Teorii efective pentru interactiile tari in fizica cuarcilor grei

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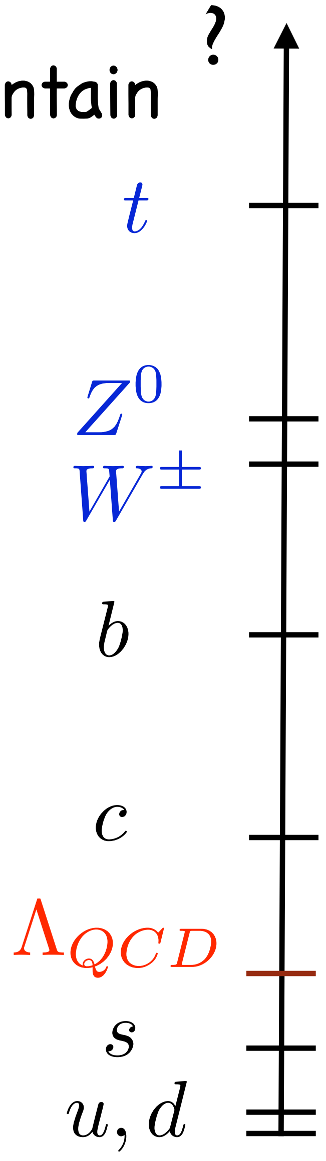
IFIN, 27 Octombrie 2008

Outline

- Effective field theories for strong and weak interactions
- Heavy quark effective theory (HQET)
- Large recoil: Soft-Collinear Effective Theory
- Introduction to SCET
    - Formalism
    - Factorization relations for exclusive B decays

Effective field theories

The weak and strong interactions of the SM contain many disparate scales



The good success of the SM -> low energy predictions must be insensitive to the high-energy theory

Effective theory approach:

identify small expansion parameters.

- $m_{u,d,s,c,b} \ll m_{W,Z,t}$

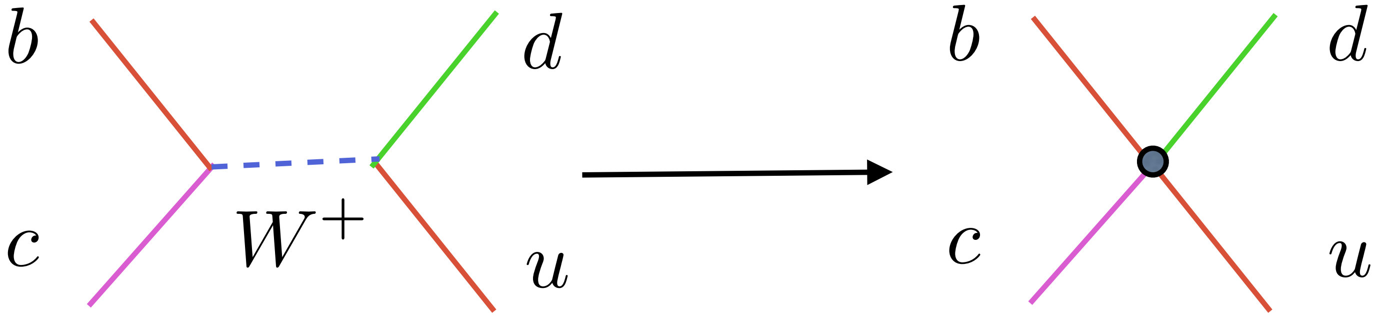
Effective theory of weak interactions

- $\Lambda_{QCD}/m_{b,c} \ll 1$

Heavy quark effective theory

Weak interactions

Fermi 4-quark interaction



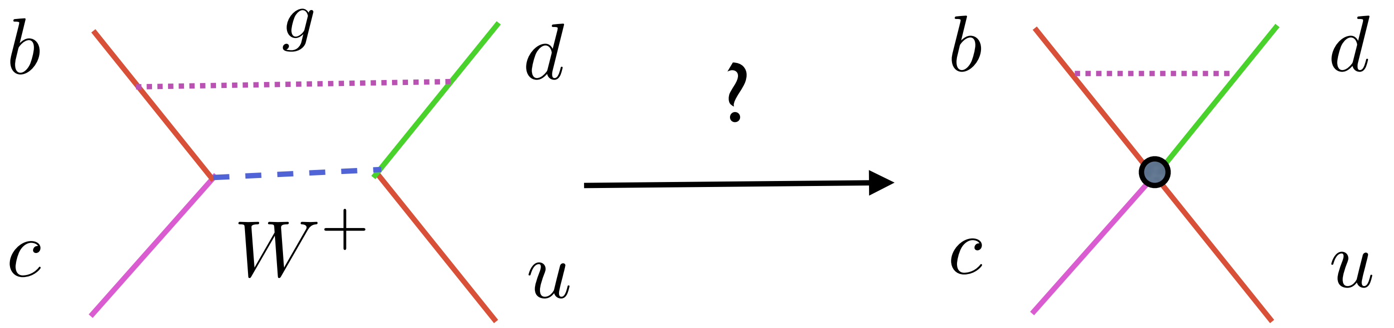
$$A = [\bar{d}\gamma_\mu P_L u]\frac{g^2}{M_W^2 - q^2}[\bar{c}\gamma^\mu P_L b] \to \boxed{\frac{g^2}{M_W^2}}[\bar{d}\gamma_\mu P_L u][\bar{c}\gamma^\mu P_L b] + O(p_q^2/M_W^2)$$

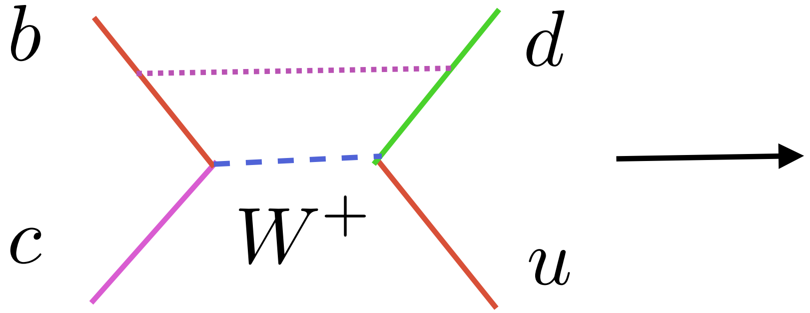
$$G_F = \frac{g^2}{M_W^2}$$ Fermi constant

Local interaction

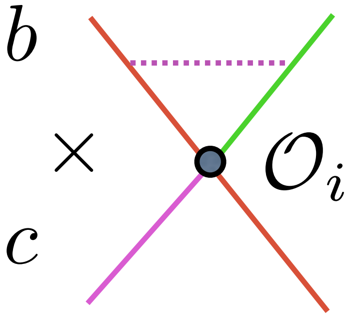
Radiative corrections

Does this picture survive the introduction of radiative corrections?





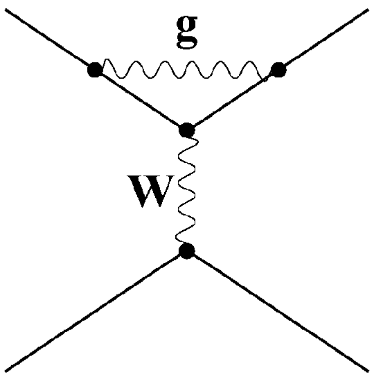
$\sum_i C_i(\frac{\mu}{M_W}) \times$



$C(\mu/M_W)$ = Wilson coefficient containing the contributions of the hard loop momenta

Can be computed in perturbation theory at any order in $\alpha_s(M_W)$ $\Longrightarrow$ Matching

Typical diagrams contributing to matching beyond tree level



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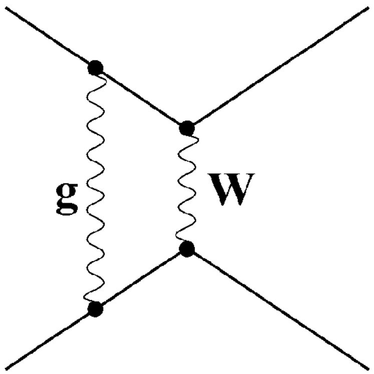
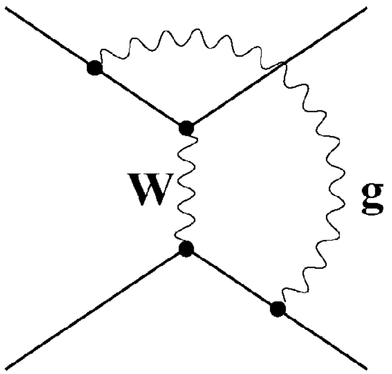
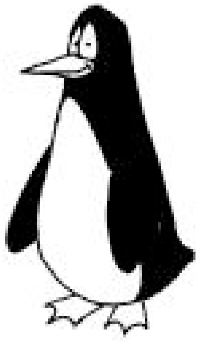
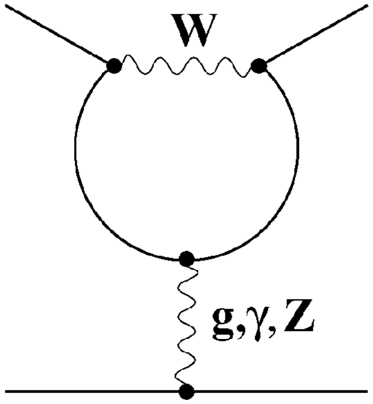


(b)

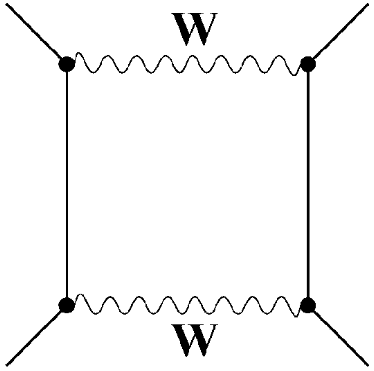


(c)





(d)



(e)

Multiple operators O_i are induced in the low energy theory through radiative corrections

The complete set of operators contains the most general local operators satisfying the conditions:

- leading order in power counting (dimension-6)
- have the correct transformation properties under the symmetries of the theory (parity, isospin, chirality)

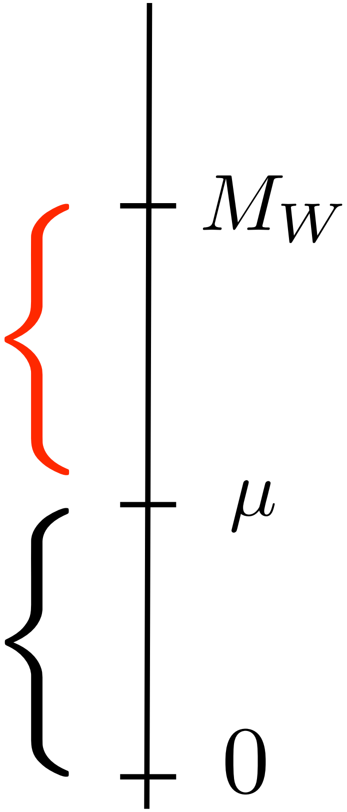
$$\boxed{\mathcal{H}_W = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* \{ C_1(\mu) [\bar{c}\gamma_\mu P_L b][\bar{d}\gamma^\mu P_L u] + C_2(\mu) (\bar{d}\gamma_\mu P_L b][\bar{c}\gamma^\mu P_L u] \}}$$

Complete effective weak Hamiltonian for $b \rightarrow c d \bar{u}$ decays

Renormalization group evolution

$$\langle \mathcal{O}_{\rm QCD}\rangle = \sum_i C_i(\frac{\mu}{m_W})\langle \mathcal{O}^i_{\rm eff}\rangle + O(\frac{m_q}{m_W})$$

power-suppressed terms



The effective theory operators reproduce the IR behaviour of the full theory (Standard Model)

The dependence on the hard scale M_W is fixed by renormalization group methods

$$\mu\frac{d}{d\mu}\mathcal{O} = 0 \longrightarrow \boxed{\mu\frac{d}{d\mu}C_i(\frac{\mu}{M_W}) = -\gamma_{ji}(\alpha_s)C_j(\frac{\mu}{M_W})}$$

$\gamma_{ji}(\alpha_s)$ = anomalous dimension matrix of the operators $\mathcal{O}_i$

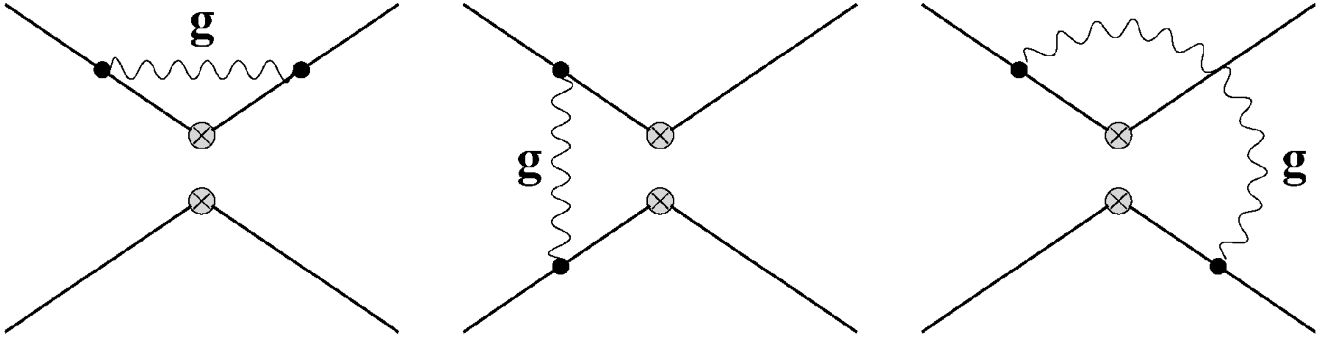
Leading log evolution for the $b \to c d\bar{u}$ effective Hamiltonian

Operator basis

$$\mathcal{O}_1 = (\bar{c}b)_{V-A}(\bar{d}u)_{V-A}$$

$$\mathcal{O}_2 = (\bar{c}T^a b)_{V-A}(\bar{d}T^a u)_{V-A}$$

$$\mu \frac{d}{d\mu} \begin{pmatrix} c_1(\mu) \\ c_2(\mu) \end{pmatrix} = -\gamma^T \begin{pmatrix} c_1(\mu) \\ c_2(\mu) \end{pmatrix} \qquad \gamma^T = \frac{\alpha_s}{2\pi} \begin{pmatrix} 0 & -\frac{4}{3} \\ -6 & 2 \end{pmatrix}$$



$$c_1(\mu) = \frac{1}{3}\left(\frac{\alpha_s(\mu)}{\alpha_s(M_W)}\right)^{12/23} + \frac{2}{3}\left(\frac{\alpha_s(\mu)}{\alpha_s(M_W)}\right)^{-6/23} = 1.12$$

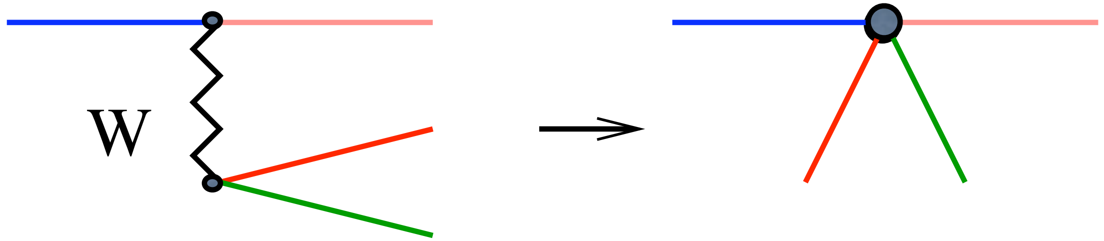
$$c_2(\mu) = -\left(\frac{\alpha_s(\mu)}{\alpha_s(M_W)}\right)^{12/23} + \left(\frac{\alpha_s(\mu)}{\alpha_s(M_W)}\right)^{-6/23} = -0.08$$

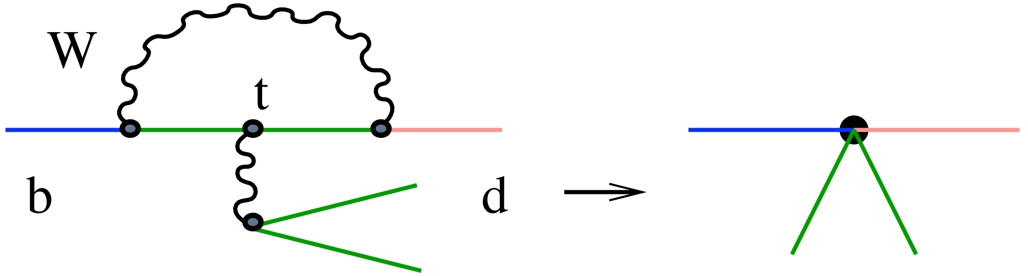
$(\mu = 4.2\ \mathrm{GeV})$

General electroweak Hamiltonian

$$\mathcal{H}_W = \frac{G_F}{\sqrt{2}} \sum_i \lambda_{CKM}^{(i)} C_i(\mu) O_i$$

Integrate out the top and W





Tree operators

$$O_1^u = (\bar{u}b)_{V-A}(\bar{d}u)_{V-A}$$

$$O_2^u = (\bar{u}_i b_j)_{V-A}(\bar{d}_j u_i)_{V-A}$$

Penguin operators

$$O_{3-6} = (\bar{d}b)_{V-A} \sum_q (\bar{q}q)_{V\pm A}$$

$$O_{7-10} = (\bar{d}b)_{V-A} \sum_q e_q (\bar{q}q)_{V\pm A}$$

Now known at NNLO

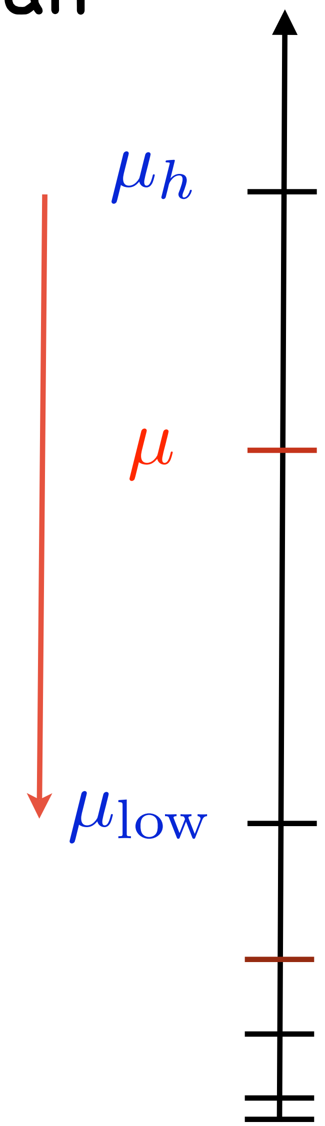
Gorbahn, Haisch - hep-ph/0411071

Gorbahn, Haisch, Misiak - hep-ph/0504194

+ new physics contributions...

Effective theories - summary

In the presence of widely disparated scales $\mu_h \gg \mu_{\rm low}$ the effects of loop momenta in this range can be accounted for using an effective theory



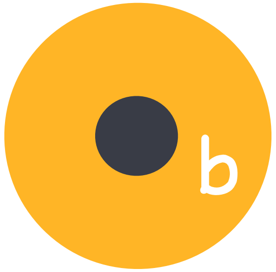
1. integrate out the degrees of freedom associated with the hard scale μ_h

$$\mathcal{H}_{eff} = \Sigma_i C_i(\frac{\mu}{\mu_h})\mathcal{O}_i + \cdots$$

2. matching at $\mu = \mu_h$ gives $C_i(1)$
3. Solve the RGE for the Wilson coeffs
4. repeat as many times as necessary

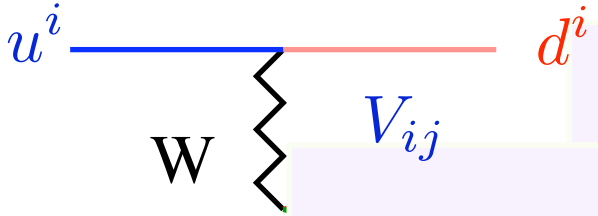
Effective theories for heavy quark physics

Heavy flavor decays



- Bound states of b and light quarks
  - mesons (B^-, B^0, B_s)
  - baryons $(\Lambda_b, \Xi_b^-, \Xi_b^0)$
- Heaviest stable bound states in QCD $(\geq 5.28 \text{ GeV})$
- Rich spectrum, many decay channels
- Important source of information about CP violation, CKM parameters, new physics

CKM matrix



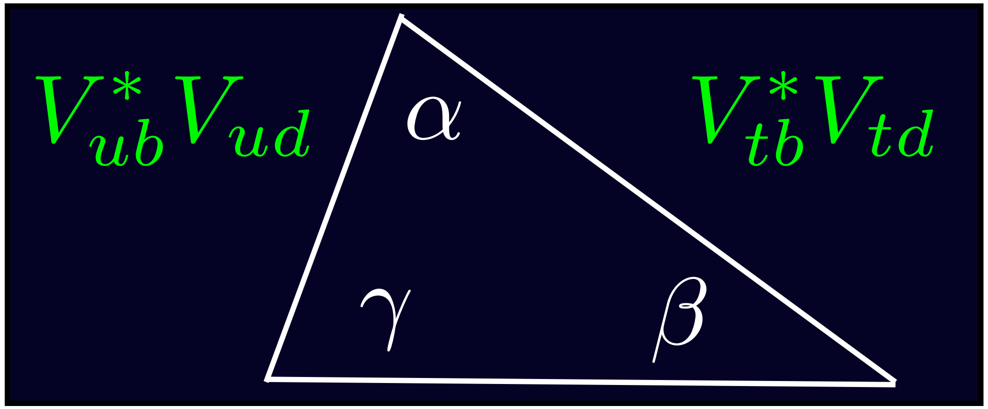
Parameterizes the strength of the charged weak couplings in the Standard model

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \simeq \begin{pmatrix} 0.975 & 0.221 & 0.003 \\ 0.221 & 0.975 & 0.040 \\ 0.005 & 0.040 & 1.000 \end{pmatrix}$$

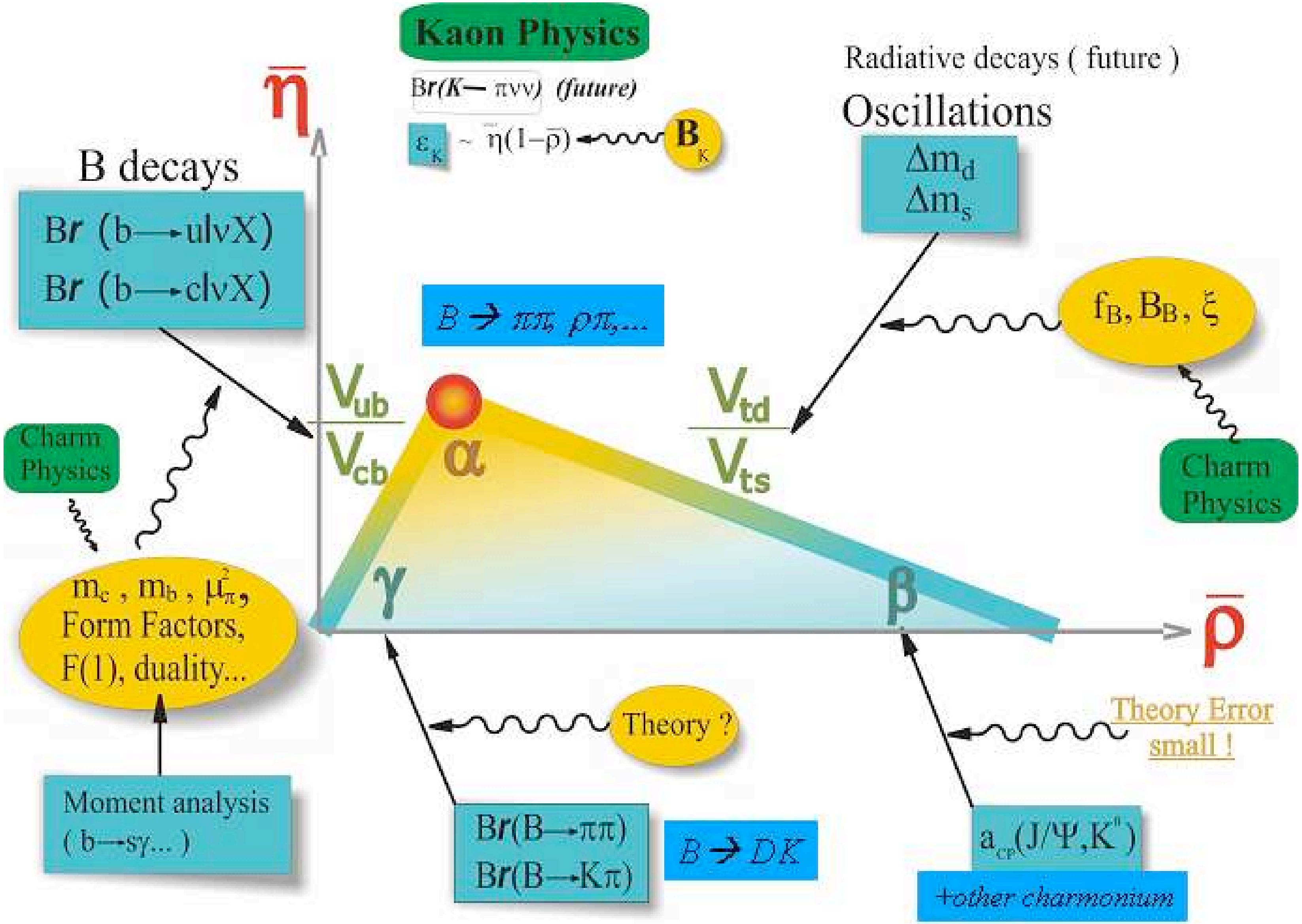
Experimental information about the smallest entries can be summarized by the unitarity relation

$$V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0$$

Unitarity triangle



Constraining the CKM triangle with B decays



Kaon Physics

Br(K→ πνν) *(future)*

$\varepsilon_K \sim \bar{\eta}(1-\bar{\rho})$ ← B_K

Radiative decays (future)

Oscillations

Δm_d
Δm_s

f_B, B_B, ξ

Charm Physics

B decays

Br (b→ulνX)
Br (b→clνX)

Charm Physics

m_c , m_b , μ_π^2,
Form Factors,
F(1), duality...

Moment analysis
(b→sγ...)

B → ππ, ρπ,...

$\bar{\eta}$

$\bar{\rho}$

$\frac{V_{ub}}{V_{cb}}$

$\frac{V_{td}}{V_{ts}}$

α

β

γ

Theory ?

Br(B→ππ)
Br(B→Kπ)

B → DK

Theory Error small !

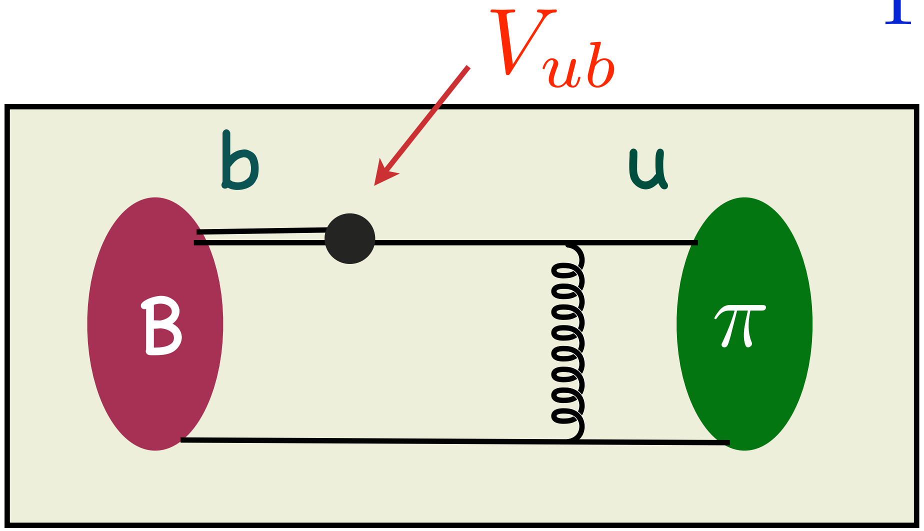
$a_{CP}(J/\Psi, K^0)$

+other charmonium

Weak semileptonic $B \to \pi \ell \bar{\nu}$ decays

- Mediated by the heavy-to-light current

$$\Gamma_\mu = \bar{u} \gamma_\mu P_L b$$



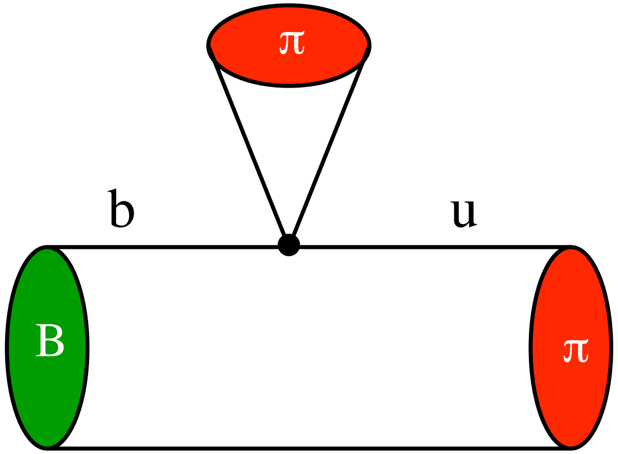
- Parameterized by two form factors

$$\langle \pi(p') | \bar{u} \gamma_\mu P_L b | \bar{B}(p) \rangle = f_+(q^2)(p+p')_\mu + f_-(q^2)(p-p')_\mu$$

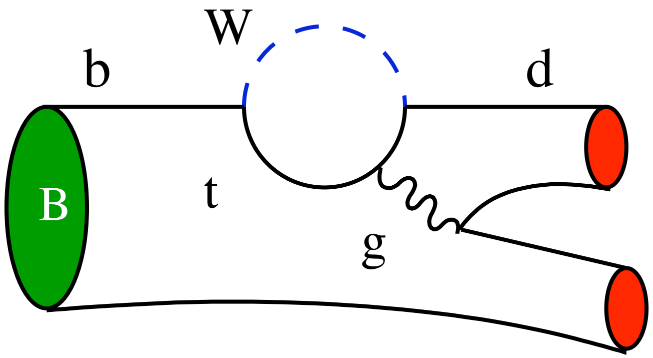
$f_\pm(q^2)$ depend on hadronic dynamics (QCD)

Nonleptonic B decays

Examples: $B^0 \to \pi^+\pi^-$, $B^0 \to K^+\pi^-$

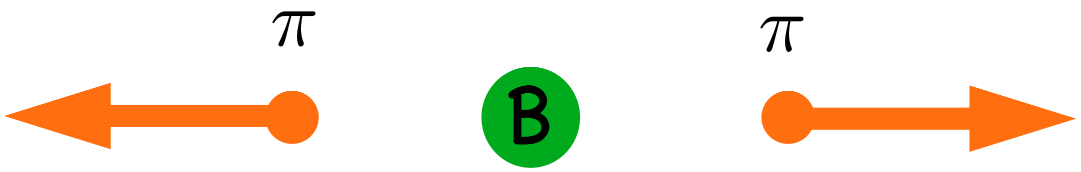


’’Tree’’



’’Penguin’’

Kinematics



$$p_\pi \sim m_B/2 \sim 2.6\ \mathrm{GeV} \gg \Lambda_{QCD}$$

Strong interaction effects

Weak interactions of quarks take place inside hadrons -> need to account for strong interaction nonperturbative effects

$$\langle M_1 M_2 | \mathcal{H}_{EW} | \bar{B} \rangle = ?$$

Controlling these effects is a central part of SM physics

- Lattice QCD
- Exploit symmetries of QCD:
  - flavor SU(3)
  - chiral symmetry
  - heavy quark symmetry

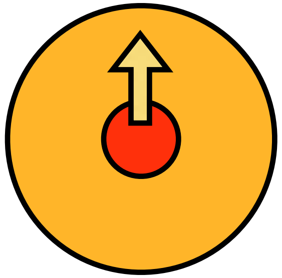
Effective field theories

- Factorization theorems of hard QCD

Recent progress from Soft-Collinear Effective Theory

Energy scales in B physics

Heavy quarks interacting with soft quarks and gluons



Relevant energy scales: $\Lambda \sim 500\ \mathrm{MeV}\,, m_b \sim 4.6\ \mathrm{GeV}$

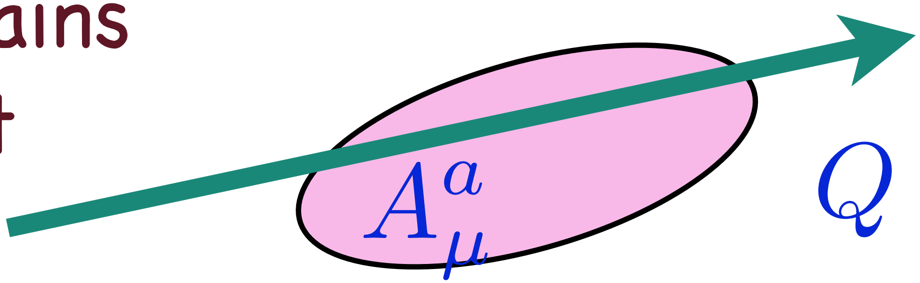
1. Small expansion parameter $\Lambda/m_b \sim 0.1$
2. Symmetries at leading order in Λ/m_b

HQET = the appropriate effective theory

Scales in HQET

Heavy quark interacting with soft gluon fields

The quark momentum contains
a large fixed component



$$p = \underbrace{m_b v}_{\text{Label}} + \boxed{k}$$

Residual momentum

$$Q(x) = e^{-im_Q v.x} h(x)$$

Modes	k	Fields
Hard	m_b	–
Soft	Λ	h_v q

HQET for a static quark

Take the heavy quark mass to infinity -> static limit

The heavy quark acts like a fixed source of color field

Propagator $$S(p) = i\frac{\not{p} + m_b}{p^2 - m_b^2}$$

$$p = m_b v + k$$

$$S(p) = i\frac{m_b \not{v} + \not{k} + m_b}{(m_b v + k)^2 - m_b^2} = \frac{i}{v \cdot k}\,\frac{1+\not{v}}{2} + O(k/m_b)$$

The heavy quark mass has disappeared!

This is the propagator corresponding to the Lagrangian

$$\mathcal{L} = \bar{h} v \cdot i D h$$ Leading order HQET Lagrangian

HQET Lagrangian

Split the heavy quark field Q into ‘large’ and ‘small’ components with respect to velocity v

$$Q(x) = e^{-im_Q v.x} h^{(+)} + e^{im_Q v.x} h^{(-)} \qquad \not{v} h^{(\pm)} = \pm h^{(\pm)}$$

Apply a sequence of field transformations which decouples them - Foldy-Wouthuysen transformation

$$\mathcal{L} = \bar{Q}(i\not{D} - m_Q)Q \quad \to \mathcal{L}_{HQET}[h^{(+)}] + \mathcal{L}_{HQET}[h^{(-)}]$$

$$\begin{aligned} Q(x) &= e^{-im_Q v.x} e^{\frac{1}{2m_Q} O_1^A} e^{\frac{1}{2m_Q^2} O_2^A} \dots h^{(+)} \\ &+ \; e^{im_Q v.x} e^{\frac{1}{2m_Q} O_1^A} e^{\frac{1}{2m_Q^2} O_2^A} \dots h^{(-)} \end{aligned}$$

HQET at first subleading order

$O_1^A = i\not{D}_\perp = i(\not{D} - \not{v}\, v.D)$

$$\mathcal{L}_{HQET} = \underbrace{\bar{h} i v.D h}_{\text{Leading order Lagrangian}} + \underbrace{\frac{1}{2m_Q}\bar{h}(iD)^2 h}_{} - \underbrace{\frac{g}{4m_Q}\bar{h}\sigma_{\mu\nu}G^{\mu\nu}h}_{} + \cdots$$

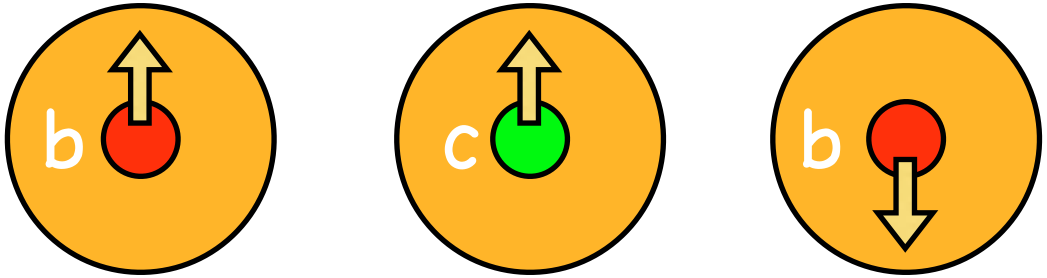
Symmetry-breaking terms

Including radiative corrections

Example: heavy-to-light current

$$\langle 0|\bar{q}\gamma_\mu\gamma_5 b|\bar{B}\rangle = \underbrace{C\left(\frac{\mu}{m_b}\right)}_{\text{Wilson coefficient}}\underbrace{\langle 0|\bar{q}\gamma_\mu\gamma_5 b_v|\bar{B}_v\rangle}_{\text{soft matrix element}} + O(\Lambda/m_b)$$

HQET - symmetries



Flavor symmetry: the physics is independent on the heavy flavor: b vs. c

Spin symmetry: the heavy quark spin orientation is irrelevant

HQET - formalism

The heavy mesons D, D^* form a spin doublet

$$j_\ell = \frac{1}{2}^- \rightarrow J^P = 0^-, 1^-$$

Combine them into a superfield

$$H^a = \frac{1+\not{v}}{2}\left[D_\mu^{a*}\gamma^\mu - D^a\gamma_5\right]$$

Transforms as:

$H^a \rightarrow S H^a$ under spin symmetry

$H^a \rightarrow U_{ab} H^b$ under flavor symm.

Construct operators using only H having the same transformation properties as the original QCD operators

Example: heavy-to-heavy decays

Semileptonic decays $\bar{B} \to D^{(*)}\ell\bar{\nu}_\ell \qquad \Rightarrow |V_{cb}|$

Strong interaction effects - hadronic matrix elements

$$\langle D^{(*)}(v')|\bar{c}\Gamma b|\bar{B}(v)\rangle$$

Heavy quark symmetry prediction:

$$\langle D^{(*)}(v')|\bar{c}\Gamma b|\bar{B}(v)\rangle = \xi(v\cdot v')\mathrm{Tr}\,[\bar{H}_{v'}^{(c)}\Gamma H_v^{(b)}] + O\left(\frac{\Lambda}{m_c}\right)$$

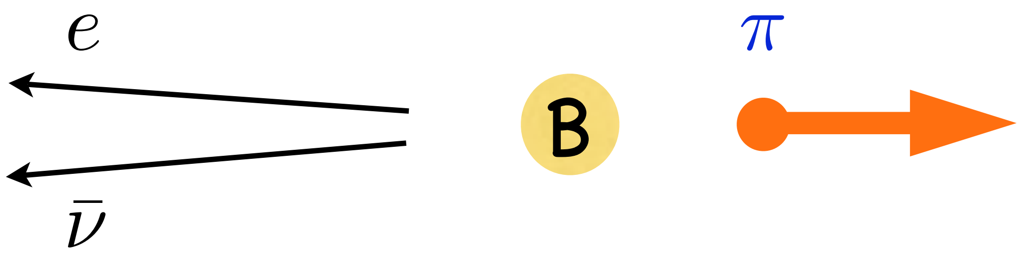
reduced matrix element
= Isgur-Wise function

1 (B->D) + 4 (B->D*) = 5 form factors fixed in terms of just one IW function

Energetic hadrons

Construct a heavy-quark expansion for processes involving both soft and energetic light hadrons

Example: semileptonic $B \to \pi \ell \bar{\nu}$ decay at large recoil



$$\boxed{E_\pi \sim 2.2 \text{ GeV}}$$

Energy scales:

- Soft $\Lambda \sim 500$ MeV
- Hard $m_b \sim 4.6$ GeV
- Hard-collinear $\sqrt{m_b \Lambda} \sim 1.4$ GeV

The soft-collinear effective theory (SCET)

- Systematic power counting in Λ/m_b implemented at the level of momenta, fields, operators
- Construct effective Lagrangians for strong and weak interactions expanded in Λ/m_b
- Guiding principles: new symmetries - soft/collinear gauge invariance, reparameterization invariance
- Nonlocal operators and Lagrangians

Soft-Collinear Effective Theory (formalism)

- Introducing the relevant modes
- Formal derivation of the SCET Lagrangian
- Power counting
- Collinear gauge invariance
- General procedure for constructing operators

Light-cone geography

Light cone coordinates $(x^0, x^1, x^2, x^3) \longrightarrow (x_+, x_-, x_\perp)$

$$x_\pm = x^0 \pm x^3 \quad , \quad x_\perp = (x^1, x^2)$$

Light cone unit vectors

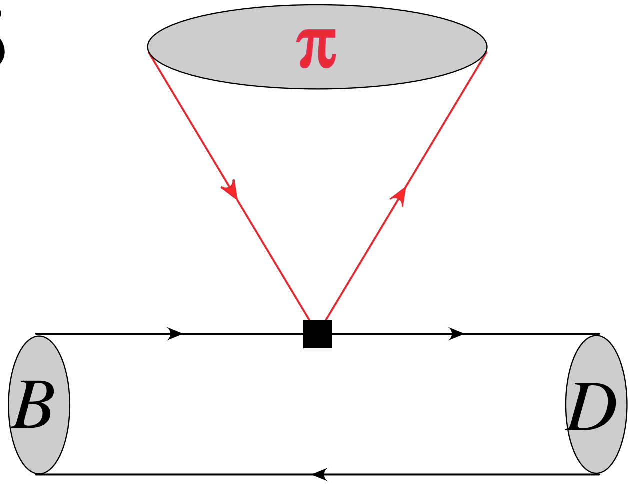
$$n^\mu = (1, 0, 0, 1)$$
$$\bar{n}^\mu = (1, 0, 0, -1)$$

$$\boxed{\begin{aligned} n \cdot n &= 0 \\ \bar{n} \cdot \bar{n} &= 0 \\ n \cdot \bar{n} &= 2 \end{aligned}}$$

vector projection

$$\begin{aligned} x_\mu &= \frac{1}{2} \underset{\downarrow}{n \cdot x}\, \bar{n}_\mu + \frac{1}{2} \underset{\swarrow}{\bar{n} \cdot x}\, n_\mu + x_\mu^\perp \\ &= \frac{1}{2} x_+ \bar{n}_\mu + \frac{1}{2} x_- n_\mu + x_\mu^\perp \end{aligned}$$

SCET - Modes



Energetic quarks and leptons ->
collinear modes

$$p_c = (p_+, p_-, p_\perp) \sim (\frac{\Lambda^2}{Q}, Q, \Lambda) = Q(\lambda^2, 1, \lambda)$$

Include also soft quarks and gluons with momenta

$$p_s = (p_+, p_-, p_\perp) \sim (\Lambda, \Lambda, \Lambda) = Q(\lambda, \lambda, \lambda)$$

Construct the effective theory as an expansion in $\lambda^2 = \frac{\Lambda}{Q}$

Fields and momentum scaling

Introduce quark and gluon fields for each relevant region of loop momenta

modes	field	$p_\mu \sim (+,-,\perp)$	p^2
hard	—	(Q,Q,Q)	Q^2
hard-collinear	$A_{n,q}, \xi_{n,p}$	$(\Lambda, Q, \sqrt{Q\Lambda})$	ΛQ
collinear	$A_{n,q}, \xi_{n,p}$	$(\Lambda^2/Q, Q, \Lambda)$	Λ^2
soft/ultrasoft	A_s, q, b_v	$(\Lambda,\Lambda,\Lambda)$	Λ^2

Can be perturbative/nonperturbative, depending on their virtuality

Power counting

$$\boxed{\lambda^2 = \frac{\Lambda}{Q}}$$

Assign a power counting in λ to the effective theory fields such that the leading action is $O(\lambda^0)$

$$S = i \int d^4x \mathcal{L}(x) \sim \lambda^{-4} \times (\underbrace{\lambda^4}_{\text{LO}} + \underbrace{\lambda^5}_{\text{NLO}} + \cdots)$$

$x_+ \sim \lambda^0$
$x_- \sim \lambda^{-2}$
$x_\perp \sim \lambda^{-1}$

$$\mathcal{L}_{\rm SCET} = \bar{\xi}_{n,p} \frac{\bar{n}\!\!\!/}{2} \Big[\underset{\lambda^2}{in\cdot D_{us}} + \underset{?}{gn\cdot A_{n,q}} + (\underset{\lambda}{p\!\!\!/_\perp} + \underset{?}{iD\!\!\!\!/_\perp}) \frac{1}{\underset{\lambda^0}{\bar{n}\cdot p} + \underset{?}{\bar{n}\cdot iD}} (\underset{\lambda}{p\!\!\!/_\perp} + \underset{?}{iD\!\!\!\!/_\perp}) \Big] \xi_{n,p}$$

Covariant derivatives can act on the collinear gluon fields, producing large factors ~ Q.

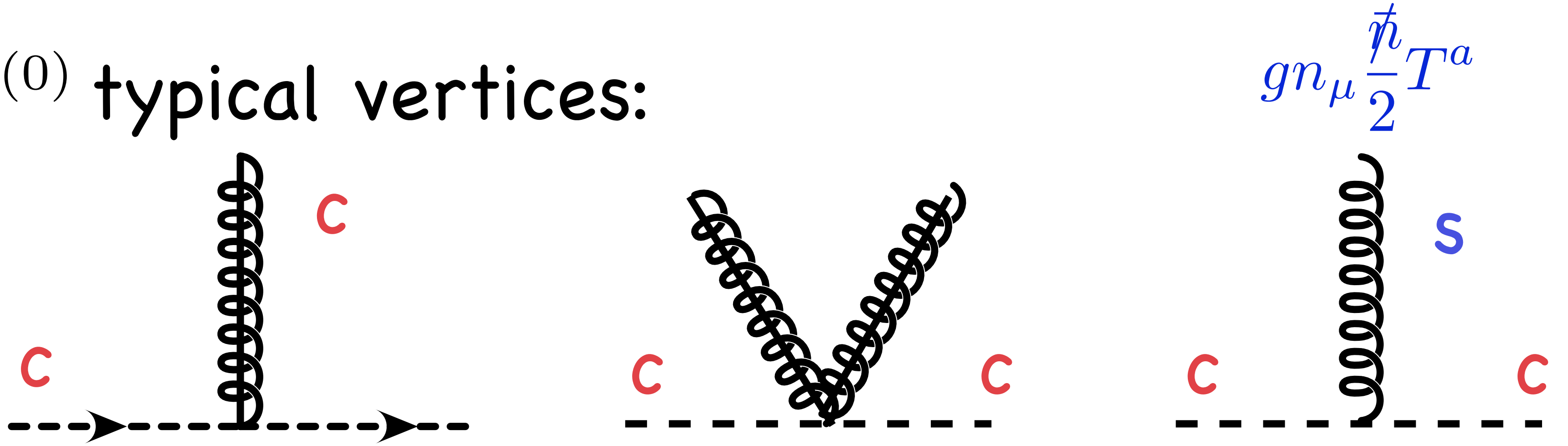
Redefine the collinear gluon field to make explicit all collinear momenta

$$\boxed{A^{a\mu}(x) = e^{-iq\cdot x} A^{a\mu}_{n,q}(x)}$$

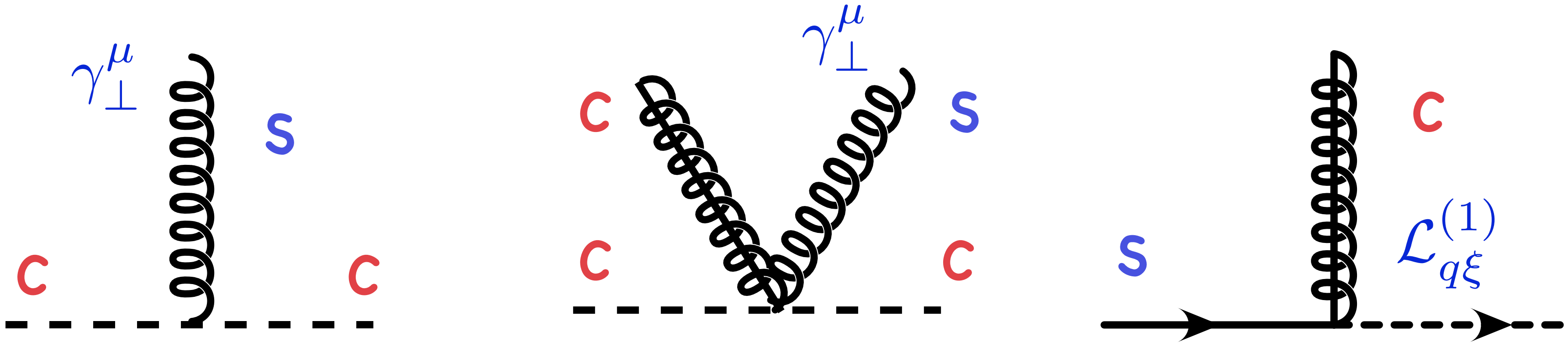
The SCET Lagrangian

$$\mathcal{L}_{\rm SCET} = \mathcal{L}^{(0)} + \mathcal{L}^{(1)} + \cdots$$

$\mathcal{L}^{(0)}$ typical vertices:

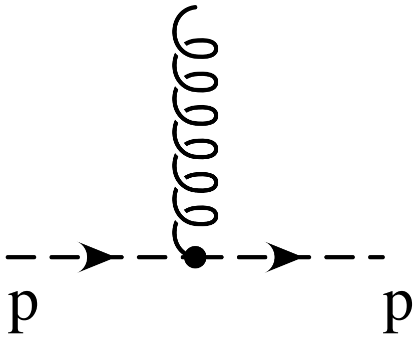


$\mathcal{L}^{(1)}$ allows also soft-collinear vertices, e.g.

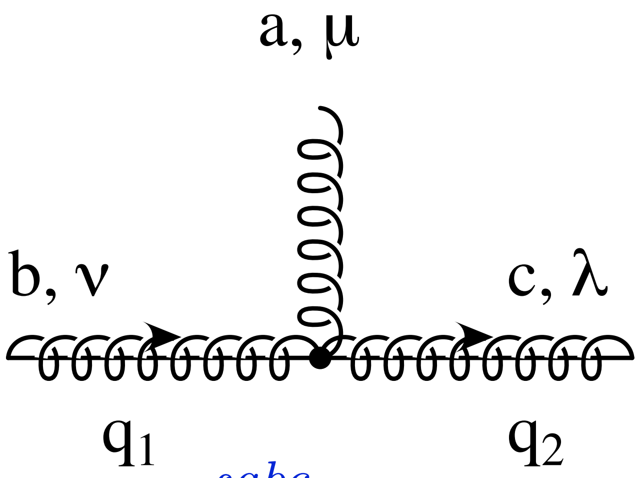


Soft-collinear factorization

Ultrasoft gluons have eikonal couplings to the collinear quarks and gluons at leading order in SCET.



$-ign_\mu T^a$



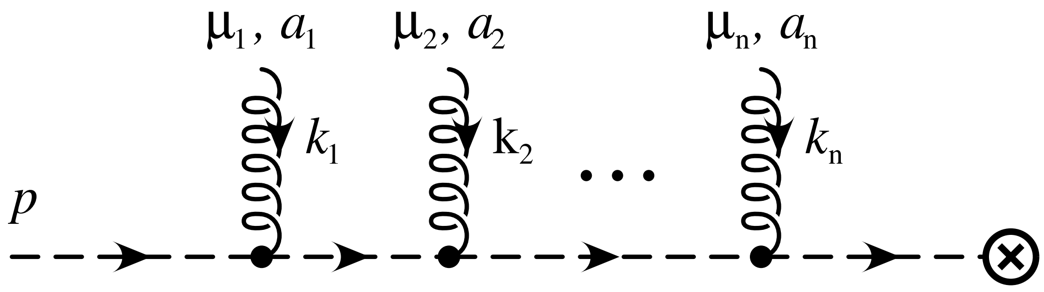
$gf^{abc}n_\mu \bar{n}\cdot q$

These couplings can be absorbed to all orders into field redefinitions

$$\xi_n = Y[n\cdot A_{\rm us}]\xi_n^{(0)} \qquad A_n^\mu = Y[n\cdot A_{\rm us}]A_n^{(0)\mu}Y^\dagger[n\cdot A_{\rm us}]$$

where

$Y[n\cdot A_{\rm us}] =$



Y[n.A] is a Wilson line of the ultrasoft gluon field

$$Y(x) = P\exp\left(ig\int_{-\infty}^{x} \mathrm{d}\lambda n\cdot A_{\mathrm{us}}(\lambda n)\right)$$

Mathematical identity

$$\boxed{n^{\mu}\frac{\partial}{\partial x_{\mu}}\left[\exp\left(\int_{-\infty}^{x} \mathrm{d}\lambda f(\lambda n)\right)\right] = f(x)}$$

$\Longrightarrow$ Y[n.A] satisfies the equation of motion

$$n\cdot iD_{\mathrm{us}}Y(x) = (n\cdot i\partial + gn\cdot A_{\mathrm{us}})Y(x) = 0$$

$\Longrightarrow$ The leading order SCET Lagrangian becomes

$$\begin{aligned}\mathcal{L}^{(0)} &= \bar{\xi}_{n,p}\frac{\not{\bar{n}}}{2}\left[in\cdot D_{us} + gn\cdot A_{n,q} + i\not{D}_{\perp}\frac{1}{\bar{n}\cdot i\mathcal{D}}i\not{D}_{\perp}\right]\xi_{n,p} \\ &= \bar{\xi}^{(0)}_{n,p}\frac{\not{\bar{n}}}{2}\left[in\cdot\partial + gn\cdot A^{(0)}_{n,q} + i\not{D}^{(0)}_{\perp}\frac{1}{\bar{n}\cdot i\mathcal{D}^{(0)}}i\not{D}^{(0)}_{\perp}\right]\xi^{(0)}_{n,p}\end{aligned}$$

General result

$$\boxed{\mathcal{L}[\xi_n, A_n^\mu, A_{\rm us}^\mu] = \mathcal{L}[\xi_n^{(0)}, A_n^{(0)\mu}, 0]}$$

The field redefinition removes all ultrasoft gluon couplings from the Lagrangian (at leading order) and shifts them into operators (currents, etc.)

Example: heavy-light current

$$\mathcal{J}(\omega) = [\bar\xi_n W]_\omega \Gamma b_v = [\bar\xi_n^{(0)} W^{(0)} Y_n^\dagger [n\cdot A_{\rm us}]]_\omega \Gamma b_v$$

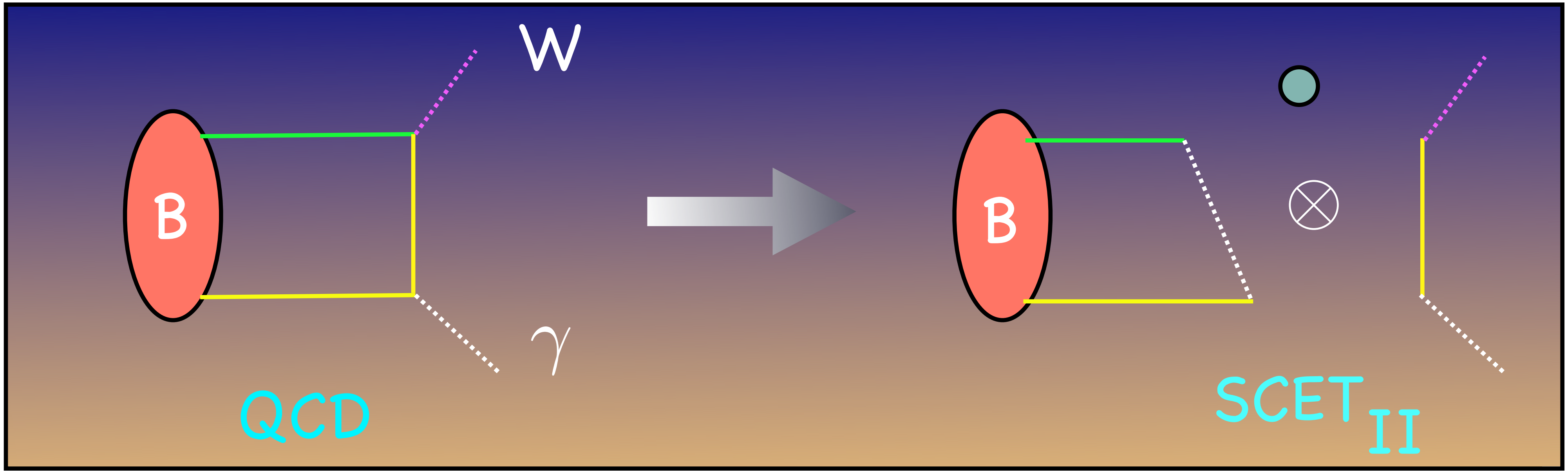
$\Longrightarrow$ Simplifies all-orders factorization proofs

Application

Factorization in radiative leptonic B decays

Factorization in $B \to \gamma \ell \bar{\nu}$

Simplest B decay, mediated by the b->u weak current

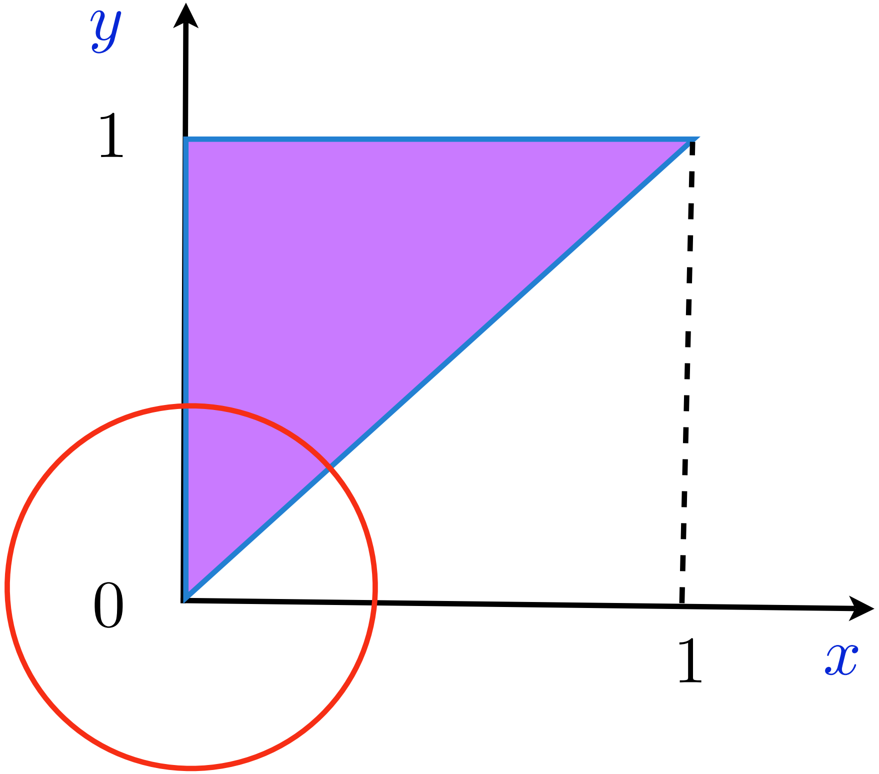


In the kinematic region w/ a hard photon $E_\gamma \gg \Lambda_{\text{QCD}}$

this amplitude can be described in factorization

Kinematics $B \to \gamma \ell \bar{\nu}$

Dalitz plot



$$x = 1 - \frac{2E_\gamma}{m_B}$$

$$y = \frac{2E_e}{m_B}$$

Validity region of the factorization relation:

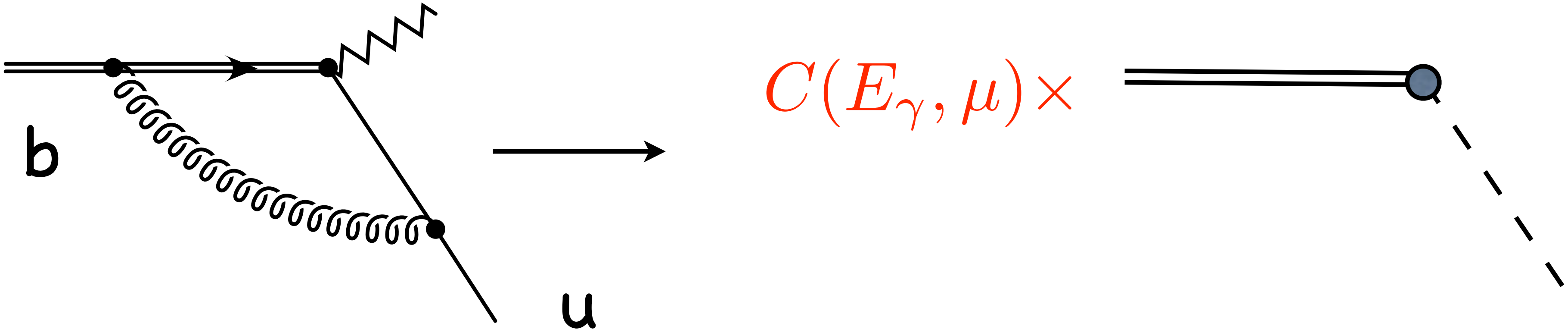
$$E_\gamma \gg \Lambda_{QCD} \sim 500 \text{ MeV}$$

$$1 - x \gg 0.2$$

Factorization relation

$$A(B \to \gamma \ell \bar{\nu}) \sim C^{(v)}(E_\gamma, \mu) \int_0^\infty dk_+ J(k_+) \phi_B^+(k_+)$$

- Wilson coefficient: contains physics effects from the hard scale $\mu = Q \equiv 2E_\gamma \sim m_b$

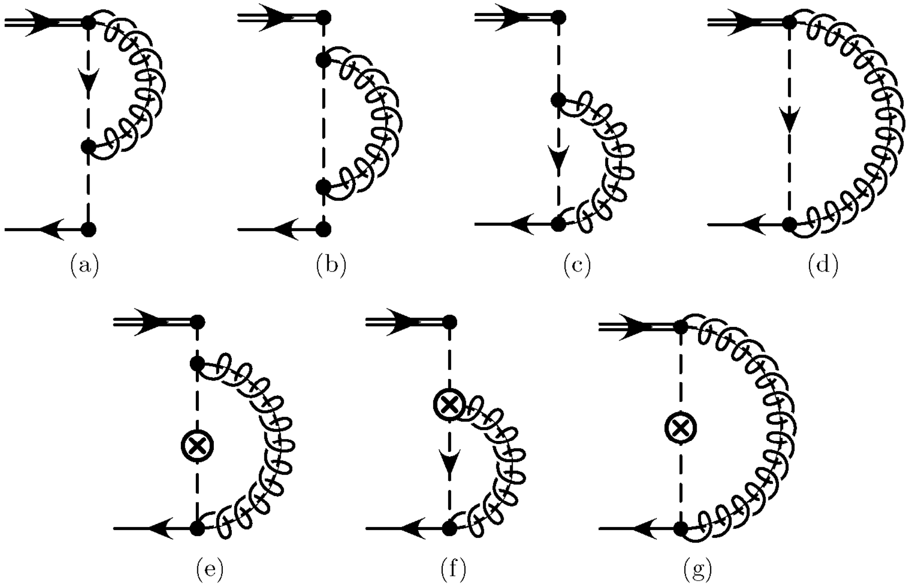


$C(E_\gamma, \mu) \times$

$$C(E_\gamma, \mu) = 1 - \frac{\alpha_s C_F}{4\pi} \left[2 \log^2 \frac{2E_\gamma}{\mu} - 5 \log \frac{2E_\gamma}{\mu} + \cdots \right]$$

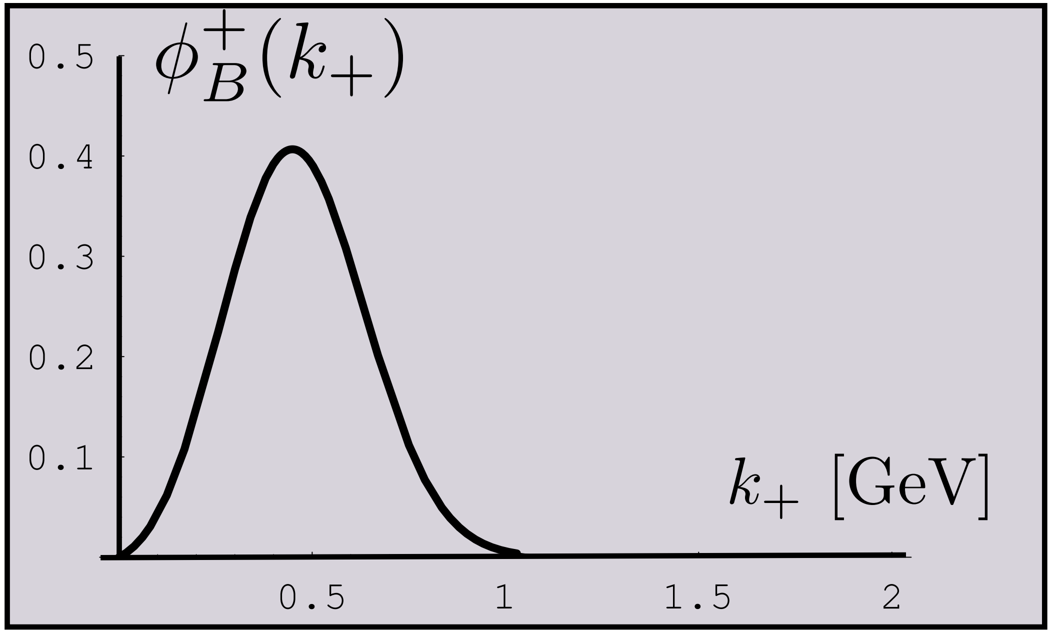
- Jet function:

$$J(k_+) = \frac{1}{k_+}(1 + \frac{\alpha_s C_F}{4\pi} f(E_\gamma k_+/\mu^2))$$



- B meson light-cone wave function

nonperturbative input



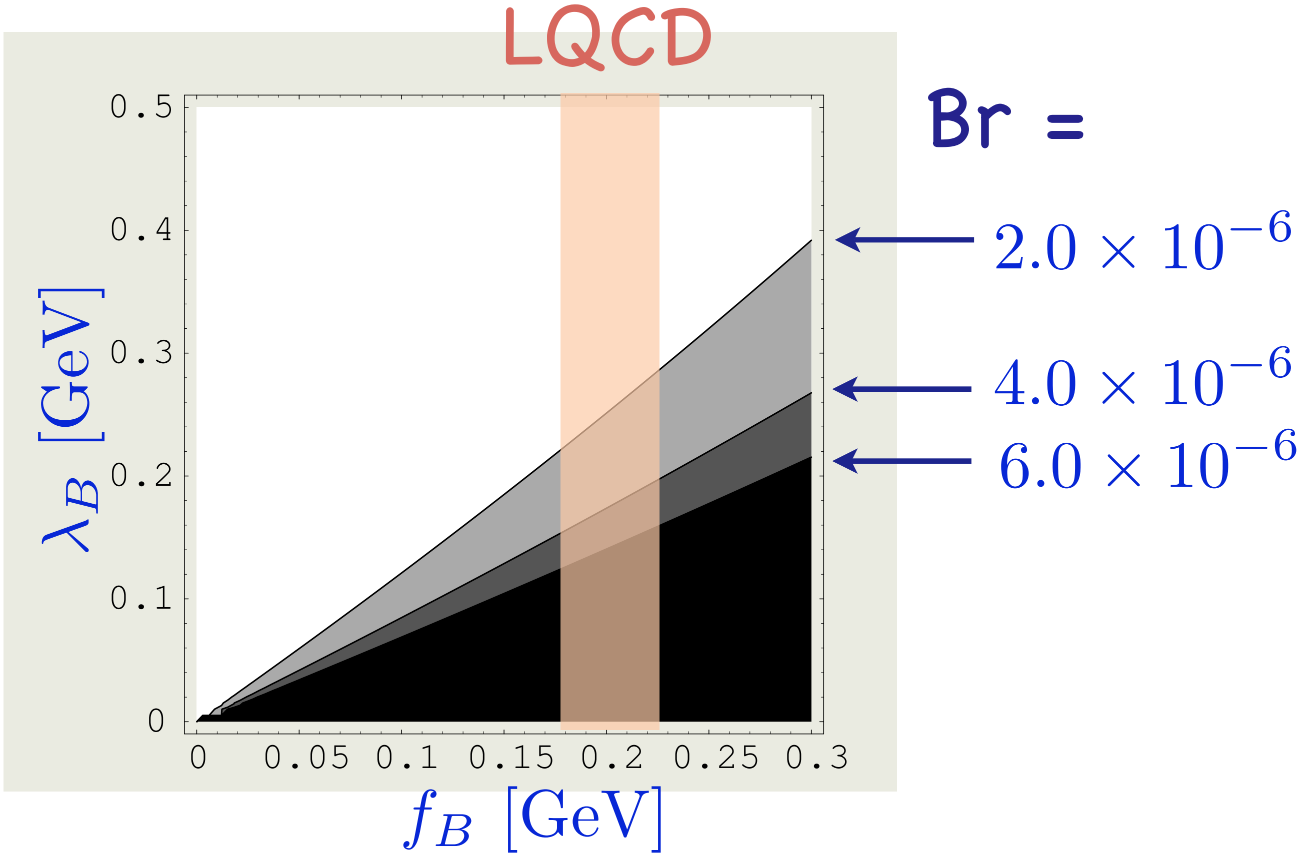
$$\langle 0|\bar{q}(\lambda n)Y_n(\lambda,0)b(0)|\bar{B}\rangle = \int dk_+ e^{-i\lambda k_+}\left[\frac{1+\not{v}}{2}(\not{\bar{n}}\phi_B^+(k_+) + \not{n}\phi_B^-(k_+))\gamma_5\right]$$

Predictions from factorization

The factorized amplitude depends (at LO in $\alpha_s(\sqrt{\Lambda Q})$) on hadronic B physics through

$$\frac{1}{\lambda_B} = \int dk_+ \frac{\phi_B^+(k_+)}{k_+}$$

Clean determination of (f_B, λ_B) from the cut rate



Br =

2.0×10^{-6}

4.0×10^{-6}

6.0×10^{-6}

Cuts (example)

$E_\gamma = (0.65\,, 2.35)$ GeV

$E_\ell = (2.0\,, 2.85)$ GeV

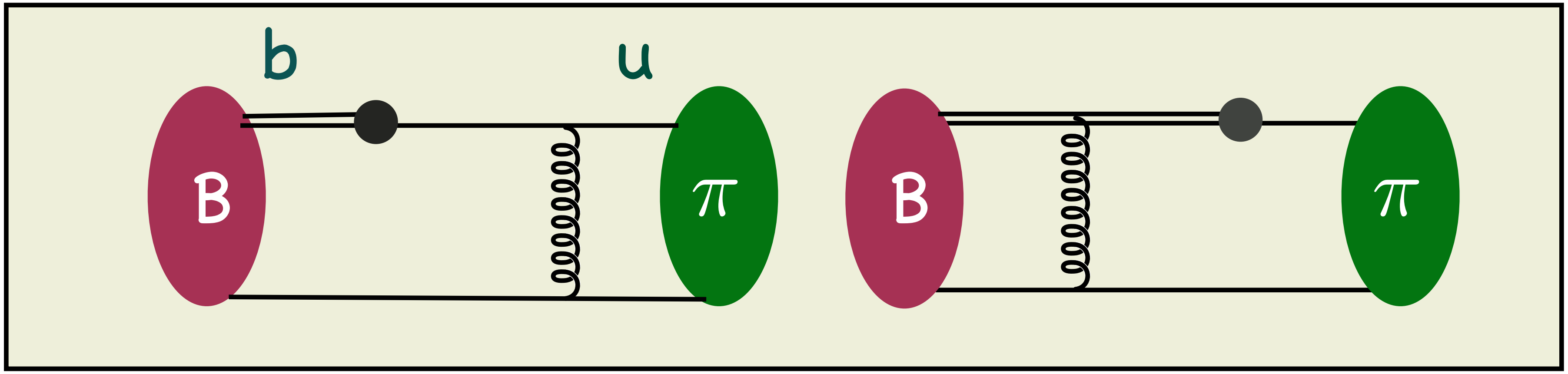
$\cos\theta_{\ell\gamma} \leq -0.42$

Decays into energetic light hadrons

Large recoil

Consider B semileptonic or rare decays into one energetic light particle

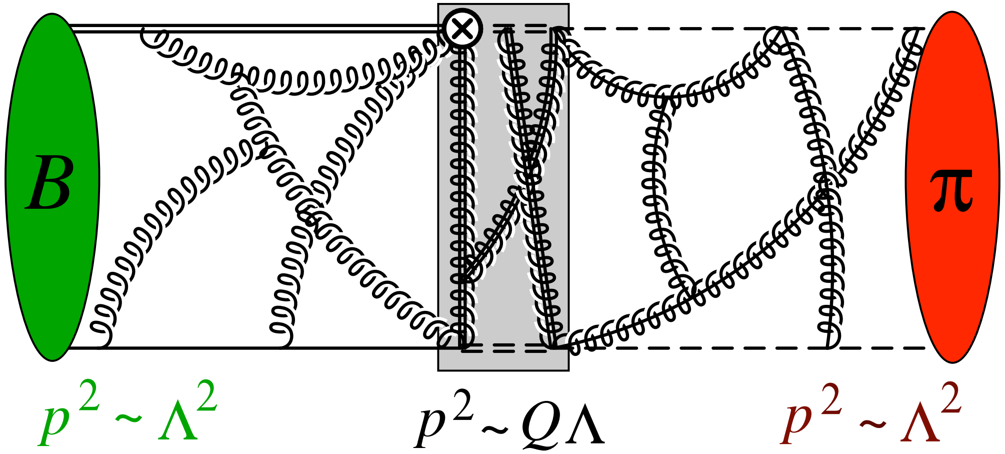
E.g. $B \to \pi \ell \bar{\nu}$ at q^2=0 $(E_\pi \sim 2.2 \text{ GeV})$



In the large recoil region, SCET gives a factorization relation for the $B \to \pi$ form factors

Factorization for heavy-light form factors

Form factors contain soft and hard scattering terms



Wilson coefficients

jet function

$$\boxed{f_i(E) = C_i(E,\mu)\zeta(E,\mu) + \int_0^1 dx dk_+ B_i(E,\mu,z) J(x,z,k_+)\phi_B^+(k_+)\phi_\pi(x)}$$

“nonfactorizable” “factorizable”

Connection to nonleptonic decays

Nonleptonic B decays into light mesons

hard $p^2 \sim Q^2$
hard-collinear $p^2 \sim Q\Lambda$
collinear
soft
$\left.\right\} p^2 \sim \Lambda^2$

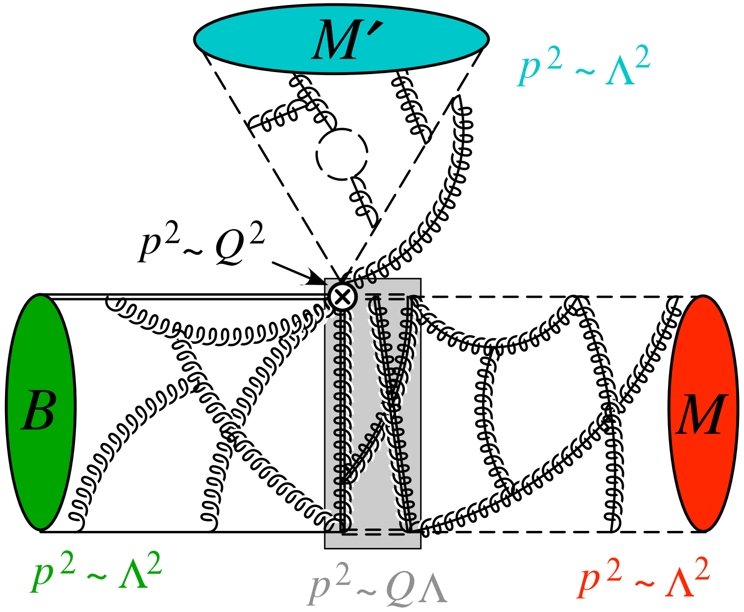
Energy scales in nonleptonic B -> MM′ decays - same as those in B -> M

Factorization relation

$$A(B \to M_1 M_2) = f_{M_2} \zeta^{BM_1} \int_0^1 du T_2(u) \phi_{M_2}(u)$$

$$+ \int_0^1 dz du T_{2J}(u,z) \zeta_J^{BM_1}(z) \phi_{M_2}(u) + (1 \leftrightarrow 2)$$

Huge simplification:
no new parameters needed!



Other applications to B physics

- Inclusive semileptonic $B \to X_u \ell \bar{\nu}$ and radiative $B \to X_s \gamma$ decays: leading order in Λ / m_b and first power corrections

- Factorization and resummation for the cut inclusive rates, e.g. $B \to X_s \gamma$ with photon energy cut

- Factorization in exclusive B decays

  - nonleptonic decays into final states with heavy mesons $B \to D^{(*)} \pi$

  - color suppressed decays $B \to D^0 \pi^0$

  - extension to multibody decays $B \to \pi \gamma \ell \nu$

    $B \to K \pi \gamma$

Applications to light quark physics

- Deep inelastic scattering, DY near x->1

- Jet physics $e^+e^- \to \bar{q}q, \bar{q}qg, \cdots$

  power corrections to jet shape variables

- Extension to unstable particles

Many more results...

- Semileptonic and radiative B decays into multibody states (one collinear + multiple soft pions)

  e.g. $B \to \pi\pi\ell\bar{\nu}$ $\quad B \to K_n\pi\ell^+\ell^-$

- Corrections to the forward-backward asymmetry in

  $B \to K_n\pi\ell^+\ell^-$

- Nonleptonic B decays into multibody final states

Larger branching ratios, more observables

Summary

- The strong interaction effects in hard processes can be described using the technique of effective field theories
- EFTs work by separating the contributions of the relevant energy scales: factorization theorems
- Heavy Quark Effective Theory (HQET): applicable to situations involving slow moving heavy hadrons
- Soft-Collinear Effective Theory (SCET): effective theory for soft and energetic quarks and gluons
- Rigorous factorization theorems for many hard processes, both at leading order and for power corrections.

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