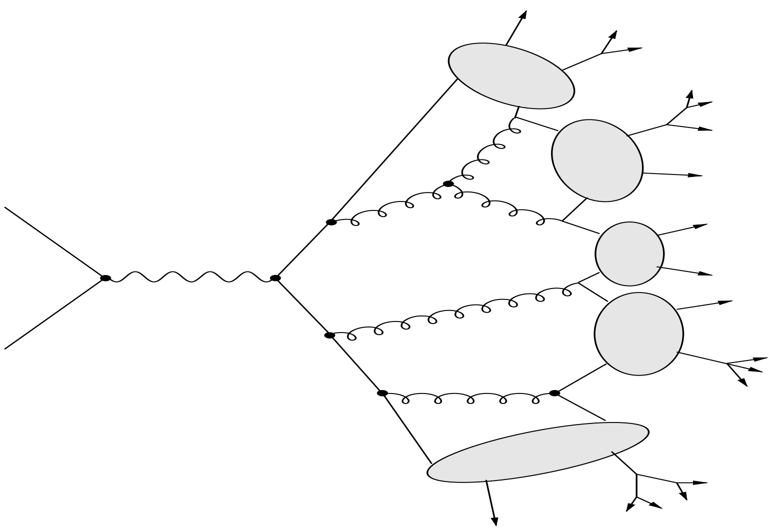
Soft or collinear limit: resummation of single logarithms $\sim \alpha_S^n L^n$

S. Catani, G. Marchesini and B.R. Webber, NPB 349 (1991) 635: HERWIG is equivalent to LL resummation, with the inclusion of some NLLs

Hadronization models

NLL calculation: Kartvelishvili model $D_{\rm np}(x_B,\gamma)=(1+\gamma)(2+\gamma)x_B(1-x_B)^\gamma$

$$\frac{d\Gamma_{\rm had}}{dx_B}(t\to B)=\frac{d\Gamma_{\rm part}}{dx_b}(t\to b)\otimes D_{np}(b\to B)$$



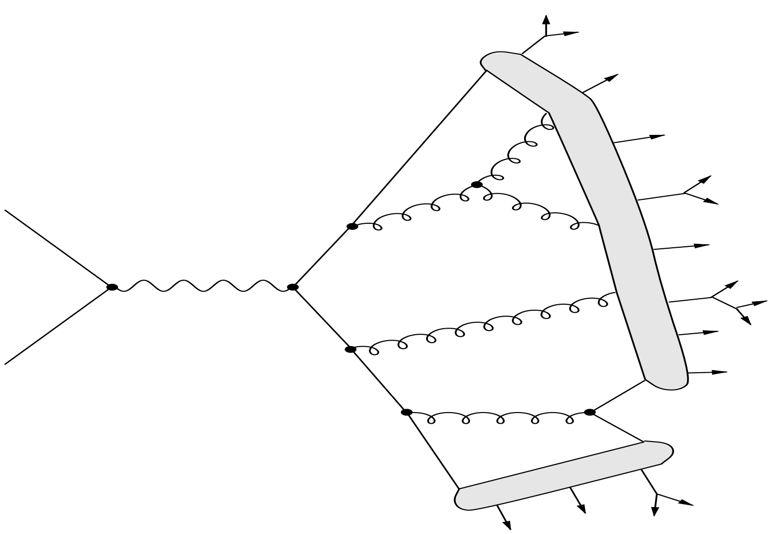
HERWIG: cluster model

Perturbative evolution ends at $Q^2=Q_0^2$

Angular ordering ⇒ colour preconfinement

Forced gluon splitting $(g\to q\bar{q})$

Colour-singlet clusters decay into the observed hadrons



PYTHIA: string model

q and $\bar{q}$ move in opposite direction

The colour field collapses into a string with uniform energy density

$q\bar{q}$ pairs are produced

The string breaks into the observed hadrons

Figures from Ellis, Stirling and Webber, 'QCD and Collider Physics'

Fits of hadronization models to $e^+e^- \to b\bar{b}$ data

NLO+NLL calculation:

$$\frac{d\sigma_{\rm had}}{dx_B}(e^+e^- \to B) = \frac{d\sigma_{\rm part}}{dx_b}(e^+e^- \to b\bar{b}) \otimes D_{np}(b \to B)$$

B-hadrons from SLD (mesons and baryons), ALEPH and OPAL (only mesons)

Kartvelishvili model ($0.18 \leq x_B \leq 0.94$): $\gamma = 17.178 \pm 0.303$, $\chi^2/\mathrm{dof} = 46.2/53$

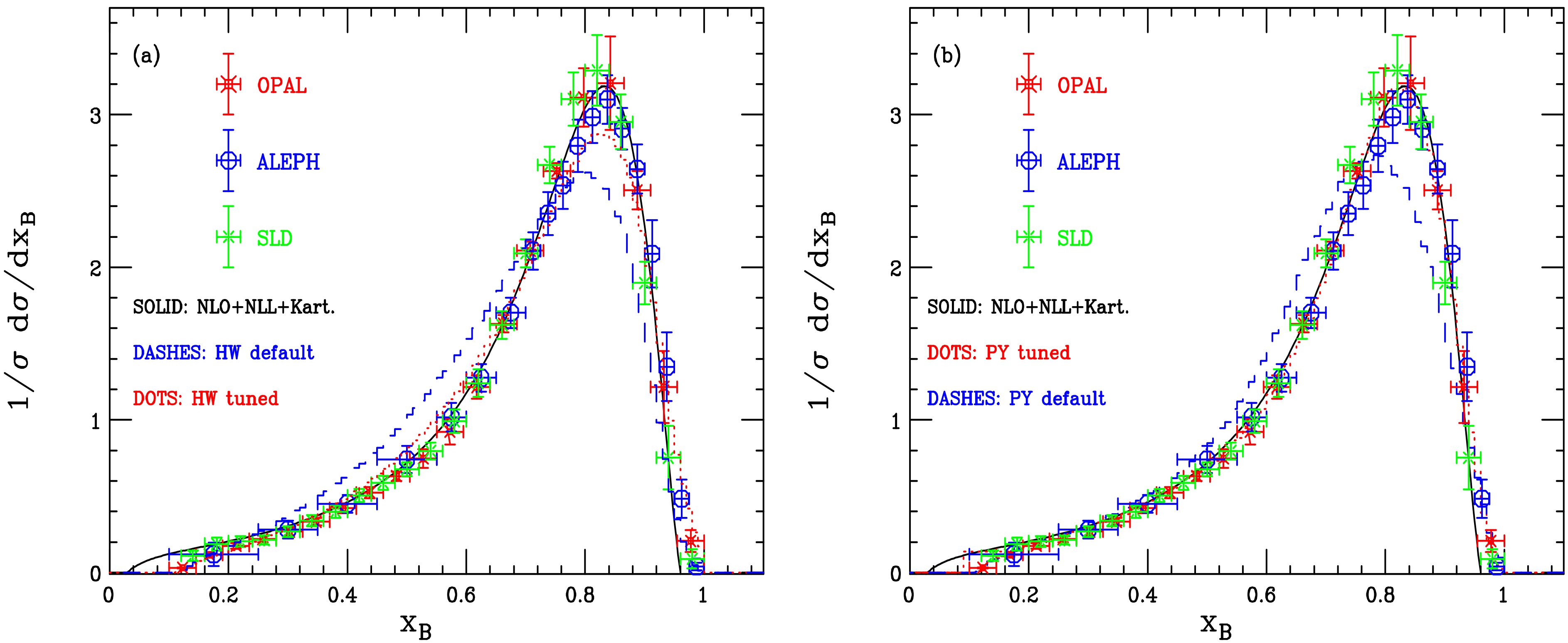
Default HERWIG: $\chi^2/\mathrm{dof} = 739.4/61$; **default PYTHIA:** $\chi^2/\mathrm{dof} = 467.9/61$

Fitting the hadronization parameters yields (all x_B range):

HERWIG 6.506	PYTHIA 6.202
CLSMR(1) = 0.4 (0.0)	
CLSMR(2) = 0.3 (0.0)	PARJ(41) = 0.85 (0.30)
DECWT = 0.7 (1.0)	PARJ(42) = 1.03 (0.58)
CLPOW = 2.1 (2.0)	PARJ(46) = 0.85 (1.00)
PSPLT(2) = 0.33 (1.00)	
χ^2/dof = 222.4/61	χ^2/dof = 45.7/61

PYTHIA 6.3: χ^2/dof **= 46.0/61**

Comparison with e^+e^- data

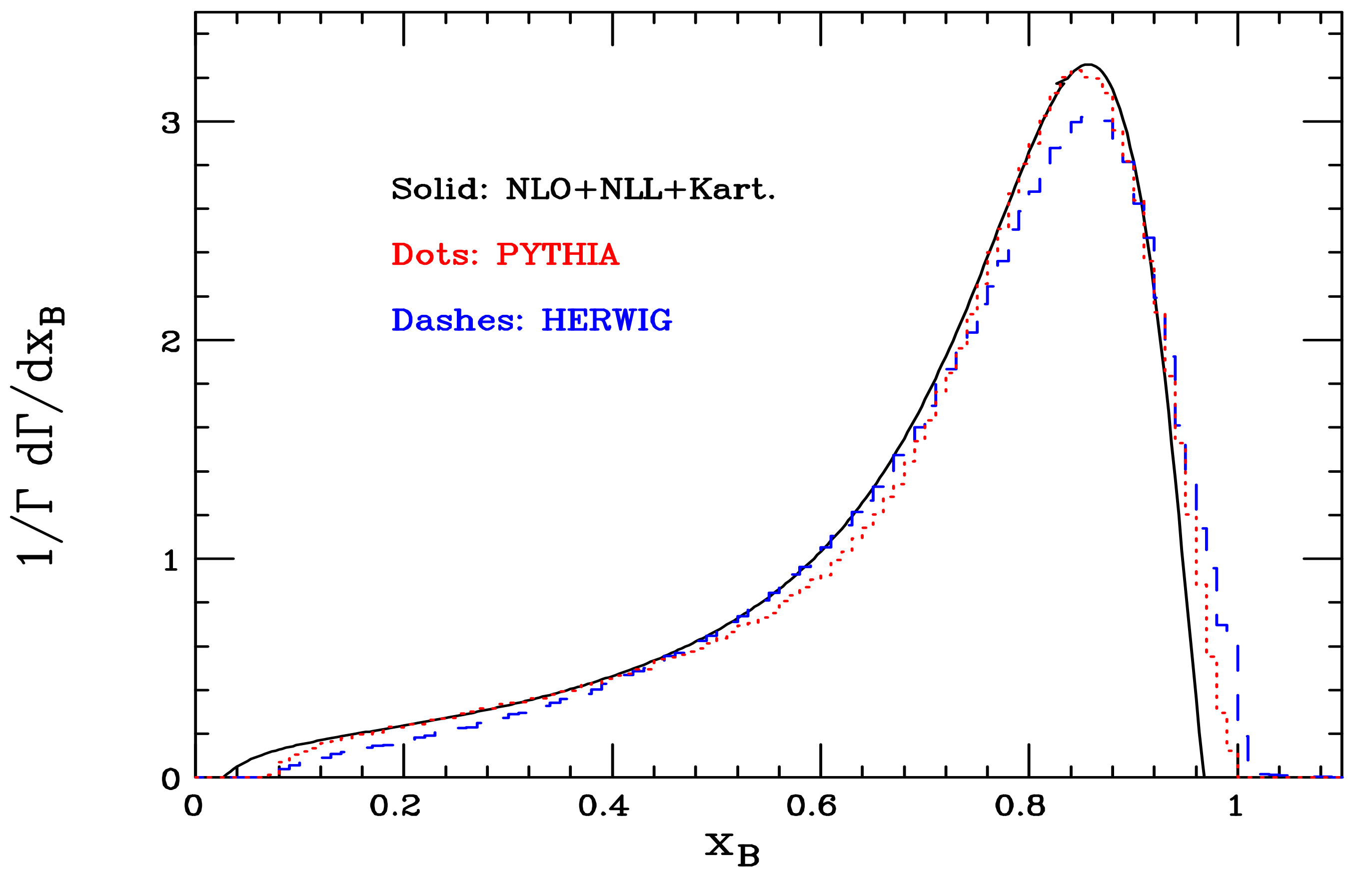


HERWIG ++: new fragmentation model ($\chi^2/\text{dof} \simeq \mathcal{O}(1)$), but hadron collisions are not yet implemented

B-hadron spectrum in top decay

Neglecting production/decay interference effects:

$$\frac{1}{\Gamma}\frac{d\Gamma}{dx_b} = \frac{1}{\sigma}\frac{d\sigma}{dx_b}$$



Results in moment space

$$\Gamma_N = \int_0^1 dz\ z^{N-1} \frac{1}{\Gamma}\frac{d\Gamma}{dz}(z)$$

e^+e^- **annihilation** $\sigma_N^B = \sigma_N^b D_N^{np}$

σ_N^B **measured ;** σ_N^b **calculated ;** D_N^{np} **fitted**

top decay: $\Gamma_N^B = \Gamma_N^b D_N^{np} = \Gamma_N^b \sigma_N^B / \sigma_N^b$

Fits to DELPHI data (ICHEP 2002 Note, DELPHI 2002-069 CONF 603)

	$\langle x \rangle$	$\langle x^2 \rangle$	$\langle x^3 \rangle$	$\langle x^4 \rangle$
e^+e^- data σ_N^B	0.7153±0.0052	0.5401±0.0064	0.4236±0.0065	0.3406±0.0064
e^+e^- NLL σ_N^b	0.7801	0.6436	0.5479	0.4755
$D_N^{\rm np}$	0.9169	0.8392	0.7731	0.7163
e^+e^- HW σ_N^B	0.7113	0.5354	0.4181	0.3353
e^+e^- PY σ_N^B	0.7162	0.5412	0.4237	0.3400
t-dec. NLL Γ_N^b	0.7883	0.6615	0.5735	0.5071
t-dec. NLL $\Gamma_N^B = \Gamma_N^b D_N^{\rm np}$	0.7228	0.5551	0.4434	0.3632
t-dec. HW Γ_N^B	0.7325	0.5703	0.4606	0.3814
t-dec. PY Γ_N^B	0.7225	0.5588	0.4486	0.3688

Conclusions

Bottom fragmentation in top decay according to resummed calculations, HERWIG and PYTHIA

Big effect of NLL soft and collinear resummations on the b-quark spectrum

Default HERWIG and PYTHIA unable to fit x_B data from LEP and SLD

Fits of Kartvelishvili, cluster and string models

NLL calculation and PYTHIA describe well the data, HERWIG marginally consistent

Predictions for B-hadron spectrum in top decay in x_B and moment spaces

In progress:

Study of other observables using resummations, HERWIG and PYTHIA with the fitted parametrizations

Impact on the top mass reconstruction