Emergent Electroweak Gravity

Bob McElrath
CERN

New Instruments for Neutrino Relics and Mass, Dec 8, 2008

Outline

- Massive cosmological relics are quantum liquids (as opposed to a classical gas)

- Interactions of quantum liquids: massive cosmological relics are superfluids

- Neutrino pairing: BCS Superconductivity and the Kohn-Luttinger effect

- The neutrino condensates: Emergent Electroweak Gravity

Brief Review on the Cosmic Neutrino Background

Cosmic neutrinos decouple from the Big Bang plasma at a temperature around 2 MeV. At that time they have a thermal Fermi-Dirac distribution.

As the universe expands, their density and temperature red-shift, leading to

$$T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma = 1.95\text{K}; \qquad n_{\nu_i} = n_{\overline{\nu}_i} = \frac{3}{22} n_\gamma = \frac{56}{\text{cm}^3}$$

where T_γ and n_γ are the measured temperature and number density of CMB photons. Thus at least two species must be non-relativistic today. If neutrinos cluster gravitationally, the density is enhanced [Singh, Ma; Ringwald, Wong].

Due to large mixing, the flavor composition is equilibrated. All three mass eigenstates have equal densities. [Lunardini, Smirnov]

The asymmetry $\eta_\nu = (n_\nu - n_{\overline{\nu}})/n_\gamma$ is related to the baryon asymmetry $\eta_b = (n_b - n_{\overline{b}})/n_\gamma \simeq 10^{-10}$, so that any asymmetry can be neglected and we will assume $n_\nu = n_{\overline{\nu}}$.

What are neutrinos doing *today*

The dynamics of the neutrino background is given just by its kinetic term and self-interaction

$$\overline{\psi}\gamma^\mu\partial_\mu\psi - m\overline{\psi}\psi + \frac{g^2}{M_Z^2}\overline{\psi}\gamma^\mu\psi\overline{\psi}\gamma_\mu\psi$$

let us ignore the interactions for a few slides and concentrate on the kinetic term. It does 2 things:

- Gives rise to the 2 point function, transporting neutrinos in space
- Causes the expansion of the neutrino's wave packet

The latter effect is normally forgotten in QFT under the assumption that we have asymptotic localized particles. Is this a good assumption for a cosmological relic?

Wave packet expansion I/III

Wave packets expand because different wave numbers move at different velocities in the presence of a mass or interaction. The wave number at $p = p_0 + \Delta p$ moves with velocity $v = (p_0 + \Delta p)/E$ while the wave number on the other side moves with velocity $v = (p_0 - \Delta p)/E$, and these wave numbers separate in space.

Thus the uncertainty of a wave packet evolves as

$$\Delta x(t)^2 = \Delta x_0^2 + \Delta v^2 t^2$$

In the relativistic case we must use

$$\Delta v = \frac{\Delta p}{E}(1 - v^2).$$

Assuming the uncertainty is given by the de Broglie wavelength

$$\Delta x_0 = \lambda / p = \lambda / \sqrt{3mkT}$$

allows us to derive the usual condition for superfluidity with $t = 0$ (or equivalently $\Delta p = 0$) from $\Delta x > n^{-1/3}$:

$$T < \frac{n^{2/3}\lambda^2}{3mk}$$

Wave packet expansion II/III

The "quantum liquid" condition is:

$$\Delta x \gg n^{1/3}.$$

The opposite limit is the "classical gas" limit, and is the limit used by scattering theory (particles are localized):

$$\Delta x \ll n^{1/3}.$$

The temperature condition is valid only if scattering occurs sufficiently often that the time dependence of the wave packet can be neglected:

$$\tau \ll \frac{\Delta x}{\Delta v} = E\frac{\Delta x}{\Delta p}$$

where $\tau = (\sigma n v)^{-1}$ is the mean time between collisions. This holds for atomic and nuclear matter at the densities usually considered.

Notice that the other assumption $\Delta p = E\Delta v = 0$ implies $\Delta x = \infty$ and vacuum calculations are not appropriate. they must be done at finite density. (i.e. we're in a momentum eigenstate but there is *no empty space*)

Wave packet expansion conclusion

Putting everything together using $t = \tau$:

$$\frac{1}{p^2} + \frac{(1-v^2)^2}{\sigma^2 n^2} > \frac{1}{\lambda^2 n^{2/3}}. \quad (1)$$

If we can neglect the first term, which is valid for decoupled relics, we obtain the quantum liquid criterion for weakly coupled relics:

$$\sigma < \frac{\lambda(1-v^2)}{n^{2/3}}. \quad (2)$$

This is very (very very) well satisfied for both relic neutrinos and dark matter ($\sigma \simeq 10^{-56}\text{eV}^{-2}$, $n^{-2/3} \simeq 10^{-8}\text{eV}^{-2}$). This means:
1) We have to worry about the dynamics of a quantum liquid for any massive cosmological relic (dark matter, at least 2 flavors of neutrinos)
2) We (probably) need to worry about quantum liquid dynamics of massless relics (lightest neutrino, axions, photons) too, due to finite density effects in the early universe, which cause wave packet expansion.
Superfluidity occurs if there is any attractive interaction. (This is model dependent → rest of talk)

Where do we go from here?

How do we deal with this kind of quantum liquid, and what are its dynamics?

- Review BCS Superconductivity

- The Kohn-Luttinger Effect

- Back to the Standard Model: Emergent Electroweak Gravity

Scales

This interactions of cosmic neutrinos are a theory of contact interactions in a quantum liquid at finite density and zero temperature. The fundamental parameters are the Fermi momentum p_F and G_F.

Let us examine the effective range expansion of neutrino self-scattering to get an idea of the scales:

$$k \cot \delta_0 = -\frac{1}{a} + \frac{1}{2} k^2 l_0 + \ldots$$

where $a = \sqrt{\sigma_{\nu\nu}/4\pi} \simeq T_\nu G_F$ is the s-wave scattering length and $l_0 = \sqrt{G_F}$ is the range of the potential. Thus we have the approximation regime $a \ll l_0$.

This is the *opposite* approximation regime to atomic and nuclear finite density systems, BEC's, and BCS superconductivity, so one must be careful when applying results from those fields, and we want to take $a \to 0$.

Therefore, the leading dynamics occurs due to this p-wave term.

Dismiss some Bad Ideas

The leading interactions at finite density come in at $\mathcal{O}(p_F^2 G_F^2)$, but one can also write $T_\nu^2 G_F^2$ which is the same order. Is it relevant?

The combination $T_\nu^2 G_F^2$ is the self-interaction cross section of neutrinos. This would seem to be a hydrodynamic theory. However then one has to confront the flux. The inverse mean free path of a neutrino is

$$\lambda^{-1} = (\sigma n)^{-1} = T_\nu^2 p_F^3 G_F^2 \simeq \mathcal{O}(p_F^5 G_F^2)$$

and much larger than the horizon size, and the interaction rate is too low to be interesting.

One can also write $m_\nu^2 G_F^2$ but this would only arise in conjunction with p_F or T_ν.

$T_\gamma^2 G_F^2$ doesn't make sense.

The finite density theory of photons has a very small self-interaction (smaller than the neutrino self-interactions).

Review of BCS Superconductivity I/V

Superconductivity is an example of a Fermi liquid which also involves symmetry breaking. The mechanism is identical for superfluids: a superconductor is a charged superfluid.

The symmetry breaking occurs because there exists an attractive interaction which causes a divergence at the Fermi surface.

The Lagrangian is

$$\mathcal{L}_{BCS} = i\psi^\dagger \partial_t \psi + \psi^\dagger \left(\frac{\nabla^2}{2m} + \mu \right) \psi + \frac{g}{2} \psi^\dagger_\beta \psi^\dagger_\alpha \psi_\alpha \psi_\beta$$

where the interaction comes from integrating out the phonon (lattice vibrations) degrees of freedom to produce an attractive contact interaction.

The BCS approximation consists in an expectation value for a fermion bilinear

$$\psi_\beta \psi_\alpha \to \langle 0 | \psi_\beta \psi_\alpha | 0 \rangle = iF(0) \epsilon_{\beta\alpha}$$

The function $F(x-y)$ is the "anomalous" Green's function. It arises by expanding the original fermion ψ to encompass it's "hole" states that exist at the Fermi surface: $\psi_M = \binom{\psi}{\psi^\dagger}$. The two point Green's function is then

$$G_{\alpha\beta}(x) = \left(\begin{array}{cc} D_{\alpha\beta}(x) & F_{\alpha\beta}(x) \\ F^\dagger_{\alpha\beta}(x) & D^\dagger_{\alpha\beta}(x) \end{array} \right)$$

where $D_{\alpha\beta}(x)$ is the "normal" Green's function and $F_{\alpha\beta}$ is the "anomalous" Green's function with spin indices α, β. $F_{\alpha\beta}$ has all momenta *incoming*, and violates fermion number.

$F_{\alpha\beta}$ can also be thought of as an effective dynamical Majorana mass. Its solutions are

$$F_{\alpha\beta}(E,p) = \frac{-\Delta\epsilon_{\alpha\beta}}{E^2 - |\Delta|^2 - \left(\frac{p^2}{2m} - \mu\right)^2 + i\epsilon}$$

The expectation value for the fermion condensate is the order parameter for the breaking of the $U(1)$ symmetry breaking. For superconductors this is the breaking of the $U(1)$ electromagnetic gauge symmetry; for superfluids it is the breaking of the global $U(1)$ fermion number conservation. Thus we can write down the "gap" equation

$$\Delta = \langle \psi_\beta \psi_\alpha \rangle = igF(0)\epsilon_{\beta\alpha} = \frac{-ig}{(2\pi)^4} \int d^4 p F(E, p)$$

or

$$1 = \frac{2\pi g m}{(2\pi)^3} \int_{E_F - \omega_D/2}^{E_F + \omega_D/2} dE \sqrt{\frac{2mE}{(E - E_F)^2 + |\Delta|^2}}$$

If there exists a solution, for any value of Δ, then the long wavelength fluctuations of the order parameter $\langle \psi_\beta \psi_\alpha \rangle$ are a physical propagating field. The solution is

$$|\Delta| \simeq \omega_D e^{-2\pi^2 / g m p_F}$$

This does not fix the phase of the field Δ, which becomes a goldstone boson associated with the symmetry breaking.

Review of BCS Superconductivity IV/V

The Fermi Surface is an abstract 2-surface in momentum space,

$$p_F = (3\pi^2 n)^{1/3}; \qquad p^\mu = (0, \hat{p} p_F)$$

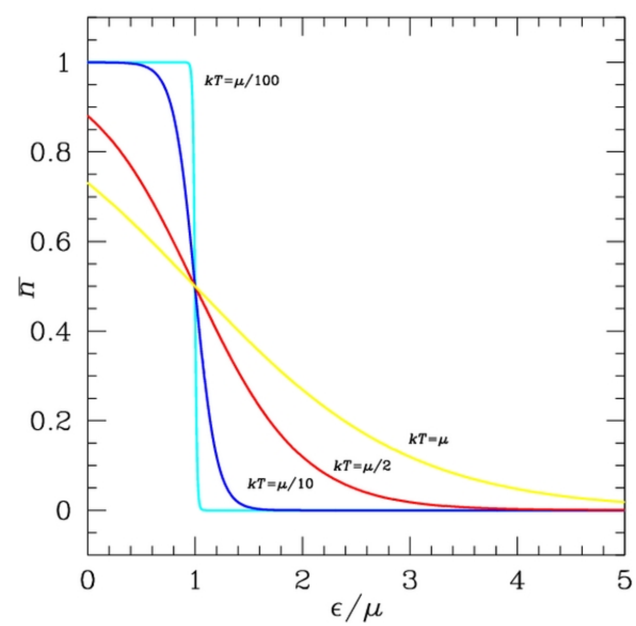
described by the unit 3-vector $\hat{p}$.

There is a massless singularity at the Fermi Surface: it costs zero energy to rotate the vector $\hat{p}$. The massless particles that appear (Cooper pair) have a momentum *relative* to the Fermi surface.

Oscillations on the Fermi surface with small non-zero energies corresponds to moving a wave number from one side of the Fermi-Dirac distribution at

$$\mu = E_F(T = 0) = \sqrt{m^2 + p_F^2}$$

to the other. ($\mu =$ chemical potential)



Review of BCS Superconductivity V/V

The critical ingredients in the BCS phenomena are:

1) The existence of an attractive interaction ($g > 0$). Without it, the Gap equation has no solution, and there is no symmetry breaking.

2) A quantum liquid: the condensate $\langle \psi_\beta \psi_\alpha \rangle$ requires two fields at the same space-time point.

At high temperatures with strong (Coulomb) interactions, particles are localized, and #2 is not satisfied.

Satisfying #1 in metals requires the phonon, since electrons are totally repulsive.

Temperature gets related to #2 only through the assumption that $\Delta x \simeq 1/p$.

Back to Cosmological Relics

I showed that #2 (quantum liquid) was satisfied for cosmological relics, now what about #1 (an attractive interaction)?

Dark Matter with an attractive self-interaction forms a superfluid in exactly the same way as a BCS superconductor. The long wavelength fluctuations of the condensate are a goldstone boson corresponding to the spontaneous breaking of the global $U(1)$ fermion number conservation.

The required self-interaction is a point operator which originates by integrating out heavy fields (e.g. squarks, higgses, sleptons, etc).

The attractive self-interaction need not be large. Any *infinitecimal* attractive coupling is sufficient to cause the symmetry breaking.

Neutrinos have *repulsive* self-interactions [Caldi, Chodos, '99]

Repulsive condensates: the Kohn-Luttinger Effect

There are only 2 options for the self-interactions: attractive or repulsive. Does a quantum fluid with repulsive interactions remain "normal" when $\Delta x \gg n^{-1/3}$?

This was answered in '65 by Kohn and Luttinger, and the answer is *no*.

Given a point-like interaction, the tree interaction scales as e^{-l^4} with partial wave number l.

The one loop correction has a new infrared divergence on the Fermi surface however, and gives rise to a correction

$$\delta V(\cos\theta) = -|V(\theta = 0)|^2 \frac{m p_F}{16\pi^2}(1 - \cos\theta)\log(1 - \cos\theta)$$

Kohn-Luttinger Effect

Let us expand this correction in partial waves

$$V_l = \int_{-1}^{1} d(\cos\theta) P_l(\cos\theta) V(\cos\theta)$$

$$V_l = (-1)^{l+1} \frac{mp_F}{4\pi^2} \frac{|V(\theta = 0)|^2}{l^4}.$$

Notice this is *attractive* for odd l.

The leading term occurs due to tree level t-channel Z exchange.

The conclusion of Kohn and Luttinger is that for fermions with any sufficiently weak interaction mediated by heavy fields, they must undergo a superfluid transition if the temperature becomes low enough, because this correction scales as l^{-4} while the direct (tree level) interaction scales as e^{-l^4}.

In other words, there *always* exists a superfluid state for a cold, weakly coupled Fermi gas.

Kohn-Luttinger, ctd. . .

Let us decompose the previous result into (non-relativistic) diagrams:

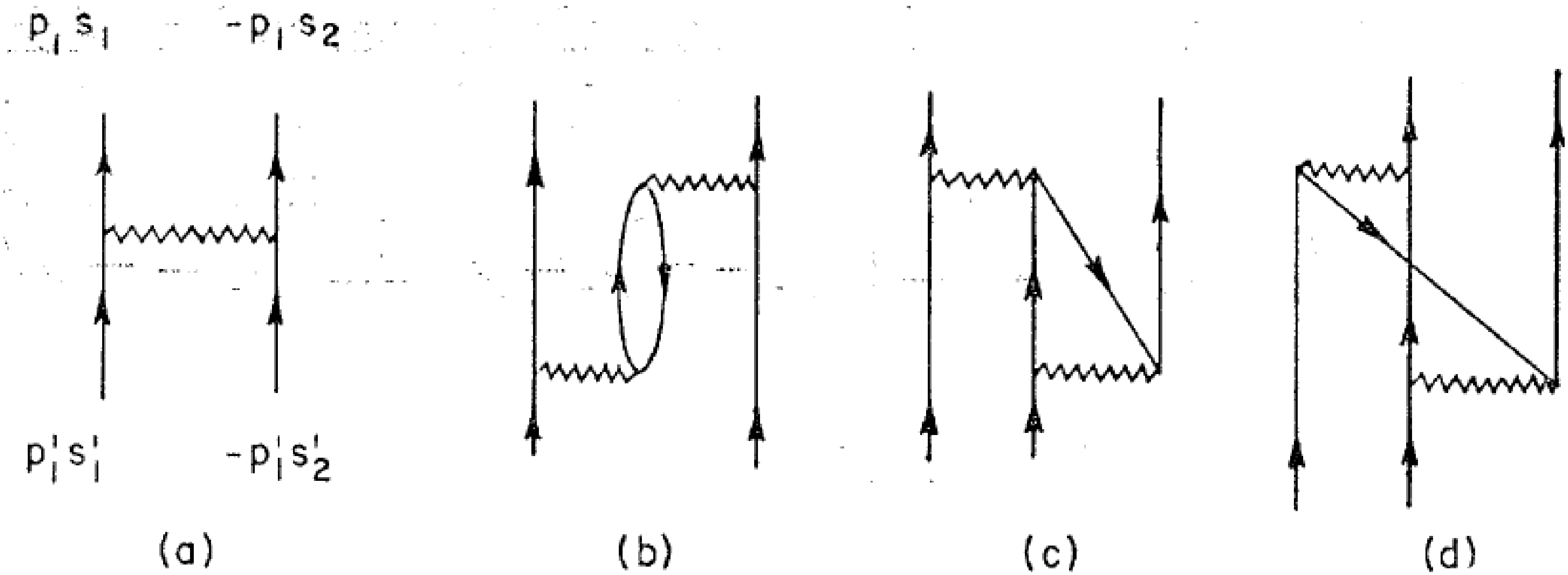


Diagram (a) is finite and repulsive. Diagrams (b) and (c) have logarithmic singularities at $\cos\theta = 1$. Diagram (d) has a logarithmic singularity at $\cos\theta = -1$.

Diagram (d) is an *exchange* interaction. That is, the propagating neutrino and the background neutrino change places.

Kohn-Luttinger, ctd. . .

The Fermi surface in this case is at $2p_F$ rather than at p_F as in the case of BCS superconductivity. This is because the particles running in the loops are what cause the divergence, and there are 2 of them.

The coefficient of this new attractive term on the Fermi surface is

$$\frac{g^4 m p_F}{4\pi^2 M_Z^4}$$

Because this condensate occurs in the p-wave, these s-wave condensates do not occur

$$\psi\psi; \qquad \psi^\dagger \overline{\sigma}^a \psi$$

the contact interaction for the latter is related to the former by a Fierz transformation. The first is a Cooper pair. The second could only form in the presence of a particle-antiparticle gas (i.e. impossible for electrons, baryons).

The actual condensate contains a derivative.

Identify the neutrino condensate

Let us examine the possible quasi-particles containing one derivative:

$$A_\mu(x,y) = \partial_\mu \chi(x)\epsilon\chi(y) - \chi(x)\epsilon\partial_\mu\chi(y)$$

$$E^a_\mu(x,y) = \partial_\mu \chi^\dagger(x)\overline{\sigma}^a\chi(y) - \chi^\dagger(x)\overline{\sigma}^a\partial_\mu\chi(y)$$

These arise from integrating out the Z and including the 1-loop corrections from the previous slide. The 4-point interactions are

$$A^\dagger_\mu A^\mu; \qquad E^{a\dagger}_\mu E^\mu_a$$

these are related to each other by a Fierz transformation, and also related to

$$\frac{g_Z^4 q^2}{M_Z^4}\left[\chi(x)\overline{\sigma}^a\chi(x)\chi(y)\overline{\sigma}_a\chi(y)\right]$$

The interaction terms on the Fermi surface are

$$-\frac{g^4 m p_F}{4\pi^2 M_Z^4} A^\dagger_\mu A^\mu; \qquad -\frac{g^4 m p_F}{4\pi^2 M_Z^4} E^{a\dagger}_\mu E^\mu_a$$

these are clearly tachyonic mass terms (though the composites need to be rescaled to make them dimension 1 and 0).

Tetrad Gap equation

In matrix notation ($\psi = (\chi, \chi^\dagger)$), the inverse fermion propagator is

$$G^{-1}(x,y) = \begin{pmatrix} \partial_\mu \overline{\sigma}^a(\delta^\mu_a + e^\mu_a) & m \\ m & \partial_\mu \overline{\sigma}^a(\delta^\mu_a + e^\mu_a) \end{pmatrix} = \begin{pmatrix} D_{\alpha\beta}(x) & F_{\alpha\beta}(x) \\ F^\dagger_{\alpha\beta}(x) & D^\dagger_{\alpha\beta}(x) \end{pmatrix}$$

where we assume in the 4-Fermi interaction $E^a_\mu E^\mu_a$ that we take the mean field of one of the E^a_μ's ($= e^a_\mu$). The quasiparticle corresponds to an expectation value for the fermion's Green's function.

However unlike the BCS cooper pair, this is a $\chi^\dagger\chi$ condensate not a $\chi\chi$ condensate. Therefore there is an expectation value for the diagonal component of $G(p)$:

$$D_{\alpha\beta}(p) = \frac{p_\mu \overline{\sigma}^a(\delta^\mu_a + e^\mu_a)}{p^\mu p^\nu \eta^{ab}(\delta^\mu_a + e^\mu_a)(\delta^\nu_b + e^\nu_b) - m^2 + i\epsilon}$$

There is a solution for its expectation value as long as the interaction is attractive.

A tale of two condensates

The A_μ condensate is a particle-particle (or antiparticle-antiparticle condensate) while E^a_μ is a particle-antiparticle condensate. Therefore the original interaction can be rewritten:

$$-\frac{g_Z^4 m p_F}{4\pi^2 M_Z^4}\int_{xy}\left[(1-\eta_\nu)E^{a\dagger}_\mu E^\mu_a + \eta_\nu A^\dagger_\mu A^\mu\right].$$

where

$$\eta_\nu = \frac{n_\nu - n_{\overline{\nu}}}{n_\nu + n_{\overline{\nu}}}$$

is the asymmetry between neutrinos and anti-neutrinos. After the phase transition has occured, the original Fermi gas is described by momentum distribution functions for A_μ and E^a_μ, rather than original one for free fermions.

Because $\eta_\nu \simeq 10^{-10}$ (it is related to the baryon-to-photon ratio), the dominant condensate is E^a_μ, and I will neglect A_μ hereafter.

Neutrino condensate symmetry breaking

The original action has a particle number global symmetry:

$$\chi \to e^{-i\theta}\chi$$

This is broken by the condensation of the field A^μ which is a $\chi\chi$ condensate. A_μ also breaks translation invariance. $\Rightarrow$ the long wavelength fluctuations around the order parameter A^μ are a gravitationally coupled goldstone vector boson.

E_μ^a is a $\chi^\dagger\chi$ condensate so does not break the $U(1)$ number conservation symmetry. However notice that it transforms as a bi-vector under $SO(3,1)_{spin} \times SO(3,1)_{space}$, and that

$$\int_x i\chi^\dagger \partial\!\!\!/ \chi = \int_x \frac{i}{2}(\partial_\mu \chi^\dagger \overline{\sigma}^\mu \chi - \chi^\dagger \overline{\sigma}^\mu \partial_\mu \chi) = \int_{xy} E_\mu^a \delta_a^\mu \delta^4(x-y)$$

in other words, the fermion kinetic term is linear in the field E_μ^a (a tadpole) and indicates that if E_μ^a is a propagating field, the original vacuum with $\chi^\dagger \partial\!\!\!/ \chi$ is not the ground state of the theory.

Shifting to the actual ground state of the theory *removes the fermion's kinetic term*.

Lorentz Symmetry Breaking

The Lorentz symmetry is actually two symmetries, spacetime and spin:

$$L_{\mu\nu} = i(x_\mu \partial_\nu - x_\nu \partial_\mu); \qquad S_{ab} = \frac{i}{2}(\gamma_a \gamma_b - \gamma_b \gamma_a)$$

the neutrino transforms as a scalar $(0,0)$ under the first group and a spinor $(\frac{1}{2},0)$ under the second group.

The Lagrangian is not symmetric under both groups. In particular, the kinetic terms for both fermions and weak bosons may relate the two groups to one another.

$$\overline{\psi}\gamma^\mu \partial_\mu \psi; \qquad (\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu)$$

Mass terms and four point interactions are symmetric under both groups.

Lorentz Symmetry Breaking at low energy

At low energies we may integrate out the Z boson, resulting in a tower of 4-point (and higher) operators. However gauge dependence may then appear in the 4-point operators. Let us examine the gauge boson propagator in the R_ξ gauge:

$$D_{\mu\nu}(q) = -i\frac{\eta_{\mu\nu}}{q^2 - M^2} + i(1-\xi)\frac{q^\mu q^\nu}{(q^2 - M^2)(q^2 - \xi M^2)}.$$

The gauge dependence of the second term gets cancelled in physical processes by the goldstone boson contribution.

The gauge parameter ξ shuffles the longitudinal portion of the gauge boson between the goldstone and the 3-dof gauge boson.

However, a *Majorana* particle cannot couple to the Z^0's goldstone boson. The gauge dependence vanishes by the Ward identity. Any $q^\mu q^\nu$ cannot contribute to physical processes (the same as in QED).

Therefore, WLOG, we may take $\xi = 1$ (Feynman-t'Hooft gauge). The couplings of a Majorana neutrino are entirely *transverse*.

Lorentz Symmetry Breaking at low energy

The gauge boson propagator is then

$$D_{\mu\nu}(q) = -i\frac{\eta_{\mu\nu}}{q^2 - M^2}$$

The metric in the numerator is both the metric of $SO(3,1)_{spin}$ and $SO(3,1)_{space}$.

Integrating out the Z boson generates a tower of point-like operators with external, transverse neutrinos. e.g. at leading order

$$\frac{\chi^\dagger(x)\overline{\sigma}^a\chi(x)\chi^\dagger(y)\overline{\sigma}^b\chi(y)\eta_{ab}}{q^2 - M^2 + i\epsilon}$$

here we take the η_{ab} to be the metric of $SO(3,1)_{spin}$.

All interaction terms are $SO(3,1) \times SO(3,1)$ symmetric! (roman a, b, c for spin and greek μ, ν, λ indices for space)

Only one term breaks this enhanced symmetry: the fermion kinetic term.

The Quasi-Particle

The fermion kinetic term is therefore a vacuum expectation value for the composite field $E_\mu^a(x,y)$. By goldstone's theorem, E_μ^a is *massless* (gapless).

E_μ^a propagates in the direction of the *unbroken generators*

$$M_{\mu\nu} = L_{\mu\nu} + S_{ab}\delta_\mu^a\delta_\nu^b$$

These unbroken generators define a *curved space*, given by the density and spin distribution of the background fermions.

Thus this theory evades the Weinberg-Witten Theorem (1980): the emergent graviton does not propagate in flat Minkowski space. It lives only in a curved space. As $p_F \to 0$, we return to Minkowski space, and in that limit, $G_N \to 0$. (Weinberg-Witten is also not applicable to "emergent" gravity)

Thus there is a conserved stress tensor for the gravitational sector of this theory, but it does not live in the same space as the gravitational theory itself.

This Quasi-Particle is a Graviton

We already know what a $SO(3,1)$ bi-vector is: the vierbein (tetrad):

$$g_{\mu\nu}(x,y) = E^a_\mu(x,y)E^b_\nu(x,y)\eta_{ab}$$

This field has an internal global $SO(3,1)$ symmetry due to the spin Lorentz invariance.

This is different from the first-order (Palatini) formulation of gravity (which uses a *local* internal Lorentz symmetry).

Thus the fermion spin dependence is not a gauge symmetry, but is a physical observable in this theory. The spin distribution of the fermion gives rise to *Torsion*.

Such a theory was explored by Hebecker and Wetterich [2003; Wetterich 2003, 2004]. They conclude that the addition of torsion, due to a global, rather than local Lorentz symmetry is at present *unobservable*.

This theory differs from that of Hebecker and Wetterich due to the presence of the $SO(3,1) \times SO(3,1)$ symmetry breaking structure, and the associated metric $\eta_{\mu\nu}$.

Conclusions

Dark matter and neutrino cosmology must take into account the fact that decoupled matter can become a superfluid when $\sigma \lesssim n^{-2/3}$.

The Standard Model contains a graviton and gravi-photon: they are quasi-particles in the Cosmic Neutrino Background.

While this may not be the theory of gravity everyone was looking for, we *must* take it into account. The Standard Model is well tested, cosmic neutrinos certainly must exist.

This theory may also contain the keys to galactic rotation curves, neutrino mass, and cosmic expansion, at the next order in $\sqrt{p_F^2 G_F}$.

This theory is *supremely testable* and *falsifiable* (unlike other gravity theories). We can make W's, Z's, and neutrinos. It contains zero free parameters.

Scales of the neutrino background

What scales do I know about? (note $p_F^3 = 3\pi^2 n$; $n = N/V$)

$p_F(\nu)$	2.34×10^{-4} eV	per flavor/anti
$\sqrt{\Delta m_{12}}$	8.94×10^{-3} eV	
$\sqrt{\Delta m_{23}}$	5.29×10^{-2} eV	
T_ν	1.68×10^{-4} eV	
G_F	$1.17 \times 10^{-23}\text{eV}^{-2}$	

What scales do I want to explain? (using p_F as representative of the low scale)

Λ	2.3×10^{-3} eV	$\mathcal{O}(p_F)$
$p_F(\chi)$	8.80×10^{-6} eV $\left(\frac{100\text{GeV}}{M_\chi}\right)$	$\mathcal{O}(p_F)$
$M_{\text{Pl}}^{-1} = \sqrt{G_N}$	$10^{-28} eV^{-1}$	$\mathcal{O}(p_F G_F)$
α_Λ	1.51×10^{-33} eV	$\mathcal{O}(p_F^3 G_F)$
α_{MOND}	2.63×10^{-34} eV	$\mathcal{O}(p_F^3 G_F)$
α_{Pioneer}	1.92×10^{-33} eV	$\mathcal{O}(p_F^3 G_F)$

Is this all a big coincidence?

My Question

ALL scales necessary to describe gravity (including its anomalies) appear to already be in the Standard Model.

Yet, we never assemble the scales in this manner. Why?

To my mind, this looks like a finite temperature/finite density theory with a small parameter ($p_F \simeq 10^{-3}$ eV) and a large parameter ($G_F^{-1/2} \simeq 10^{11}$ eV), and at each ratio of these scales, new dynamics arises.

Therefore I am led to the following question:

What is the Standard Model dynamics which arises proportional to: p_F, $p_F G_F$, $p_F^2 G_F^2$, $p_F^3 G_F$?

We *must* answer this question, because the answer may give corrections to gravitational dynamics, dark matter dynamics, the cosmological constant, and other anomalies such as Pioneer. Either these dynamics exists, or we should prove that it does not, before assuming that G_N, Λ (etc) are unrelated to the weak scale.

The Cutoff and Quasi-Particle Lifetime

This theory has a cutoff when the neutrino becomes strongly interacting enough to destroy its quantum liquid state.

For the condensed theory to be valid, scattering must not occur often enough to destroy the condensate. Scattering becomes strong again when the CM energy puts the Z on pole. For a probe with energy E, this occurs when

$$E = M_Z^2/T_\nu \simeq M_{Pl}$$

Quasiparticles have a lifetime: scattering can cause them to decay into their constituents. For the present theory,

$$\tau \propto \left[(E_p - E_F)^2 p_F G_F\right]^{-1} \simeq \frac{M_{\mathrm{Pl}}}{(E_p - E_F)^2}$$

Soft gravitons, close to the Fermi surface, are long lived. For $E_p = 0$ this is $\sim 10^{11}$y.

Hierarchy Problems

This theory has neither the Gauge hierarchy problem nor the cosmological constant problem.

The gravitational theory undergoes a phase transition at M_Z. Thus, scalar masses are pulled by radiative corrections up to M_Z, not M_{Pl}.

Zero-point vacuum diagrams contribute constants to the effective action. However constants are *non-dynamical*. The cosmological constant is related to the physical mass and density of the theory (and as such, is a "rolling" CC). Also, it could not be negative or zero.

Given this, I prefer to discard the notion of classical gravity (and the two hierarchy problems along with it), and let's see if this theory can fit gravitational data, before we start adding new fundamental fields.