

# Mössbauer neutrinos

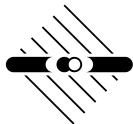
Joachim Kopp

Max-Planck-Institut für Kernphysik, Heidelberg

Workshop on new instruments for neutrino relics and mass  
CERN, 8 December 2008



in collaboration with E. Kh. Akhmedov and M. Lindner  
based on JHEP **0805** (2008) 005 (arXiv:0802.2513)  
and arXiv:0803.1424



MAX-PLANCK-INSTITUT  
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# Outline

- 1 The Mössbauer neutrino experiment
- 2 Oscillations of Mössbauer neutrinos: Qualitative arguments
- 3 Mössbauer neutrinos in QFT
- 4 The time-energy uncertainty relation
- 5 Conclusions

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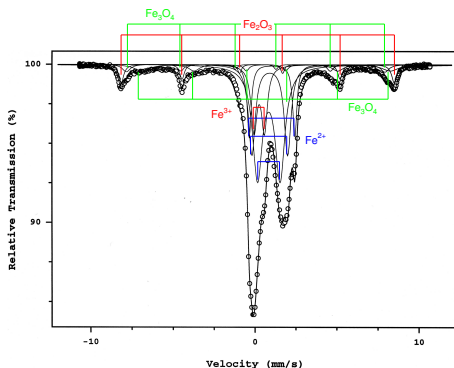
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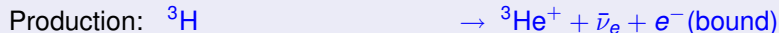
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Proposed experiment:

Production:  ${}^3\text{H} \rightarrow {}^3\text{He}^+ + \bar{\nu}_e + e^- (\text{bound})$

Detection:  ${}^3\text{He}^+ + e^- (\text{bound}) + \bar{\nu}_e \rightarrow {}^3\text{H}$

${}^3\text{H}$  and  ${}^3\text{He}$  are embedded in metal crystals (metal hydrides).

Physics goals:

- Neutrino oscillations on a laboratory scale:  $E = 18.6 \text{ keV}$ ,  $L_{\text{atm}}^{\text{osc}} \sim 20 \text{ m}$ .
- Gravitational interactions of neutrinos
- Study of solid state effects with unprecedented precision

# Mössbauer neutrinos (2)

Mössbauer neutrinos have very special properties:

- Neutrino receives *full* decay energy:  $Q = 18.6 \text{ keV}$
- Natural line width:  $\gamma \sim 1.17 \times 10^{-24} \text{ eV}$
- Actual line width:  $\gamma \gtrsim 10^{-11} \text{ eV}$ 
  - ▶ Inhomogeneous broadening (Impurities, lattice defects)
  - ▶ Homogeneous broadening (Spin interactions)

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## Experimental challenges:

- Is the Lamb-Mössbauer factor (fraction of recoil-free emissions/absorptions) large enough?
- Can a linewidth  $\gamma \gtrsim 10^{-11} \text{ eV}$  be achieved?
- Can the resonance condition be fulfilled?

# Mössbauer neutrinos (3)

Recent controversy:

- Does the small energy uncertainty prohibit oscillations of Mössbauer neutrinos?
- Do oscillating neutrinos need to have equal energies resp. equal momenta?

S. M. Bilenky, F. v. Feilitzsch, W. Potzel, J. Phys. **G34** (2007) 987, hep-ph/0611285

- Does the time-energy uncertainty relation prevent oscillations?

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- ⇒ Careful treatment with as few assumptions as possible is needed
- ⇒ Answer to the above questions will be No.

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# Equal energies or equal momenta?

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These are *assumptions* or *approximations*, not fundamental principles!

## Equal energies or equal momenta? (2)

- In general, neither the equal energy assumption nor the equal momentum assumption is physically justified because both violate energy-momentum conservation in the production and detection processes.

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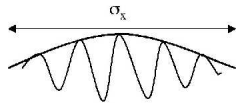
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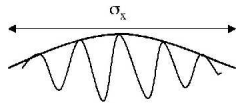
Beuthe, Giunti, Grimus, Kiers, Kim, Lee, Mohanty, Nussinov, Stockinger, Weiss, ...

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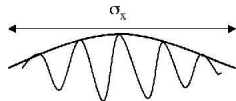


# Conditions for oscillations in a wave packet approach

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Neutrino oscillations are caused by the superposition of different mass eigenstates.

- ⇒ If an experiment can distinguish different mass eigenstates, oscillations will vanish.

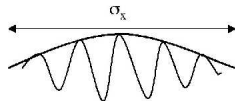


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Requirement for mass resolution  $\sigma_m$ :

$$\sigma_m^2 = \sqrt{(2E\sigma_E)^2 + (2p\sigma_p)^2} > \Delta m^2$$

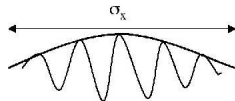
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This is easily fulfilled for Mössbauer neutrinos, since

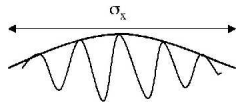
$$\sigma_E \sim 10^{-11} \text{ eV}$$

$$\sigma_p = 1/2\sigma_x \sim 1/\text{interatomic distance} \sim 10 \text{ keV}$$

$$E = p = 18.6 \text{ keV}$$

# Conditions for oscillations in a wave packet approach

- Coherence in production and detection processes
- Coherence maintained during propagation

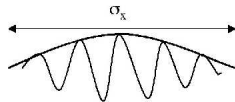




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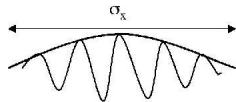
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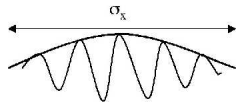
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$$L^{\text{osc}} \ll L^{\text{coh}}.$$

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$$L^{\text{osc}} \ll L^{\text{coh}}.$$

⇒ Standard oscillation formula is approximately recovered:

$$P_{ee} = \sum_{j,k} |U_{ej}|^2 |U_{ek}|^2 \exp \left[ -2\pi i \frac{L}{L_{jk}^{\text{osc}}} \right]$$

$$L_{jk}^{\text{osc}} = \frac{4\pi E}{\Delta m_{jk}^2}$$

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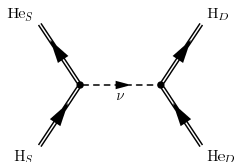
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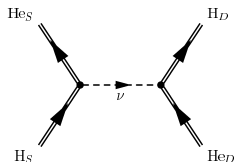
**Idea:** Treat neutrino as an internal line in a tree level Feynman diagram:



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**Idea:** Treat neutrino as an internal line in a tree level Feynman diagram:



External particles reside in harmonic oscillator potentials.  
E.g. for  ${}^3\text{H}$  atoms in the source:

$$\psi_{H,S}(\vec{x}, t) = \left[ \frac{m_H \omega_{H,S}}{\pi} \right]^{\frac{3}{4}} \exp \left[ -\frac{1}{2} m_H \omega_{H,S} |\vec{x} - \vec{x}_S|^2 \right] \cdot e^{-iE_{H,S}t}$$

# Oscillation amplitude

$$\begin{aligned}
 i\mathcal{A} = & \int d^3x_1 dt_1 \int d^3x_2 dt_2 \left( \frac{m_H \omega_{H,S}}{\pi} \right)^{\frac{3}{4}} \exp \left[ -\frac{1}{2} m_H \omega_{H,S} |\vec{x}_1 - \vec{x}_S|^2 \right] e^{-iE_{H,S} t_1} \\
 & \cdot \left( \frac{m_{He} \omega_{He,S}}{\pi} \right)^{\frac{3}{4}} \exp \left[ -\frac{1}{2} m_{He} \omega_{He,S} |\vec{x}_1 - \vec{x}_S|^2 \right] e^{+iE_{He,S} t_1} \\
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 & \cdot \sum_j \mathcal{M}^\mu \mathcal{M}^{\nu*} |U_{ej}|^2 \int \frac{d^4p}{(2\pi)^4} e^{-ip_0(t_2-t_1) + i\vec{p}(\vec{x}_2 - \vec{x}_1)} \\
 & \cdot \bar{u}_{e,S} \gamma_\mu (1 - \gamma^5) \frac{i(\not{p} + m_j)}{p_0^2 - \vec{p}^2 - m_j^2 + i\epsilon} (1 + \gamma^5) \gamma_\nu u_{e,D}.
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Evaluation:

- $dt_1 dt_2$ -integrals  $\rightarrow$  energy-conserving  $\delta$  functions  $\rightarrow p_0$ -integral trivial
- $d^3x_1 d^3x_2$ -integrals are Gaussian
- $d^3p$ -integral: Use **Grimus-Stockinger theorem** (for large  $L = |\vec{x}_D - \vec{x}_S|$ ).

# From the amplitude to the transition rate

Amplitude:

$$i\mathcal{A} = \frac{-i}{2L} \mathcal{N} \delta(E_S - E_D) \exp\left[-\frac{E_S^2 - m_j^2}{2\sigma_p^2}\right] \sum_j \mathcal{M}^\mu \mathcal{M}^{\nu*} |U_{ej}|^2 e^{i\sqrt{E_S^2 - m_j^2}L}$$
$$\cdot \bar{u}_{e,S} \gamma_\mu \frac{1-\gamma^5}{2} (\not{p}_j + m_j) \frac{1+\gamma^5}{2} \gamma_\nu u_{e,D},$$
$$\sigma_p^{-2} = (m_H \omega_{H,S} + m_{He} \omega_{He,S})^{-1} + (m_H \omega_{H,D} + m_{He} \omega_{He,D})^{-1}$$

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Transition rate: Integrate  $|\mathcal{A}|^2$  over densities of initial and final states

$$\Gamma \propto \int_0^\infty dE_{H,S} dE_{He,S} dE_{He,D} dE_{H,D} \cdot \delta(E_S - E_D) \rho_{H,S}(E_{H,S}) \rho_{He,D}(E_{He,D}) \rho_{He,S}(E_{He,S}) \rho_{H,D}(E_{H,D}) \cdot \sum_{j,k} |U_{ej}|^2 |U_{ek}|^2 \underbrace{\exp\left[-\frac{2E_S^2 - m_j^2 - m_k^2}{2\sigma_\rho^2}\right]}_{\text{Analogue of Lamb-Mössbauer factor (Recoil-free fraction)}} \underbrace{e^{i(\sqrt{E_S^2 - m_j^2} - \sqrt{E_S^2 - m_k^2})L}}_{\text{Oscillation phase}}$$

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Convenient reformulation:

$$\exp \left[ - \frac{2E_S^2 - m_j^2 - m_k^2}{2\sigma_p^2} \right] = \exp \left[ - \frac{(p_{jk}^{\min})^2}{\sigma_p^2} \right] \exp \left[ - \frac{|\Delta m_{jk}^2|}{2\sigma_p^2} \right]$$

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⇒ **Localization condition**

$$4\pi\sigma_x E / \sigma_p \lesssim L_{jk}^{\text{osc}},$$

(with  $\sigma_x = 1/2\sigma_p$ ) is satisfied if  $L_{jk}^{\text{osc}} \gtrsim 2\pi\sigma_x$ , which is easily fulfilled in realistic situations.

# Line broadening

Energy levels of  $^3\text{H}$  and  $^3\text{He}$  in the source and detector are smeared e.g. due to spin-spin interactions, crystal impurities, lattice defects, etc.

R. S. Raghavan, hep-ph/0601079

W. Potzel, Phys. Scripta **T127** (2006) 85

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$$\rho_{A,B}(E_{A,B}) = \frac{\gamma_{A,B}/2\pi}{(E_{A,B} - E_{A,B,0})^2 + \gamma_{A,B}^2/4}$$

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Result for two neutrino flavours:

$$\Gamma \propto \frac{(\gamma_S + \gamma_D)/2\pi}{(E_{S,0} - E_{D,0})^2 + \frac{(\gamma_S + \gamma_D)^2}{4}} \cdot \left\{ 1 - 2s^2 c^2 \left[ 1 - \frac{1}{2} (e^{-L/L_S^{\text{coh}}} + e^{-L/L_D^{\text{coh}}}) \cos \left( \pi \frac{L}{L_{\text{osc}}} \right) \right] \right\}$$

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In realistic cases:  $L_{S,D}^{\text{coh}} \gg L^{\text{osc}} \Rightarrow$  Decoherence is not an issue.

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# The time-energy uncertainty relation

Mandelstam-Tamm relation:

$$\Delta E \Delta O \geq \frac{1}{2} \left| \frac{d}{dt} \overline{O}(t) \right|.$$

Here,  $\overline{O}(t) = \langle \psi(t) | O | \psi(t) \rangle$ , for any operator  $O$  and QFT Fock state  $|\psi(t)\rangle$ .

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Choose  $O \equiv |\nu_\alpha\rangle\langle\nu_\alpha|$  (projection in 3d flavour space) and  $\psi(t)$  a neutrino state

$$\Delta E \geq \frac{1}{2} \frac{\left| \frac{d}{dt} P(t) \right|}{\sqrt{P(t) - P^2(t)}},$$

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S. M. Bilenky, F. v. Feilitzsch, W. Potzel, J. Phys. **G35** (2008) 095003 (arXiv:0803.0527)

**Problem:** Interpretation of  $P(t)$ .

- Would be the oscillation probability **for a completely delocalized detector**.
- Imagine wave packet which is large compared to  $L^{\text{osc}}$ : Delocalized detector averages out oscillations.
- More realistic: **Projection in flavour space and in coordinate space:**

$$O_{\vec{x}} \equiv |\nu_\alpha\rangle |\vec{x}\rangle \langle \vec{x}| \langle \nu_\alpha|$$

## The time-energy uncertainty relation (2)

Mandelstam-Tamm relation now reads:

$$\Delta E \geq \frac{1}{2} \frac{\left| \frac{d}{dt} P(\vec{x}, t) \right|}{\sqrt{P(\vec{x}, t) - P^2(\vec{x}, t)}},$$

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Two-flavour approximation:  $P(\vec{x}, t) = 1 - \sin^2 2\theta \sin^2 \phi(\vec{x}, t)$

$$\Rightarrow \Delta E \geq |E_1 - E_2| \frac{\sin 2\theta \cos \phi(\vec{x}, t)}{\sqrt{1 - \sin^2 2\theta \sin^2 \phi(\vec{x}, t)}}.$$

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Certainly fulfilled, if fulfilled for  $\sin^2 2\theta = 1$ .

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$$\Rightarrow \Delta E \geq |E_1 - E_2|.$$

“Energy difference of mass eigenstates must be smaller than energy uncertainty.”

Easily fulfilled for Mössbauer neutrinos due to large *momentum* uncertainty.

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  - ▶ Both conditions are easily fulfilled in realistic experiments.
- The time energy uncertainty relation does not inhibit oscillations of Mössbauer neutrinos.

Thank you!



# Textbook derivation of the oscillation formula

Diagonalization of the mass terms of the charged leptons and neutrinos gives

$$\mathcal{L} \supset -\frac{g}{\sqrt{2}} (\bar{e}_{\alpha L} \gamma^\mu U_{\alpha j} \nu_{jL}) W_\mu^- + \text{diag. mass terms} + h.c.$$

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Assume, at time  $t = 0$  and location  $\vec{x} = 0$ , a flavour eigenstate

$$|\nu(0, 0)\rangle = |\nu_\alpha\rangle = \sum_j U_{\alpha j}^* |\nu_j\rangle$$

is produced. At time  $t$  and position  $\vec{x}$ , it has evolved into

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Oscillation probability:

$$P(\nu_\alpha \rightarrow \nu_\beta) = \left| \langle \nu_\beta | \nu(t, \vec{x}) \rangle \right|^2 = \sum_{j,k} U_{\alpha j}^* U_{\beta j} U_{\alpha k} U_{\beta k}^* e^{-i(E_j - E_k)t + i(\vec{p}_j - \vec{p}_k) \vec{x}}$$

# Problems with the textbook derivation

- In general, neither the equal energy assumption nor the equal momentum assumption is physically justified because both violate energy-momentum conservation in the production and detection processes.

R. G. Winter, Lett. Nuovo Cim. **30** (1981) 101

C. Giunti, W. Kim, Found. Phys. Lett. **14** (2001) 213, hep-ph/0011072

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- Example: Pion decay at rest:  $\pi^+ \rightarrow \mu^+ + \nu_\mu$ ,  $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$

Energy-momentum conservation for emission of mass eigenstate  $|\nu_i\rangle$ :

$$E_i^2 = \frac{m_\pi^2}{4} \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2 + \frac{m_i^2}{2} \left(1 - \frac{m_\mu^2}{m_\pi^2}\right) + \frac{m_i^4}{4m_\pi^2}$$

$$p_i^2 = \frac{m_\pi^2}{4} \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2 - \frac{m_i^2}{2} \left(1 - \frac{m_\mu^2}{m_\pi^2}\right) + \frac{m_i^4}{4m_\pi^2}$$

For massless neutrinos:  $E_i = p_i = E \equiv \frac{m_\pi}{2} \left(1 - \frac{m_\mu^2}{m_\pi^2}\right) \simeq 30 \text{ MeV}$ .

To first order in  $m_i^2$ :

$$E_i \simeq E + \xi \frac{m_i^2}{2E}, \quad p_i \simeq E - (1 - \xi) \frac{m_i^2}{2E}, \quad \xi \approx \frac{1}{2} \left(1 - \frac{m_\mu^2}{m_\pi^2}\right) \approx 0.2$$

# Oscillation formula for neutrino wave packets

Assume Gaussian wave packets:

$$|\nu_\alpha(x, t)\rangle = \frac{1}{(2\pi\sigma_{pS}^2)^{1/4}} \sum_j U_{\alpha j}^* \int \frac{dp}{\sqrt{2\pi}} e^{-(p-p_{jS})^2/4\sigma_{pS}^2} e^{-i\sqrt{p^2+m_j^2}t+ipx} |\nu_j\rangle$$

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Oscillation formula:

$$P_{ee} = \sum_{j,k} |U_{ej}|^2 |U_{ek}|^2 \exp \left[ -2\pi i \frac{L}{L_{jk}^{\text{osc}}} - \left( \frac{L}{L_{jk}^{\text{coh}}} \right)^2 - 2\pi^2 \xi^2 \left( \frac{1}{2\sigma_p L_{jk}^{\text{osc}}} \right)^2 \right]$$

C. Giunti, C. W. Kim, U. W. Lee, Phys. Rev. **D44** (1991) 3635; C. Giunti, C. W. Kim, Phys. Lett. **B274** (1992) 87  
K. Kierns, S. Nussinov, N. Weiss, Phys. Rev. **D53** (1996) 537, hep-ph/9506271

C. Giunti, C. W. Kim, Phys. Rev. **D58** (1998) 017301, hep-ph/9711363, C. Giunti, Found. Phys. Lett. **17** (2004) 103, hep-ph/0302026

with

$$L_{jk}^{\text{osc}} = 4\pi E / \Delta m_{jk}^2$$

$$L_{jk}^{\text{coh}} = 2\sqrt{2}E^2 / \sigma_p |\Delta m_{jk}^2|$$

$E$

$\xi$

$\sigma_p$

Oscillation length

Coherence length

Energy that a massless neutrino would have  
quantifies the deviation from  $E$   
(tiny for Mössbauer neutrinos)

Effective wave packet width  
(tiny for Mössbauer neutrinos)



# Results from the wave packet treatment

- Decoherence term

$$\exp \left[ - \frac{L}{L_{jk}^{\text{coh}}} \right]$$

cannot inhibit oscillations because

$$L_{jk}^{\text{coh}} / L_{jk}^{\text{osc}} \sim E / \sigma_p \sim 10^{15}.$$

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- **Localization term**

$$\exp \left[ - 2\pi^2 \xi^2 \left( \frac{1}{2\sigma_p L_{jk}^{\text{osc}}} \right)^2 \right]$$

cannot inhibit oscillations, because

$$\xi / \sigma_p L_{jk}^{\text{osc}} \ll 1.$$

# Results from the wave packet treatment

- Decoherence term

$$\exp \left[ - \frac{L}{L_{jk}^{\text{coh}}} \right]$$

cannot inhibit oscillations because

$$L_{jk}^{\text{coh}} / L_{jk}^{\text{osc}} \sim E / \sigma_p \sim 10^{15}.$$

- Localization term

$$\exp \left[ - 2\pi^2 \xi^2 \left( \frac{1}{2\sigma_p L_{jk}^{\text{osc}}} \right)^2 \right]$$

cannot inhibit oscillations, because

$$\xi / \sigma_p L_{jk}^{\text{osc}} \ll 1.$$

⇒ Our expectation is confirmed:

$$P(\nu_\alpha \rightarrow \nu_\beta) = \sum_{j,k} U_{\alpha j}^* U_{\beta j} U_{\alpha k} U_{\beta k}^* e^{-i \frac{\Delta m_{jk}^2 L}{2E}}.$$

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We are looking for a formalism, in which these quantities are automatically determined from the properties of the source and the detector.

# The Grimus-Stockinger theorem

Let  $\psi(\vec{p})$  be a three times continuously differentiable function on  $\mathbb{R}^3$ , such that  $\psi$  itself and all its first and second derivatives decrease at least like  $1/|\vec{p}|^2$  for  $|\vec{p}| \rightarrow \infty$ . Then, for any real number  $A > 0$ ,

$$\int d^3p \frac{\psi(\vec{p}) e^{i\vec{p}\vec{L}}}{A - \vec{p}^2 + i\epsilon} \xrightarrow{|\vec{L}| \rightarrow \infty} -\frac{2\pi^2}{L} \psi(\sqrt{A}\frac{\vec{L}}{L}) e^{i\sqrt{A}L} + \mathcal{O}(L^{-\frac{3}{2}}).$$

⇒ Quantification of requirement of on-shellness for large  $L = |\vec{L}|$ .

# Amplitude for broadening by natural line width

Take into account the instability of  ${}^3\text{H}$  in the source and the detector.

$$\begin{aligned}
 i\mathcal{A} = & \int d^3x_1 \int_0^T dt_1 \int d^3x_2 \int_0^T dt_2 \left( \frac{m_{\text{H}}\omega_{\text{H},\text{S}}}{\pi} \right)^{\frac{3}{4}} \exp \left[ -\frac{1}{2} m_{\text{H}}\omega_{\text{H},\text{S}} |\vec{x}_1 - \vec{x}_{\text{S}}|^2 \right] e^{-iE_{\text{H},\text{S}}t_1} \\
 & \cdot \left( \frac{m_{\text{He}}\omega_{\text{He},\text{S}}}{\pi} \right)^{\frac{3}{4}} \exp \left[ -\frac{1}{2} m_{\text{He}}\omega_{\text{He},\text{S}} |\vec{x}_1 - \vec{x}_{\text{S}}|^2 \right] e^{+iE_{\text{He},\text{S}}t_1} \\
 & \cdot \left( \frac{m_{\text{He}}\omega_{\text{He},\text{D}}}{\pi} \right)^{\frac{3}{4}} \exp \left[ -\frac{1}{2} m_{\text{He}}\omega_{\text{He},\text{D}} |\vec{x}_2 - \vec{x}_{\text{D}}|^2 \right] e^{-iE_{\text{He},\text{D}}t_2} \\
 & \cdot \left( \frac{m_{\text{H}}\omega_{\text{H},\text{D}}}{\pi} \right)^{\frac{3}{4}} \exp \left[ -\frac{1}{2} m_{\text{H}}\omega_{\text{H},\text{D}} |\vec{x}_2 - \vec{x}_{\text{D}}|^2 \right] e^{+iE_{\text{H},\text{D}}t_2} \\
 & \cdot \sum_j \mathcal{M}^\mu \mathcal{M}^{\nu*} |U_{ej}|^2 \int \frac{d^4p}{(2\pi)^4} e^{-ip_0(t_2-t_1) + i\vec{p}(\vec{x}_2-\vec{x}_1)} \\
 & \cdot \bar{u}_{e,\text{S}} \gamma_\mu \frac{1-\gamma^5}{2} \frac{i(\not{p} + m_j)}{p_0^2 - \vec{p}^2 - m_j^2 + i\epsilon} \frac{1+\gamma^5}{2} \gamma_\nu u_{e,\text{D}}
 \end{aligned}$$



# Amplitude for broadening by natural line width

Take into account the instability of  ${}^3\text{H}$  in the source and the detector.

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 & \cdot \left( \frac{m_{\text{He}}\omega_{\text{He},\text{S}}}{\pi} \right)^{\frac{3}{4}} \exp \left[ -\frac{1}{2} m_{\text{He}}\omega_{\text{He},\text{S}} |\vec{x}_1 - \vec{x}_{\text{S}}|^2 \right] e^{+iE_{\text{He},\text{S}}t_1} \\
 & \cdot \left( \frac{m_{\text{He}}\omega_{\text{He},\text{D}}}{\pi} \right)^{\frac{3}{4}} \exp \left[ -\frac{1}{2} m_{\text{He}}\omega_{\text{He},\text{D}} |\vec{x}_2 - \vec{x}_{\text{D}}|^2 \right] e^{-iE_{\text{He},\text{D}}t_2} \\
 & \cdot \left( \frac{m_{\text{H}}\omega_{\text{H},\text{D}}}{\pi} \right)^{\frac{3}{4}} \exp \left[ -\frac{1}{2} m_{\text{H}}\omega_{\text{H},\text{D}} |\vec{x}_2 - \vec{x}_{\text{D}}|^2 \right] e^{+iE_{\text{H},\text{D}}t_2 - \frac{1}{2}\gamma(T-t_2)} \\
 & \cdot \sum_j \mathcal{M}^\mu \mathcal{M}^{\nu*} |U_{ej}|^2 \int \frac{d^4p}{(2\pi)^4} e^{-ip_0(t_2-t_1) + i\vec{p}(\vec{x}_2 - \vec{x}_1)} \\
 & \cdot \bar{u}_{e,\text{S}} \gamma_\mu \frac{1-\gamma^5}{2} \frac{i(\not{p} + m_j)}{p_0^2 - \vec{p}^2 - m_j^2 + i\epsilon} \frac{1+\gamma^5}{2} \gamma_\nu u_{e,\text{D}}
 \end{aligned}$$

# Probability for broadening by natural line width

$$\begin{aligned} \mathcal{P} \propto & \sum_{j,k} \theta(T_{jk}) |U_{ej}|^2 |U_{ek}|^2 \\ & \cdot \exp \left[ -\frac{(p_{jk}^{\min})^2}{\sigma_p^2} \right] \exp \left[ -\frac{|\Delta m_{jk}^2|}{2\sigma_p^2} \right] e^{i(\sqrt{E^2 - m_j^2} - \sqrt{E^2 - m_k^2})L} \\ & \cdot e^{-\gamma T_{jk}} e^{-L/L_{jk}^{\text{coh}}} \frac{\sin \left[ \frac{1}{2}(E_S - E_D)(T - \frac{L}{v_j}) \right] \sin \left[ \frac{1}{2}(E_S - E_D)(T - \frac{L}{v_k}) \right]}{(E_S - E_D)^2} \end{aligned}$$

$$T_{jk} = \min \left( T - \frac{L}{v_j}, T - \frac{L}{v_k} \right)$$

$$L_{jk}^{\text{coh}} = \frac{4\bar{E}^2}{\gamma |\Delta m_{jk}^2|}$$