Automatic Loop Calculations in the MSSM with Non-minimal Flavor Violation

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CERN, January 2005

on collaboration with

S. Heinemeyer and *T. Hahn*

- FeynArts, FormCalc, LoopTools with NMFV
- FeynHiggs2.2 with NMFV
- E.g.: Corrections to M_W , $\sin^2\theta_{\text{eff}}$, m_h
- Conclusions

NMFV ⇒ Connection to Flavour Physics

FEYNARTS

FORMCALC

and

LOOPTOOLS

FeynArts

T. Hahn, Comput. Phys. Commun. 140 (2001) 418, hep-ph/0012260

The MSSM model files are documented in:

Comput. Phys. Commun. 143 (2002) 54, hep-ph/0105349

FormCalc, LoopTools

T. Hahn, M. Perez-Victoria,

Comput. Phys. Commun. 118 (1999) 153, hep-ph/9807565

Latest version:

http://www.feynarts.de - (10 Jan 05)

http://www.feynarts.de/formcalc - (20 Jan 05)

http://www.feynarts.de/looptools - (7 Dec 04)

Automatic Installation with `FeynInstall`

Evaluating Feynman Diagrams

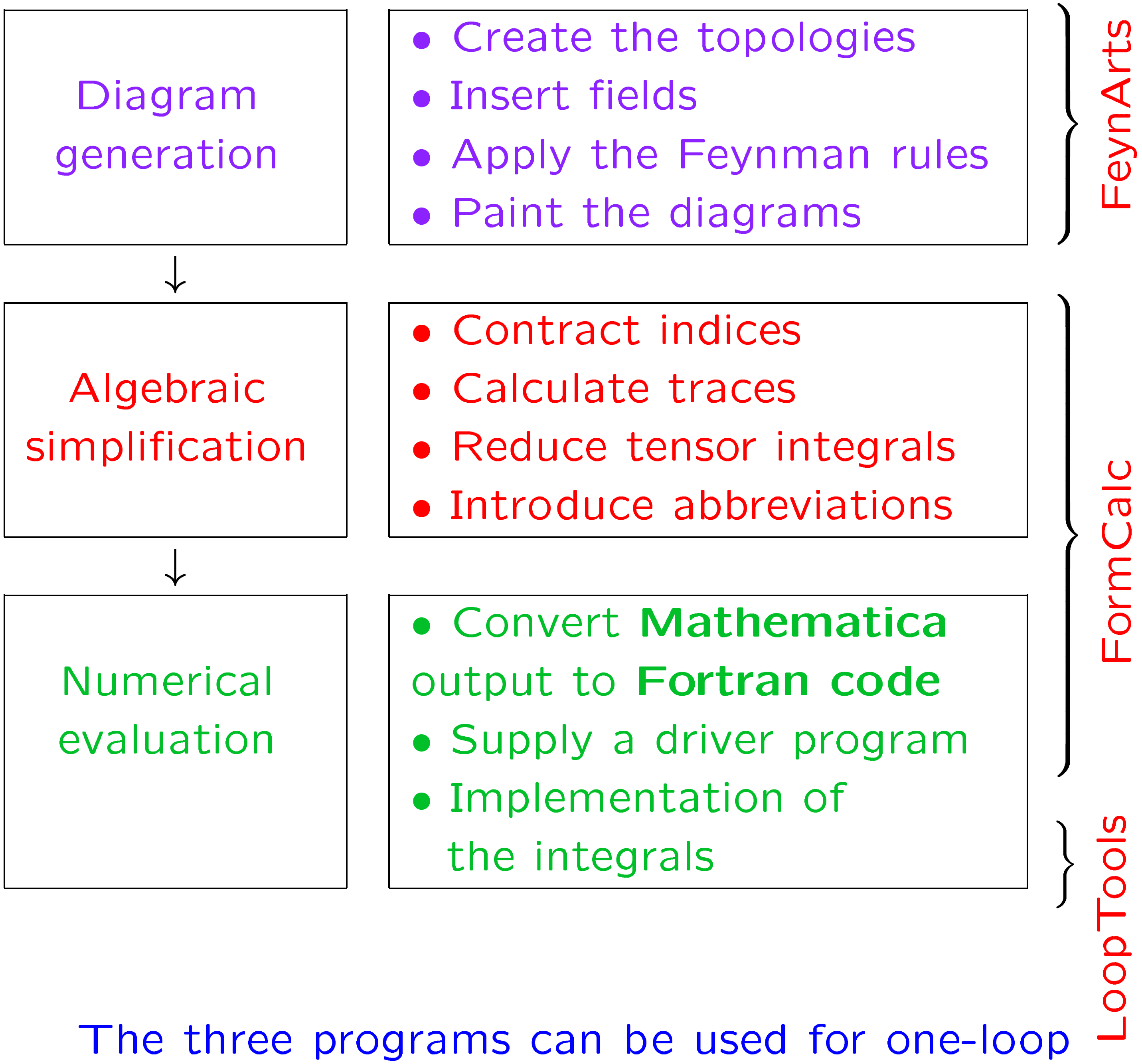


Diagram generation

- Create the topologies
- Insert fields
- Apply the Feynman rules
- Paint the diagrams

FeynArts

↓

Algebraic simplification

- Contract indices
- Calculate traces
- Reduce tensor integrals
- Introduce abbreviations

↓

Numerical evaluation

- Convert **Mathematica** output to **Fortran code**
- Supply a driver program
- Implementation of the integrals

FormCalc

LoopTools

The three programs can be used for one-loop calculations and they work together smoothly

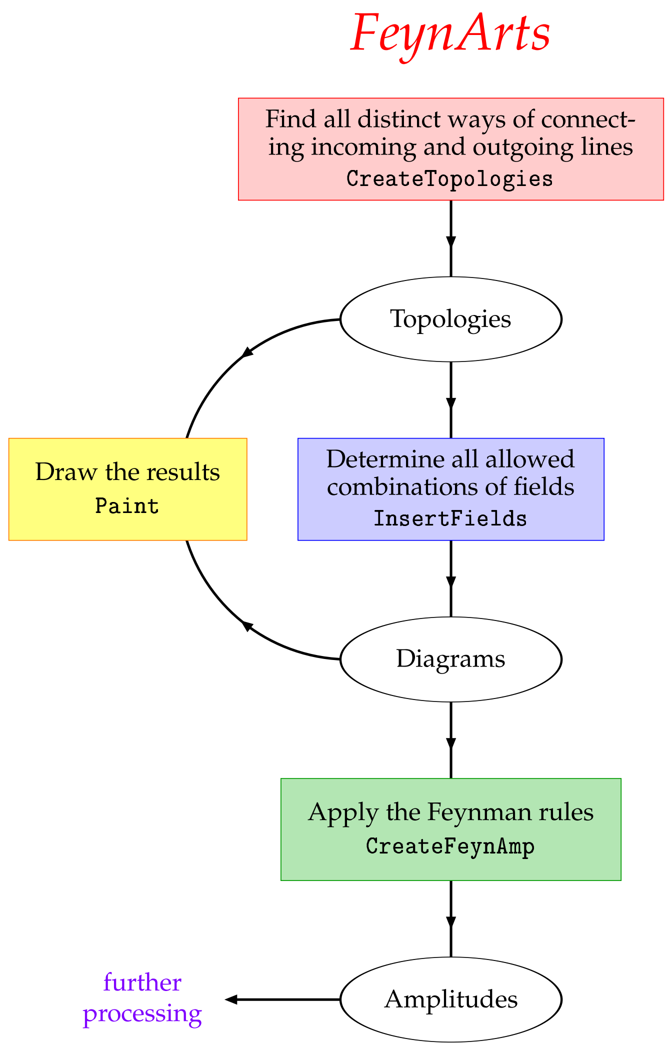
FeynArts is a Mathematica package

FormCalc is a Mathematica package, uses FORM

J. Vermasseren, math-ph/0010025, http://www.nikhef.nl/∼form/

LoopTools implementation in Fortran, can be used in Fortran/Mathematica/C++

How FeynArts works ?



FeynArts

Find all distinct ways of connecting incoming and outgoing lines
`CreateTopologies`

Topologies

Determine all allowed combinations of fields
`InsertFields`

Diagrams

Apply the Feynman rules
`CreateFeynAmp`

Amplitudes

Draw the results
`Paint`

further processing

EXAMPLE: generating the Higgs self-energy

```
top = CreateTopologies[ 1, 1 -> 1 ]
```

one loop

one incoming particle

one outgoing particle

```
Paint[top]
```

```
ins = InsertFields[ top, S[1] -> S[1],
        Model -> SM ]
```

use the Standard Model

the name of the Higgs boson in the "SM" model file

```
Paint[ins]
```

```
amp = CreateFeynAmp[ins]
```

```
amp >> HiggsSelfEnergy.amp
```

Generating diagrams in just a few lines - aren't there any strings attached ?

Yes, one has to set up, once and for all, a MODEL FILE containing the particles and the allowed couplings .

★ FeynArts Model files can be supplied/modified by the user

Model Files Currently included:

- Standard Model (SM) [w/QCD]
  All one-loop counter terms included.
- Minimal Supersymmetric Standard Model (MSSM) [w/QCD]
  Counter terms by T. Fritzsche.
- Two-Higgs-Doublet Model (THDM)
  Counter terms not included yet.

In preparation: FVMSSM

by T.Hahn and J.I.Illana

FVMSSM.mod
Model file for the flavour-violating MSSM

FeynArts-3.2/Models/FVMSSM.mod -
last modified 7/10/04

• Feynman rules with FV are included by modifying an existing model file MSSMQCD.mod
⟶ FVMSSM.mod is technically not a full model file, but loads MSSM.mod and applies changes

• FormCalc files must be modified as well
⟶ Set up the model.h and mssm-ini.F files which declares and computes the squarks mixing matrices

FVMSSM.mod

Feynman rules included:

[FFS] 2 Leptons - Higgs
- 2 Quarks - Higgs
- Chargino - Lepton -Slepton
- Chargino - Quark - Squark
- Gluino - Quark - Squark
- Lepton - Neutralino - Slepton
- Neutralino - Quark - Squark

[FFV] 2 Leptons - Gauge boson
- 2 Quarks - Gauge boson
- 2 Quarks - Gluon

[SSS] Higgs - 2 Sleptons
- Higgs - 2 Squarks

[SSSS] 2 Higgs - 2 Sletons
- 2 Higgs - 2 Squarks
- 2 Sleptons - 2 Squarks
- 4 Sleptons
- 4 Squarks

[SSV] 2 Sleptons - Gauge bosons
- 2 Squarks - Gauge bosons
- 2 Squarks - Gluon

[SSVV] 2 Sleptons - 2 Gauge bosons
- 2 Squarks - 2 Gauge bosons
- 2 Squarks - 2 Gluons
- 2 Squarks - Gauge boson - Gluons

E.g. the SSS coupling is declared by:

- in the Classes model file (here for $h_0\,\tilde{u}_g^s\,\bar{\tilde{u}}_g^s$)

$$
\begin{gathered}
C[\,S[1], S[13, s1, j1, o1], -S[13, s2, j2, o2]\,] == \\
((-I/6) * EL * IndexDelta[j1, j2] * IndexDelta[o1, o2]* \\
(Conjugate[USf[3, j1][s1, 1]] * \\
((MW * MZ * SAB * SB * (-3 + 4 * SW^2) + \\
6 * CA * CW * Mass[F[3, j1]]^2) * USf[3, j1][s2, 1] + \\
3 * CW * (CA * Af[3, j1] + SA * Conjugate[MUE]) * \\
Mass[F[3, j1]] * USf[3, j1][s2, 2]) + \\
Conjugate[USf[3, j1][s1, 2]] * \\
(3 * CW * (MUE * SA + CA * Conjugate[Af[3, j1]]) * \\
Mass[F[3, j1]] * USf[3, j1][s2, 1] - \\
4 * MW * MZ * SAB * SB * SW^2 * USf[3, j1][s2, 2] + \\
6 * CA * CW * Mass[F[3, j1]]^2 * USf[3, j1][s2, 2]))) \\
/(CW * MW * SB * SW)
\end{gathered}
$$

USf - sfermion mixing matrices (6x6)

$Mass[F]$ - sfermion masses

NMFV in the MSSM

→ Mixing of scalar quark families (beyond CKM)

E.g.: General case of mixing between the third and second generation of squarks ($\tilde{t}/\tilde{c}, \tilde{b}/\tilde{s}$ sectors)

Squark Generation Mixing via Soft Breaking

Parametrization of non-diagonal squark mass matrices

$$M^2_{\tilde{u}} = \begin{pmatrix} M^2_{\tilde{L}_c} & \Delta^t_{LL} & m_c X_c & \Delta^t_{LR} \\ \Delta^t_{LL} & M^2_{\tilde{L}_t} & \Delta^t_{RL} & m_t X_t \\ m_c X_c & \Delta^t_{RL} & M^2_{\tilde{R}_c} & \Delta^t_{RR} \\ \Delta^t_{LR} & m_t X_t & \Delta^t_{RR} & M^2_{\tilde{R}_t} \end{pmatrix}$$

$$M^2_{\tilde{d}} = \begin{pmatrix} M^2_{\tilde{L}_s} & \Delta^b_{LL} & m_s X_s & \Delta^b_{LR} \\ \Delta^b_{LL} & M^2_{\tilde{L}_b} & \Delta^b_{RL} & m_b X_b \\ m_s X_s & \Delta^b_{RL} & M^2_{\tilde{R}_s} & \Delta^b_{RR} \\ \Delta^b_{LR} & m_b X_b & \Delta^b_{RR} & M^2_{\tilde{R}_b} \end{pmatrix}$$

$$\begin{aligned} M^2_{\tilde{L}_q} &= M^2_{\tilde{Q}_q} + m_q^2 + \cos 2\beta\, M_Z^2 (T_3^q - Q_q s_{\rm w}^2) \\ M^2_{\tilde{R}_q} &= M^2_{\tilde{U}_q} + m_q^2 + \cos 2\beta\, M_Z^2 Q_q s_{\rm w}^2 \;\; (q = t, c) \\ M^2_{\tilde{R}_q} &= M^2_{\tilde{D}_q} + m_q^2 + \cos 2\beta\, M_Z^2 Q_q s_{\rm w}^2 \;\; (q = b, s) \\ X_q &= A_q - \mu (\tan\beta)^{-2T_3^q} \end{aligned}$$

E.g. $\Delta^t_{LL} = \lambda^t M_{\tilde{L}_t} M_{\tilde{L}_c}$, $\Delta^b_{LL} = \lambda^b M_{\tilde{L}_b} M_{\tilde{L}_s}$, ...

→ λ^t and λ^b correspond to $(\delta^u_{LL})_{23}$ and $(\delta^d_{LL})_{23}$

Mass eigenstates : To diagonalize the 4 × 4 squark mass matrices, two 4 × 4 rotation matrices, $R_{\tilde{u}}$ and $R_{\tilde{d}}$, are needed

model.h

Common blocks for the model parameters

- SM parameters
- MSSM parameters
- flavour-violating parameters

mssm-ini.F

initialize the flavour-violating MSSM
based on FormCalc's mssm-ini.F

- Input parameters
- All parameters appearing in the Model File
- Exclusion limits
- Compute the squark masses:

  1) "regular" ones with only left-right mixing (needed for computing the Higgs mass)

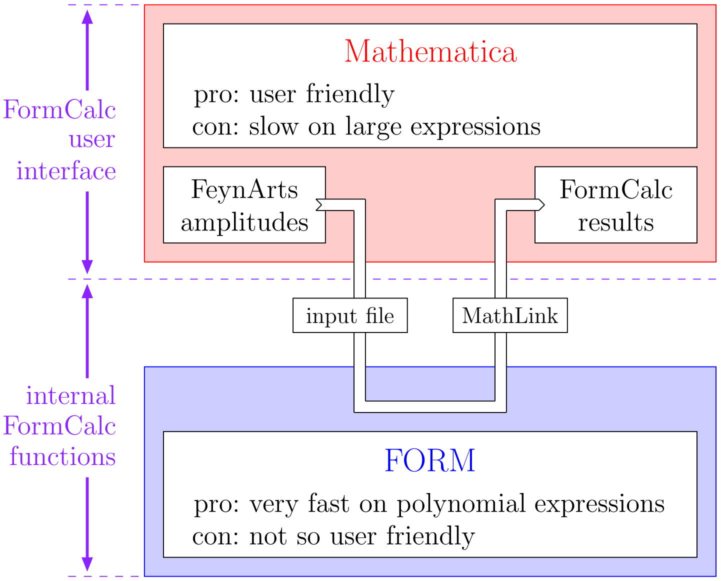
  2) fully 6x6 generalized mixing matrices

Modified versions are not (yet) public : –(

Checks are being done !

Files available on request !!

How FormCalc works ?



Example: Calculating Higgs self-energy

```
In[1]:= << FormCalc`

FormCalc 4
by Thomas Hahn
last revised 11 Nov 04

In[2]:= CalcFeynAmp[<<HiggsSelfEnergy.amp]

preparing FORM code in /tmp/m6.frm
running FORM...  ok

                                 MW2
                         Alfa (-(---) + MZ2)
                                 CW2
Out[2]= Amp[{MH} -> {MH}][------------------- -
                            4 CW2 Pi SW2

3 Alfa MH2 A0[MH2]
------------------ -
 16 MW2 Pi SW2

                 MH2                         3     MH2
 Alfa (3 + -----)A0[MW2]     Alfa (--- + ----) A0[MZ2]
           2 MW2                          CW2   2MW2
 ----------------------- - ------------------------- +
        4 Pi SW2                   8 Pi SW2

            2
9 Alfa MH2  B0[MH2, MH2, MH2]
----------------------------- +
       32 MW2 Pi SW2

                         2
        -MH2      MH2
Alfa (------ + ----- + 3 MW2) B0[MH2, MW2, MW2]
          2       4 MW2
----------------------------------------------- +
                    4 Pi SW2

                      2
            MH2     MH2     7 MW2     MZ2
Alfa (-(---) + ----- + ------ - ---) B0[MH2, MZ2, MZ2]
            CW2    2MW2        2      MW2
                            CW2
------------------------------------------------------ +
                    16 Pi SW2

Alfa MH2 B1[MH2, MW2, MW2]   Alfa MH2 B1[MH2, MZ2, MZ2]
-------------------------- + -------------------------- ,
       4 Pi SW2                    8 CW2 Pi SW2

+ Fermionic Contributions
```

Implemented constraints

- Exclusion limits:
  lower bound on $m_{\tilde{f}}, m_{\tilde{\chi}}, m_{\tilde{g}}, m_h$
- $\Delta\rho \leq 2 \times 10^{-3}$

Example: Calculating Higgs self-energy

```
In[1]:= << FormCalc`

FormCalc 4
by Thomas Hahn
last revised 11 Nov 04

In[2]:= CalcFeynAmp[<<HiggsSelfEnergy.amp]

preparing FORM code in /tmp/m6.frm
running FORM...  ok

                                          MW2
                          Alfa (-(---) + MZ2)    3 Alfa MH2 A0[MH2]
                                          CW2
Out[2]= Amp[{MH} -> {MH}][------------------- - ------------------ -
                            4 CW2 Pi SW2         16 MW2 Pi SW2

             MH2                     3     MH2
 Alfa (3 + -----)A0[MW2]     Alfa (--- + ----) A0[MZ2]
            2 MW2                   CW2   2MW2
 ----------------------- - ------------------------- +
        4 Pi SW2                  8 Pi SW2

         2
 9 Alfa MH2  B0[MH2, MH2, MH2]
 ----------------------------- +
        32 MW2 Pi SW2

        -MH2     MH2^2
 Alfa (----- + -----+ 3 MW2) B0[MH2, MW2, MW2]
         2     4 MW2
 --------------------------------------------- +
                    4 Pi SW2

          MH2     MH2^2    7 MW2    MZ2
 Alfa (-(---) + ----- + ------ - ---) B0[MH2, MZ2, MZ2]
          CW2     2MW2    CW2^2    MW2
 ------------------------------------------------------ +
                      16 Pi SW2

 Alfa MH2 B1[MH2, MW2, MW2]   Alfa MH2 B1[MH2, MZ2, MZ2]
 -------------------------- + -------------------------- ,
          4 Pi SW2                  8 CW2 Pi SW2

  Alfa A0[Mf2[2, Gen1]] Mf2[2, Gen1]
 (---------------------------------- -
            2 MW2 Pi SW2

 Alfa MH2 B1[MH2, Mf2[2, Gen1], Mf2[2, Gen1]] Mf2[2, Gen1]
 --------------------------------------------------------- -
                      4 MW2 Pi SW2

                                                      2
 Alfa B0[MH2, Mf2[2, Gen1], Mf2[2, Gen1]] Mf2[2, Gen1]
 ------------------------------------------------------ +
                      2 MW2 Pi SW2

 3 Alfa A0[Mf2[3, Gen1]] Mf2[3, Gen1]
 ------------------------------------ -
            2 MW2 Pi SW2

 3 Alfa MH2 B1[MH2, Mf2[3, Gen1], Mf2[3, Gen1]] Mf2[3, Gen1]
 ----------------------------------------------------------- -
                      4 MW2 Pi SW2

                                                        2
 3 Alfa B0[MH2, Mf2[3, Gen1], Mf2[3, Gen1]] Mf2[3, Gen1]
 -------------------------------------------------------- +
                      2 MW2 Pi SW2

 3 Alfa A0[Mf2[4, Gen1]] Mf2[4, Gen1]
 ------------------------------------ -
            2 MW2 Pi SW2

 3 Alfa MH2 B1[MH2, Mf2[4, Gen1], Mf2[4, Gen1]] Mf2[4, Gen1]
 ----------------------------------------------------------- -
                      4 MW2 Pi SW2

                                                        2
 3 Alfa B0[MH2, Mf2[4, Gen1], Mf2[4, Gen1]] Mf2[4, Gen1]
 --------------------------------------------------------) SumOver[Gen1, 3]]
                      2 MW2 Pi SW2
```

Results for

M_W , $\sin^2\theta_{\text{eff}}$ and m_h

S. Heinemeyer, W. Hollik, F. Merz, S.P.

Eur.Phys.J.C37, 481-493, 2004, hep-ph/0403228

Simplest scenario (Numerical result)

Mixing only between the left-handed components of $\tilde{t}, \tilde{c}$ and $\tilde{b}, \tilde{s}$

$$\boxed{\begin{aligned} \Delta^t_{LL} &= \lambda^t M_{\tilde{L}_t} M_{\tilde{L}_c}\,, & \Delta^t_{LR} &= \Delta^t_{RL} = \Delta^t_{RR} = 0\,, \\ \Delta^b_{LL} &= \lambda^b M_{\tilde{L}_b} M_{\tilde{L}_s}\,, & \Delta^b_{LR} &= \Delta^b_{RL} = \Delta^b_{RR} = 0\,. \end{aligned}}$$

The same flavor mixing parameter in the $\tilde{t}/\tilde{c}$ and $\tilde{b}/\tilde{s}$ sectors is assumed : $\lambda = \lambda^t = \lambda^b$

→ $\lambda^t = \lambda^b = 0$ corresponds to the MSSM with MFV

→ λ^t and λ^b correspond to $(\delta^u_{LL})_{23}$ and $(\delta^d_{LL})_{23}$

Numerical analysis performed in 2 benchmark scenarios, but with a free scale M_{SUSY} :

M. Carena et. al, hep-ph/0202167

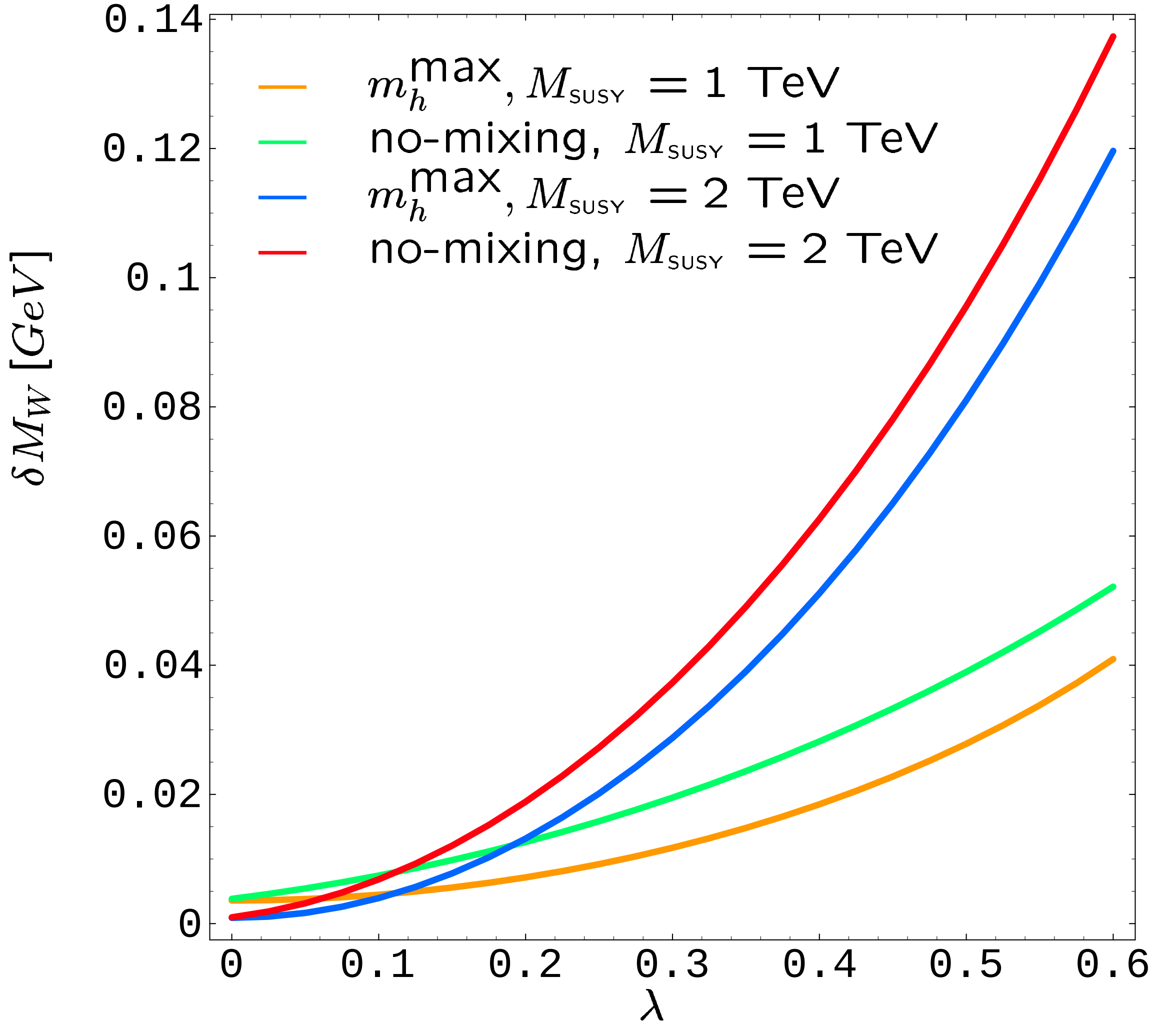
– m_h^{max} (A_t is not a free parameter,
obeying $X_t = 2M_{\text{SUSY}}$, with $X_t = A_t - \mu \cot\beta$)

– no-mixing (no mixing in the MFV $\tilde{t}$ sector ($X_t = 0$))

Analytical result:

evaluation with arbitrary NMFV couplings

δM_W as a function of λ:



→ The induced shifts in M_W can become as large as 0.14 GeV for no-mixing, $M_{\rm SUSY} = 2$ TeV, $\lambda = 0.6$.

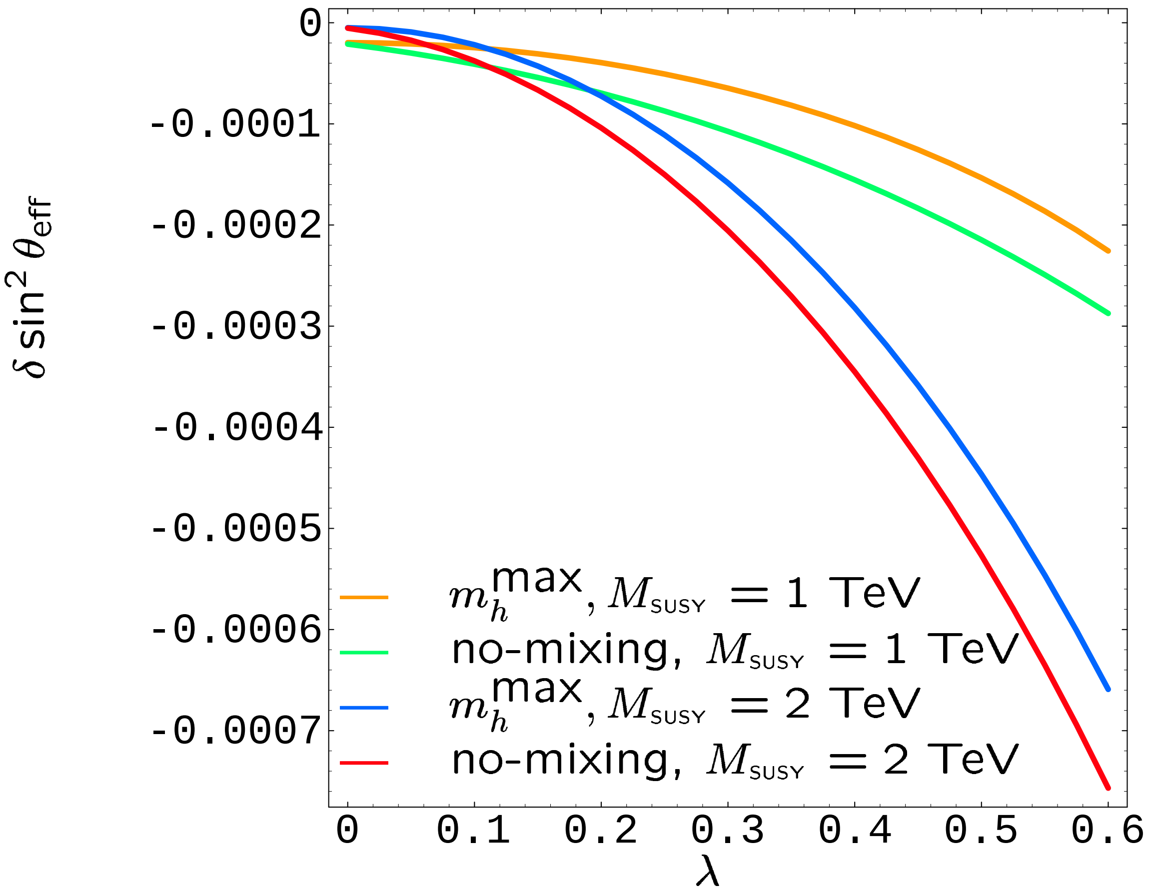
→ $\delta M_W \lesssim 0.05$ GeV in the $m_h^{\max}$ scenario, but still sizeable.

$$\delta M_W^{\rm exp,today} = 34 \text{ MeV}$$

$$\delta M_W^{\rm exp,future} = 7 \text{ MeV}$$

⇒ extreme parameter regions already ruled out

$\delta \sin^2\theta_{\text{eff}}$ as a function of λ:



→ The shifts $\delta \sin^2\theta_{\text{eff}}$ can reach values up 7×10^{-4} for no-mixing scenario, $M_{\text{SUSY}} = 2$ TeV, $\lambda = 0.6$,

→ smaller, but still sizeable, for the other scenarios.

$$\delta \sin^2\theta_{\text{eff}}^{\text{exp,today}} = 17 \times 10^{-5}$$

$$\delta \sin^2\theta_{\text{eff}}^{\text{exp,future}} = 1.3 \times 10^{-5}$$

$\Rightarrow$ extreme parameter
regions already ruled out

Higgs mass corrections

Contrary to the SM: m_h is not a free parameter

- Large radiative corrections:

  Dominant one-loop corrections: $\sim G_\mu m_t^4 \ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right)$

  MSSM with MFV : Dominant one-loop contributions involves third-generation quarks and squarks.

- MSSM with NMFV

  The squark loops have to be modified by introducing the generation-mixed squarks.

- Measurement of m_h, Higgs couplings
  $\Rightarrow$ test of the theory

  LHC: $\Delta m_h \approx 0.2$ GeV

  LC: $\Delta m_h \approx 0.05$ GeV

Higgs boson sector analysis performed in 5 benchmark scenarios : *M. Carena et al, hep-ph/0202167*

– m_h^{max} : to maximize the lightest Higgs boson mass

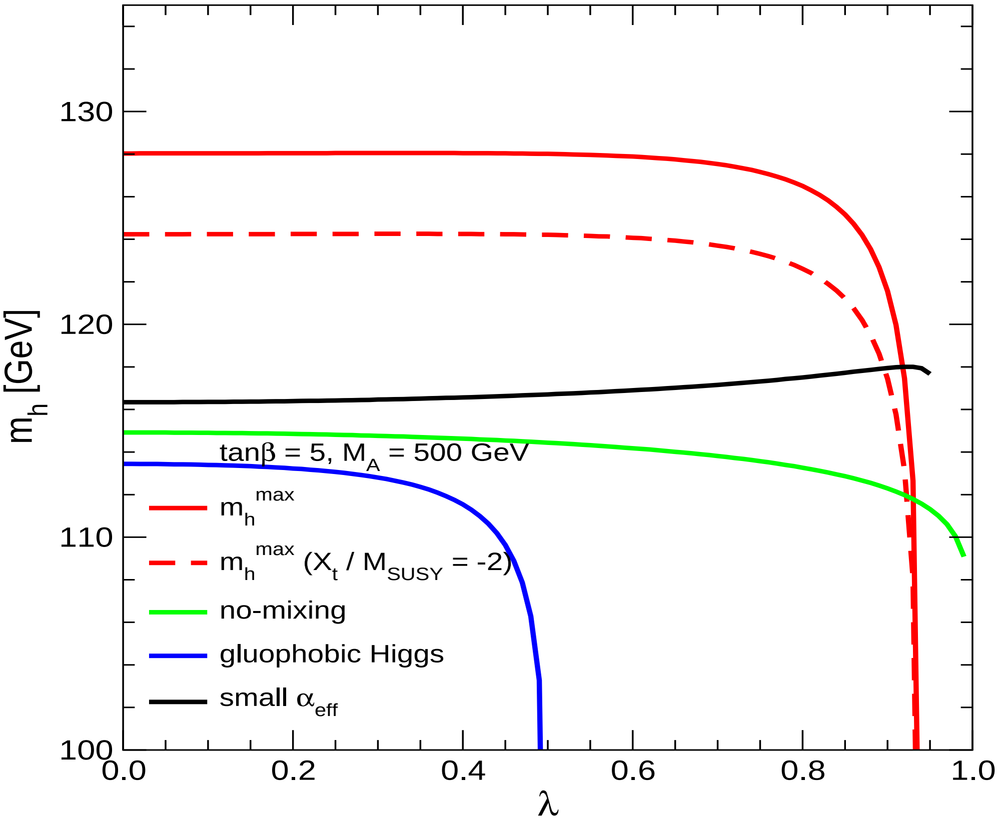
– constrained m_h^{max} : with $X_t/M_{\text{SUSY}} = -2$ for $b \to s\gamma$

– no-mixing in the MFV $\tilde{t}$ sector

– gluophobic Higgs : with reduced ggh coupling

– small α_{eff} : with reduced $h\bar{b}b$ and $h\tau^+\tau^-$ coupling

m_h as a function of λ:



$$M_{\text{SUSY}} = M_{\tilde{Q}_q} = M_{\tilde{U}_q} = M_{\tilde{D}_q}\,, \quad A_s = A_b = A_c = A_t$$

All scenarios show a similar behavior

$\Rightarrow$ small effects for small/moderate λ

$\Rightarrow$ $\delta m_h = \mathcal{O}\,(5\text{ GeV})$ only for very large λ

(around 0.5 in the gluophobic Higgs scenario, and around 0.9 in the other four scenarios)

$\Rightarrow$ mostly decreasing m_h, but also increase possible

(in small α_{eff}-scenario it can be enhanced by up to 2 GeV)

$\Rightarrow$ the impact of NMFV on m_h is in general rather small

FEYNHIGSS

FeynHiggs

S. Heinemeyer, W. Hollik, G. Weiglein,
Comput.Phys.Commun. 124 (2000) 76-89, hep-ph/9812320

T. Hahn, S. Heinemeyer, W. Hollik, G. Weiglein,
MPP-2003-147, see: hep-ph/0406152

Latest version:

hhttp://www.feynhiggs.de - (Jan 05)

FeynHiggs2.2

- A high precision tool for Higgs physics at colliders

- Provides Higgs boson masses, mixing angles, couplings, branching ratios, ...

  in the MSSM with/without complex parameters

  AND FOR NMFV

- Additional constraints:

  - $\Delta\rho \leq 2 \times 10^{-3}$, $\Delta\rho$ at $\mathcal{O}(\alpha, \alpha\alpha_s)$

  - $(g_\mu - 2)_{\text{SUSY}}$: Full one-, leading/subleading two-loop SUSY corrections.

- SUSY Les Houches Accord can also be used

- FeynHiggs with NMFV MSSM

  → Evaluation of

  Higgs masses and the mixing angle α

  → $\Delta\rho$ evaluation is included

  [S. Heinemeyer, W. Hollik, F.Merz, S.P '04]

Subroutines contained in *FeynHiggs*2.2:

- FHSetFlags
  – set the flags of the calculation,
- FHSetPara
  – set the MSSM input parameters directly, *or:*
  FHSetSLHA
  – extract the input parameters from an SLHA data structure,
- FHGetPara
  – retrieve (some of) the derived parameters (e.g. chargino masses),
- FHHiggsCorr
  – compute the Higgs masses and mixings,
- FHCouplings
  – compute the Higgs couplings and BRs,
- FHConstraints
  – evaluate $\Delta\rho$ and $(g_\mu - 2)$ constraints.

Conclusions

- FeynHiggs2.2

  MSSM with NMFV:

  general 4×4 mixing in $\tilde{t}/\tilde{c}$ and $\tilde{b}/\tilde{s}$ sectors

  m_h, mixing angle α and $\Delta\rho$ included

- FeynArts, FormCalc and LoopTools

  MSSM with NMFV:

  6×6 generalized squarks mixing matrices

- Several checks have been done, e.g.:

  - Results for M_W, $\sin^2\theta_{\text{eff}}$ and $\Delta\rho$ reproduced independently

  - Results for $BR(H \to bs,\, tc)$

- In preparation:

  $BR(b \to s\gamma)$, $B_s \to \mu^+\mu^-$, ...

  [T. Hahn, W. Hollik, J.I.Illana, S.P '05]

  [S.Heinemeyer, W. Hollik, J.I.Illana, S.P '05]

Evaluation of M_W, $\sin^2\theta_{\rm eff}$, m_h

- Analytical results: for arbitrary mixing

  Numerical results: only for LL mixing,
  parametrized with λ ($(\delta_{LL})_{23}$)

- Large effects possible for M_W, $\sin^2\theta_{\rm eff}$:

  $\lambda \lesssim 0.2 \Rightarrow \delta M_W \lesssim 20$ MeV

  $\lambda \lesssim 0.2 \Rightarrow \delta \sin^2\theta_{\rm eff} \lesssim 10^{-4}$

  → We have shown that the effects of scalar quark generation mixing enters essentially through $\Delta\rho$

- Moderate effects possible for m_h only for large λ

- Scenarios for B physics can now be tested,
  whether they are compatible with electroweak precision observables
  (where the effects are large)
  and with Higgs physics
  (where the effects are small)