Neutrino masses and LHC

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Introduction

Within the Standard Model neutrinos are massless.

We know from oscillation experiments that neutrinos do have mass.

$\Rightarrow$ Neutrino mass implies physics beyond the SM.

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We know from oscillation experiments that neutrinos do have mass.

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Can we learn something about the new physics responsible for neutrino mass at LHC?

A random selection of references on this topic:

G. Senjanovic, J. Phys. Conf. Ser. **136** (2008) 022039. A. Datta and S. Poddar, 0901.1619 . Z. Z. Xing, 0901.0209 S. Blanchet, Z. Chacko and R. N. Mohapatra, 0812.3837 P. Fileviez Perez, T. Han, T. Li and M. J. Ramsey-Musolf, 0810.4138 F. del Aguila and J. A. Aguilar-Saavedra, Phys. Lett. B **672** (2009) 158 T. Ibrahim and P. Nath, Phys. Rev. D **78** (2008) 075013 P. Fileviez Perez, T. Han, G. y. Huang, T. Li and K. Wang, Phys. Rev. D **78** (2008) 015018 S. Gabriel, B. Mukhopadhyaya, S. Nandi and S. K. Rai, Phys. Lett. B **669** (2008) 180 F. de Campos *et al.* Phys. Rev. D **77** (2008) 115025 P. Fileviez Perez, T. Han, G. Y. Huang, T. Li and K. Wang, Phys. Rev. D **78** (2008) 071301 S. Bar-Shalom, G. Eilam, T. Han and A. Soni, Phys. Rev. D **77** (2008) 115019 A. G. Akeroyd, M. Aoki and H. Sugiyama, Phys. Rev. D **77** (2008) 075010 J. Garayoa and T. Schwetz, JHEP **0803** (2008) 009 C. S. Chen, C. Q. Geng, J. N. Ng and J. M. S. Wu, JHEP **0708** (2007) 022 T. Han, B. Mukhopadhyaya, Z. Si and K. Wang, Phys. Rev. D **76** (2007) 075013 J. Kersten and A. Y. Smirnov, Phys. Rev. D **76** (2007) 073005 A. Hektor, M. Kadastik, M. Muntel, M. Raidal and L. Rebane, Nucl. Phys. B **787** (2007) 198 F. M. L. de Almeida *et al.*, Phys. Rev. D **75** (2007) 075002 S. Bray, J. S. Lee and A. Pilaftsis, Nucl. Phys. B **786** (2007) 95 L. D. Ninh and L. N. Hoang, Phys. Rev. D **72** (2005) 075004 W. Porod and P. Skands, hep-ph/0401077. S. N. Gninenko, M. M. Kirsanov, N. V. Krasnikov and V. A. Matveev, hep-ph/0301140. A. Ali, A. V. Borisov and N. B. Zamorin, Eur. Phys. J. C **21** (2001) 123 E. Fernandez, Nucl. Phys. Proc. Suppl. **31** (1993) 326. A. Datta, M. Guchait and D. P. Roy, Phys. Rev. D **47** (1993) 961 J. N. Esteves *et al.* 0903.1408 A. Villanova del Moral, 0810.3270 I. Gogoladze, N. Okada and Q. Shafi, Phys. Lett. B **672**, 235 (2009) F. del Aguila and J. A. Aguilar-Saavedra, Nucl. Phys. B **813**, 22 (2009) M. Hirsch, S. Kaneko and W. Porod, Phys. Rev. D **78**, 093004 (2008) M. Hirsch *et al.*, Phys. Rev. D **78**, 013006 (2008) I. Gogoladze, N. Okada and Q. Shafi, Phys. Rev. D **78**, 085005 (2008) B. Bajc, M. Nemevsek and G. Senjanovic, Phys. Rev. D **76**, 055011 (2007) B. Bajc and G. Senjanovic, JHEP **0708**, 014 (2007)

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there is a huge literature

I will be very sloppy with citations and apologize for omissions

Seesaw

Weinberg 1979: there is one dim-5 operator in the SM, which will lead to a Majorana mass term for neutrinos after EWSB:

$$\frac{L^T \tilde{\phi}^* \, \tilde{\phi}^\dagger L}{\Lambda} \quad \longrightarrow \quad m_\nu \sim \frac{v^2}{\Lambda}$$

This implies that the physics responsible for neutrino masses lives at the very high scale

$$\Lambda \sim 10^{14}\,\mathrm{GeV}$$

which is impossible to probe at LHC or any other imaginable collider experiment.

Neutrino masses from the TeV scale

Maybe the BSM physics expected[a] around TeV and searched for at LHC can also be responsible for the generation of neutrino masses:

$\Rightarrow$ TeV scale neutrino mass models

Generically in such models seesaw suppression is not sufficient, one needs additional means to obtain small neutrino masses:

- putting small numbers by hand
- cancellations between large terms
- radiative neutrino masses
- …

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[a] stabilizing the Higgs mass, Dark Matter,...

Type I, II, III seesaw

3 realizations of the Weinberg operator:

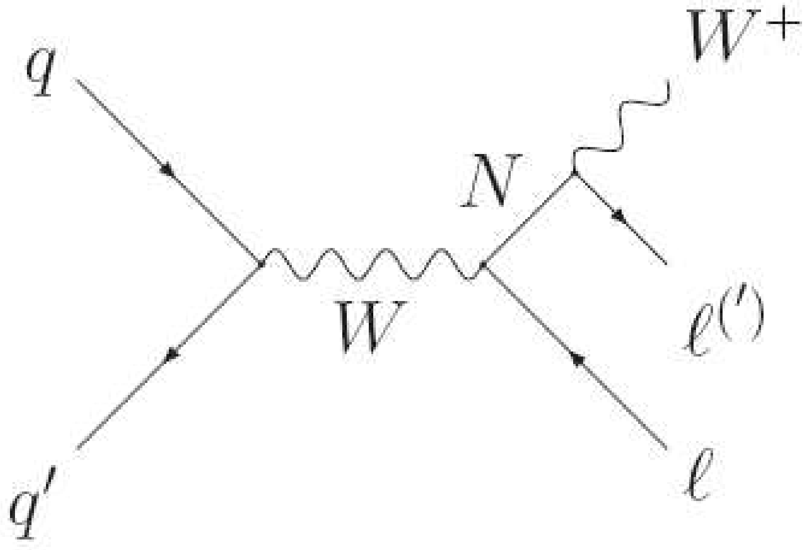
UV completion by

- Type I: fermionic singlet (right-handed neutrinos)
- Type II: scalar triplet
- Type III: fermionic triplet

What happens if these new particles do not have masses of order 10^{14} GeV but only few 100 GeV, within the LHC reach?

Type I seesaw at LHC

e.g., Han, Zhang, 06; Del Aguila, Aguilar-Saavedra, Pittau 07; Kersten Smirnov 07; ...
see also talk by S. Pascoli



$\Rightarrow$ dilepton (or multi-lepton) events, e.g.:

lepton number violating: $\ell^\pm\ell^\pm$ + jets or
lepton flavour violating: $\ell^\pm_\alpha\ell^\mp_\beta$ + jets

Type I seesaw at LHC

$$m^{\nu}_{\alpha\beta} = v^2 \sum_i \frac{Y_{\alpha i} Y_{\beta i}}{M_i}$$

If heavy neutrino masses M_i are not so heavy there are two possibilities to obtain small neutrino masses:

1. small Yukawas $Y_{\alpha i} \sim 10^{-6}$ (electron Yukawa)
2. cancellations in the sum over N_i

Buchmüller, Wyler, 1990; Pilaftsis, 1992

Type I seesaw at LHC

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2. cancellations in the sum over N_i

add 1: since N_i are SM singlets they interact only via Yukawas $\Rightarrow$ tiny Yukawas imply negligible production rate at LHC.

Type I seesaw at LHC

$$m^{\nu}_{\alpha\beta} = v^2 \sum_i \frac{Y_{\alpha i} Y_{\beta i}}{M_i}$$

If heavy neutrino masses M_i are not so heavy there are two possibilities to obtain small neutrino masses:

1. small Yukawas $Y_{\alpha i} \sim 10^{-6}$ (electron Yukawa)
2. cancellations in the sum over N_i

add 2: cancellations could be motivated by symmetries, but decouple LHC signatures from light neutrino mass matrix Kersten, Smirnov, 07

Seesaw at LHC with tiny Yukawas

Way out: give N_i gauge interaction, such that new production channels at LHC open:

- Low scale Left-Right symmetry
  $q\bar{q} \to W_R \to N\ell \to \ell\ell$ + jets
  e.g., Keung, Senjanovic, 1983
- Type II seesaw: $N \to \Delta$ scalar triplet
  $q\bar{q} \to Z^0(\gamma) \to \Delta^{--}\Delta^{++} \to \ell^-\ell^-\ell^+\ell^+$
  see below
- Type III seesaw: $N \to T$ fermionic triplet
  $q\bar{q} \to W^- \to T^-T^0 \to \ell^-\ell^-$ + jets
  e.g., Bajc, Senjanovic, 07; Franceschini, Hambye, Strumia, 08
- provide some new BSM interaction for N

Other examples of TeV scale ν masses

- R-parity violating SUSY: neutrino mass generation is related to lepton number violating terms in superpotential $\Rightarrow$ can study neutrino properties by observing R-parity violating decays of the LSP (neutralino) at LHC

  Hirsch, Porod, Romao, Valle, Dedes, Allanach, many many others

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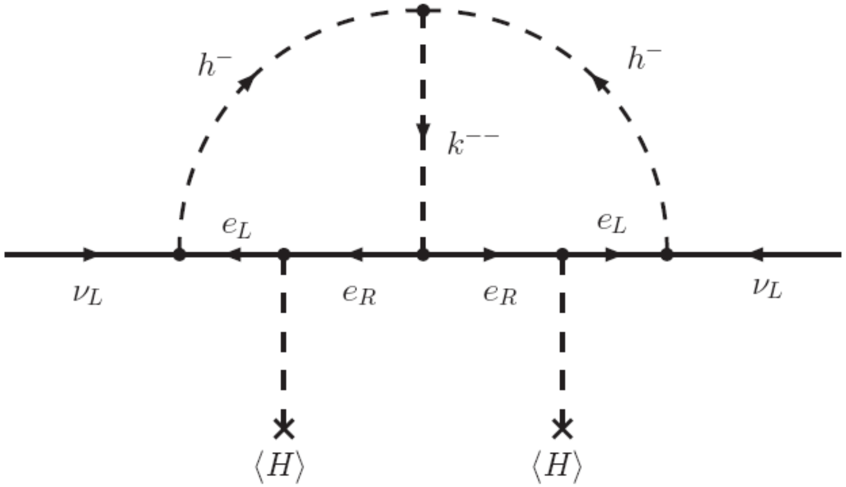
- radiative neutrino mass generation, ex.: Zee-Babu



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- radiative neutrino mass generation, ex.: Zee-Babu good prosprects to see doubly-charged scalar at LHC $\rightarrow$ like-sign lepton events; if k^{++} is within reach for LHC the model is tightly constrained by perturbativity requirements and bounds from LFV

  Babu, Macesanu, 02; Aristizabal, Hirsch, 06; Nebot et al., 07

The Higgs triplet model and LHC

(I prefer not to call it Type-II seesaw)

based on J. Garayoa and T. Schwetz, JHEP **0803** (2008) 009 [0712.1453]

other recent works:

A. Hektor et al., Nucl. Phys. B **787** (2007) 198 [0705.1495].

T. Han, B. Mukhopadhyaya, Z. Si and K. Wang, Phys. Rev. D **76** (2007) 075013 [0706.0441].

A. G. Akeroyd, M. Aoki and H. Sugiyama, Phys. Rev. D **77** (2008) 075010 [0712.4019].

M. Kadastik, M. Raidal and L. Rebane, Phys. Rev. D **77** (2008) 115023 [0712.3912].

P. Fileviez Perez et al., Phys. Rev. D **78** (2008) 015018 [0805.3536].

The model

add a triplet Δ under SU(2)$_L$ to the SM:

$$\mathcal{L}_\Delta = f_{ab}\, L_a^T C^{-1}\, i\tau_2 \Delta\, L_b + \text{h.c.}\,,$$

$$\Delta = \begin{pmatrix} H^+/\sqrt{2} & H^{++} \\ H^0 & -H^+/\sqrt{2} \end{pmatrix}$$

The VEV of the neutral component $\langle H^0 \rangle \equiv v_T/\sqrt{2}$ induces a Majorana mass term for the neutrinos:

$$\frac{1}{2}\nu_{La}^T C^{-1} m^\nu_{ab}\, \nu_{Lb} + \text{h.c.} \qquad \text{with} \qquad m^\nu_{ab} = \sqrt{2}\, v_T\, f_{ab}$$

Neutrino masses

$$m^{\nu}_{ab} = \sqrt{2}\, v_T\, f_{ab} \lesssim 10^{-10}\,\mathrm{GeV}$$

Neutrino masses are small because of

- a small triplet VEV v_T
- small Yukawas f_{ab}
- or a combination of these two

Neutrino masses

$$m^{\nu}_{ab} = \sqrt{2}\, v_T\, f_{ab} \lesssim 10^{-10}\,\mathrm{GeV}$$

Lepton number violating term in Higgs potential: $\mu\phi^\dagger\Delta\tilde{\phi}$

minimisation of the potential gives

$$v_T \sim \mu\frac{v^2}{M_\Delta^2}$$

Neutrino masses

$$m^\nu_{ab} = \sqrt{2}\, v_T\, f_{ab} \lesssim 10^{-10}\,\mathrm{GeV}$$

Lepton number violating term in Higgs potential: $\mu\phi^\dagger\Delta\tilde{\phi}$
minimisation of the potential gives

$$v_T \sim \mu\frac{v^2}{M_\Delta^2}$$

Type II seesaw: heavy triplet

$$\mu \sim M_\Delta \sim 10^{14}\,\mathrm{GeV} \qquad \Rightarrow \qquad v_T \sim m^\nu\,,\ f_{ab} \sim \mathcal{O}(1)$$

Neutrino masses

$$m^{\nu}_{ab} = \sqrt{2}\, v_T\, f_{ab} \lesssim 10^{-10}\, \mathrm{GeV}$$

Lepton number violating term in Higgs potential: $\mu \phi^\dagger \Delta \tilde{\phi}$

minimisation of the potential gives

$$v_T \sim \mu \frac{v^2}{M_\Delta^2}$$

if we want to see the triplet at LHC we need a light triplet:

$M_\Delta \sim v \sim 100\, \mathrm{GeV} \quad \Rightarrow \quad v_T \sim \mu$

light neutrinos require a small μ

put “by hand” (technically natural → Lepton number)

The triplet at LHC

$$pp \to Z^*(\gamma^*) \to H^{++}H^{--} \to \ell^+\ell^+\,\ell^-\ell^-$$

doubly charged component of the triplet:

$$\Delta = \begin{pmatrix} H^+/\sqrt{2} & H^{++} \\ H^0 & -H^+/\sqrt{2} \end{pmatrix}$$

very clean signature:

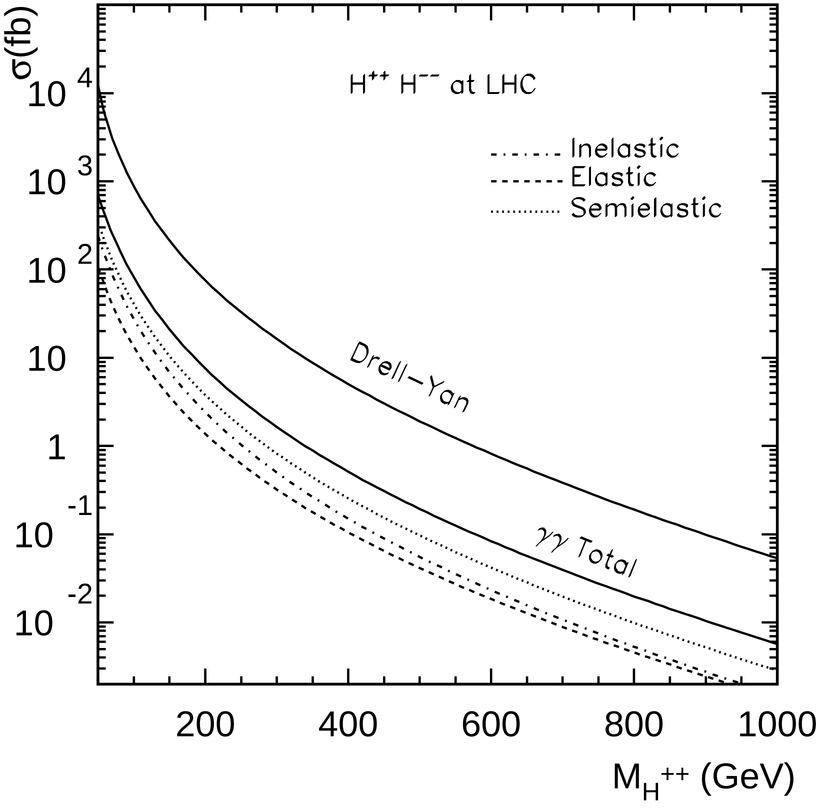
two like-sign lepton paris with the same invariant mass and no missing transverse momentum

practically no SM background

The triplet at LHC

$$pp \to Z^*(\gamma^*) \to H^{++}H^{--} \to \ell^+\ell^+\,\ell^-\ell^-$$

promising production rate: Han, Mukhopadhyaya, Si, Wang, 0706.0441



Decays of the triplet

remember: $\mathcal{L}_\Delta = f_{ab}\, L_a^T C^{-1}\, i\tau_2 \Delta\, L_b\,, \quad m^\nu_{ab} = \sqrt{2}\, v_T\, f_{ab}$

$$\Gamma(H^{++} \to \ell_a^+ \ell_b^+) = \frac{1}{4\pi(1+\delta_{ab})}\, |f_{ab}|^2 M_{H^{++}}\,,$$

$\Rightarrow$ Decays of doubly charged Higgs are proportional to the elements of the neutrino mass matrix!

Decays of the triplet

other decay channels of the doubly-charged Higgs:

$H^{++} \to H^+H^+$
$H^{++} \to H^+W^+$
$H^{++} \to W^+W^+$

The first two decay modes depend on the mass splitting within the triplet $\longrightarrow$ assume that they are kinematically suppressed.

Decays of the triplet

other decay channels of the doubly-charged Higgs:

$H^{++} \rightarrow H^+ H^+$
$H^{++} \rightarrow H^+ W^+$
$H^{++} \rightarrow W^+ W^+$

rate for the WW mode depends on triplet VEV v_T

$$\Gamma(H^{++} \rightarrow W^+ W^+) \approx v_T^2 M_{H^{++}}^3 / (2\pi v^4)$$

require $\Gamma(H^{++} \rightarrow W^+ W^+) \lesssim \Gamma(H^{++} \rightarrow \ell_a^+ \ell_b^+) \quad \Rightarrow$

$$\frac{v_T}{v} \lesssim 10^{-6} \left(\frac{100\,\text{GeV}}{M_{H^{++}}} \right)^{1/2}$$

$\longrightarrow$ save from LEP EW precision tests

Range for Yukawas and triplet VEV

bounds on LFV constrain triplet Yukawas f_{ab}
most stringent from $\mu \to eee$ (tree level in this model)

$$4 \times 10^{-7} \left(\frac{M_{H^{++}}}{100\,\text{GeV}} \right)^{1/2} \lesssim f_{ab} \lesssim 5 \times 10^{-4} \left(\frac{M_{H^{++}}}{100\,\text{GeV}} \right)$$

and with $v_T f_{ab} \sim 0.1\,\text{eV}$:

$$0.2\,\text{keV} \left(\frac{M_{H^{++}}}{100\,\text{GeV}} \right)^{-1} \lesssim v_T \lesssim 0.2\,\text{MeV} \left(\frac{M_{H^{++}}}{100\,\text{GeV}} \right)^{-1/2}$$

The branchings $H^{++} \to \ell_a^+ \ell_b^+$

$$m^\nu_{ab} = \sqrt{2}\, v_T\, f_{ab}\,, \qquad \Gamma(H^{++} \to \ell_a^+ \ell_b^+) = \frac{1}{4\pi(1+\delta_{ab})}\, |f_{ab}|^2 M_{H^{++}}\,,$$

$$\mathrm{BR}_{ab} \equiv \frac{\Gamma(H^{++} \to \ell_a^+ \ell_b^+)}{\sum_{cd} \Gamma(H^{++} \to \ell_c^+ \ell_d^+)} = \frac{2}{(1+\delta_{ab})} \frac{|M_{ab}|^2}{\sum_{cd} |M_{cd}|^2}$$

and

$$\sum_{cd} |M_{cd}|^2 = \sum_{i=1}^{3} m_i^2 = \begin{cases} 3m_0^2 + \Delta m^2_{21} + \Delta m^2_{31} & \text{(NH)} \\ 3m_0^2 + \Delta m^2_{21} + 2|\Delta m^2_{31}| & \text{(IH)} \end{cases}$$

The branchings $H^{++} \to \ell_a^+ \ell_b^+$

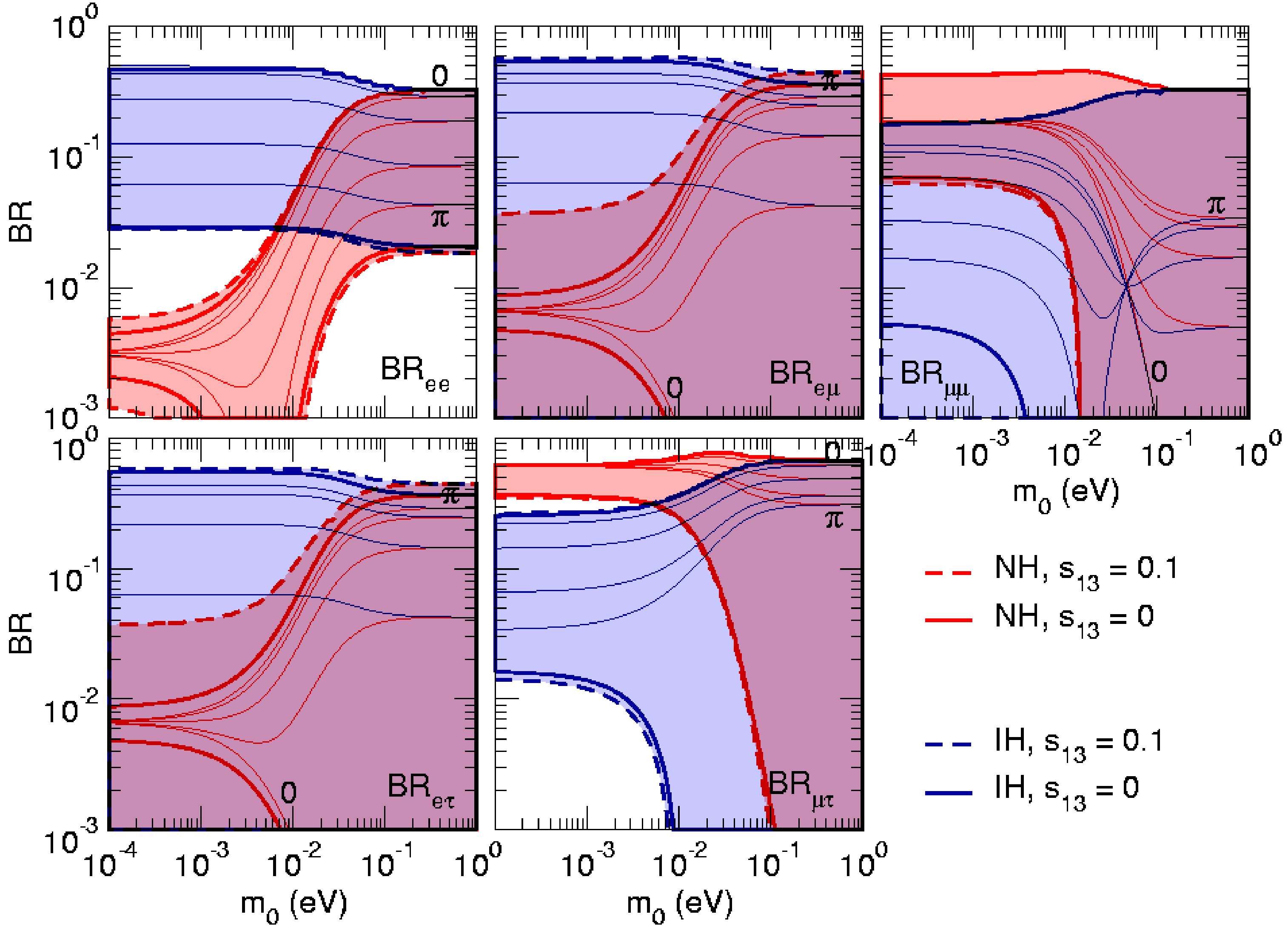
the branchings

$$\mathrm{BR}_{ab} = \frac{2}{(1+\delta_{ab})} \frac{|M_{ab}|^2}{\sum_{i=1}^{3} m_i^2}$$

depend on

- the lightest neutrino mass m_0
- the type of the neutrino mass ordering (NH/IH)
- Majorana CP phases

The branchings $H^{++} \to \ell_a^+ \ell_b^+$



Numerical analysis

Assume two cases: $\epsilon N_{2H} = 100$ or 1000

ϵ : detection efficiency

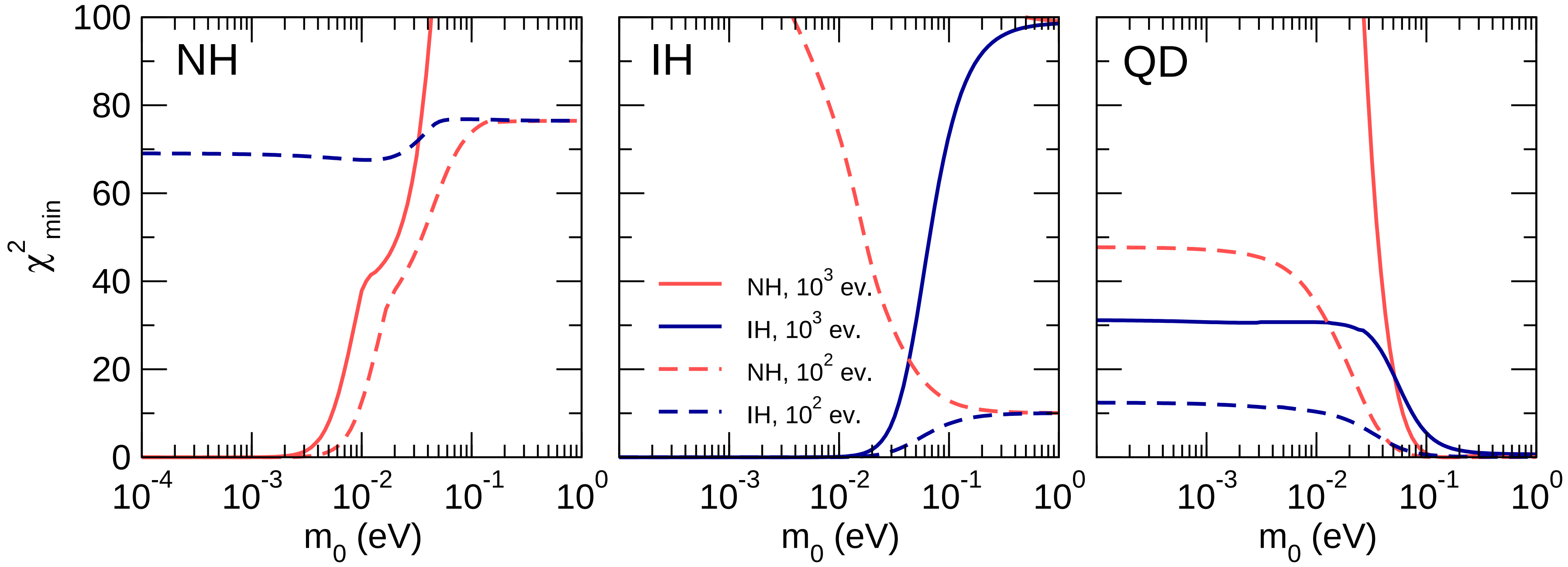
N_{2H} : total number of doubly charged Higges decaying to 4ℓ

for 100 fb^{-1} at LHC we will have roughly 1000 (100) events for $M_{H^{++}} \simeq 350$ GeV (600 GeV).

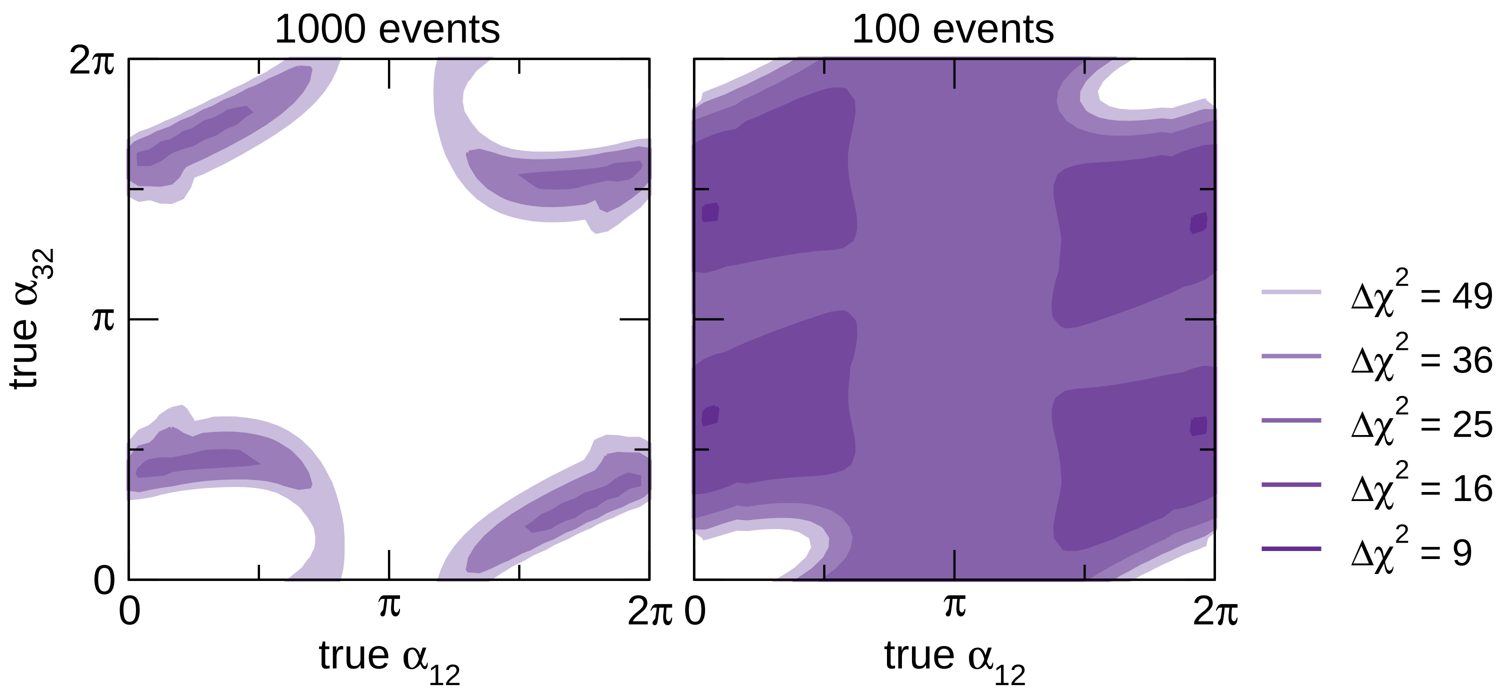
consider all possible flavour combinations of 4 lepton events allowing for at most one τ among the 4 leptons

perform a χ^2 fit for the 5 observables corresponding to the number of like-sign lepton pairs with the flavour combinations $(ee), (e\mu), (\mu\mu), (e\tau), (\mu\tau)$.

Determining the hierarchy

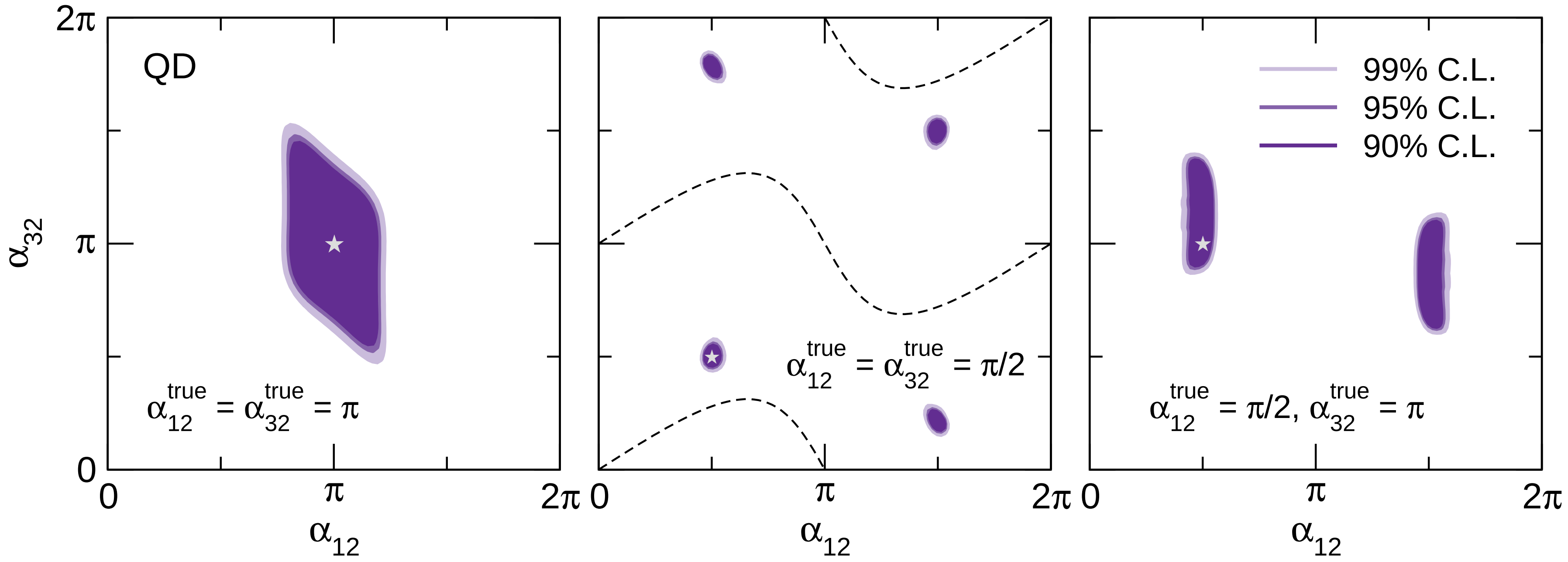


IH versus QD

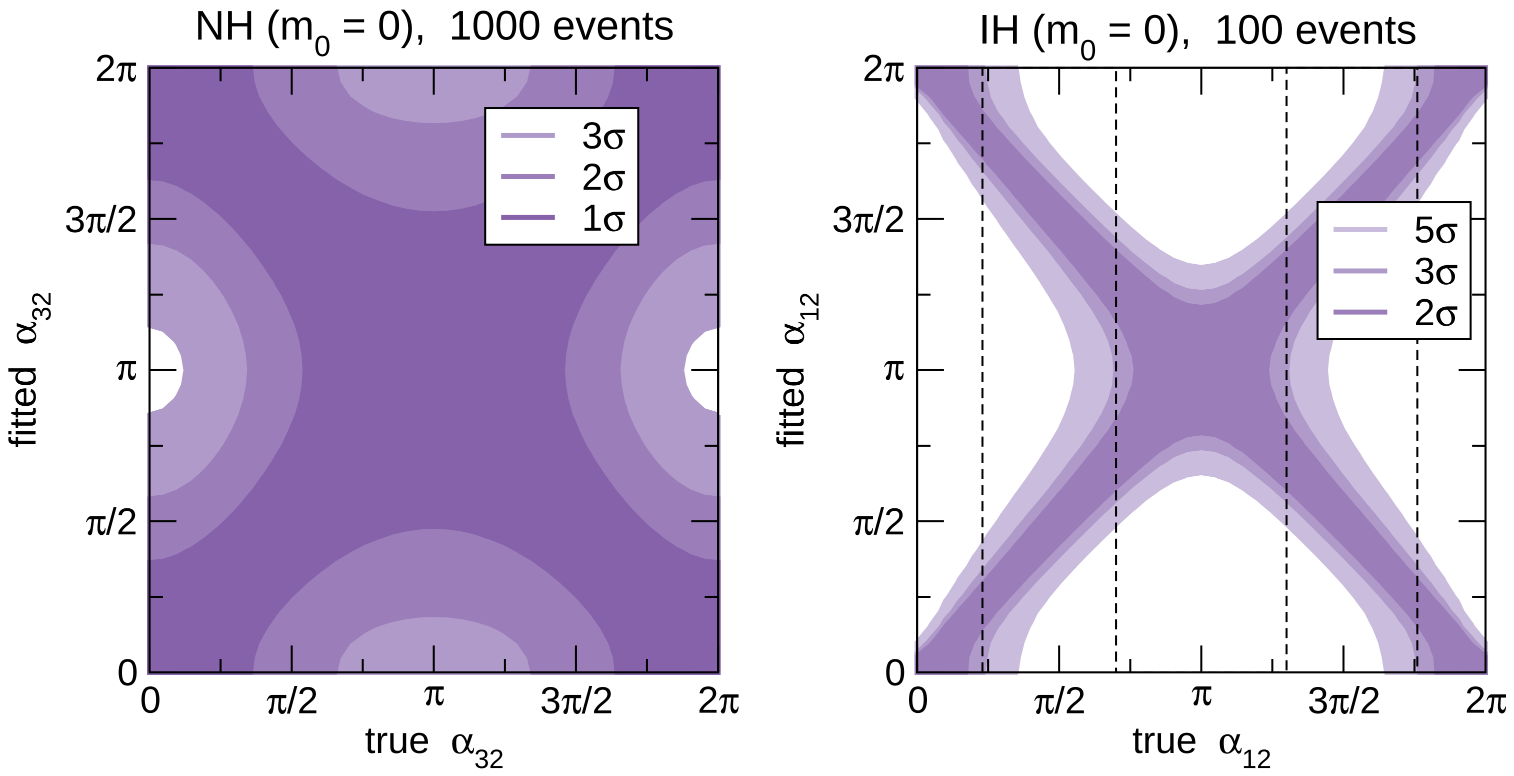


excluding IH with $m_0 = 0$ in case of true QD

Measuring Majorana phases for QD



Measuring Majorana phases for $m_0 = 0$



Comment on CP violation

there is no CP-odd observable

$$\Gamma(H^{++} \to \ell_a^+ \ell_b^+) = \Gamma(H^{--} \to \ell_a^- \ell_b^-) \propto |m_{ab}^\nu|^2$$

$\Rightarrow$ only $\cos(\alpha_{ij})$ can be measured

nevertheless one can (in principle) confine the Majorana phases to CP violating values

(like in neutrino-less double beta decay)

Summary

Summary

- typical seesaw models for neutrino mass are very hard to test at LHC
- no chance to test neutrino properties at LHC for type-I seesaw even if right-handed neutrinos have TeV masses
- but there are many examples for models where neutrino masses are generated by physics at the TeV scale, testable at LHC
- the typical signature of TeV scale neutrino mass models are like-sign lepton events
- the relation of LHC observables to neutrino properties is very model dependent

Summary

I have discussed one particular example where neutrino mass emerges from the VEV of a Higgs triplet at the TeV scale

- very simple extension of the SM
- Triplets occur in many BSM theories
  e.g., L-R symmetric models, Little Higgs theories
- need Yukawas $\mathcal{O}(10^{-6})$ and a triplet VEV $\ll$ EW scale
- clean signature at LHC: di-lepton events and nothing else
- can directly probe the neutrino mass matrix through the branching ratios of the decays $H^{\pm\pm} \to \ell_a^\pm \ell_b^\pm$
- can even measure Majorana phases

Let’s hope for doubly-charged Higges at LHC ...

Thank you for your attention!

Additional slides

The branchings $H^{++} \to \ell_a^+ \ell_b^+$ for NH

$$
\begin{aligned}
\mathrm{BR}_{ee}^{\mathrm{NH},m_0=0} &\approx s_{12}^4 r + 2 s_{12}^2 s_{13}^2 \sqrt{r} \cos(\alpha_{32} - 2\delta)\,, \\
\mathrm{BR}_{e\mu}^{\mathrm{NH},m_0=0} &\approx 2\left[s_{12}^2 c_{12}^2 c_{23}^2 r + s_{23}^2 s_{13}^2 + 2 s_{12} c_{12} s_{23} c_{23} s_{13} \sqrt{r} \cos(\alpha_{32} - \delta)\right]\,, \\
\mathrm{BR}_{\mu\mu}^{\mathrm{NH},m_0=0} &\approx s_{23}^4 + 2 s_{23}^2 c_{23}^2 c_{12}^2 \sqrt{r} \cos\alpha_{32} + c_{23}^4 c_{12}^4 r \\
&\qquad\qquad -4 s_{23}^3 c_{23} s_{12} c_{12} s_{13} \sqrt{r} \cos(\alpha_{32} - \delta)\,, \\
\mathrm{BR}_{e\tau}^{\mathrm{NH},m_0=0} &\approx 2\left[s_{12}^2 c_{12}^2 s_{23}^2 r + c_{23}^2 s_{13}^2 - 2 s_{12} c_{12} s_{23} c_{23} s_{13} \sqrt{r} \cos(\alpha_{32} - \delta)\right]\,, \\
\mathrm{BR}_{\mu\tau}^{\mathrm{NH},m_0=0} &\approx 2 s_{23}^2 c_{23}^2 \left(1 - 2 c_{12}^2 \sqrt{r} \cos\alpha_{32} + c_{12}^4 r\right)\,.
\end{aligned}
$$

The branchings $H^{++} \to \ell_a^+ \ell_b^+$ for IH

$$
\begin{aligned}
\mathsf{BR}_{ee}^{\mathrm{IH},m_0=0} &= \frac{1}{2}\left(1 - \sin^2 2\theta_{12} \sin^2 \frac{\alpha_{12}}{2}\right) \,, \\
\mathsf{BR}_{e\mu}^{\mathrm{IH},m_0=0} &= c_{23}^2 \sin^2 2\theta_{12} \sin^2 \frac{\alpha_{12}}{2} \,, \\
\mathsf{BR}_{\mu\mu}^{\mathrm{IH},m_0=0} &= \frac{c_{23}^4}{2}\left(1 - \sin^2 2\theta_{12} \sin^2 \frac{\alpha_{12}}{2}\right) \,, \\
\mathsf{BR}_{e\tau}^{\mathrm{IH},m_0=0} &= s_{23}^2 \sin^2 2\theta_{12} \sin^2 \frac{\alpha_{12}}{2} \,, \\
\mathsf{BR}_{\mu\tau}^{\mathrm{IH},m_0=0} &= \frac{1}{4} \sin^2 2\theta_{23} \left(1 - \sin^2 2\theta_{12} \sin^2 \frac{\alpha_{12}}{2}\right) \,,
\end{aligned}
$$

The branchings $H^{++} \to \ell_a^+ \ell_b^+$ for QD

$$\begin{aligned}
\mathrm{BR}_{ee}^{\mathrm{QD}} &= \frac{1}{3}\left(1 - \sin^2 2\theta_{12}\,\sin^2\frac{\alpha_{12}}{2}\right) = \frac{2}{3}\,\mathrm{BR}_{ee}^{\mathrm{IH},m_0=0}\,,\\
\mathrm{BR}_{e\mu}^{\mathrm{QD}} &= \frac{2}{3}\,c_{23}^2\,\sin^2 2\theta_{12}\,\sin^2\frac{\alpha_{12}}{2} = \frac{2}{3}\,\mathrm{BR}_{e\mu}^{\mathrm{IH},m_0=0}\,,\\
\mathrm{BR}_{\mu\mu}^{\mathrm{QD}} &= \frac{1}{3}\left[1 - \frac{1}{2}\sin^2 2\theta_{23}\left(1 - s_{12}^2\cos\alpha_{31} - c_{12}^2\cos\alpha_{32}\right) - c_{23}^4\sin^2 2\theta_{12}\sin^2\frac{\alpha_{12}}{2}\right],\\
\mathrm{BR}_{e\tau}^{\mathrm{QD}} &= \frac{2}{3}\,s_{23}^2\,\sin^2 2\theta_{12}\,\sin^2\frac{\alpha_{12}}{2} = \frac{2}{3}\,\mathrm{BR}_{e\tau}^{\mathrm{IH},m_0=0}\,,\\
\mathrm{BR}_{\mu\tau}^{\mathrm{QD}} &= \frac{1}{3}\sin^2 2\theta_{23}\left(1 - s_{12}^2\cos\alpha_{31} - c_{12}^2\cos\alpha_{32} - \frac{1}{2}\sin^2 2\theta_{12}\sin^2\frac{\alpha_{12}}{2}\right).
\end{aligned}$$