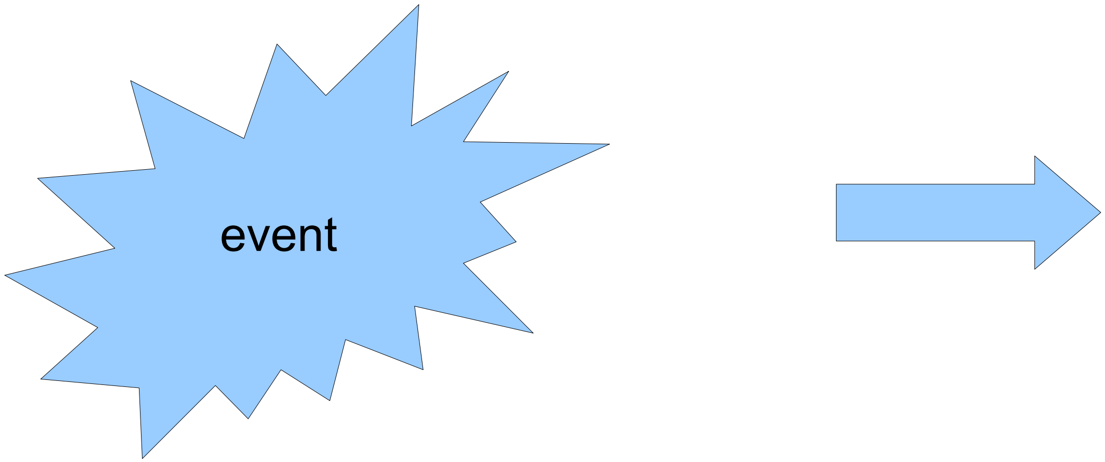
Nuclear electronics

A simple introduction to some basic aspects of the analogue front end part of detector electronics

Patrick Van Esch (ILL)

Detection, current, charge I



Number
Or
Decision

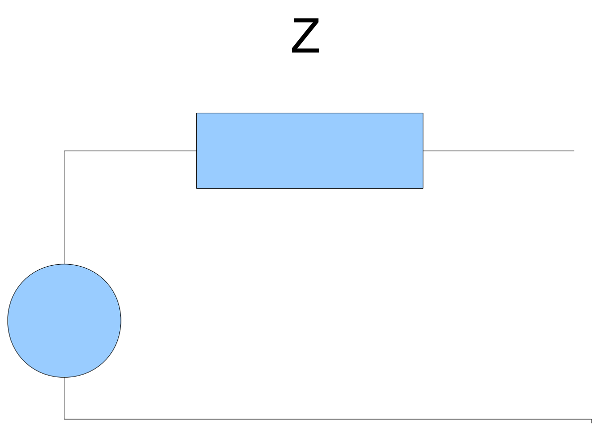
Physical detection → movement of charges in strong E field
- Induction of currents in conductors (electrodes)

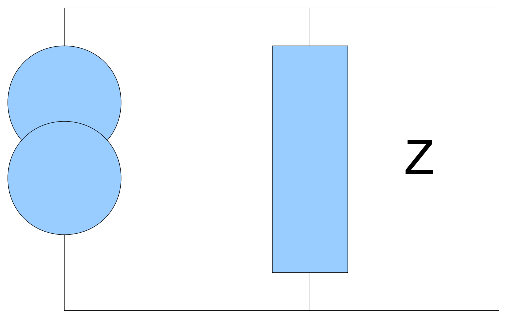
Most other instrumentation:
- Change in impedance
- Change in electro(chemical) potential

Thevenin and Norton

A « source » is represented by an electric circuit, which can be simplified as:

- An imperfect voltage source (Thevenin)
- An imperfect current source (Norton)





Amplifiers

Amplifiers are « matched » to sources.

- Ideal voltage sources with ideal voltage amplifiers
- Ideal current sources with ideal current amplifiers

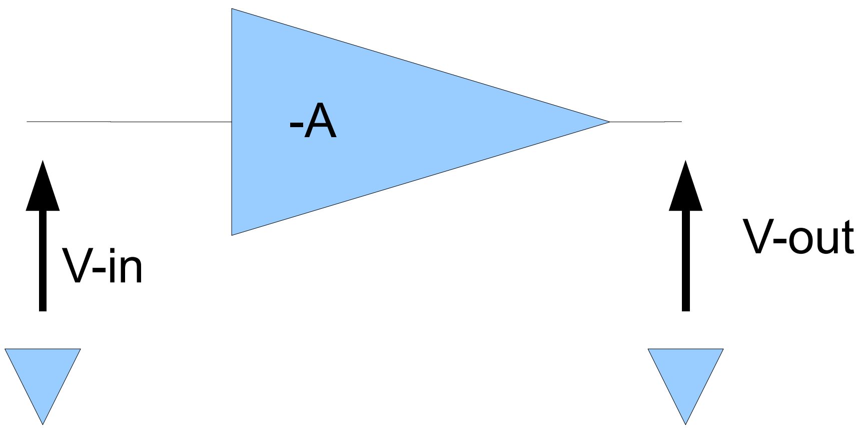
Voltage amplifier: high impedance

Current amplifier: low impedance

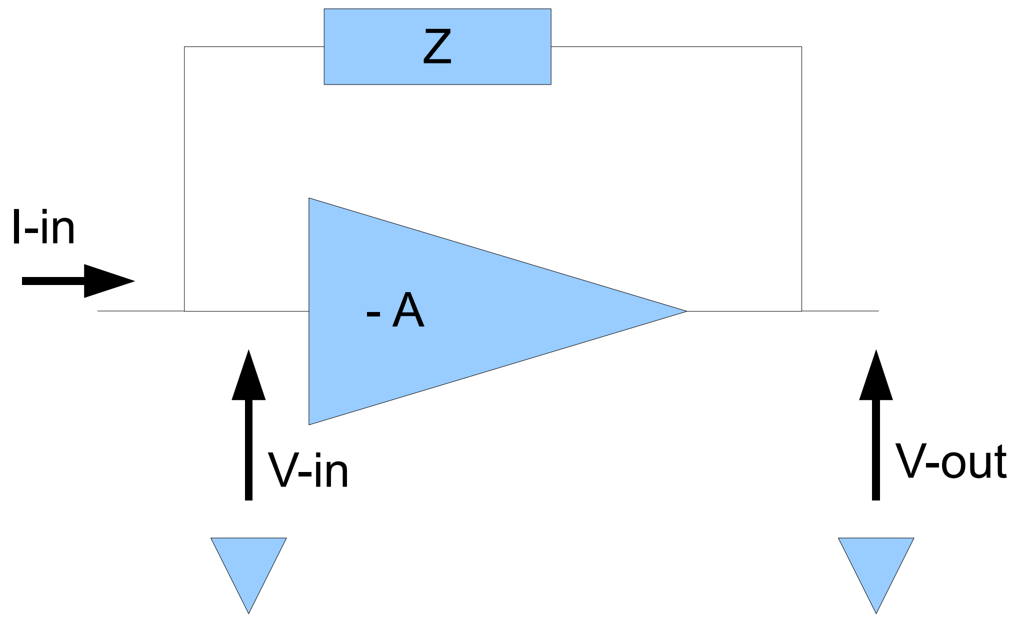
In fact, the output is always a voltage:

- Voltage – voltage amplifier (« normal » amplifier)
- Current – voltage amplifier (transimpedance amp.)

How to make a transimpedance amplifier ?



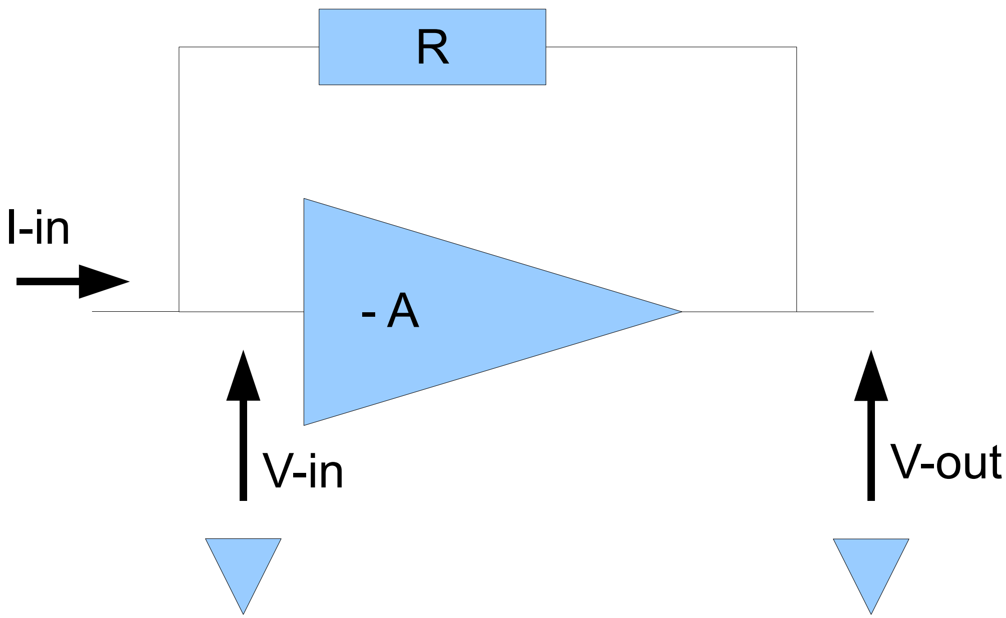
V-out = - A V-in

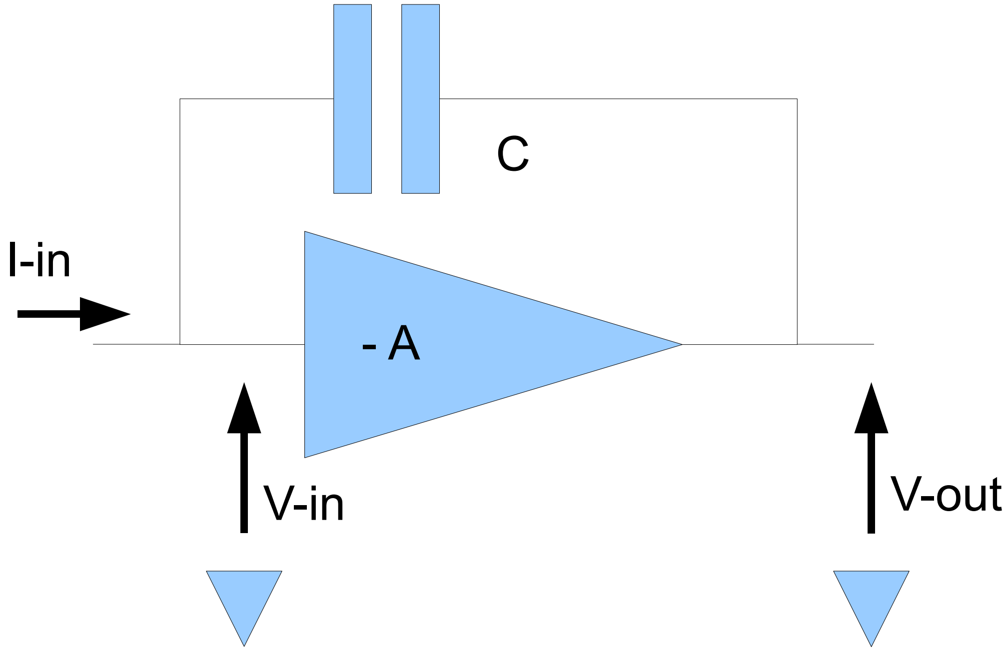


V-out = - A V-in ;
V-in – V-out = I-in Z ;
A very big, hence V-in much smaller than V-out, hence:

V-out = - Z I-in

« Current » vs. « Charge »





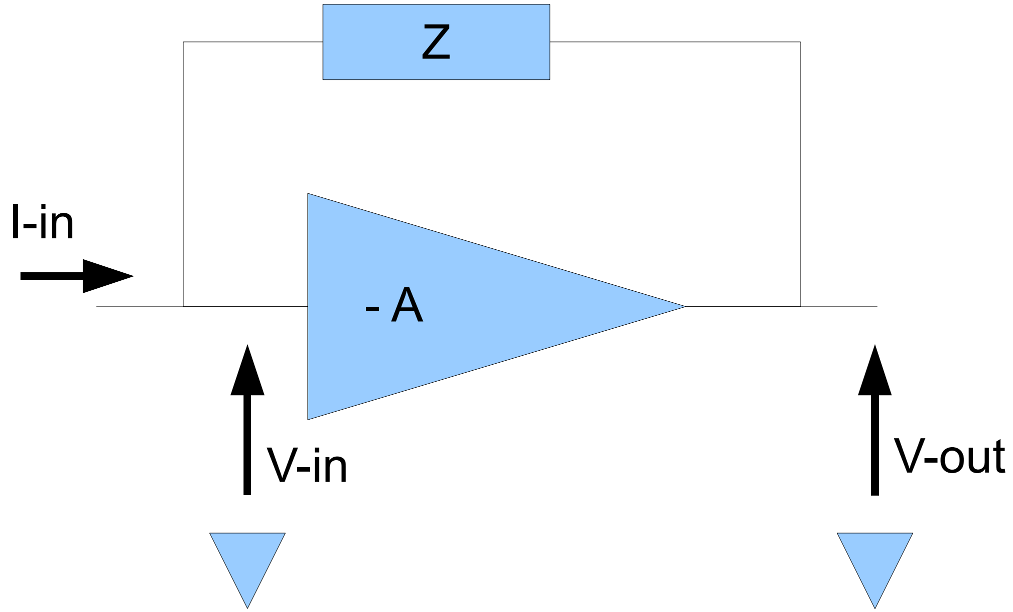
$$V_{out}(s) = -R\,I_i(s)$$

$$v_{out}(t) = -R\,i_i(t)$$

$$V_{out}(s) = \frac{-1}{s.C} I_i(s)$$

$$V_{out}(t) = \frac{-1}{C} \int i_i(t)\,dt = \frac{-Q(t)}{C}$$

Input impedance



$$V_i = V_{out} + i_i . Z$$

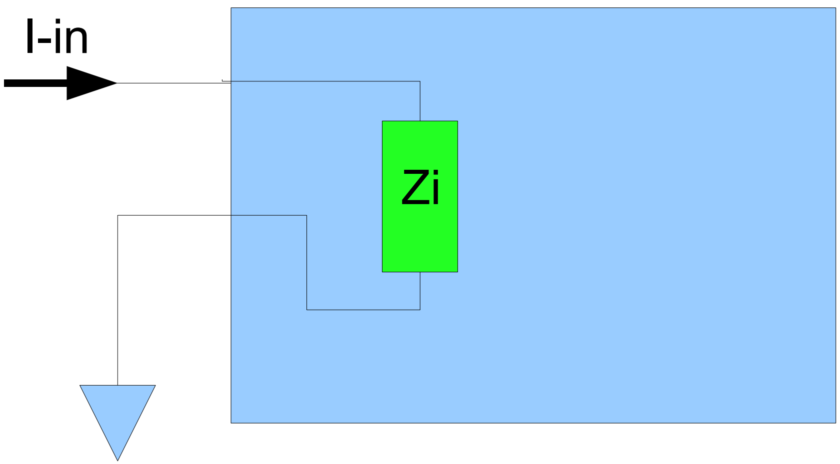
$$V_{out} = -A V_i$$

$$V_i = -A V_i + i_i Z$$

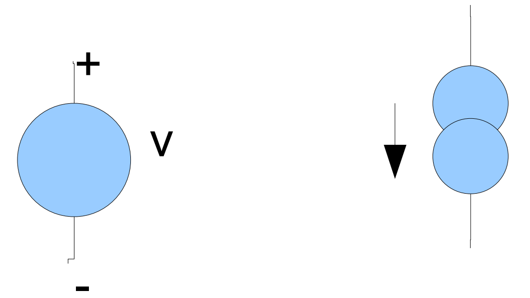
$$Z_i = -A . Z_i + Z$$

$$(1+A) Z_i = Z$$

$$Z_i = \frac{Z}{A+1}$$



Noise sources I



Represented by voltage or current sources

V or I is a random process with average 0 and given autocorrelation.

$$f(t_1) = X_1 ; f(t_2) = X_2$$

$$\langle X_1 \rangle = 0$$

$$R(\tau) = \langle X_1 X_2 \rangle ; \tau = t_2 - t_1$$

$$R(0) = \langle X_1^2 \rangle = f_{rms}^2$$

Wiener-Khinchine: power spectral density (one-sided)

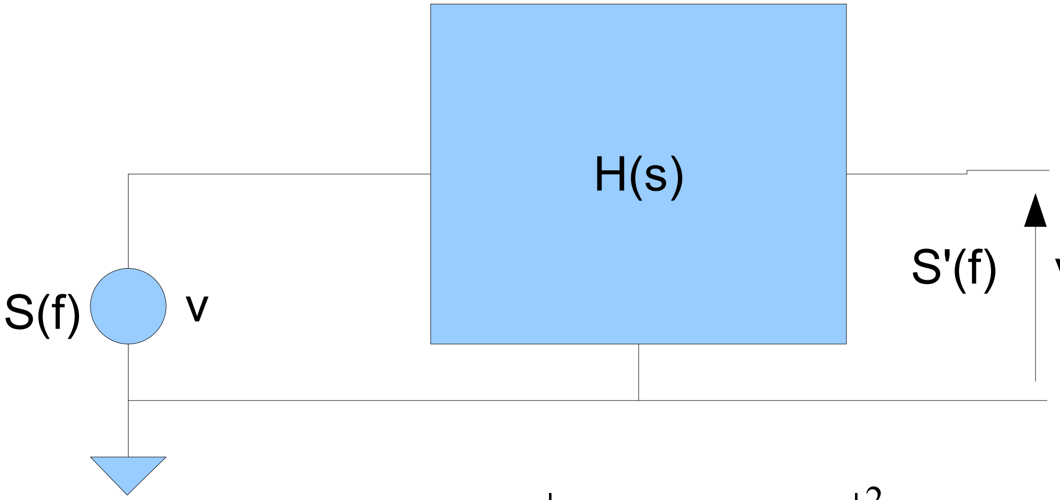
$$S(f) = 4 \int_{\tau=0}^{\infty} R(\tau) \cos(2\pi f \tau) d\tau$$

Noise sources II

- Power spectral density and RMS

$$v_{RMS} = \sqrt{R(0)} = \sqrt{\int_{f=0}^{\infty} S(f)\,df}$$

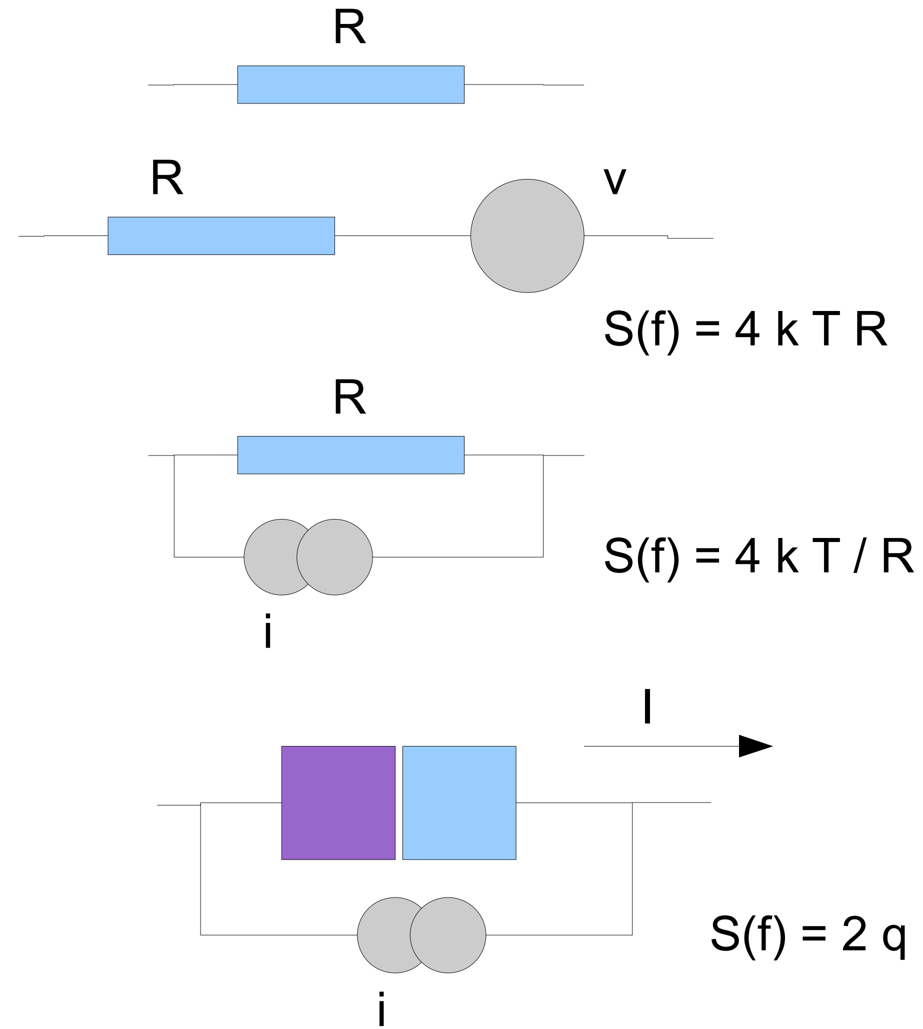
- Noise and linear filter
- White noise
  - S(f) = constant
  - R ~ Dirac function



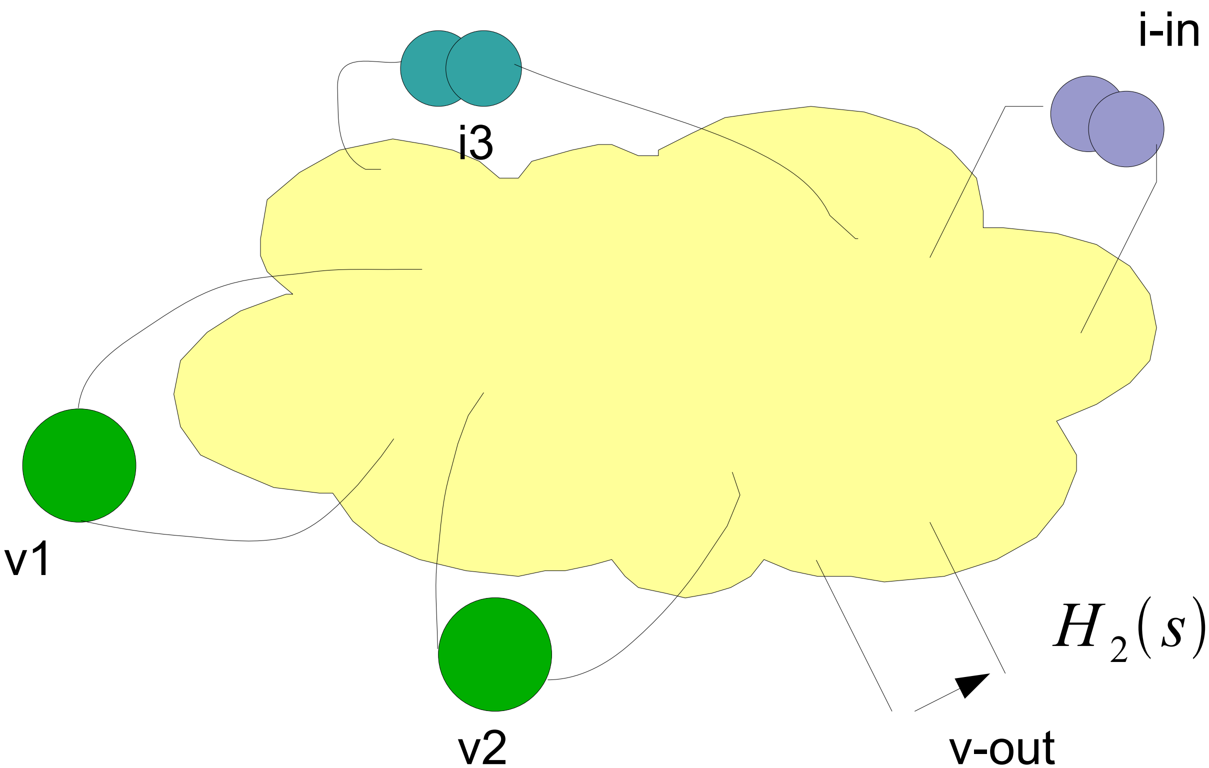
$$S'(f) = |H(j2\pi f)|^2 S(f)$$

Physical noise sources

- Thermal (Johnson or Nyquist) noise in resistors (white noise)
- Shot noise in PN barriers (white noise)
- Flicker noise, 1/f noise, pink noise



Networks with noise sources.

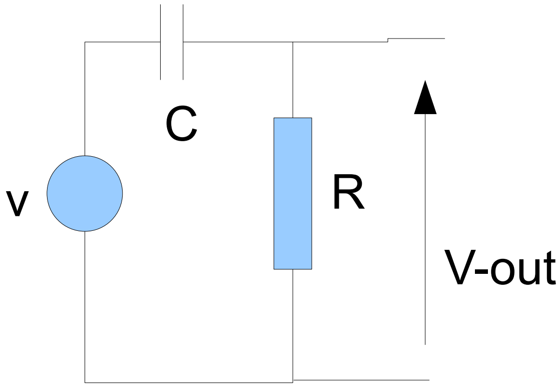


$$V_{out}(s) = H_1(s)V_1(s) + \ldots$$
$$H_2(s)V_2(s) + Z_3(s)I_3(s) + \underline{Z(s)I_i(s)}$$

$$S_{out}(f) = |H_1(j2\pi f)|^2 S_1(f) + |H_2(j2\pi f)|^2 S_2(f) + |Z_3(j2\pi f)|^2 S_3(f)$$

$$S_{equi}(f) = \frac{S_{out}(f)}{|Z(j2\pi f)|^2}$$

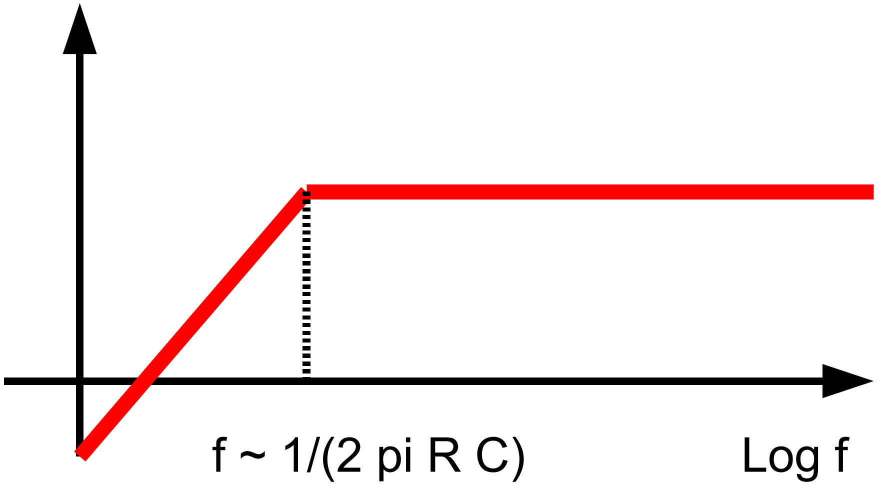
Example 1



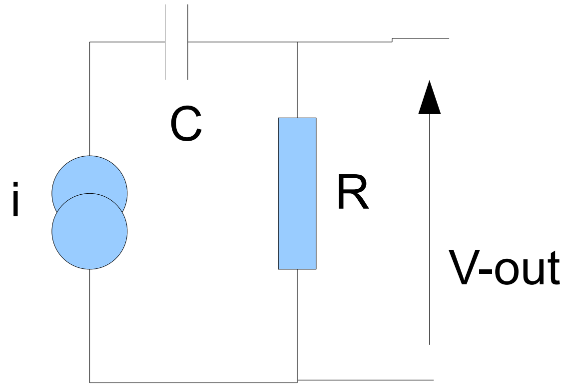
$$V_{out}(s) = \frac{R}{R + \frac{1}{sC}} V_i = \frac{RCs}{1 + RCs} V_i$$

$$|H(j2\pi f)|^2 = \left|\frac{j2\pi f RC}{1 + j2\pi f RC}\right|^2 = \frac{4\pi^2 f^2 R^2 C^2}{1 + 4\pi^2 f^2 R^2 C^2}$$

$$S'(f) = \frac{4\pi^2 f^2 R^2 C^2}{1 + 4\pi^2 f^2 R^2 C^2} S(f)$$



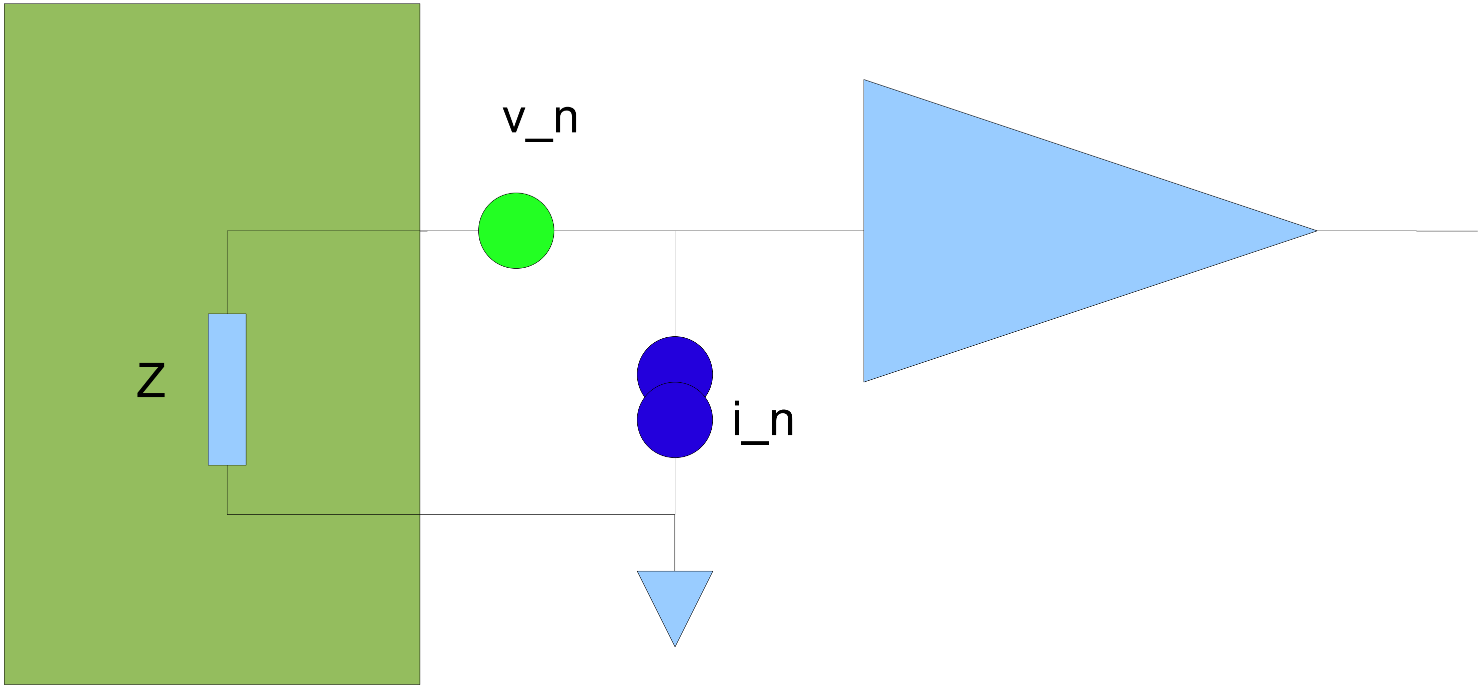
Example 2 (exercise)



$$V_{out}(s) = R\,I(s)$$

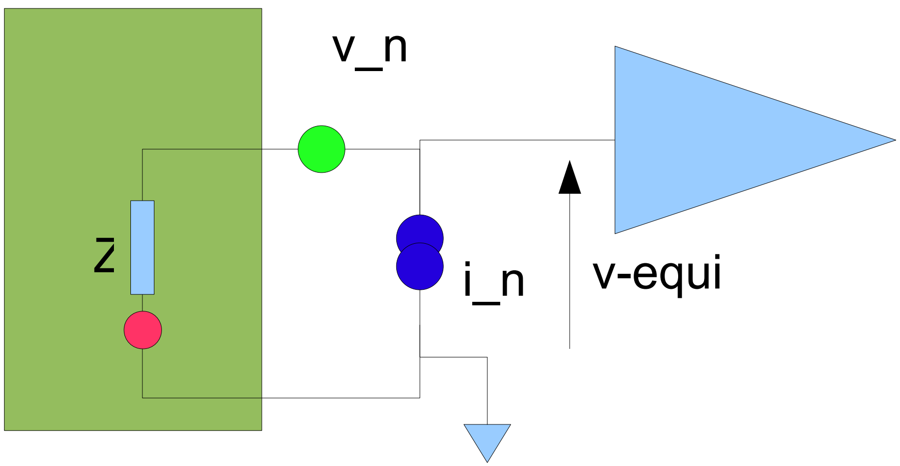
S'(f) = ?

Equivalent noise of amplifier



Equivalent noise of amplifier II

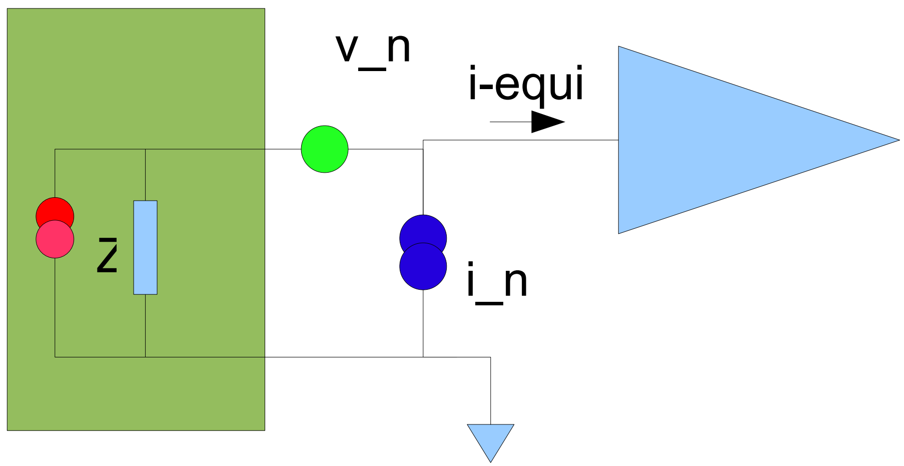
Case of voltage amplifier (Thevenin)



$$v_{equi}(s) = v_n(s) + Z(s) i_n(s)$$

$$S_{equi}(f) = S_{v_n}(f) + |Z(j 2\pi f)|^2 S_{i_n}(f)$$

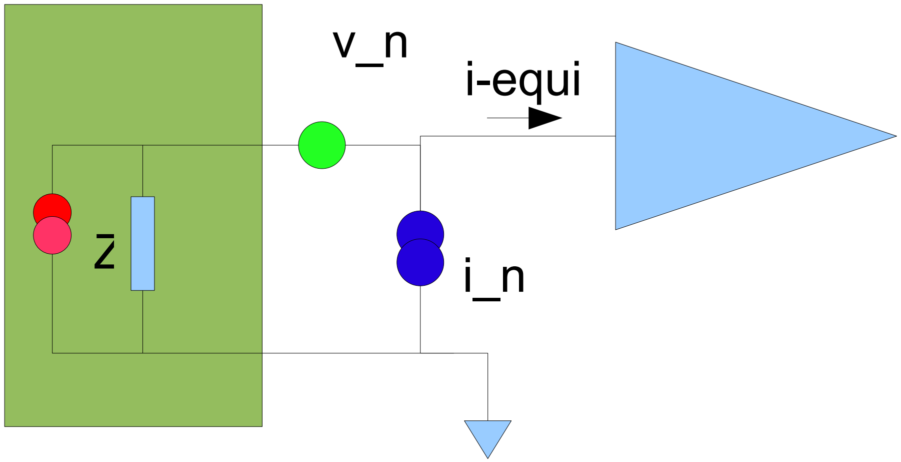
Case of current amplifier (Norton)



$$i_{equi}(s) = i_n(s) + \frac{v_n(s)}{Z(s)}$$

$$S_{equi}(f) = S_{i_n}(f) + \frac{S_{v_n}(f)}{|Z(j 2\pi f)|^2}$$

Equivalent noise of amplifier III



$$S_{equi}(f) = S_{i_n}(f) + \frac{S_{v_n}(f)}{|Z(j2\pi f)|^2}$$

Load is capacitive: $Z = \frac{1}{sC}$

$$S_{equi}(f) = S_{i_n}(f) + 4\pi^2 f^2 S_{v_n}(f)$$

Load is resistive: Z = R

$$S_{equi}(f) = S_{i_n}(f) + \frac{S_{v_n}(f)}{R^2}$$

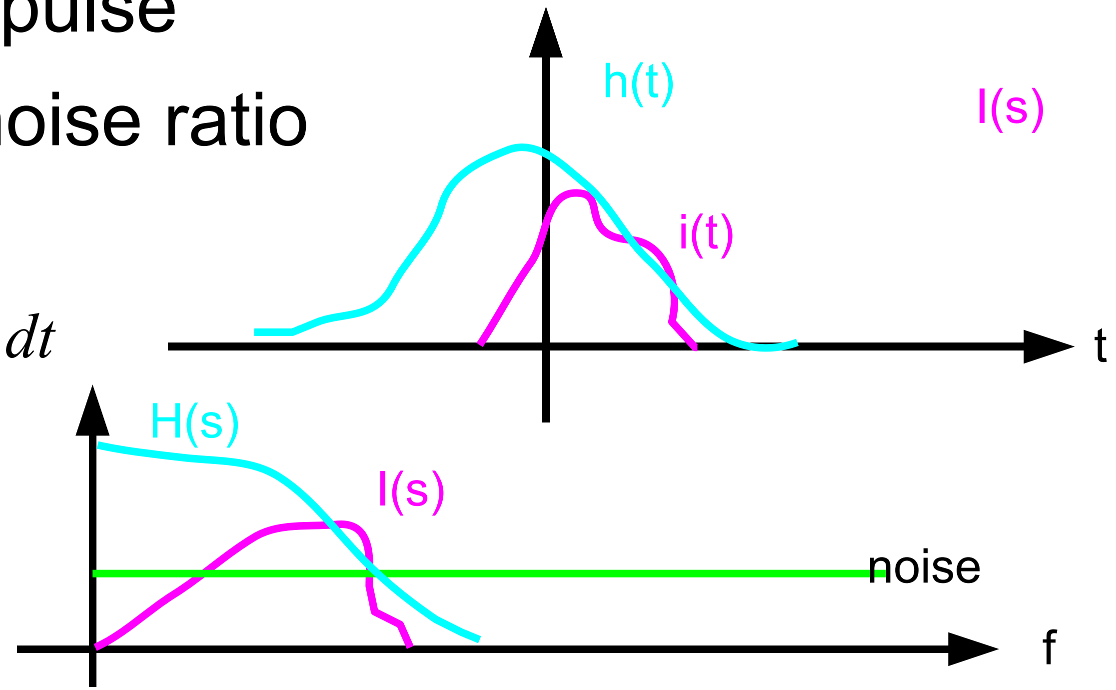
Shaping

Shaping for charge: impulse response of overall circuit

- long enough to « integrate » the charge
- short enough to limit « dead time »
- signal = maximum of pulse
- best possible signal/noise ratio

$$v_{out}(t)=\int_{\tau=-\infty}^{t} i(\tau)\,h(t-\tau)\,dt$$

$$V(s)=H(s)\,I(s)$$



Shaping and scaling

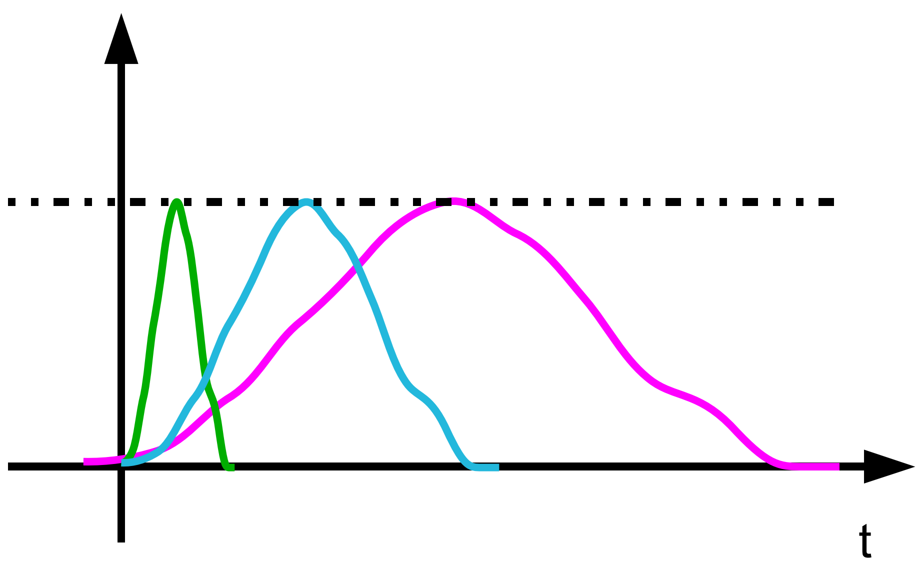
A « unit charge » will generate an impulse response of which the height is the « gain » of the amplifier chain.

Scaling the time axis (k times faster), but keeping the gain (maximum value):

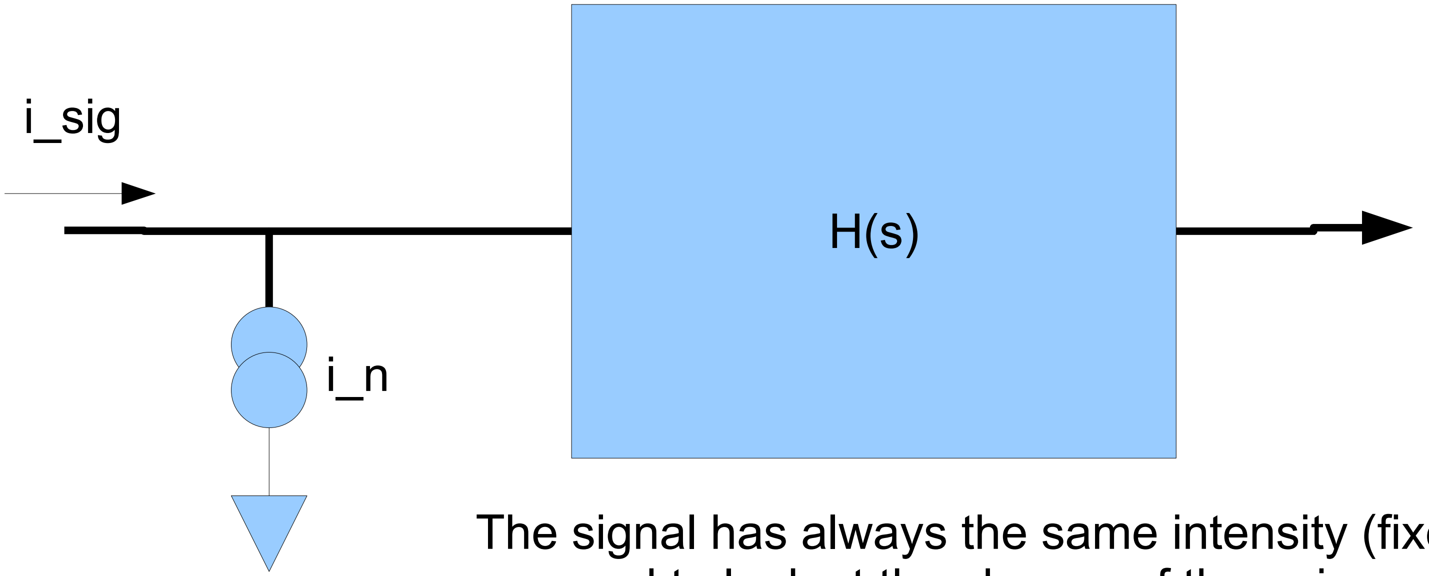
$$h(t) \rightarrow h'(t) = h(k\,t)$$

$$H(s) = \int_0^\infty h(t)\, e^{-s\,t}\, dt$$

$$H(s) \rightarrow H'(s) = \frac{1}{|k|} H\left(\frac{s}{k}\right)$$



Application: scaling of S/N 1



The signal has always the same intensity (fixed gain)
we need to look at the change of the noise as a function of time scale.
Noise = RMS value of noise (to be compared with peak signal)

$$N_1 = \sqrt{\int_{f=0}^{\infty} |H(j2\pi f)|^2 S(f)\,df}$$

$$N_2 = \frac{1}{k}\sqrt{\int_{f=0}^{\infty} \left|H\left(j2\pi \frac{f}{k}\right)\right|^2 S(f)\,df}$$

$$= \frac{1}{\sqrt{k}}\sqrt{\int_{u=0}^{\infty} |H(j2\pi u)|^2 S(k u)\,du}$$

f/k = u

Scaling of S/N 2

Let us assume a very simple filter: perfect lowpass filter (with sinc response):

f < f0 then |H(f)| = A

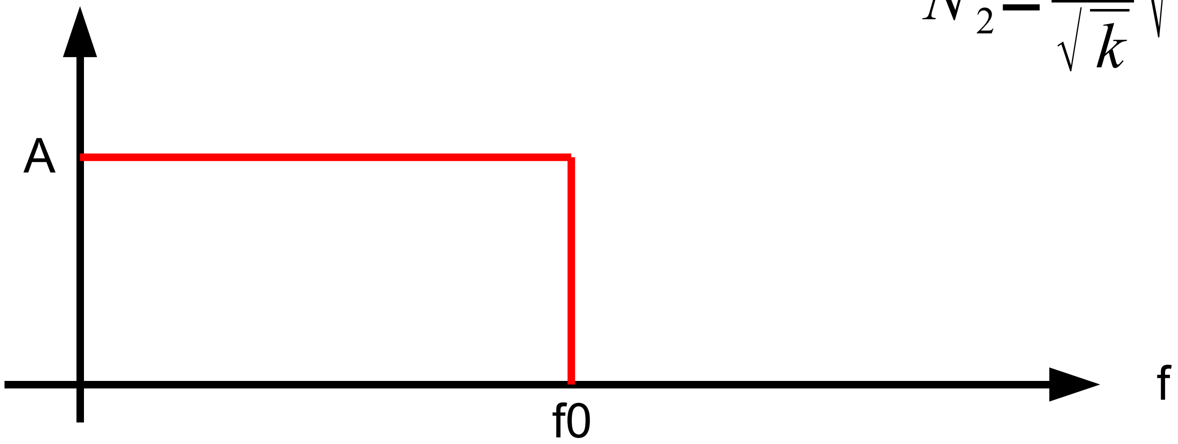
f > f0 then H(f) = 0

$$N_1 = \sqrt{\int_{f=0}^{\infty} |H(j2\pi f)|^2 S(f)\,df}$$

$$N_1 = \sqrt{\int_{f=0}^{f_0} A^2 S(f)\,df}$$

$$N_2 = \frac{1}{\sqrt{k}} \sqrt{\int_{u=0}^{\infty} |H(j2\pi u)|^2 S(ku)\,du}$$

$$N_2 = \frac{1}{\sqrt{k}} \sqrt{\int_{u=0}^{f_0} A^2 S(ku)\,du}$$



Scaling of S/N 3

Case of resistive load

$$S_{equi}(f) = S_{i_n}(f) + \frac{S_{v_n}(f)}{R^2} = S_n$$

assume white equivalent noise sources

$$N_1 = \sqrt{\int_{f=0}^{f_0} A^2 S(f)\, df}$$

$$N_2 = \frac{1}{\sqrt{k}} \sqrt{\int_{u=0}^{f_0} A^2 S(k\,u)\, du}$$

$$N_1 = \sqrt{f_0 A^2 S_n}$$

$$N_2 = \frac{1}{\sqrt{k}} \sqrt{f_0 A^2 S_n} = \frac{N_1}{\sqrt{k}}$$

A k times FASTER amplifier has sqrt(k) times LESS noise ;
the signal-to-noise ratio IMPROVES for faster amplifiers!

Scaling of S/N 4

Capacitive load

Assume only voltage noise,

assume it to be white

current noise will behave as in the « resistive case »

$$S_{equi}(f) = S_{i_n}(f) + 4\pi^2 f^2 S_{v_n}(f)$$

$$S_{equi}(f) = 4\pi^2 f^2 S_{v_n}$$

$$N_1 = \sqrt{\int_{f=0}^{f_0} A^2 S(f)\, df}$$

$$N_2 = \frac{1}{\sqrt{k}} \sqrt{\int_{u=0}^{f_0} A^2 S(ku)\, du}$$

$$N_1 = \sqrt{\int_{f=0}^{f_0} A^2 4\pi^2 f^2 S_{v_n}\, df} = 2\pi A \sqrt{S_{v_n} \frac{f_0^3}{3}}$$

$$N_2 = \frac{1}{\sqrt{k}} \sqrt{\int_{u=0}^{f_0} A^2 4\pi^2 k^2 u^2 S_{v_n}\, du} = \sqrt{k} \sqrt{\int_{u=0}^{f_0} A^2 4\pi^2 u^2 S_{v_n}\, du} = \sqrt{(k)}\, N_1$$

This time, a k times FASTER amplifier has sqrt(k) times MORE noise ;
a faster amplifier DETERIORATES the S/N ratio.

Scaling of S/N 5

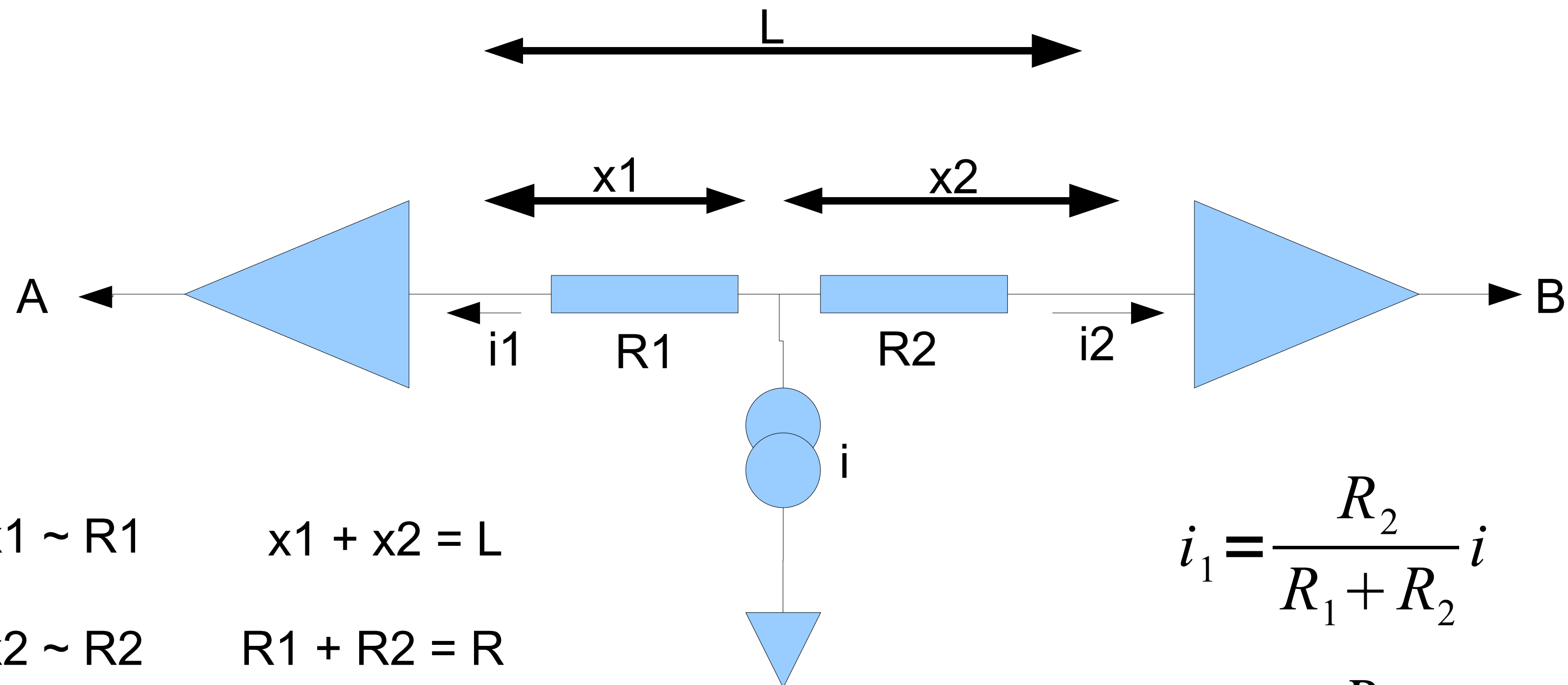
Current noise at the input: a faster amplifier IMPROVES S/N

Voltage noise with resistive load: a faster amplifier IMPROVES S/N

Voltage noise with capacitive load: a faster amplifier DETERIORATES S/N

Mixed case: it depends on the relative contributions: *there will be an optimum amplifier speed.*

Resistive charge division: principle



x1 ~ R1

x2 ~ R2

x1 + x2 = L

R1 + R2 = R

$$i_1 = \frac{R_2}{R_1 + R_2} i$$

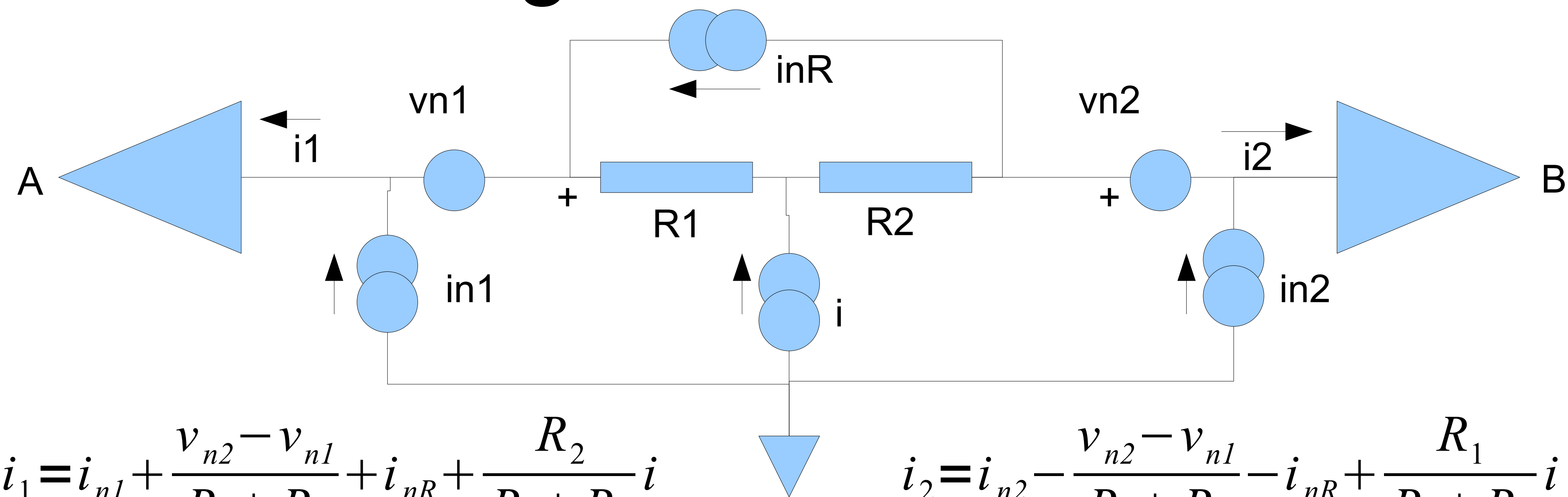
$$i_2 = \frac{R_1}{R_1 + R_2} i$$

A ~ i1

B ~ i2

$$\frac{x_2 - x_1}{L} = \frac{A - B}{A + B}$$

Charge division: noise.



$$i_1 = i_{n1} + \frac{v_{n2} - v_{n1}}{R_1 + R_2} + i_{nR} + \frac{R_2}{R_1 + R_2} i$$

$$i_2 = i_{n2} - \frac{v_{n2} - v_{n1}}{R_1 + R_2} - i_{nR} + \frac{R_1}{R_1 + R_2} i$$

$$i_1 - i_2 = i_{n1} - i_{n2} + 2\frac{v_{n2} - v_{n1}}{R_1 + R_2} + 2 i_{nR} + \frac{R_2 - R_1}{R_1 + R_2} i$$

$$i_1 + i_2 = i_{n1} + i_{n2} + i$$

$$S_{min} = S_i + S_i + \frac{4}{R^2}(S_v + S_v) + 4 S_R = 2 S_i + \frac{8 S_v}{R^2} + \frac{16 k T}{R}$$

$$S_{plus} = 2 S_i$$

Charge division: resolution

$$\delta_{rms} D = \sqrt{\int_{f=0}^{\infty} |H|^2 S_{min} df}$$

$$\delta_{rms} S = \sqrt{\int_{f=0}^{\infty} |H|^2 S_{plus} df}$$

$$p = \frac{D}{S}$$

Relative position from -1 to 1.

$$dp = \frac{dD}{S} - \frac{D}{S}\frac{dS}{S} = \frac{1}{S}(dD - p\, dS)$$

$$x = \frac{p}{2} L$$

Physical position

$$\delta_{rms} p = \frac{1}{S}\sqrt{(\delta_{rms} D)^2 + p^2 (\delta_{rms} S)^2}$$

$$\delta_{rms} x = \frac{L}{2S}\sqrt{(\delta_{rms} D)^2 + p^2 (\delta_{rms} S)^2}$$

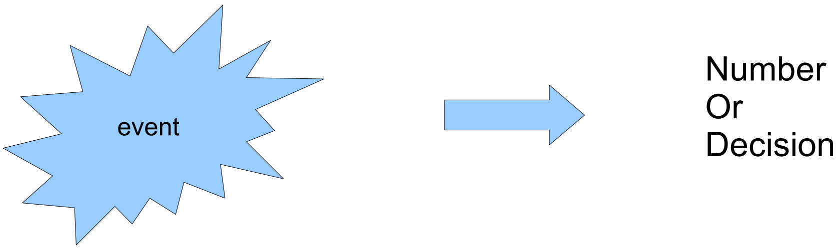
Position resolution (1 sigma)

Nuclear electronics

A simple introduction to some basic aspects of the analogue front end part of detector electronics

Patrick Van Esch (ILL)

Detection, current, charge I



Physical detection → movement of charges in strong E field
Induction of currents in conductors (electrodes)
Most other instrumentation:
Change in impedance
Change in electro(chemical) potential

2

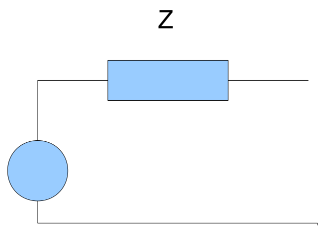
A nuclear detection event usually ressorts ultimately in the movement of charges (photodetectors, gas detectors, silicon detectors,...) which induce currents in electrodes.
Most other instrumentation, most other sensors are rather based upon a change in impedance, or a change in electrochemical potential. Examples: thermocouples, strain gauges, reflectometry, …

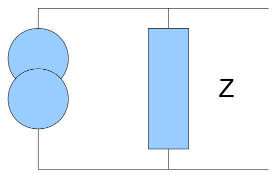
This characterises the nuclear instrumentation front end as particular.

Thevenin and Norton

A « source » is represented by an electric circuit, which can be simplified as:

- An imperfect voltage source (Thevenin)
- An imperfect current source (Norton)





In order to use a physical detection device in an electronic circuit, one has to model it with circuit elements which represent as accurately as possible the electric behaviour of the device. So these devices are modeled as an electrical circuit containing sources and impedances.
Ideal sources are voltage sources or current sources. In practice, a source that acts as a voltage source, will nevertheless lower its voltage when a current is drawn, and a current source will lower its current when it has to drive a high potential difference, due to physical phenomena in whatever the source. This is described by several network elements.
As seen from their « connection point », any such circuit can be represented by a single source and a single impedance. For a voltage source, this is a series impedance (Thevenin), for a current source this is a parallel impedance (Norton). Both are equivalent.

Amplifiers

Amplifiers are « matched » to sources.

- Ideal voltage sources with ideal voltage amplifiers
- Ideal current sources with ideal current amplifiers

Voltage amplifier: high impedance

Current amplifier: low impedance

In fact, the output is always a voltage:

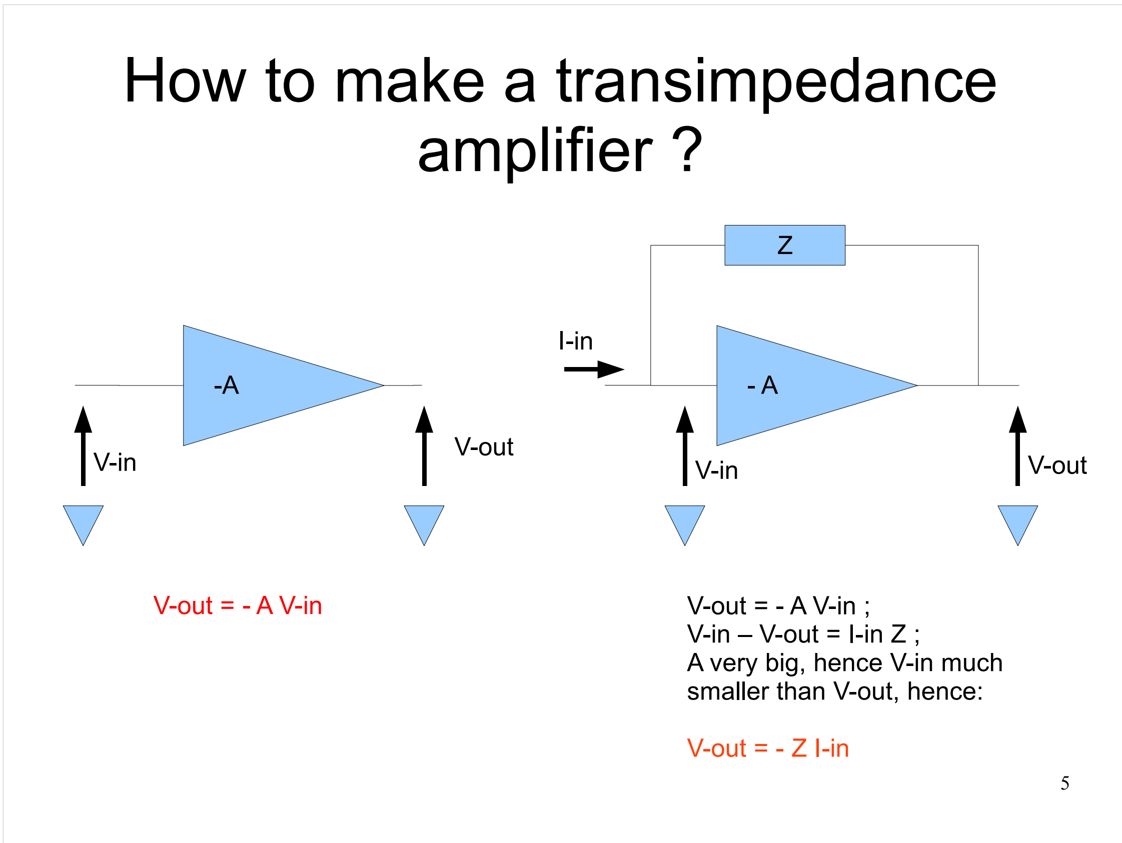
- Voltage – voltage amplifier (« normal » amplifier)
- Current – voltage amplifier (transimpedance amp.)

An amplifier tries to minimise the effects of « imperfections » of the source. This means that a voltage amplifier is not supposed to draw any current, and that a current amplifier is not supposed to show any voltage at its input.

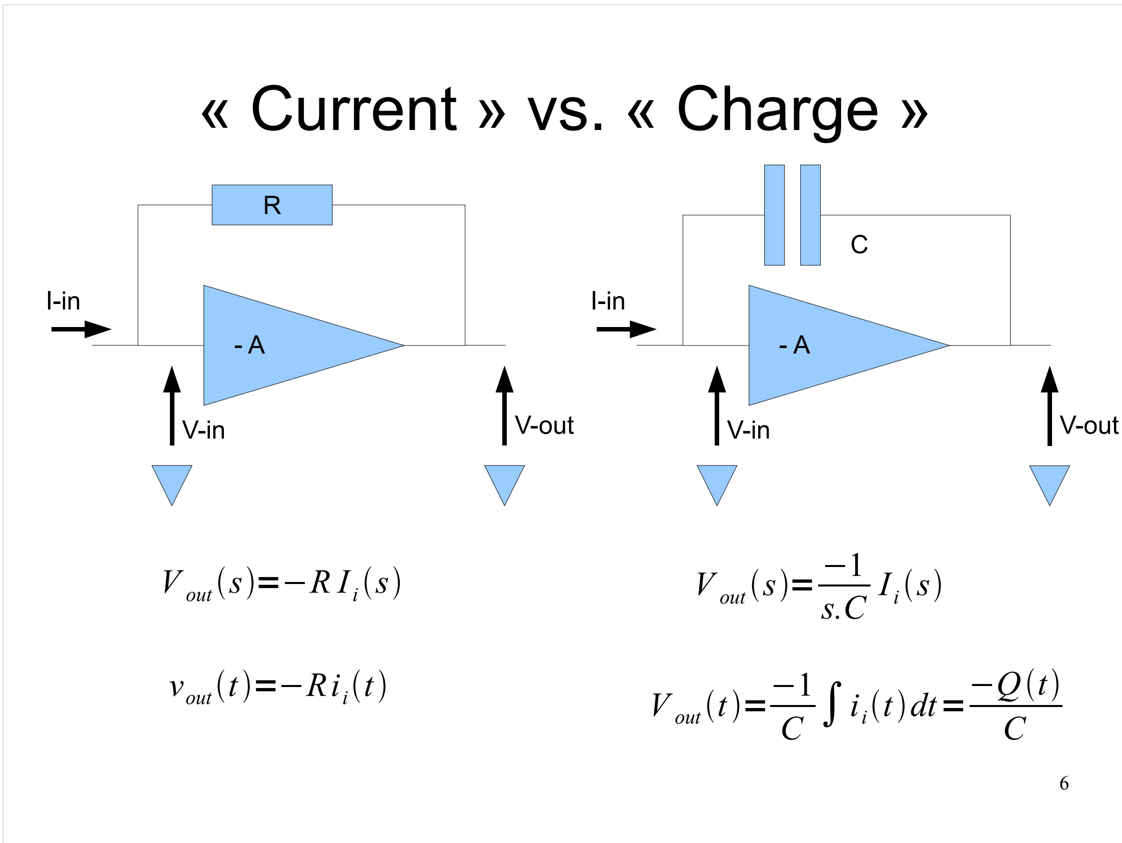
Ideally, a voltage amplifier has an « open » input, and a current amplifier has a « shorted » input. In practice, one tries to have as high as possible the input impedance of a voltage amplifier and as low as possible the input impedance of a current amplifier.

Normally, amplifiers are « voltage amplifiers ». They have a dimensionless amplification (voltage (out) over voltage (in))

Current amplifiers have an impedance (ohms) as amplification: (voltage (out) over current (in)).



A normal amplifier has an input voltage V-in and an output voltage V-out, proportional to the input voltage. Amplification factor (minus) A.
The input impedance is very high (no current flows into the amplifier input)
Consider that we have such an amplifier, with a very large amplification factor A. By connecting an impedance between the input and the output, the input current will flow through this impedance (cannot flow into the amplifier directly). We find that the output voltage is (minus) the impedance times the input current.
The minus sign is necessary for stability.
The calculation is also valid in the frequency domain of course.



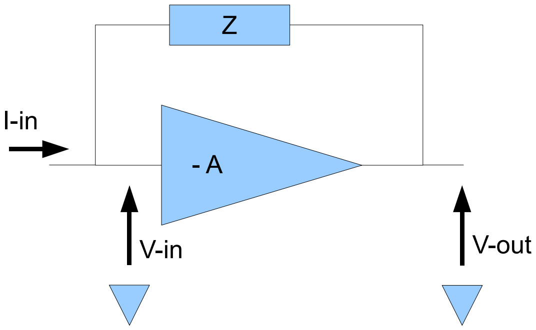
The application to two idealised amplifiers:
- the perfect current amplifier
- the perfect charge amplifier

The output voltage is proportional to the current with the perfect current amplifier.
Amplification is given in « volt per (nano) ampere » or (giga) ohm.

The output voltage is proportional to the integrated charge with the perfect charge amplifier.
Amplification is given in « volt per (pico) coulomb » or « one over (pico) farad ».

Practical limitations: finite bandwidth of amplifier for the current amplifier, drift and bias current for the charge amplifier.

Input impedance



$$V_i = V_{out} + i_i . Z$$

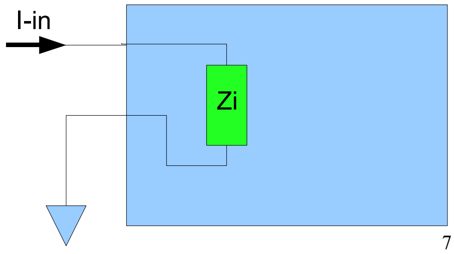
$$V_{out} = -A V_i$$

$$V_i = -A V_i + i_i Z$$

$$Z_i = -A . Z_i + Z$$

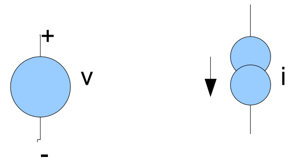
$$(1+A) Z_i = Z$$

$$Z_i = \frac{Z}{A+1}$$



The input impedance is the voltage-current relationship « seen » by the source. It can easily be calculated for our trans-impedance amplifier built around a voltage amplifier.
The aim is to have a low input impedance, and we see that it is proportional to the « gain » of the amplifier (Z), and inversely proportional to the amplification factor of the voltage amplifier we started out with.

Noise sources I



- Represented by voltage or current sources
- V or I is a random process with average 0 and given autocorrelation.
- Wiener-Khinchine: power spectral density (one-sided)

$$f(t_1) = X_1 ; f(t_2) = X_2$$

$$\langle X_1 \rangle = 0$$

$$R(\tau) = \langle X_1 X_2 \rangle ; \tau = t_2 - t_1$$

$$R(0) = \langle X_1^2 \rangle = f_{rms}^2$$

$$S(f) = 4 \int_{\tau=0}^{\infty} R(\tau) \cos(2\pi f \tau) d\tau$$

Noise sources are represented by time dependent voltage sources or current sources in a network. Their time dependence is given by a random process (a randomly drawn function from a set of possible functions with statistical properties).
For each individual time t1, the value of such a random process is a random number with a statistical distribution. We assume that the average is 0.
For each pair of times t1 and t2, the values of the random process at these times is a couple of random numbers. The correlation between the two values as a function of the time difference, is the auto correlation function. For tau = 0, this is nothing else but the square of the RMS value
The (double of the) Fourier transform of the autocorrelation function is the spectral power density.

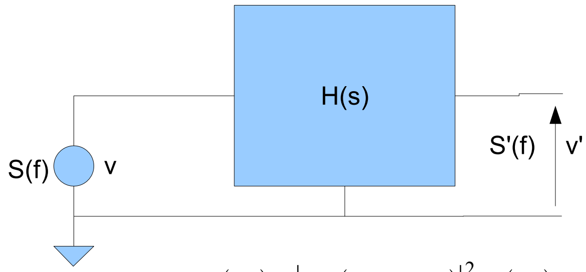
Noise sources II

Power spectral density and RMS $v_{RMS} = \sqrt{R(0)} = \sqrt{\int_{f=0}^{\infty} S(f)\,df}$

Noise and linear filter

White noise

- S(f) = constant
- R ~ Dirac function



$$S'(f) = |H(j2\pi f)|^2 S(f)$$

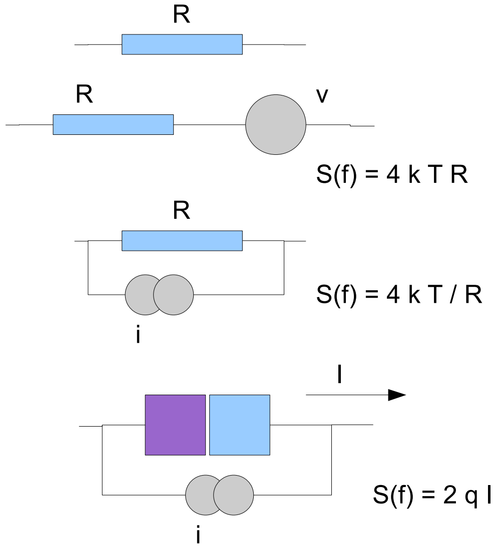
Noise sources are described by their autocorrelation function, or, equivalently, by their power spectral density, which will turn out to be quite useful in circuits.
There is a simple relationship between the rms value of the noise, and the power spectral density: the rms value is the square root of the integral of the PSD.
When a noise signal passes through a linear filter with transfer function H(s), the PSD at the output is related to the PSD of the signal at the input. This is the fundamental property that will allow us to « calculate networks with noise sources »
White noise is an idealized noise source with flat power spectral density (inifinite power). Several physical processes can be approximated with white noise.

Physical noise sources

Thermal (Johnson or Nyquist) noise in resistors (white noise)

Shot noise in PN barriers (white noise)

Flicker noise, 1/f noise, pink noise

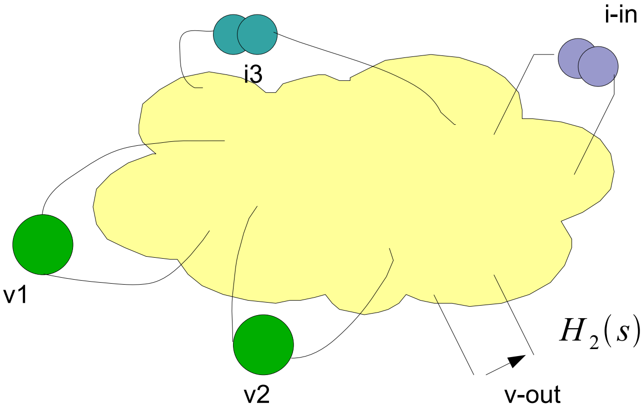


Thermodynamics determines that a resistor must be associated with a noise source: thermal, Johnson or Nyquist noise. There are two possible « implementations »: one in a Thevenin equivalent, the other in a Norton equivalent. As the spectral power density is not dependent on frequency, this is white noise. This noise is also present in the channel of a field effect transistor.

Shot noise is noise due to electrical current represented as an uncorrelated Poisson stream of discrete charges (q). This occurs when a current crosses a PN junction for instance. It is also white noise.

Flicker noise is pink noise: the spectral density goes (more or less) as 1/f. It is technology related, and finds its origin in chaotic phenomena. It occurs for instance in MOSFETs or in carbon resistors.

Networks with noise sources.



$$V_{out}(s) = H_1(s)V_1(s) + \ldots$$
$$H_2(s)V_2(s) + Z_3(s)I_3(s) + \underline{Z(s)I_i(s)}$$

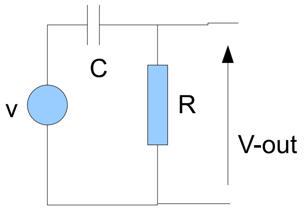
$$S_{out}(f) = |H_1(j2\pi f)|^2 S_1(f) + |H_2(j2\pi f)|^2 S_2(f) + |Z_3(j2\pi f)|^2 S_3(f)$$

$$S_{equi}(f) = \frac{S_{out}(f)}{|Z(j2\pi f)|^2}$$

A general linear network containing several independent noise sources and one input and one output can be treated as follows:

- find the transfer function from each individual source to the output (using the superposition principle)
- the spectral density of the output noise is given by the sum of the spectral densities of the different sources, weighted with the transfer functions, absolute squared.
- one can reduce the output noise to an equivalent input noise source (which has the same effect as all the noise sources together).

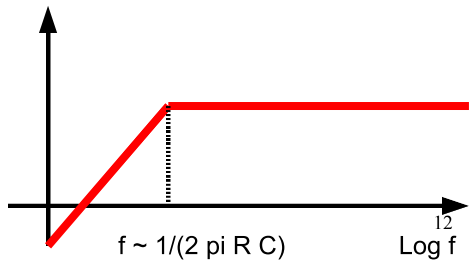
Example 1



$$V_{out}(s)=\frac{R}{R+\frac{1}{sC}}V_i=\frac{RCs}{1+RCs}V_i$$

$$|H(j2\pi f)|^2=\left|\frac{j2\pi fRC}{1+j2\pi fRC}\right|^2=\frac{4\pi^2f^2R^2C^2}{1+4\pi^2f^2R^2C^2}$$

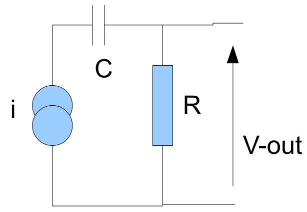
$$S'(f)=\frac{4\pi^2f^2R^2C^2}{1+4\pi^2f^2R^2C^2}S(f)$$



We start with a very simple example. A voltage noise source with power spectral density S(f) is AC coupled to a resistor R through a capacitor C.
We calculate the output voltage as a function of the noise source as if it were a normal voltage source.
From that, we deduce the transfer function H(s).
We calculate the transfer function absolute squared.

This gives us the factor by which we have to weight the power spectral density of the source to find the power spectral density at the output.

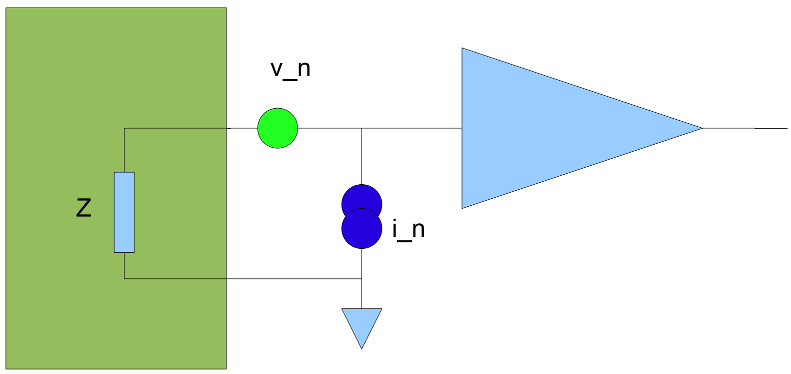
Example 2 (exercise)



$$V_{out}(s) = R\,I(s)$$

S'(f) = ?

Equivalent noise of amplifier



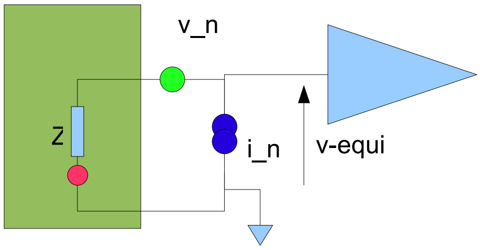
14

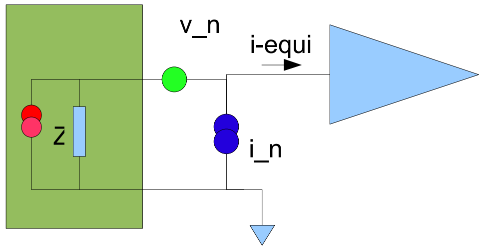
It can be shown that the full effect of the noise sources in an amplifier can be represented by a combination of a voltage noise source and a current noise source (we make the approximation that they are uncorrelated here).
This takes into account the effect of the input load (the Thevenin or Norton equivalent impedance of the source as seen from the input of the amplifier).
In the case the amplifier is driven from a pure current source, only the current noise source matters, and in the case of a pure voltage source, only the voltage source matters.
The reason why we need TWO noise sources is that we need to accomodate the impedance of the source.

Equivalent noise of amplifier II

Case of voltage amplifier (Thevenin)

Case of current amplifier (Norton)





$$v_{equi}(s) = v_n(s) + Z(s)\, i_n(s)$$

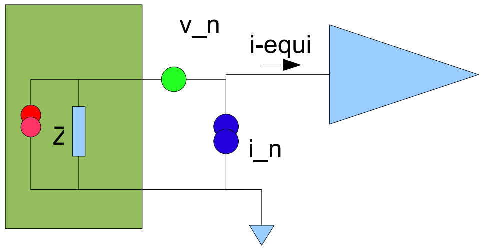
$$S_{equi}(f) = S_{v_n}(f) + |Z(j2\pi f)|^2 S_{i_n}(f)$$

$$i_{equi}(s) = i_n(s) + \frac{v_n(s)}{Z(s)}$$

$$S_{equi}(f) = S_{i_n}(f) + \frac{S_{v_n}(f)}{|Z(j2\pi f)|^2}$$

We calculate the « output signal » v-equi from the network, assuming the amplifier is perfect and hence an « open circuit » (infinite input impedance). The current through Z comes from the current source, and hence the output voltage is the sum of the voltage source and the impedance times the current source. We now analyse this in terms of the superposition principle, in order to find the contribution of each (independent) noise source individually, to find the total power spectral density.
We do the same for the « output signal » i-equi, this time assuming a short circuit for the amplifier (0 input impedance).

Equivalent noise of amplifier III



$$S_{equi}(f) = S_{i_n}(f) + \frac{S_{v_n}(f)}{|Z(j2\pi f)|^2}$$

Load is capacitive: $Z = \frac{1}{sC}$

$$S_{equi}(f) = S_{i_n}(f) + 4\pi^2 f^2 S_{v_n}(f)$$

Load is resistive: Z = R

$$S_{equi}(f) = S_{i_n}(f) + \frac{S_{v_n}(f)}{R^2}$$

16

We apply the result of the previous slide in the case of the perfect current amplifier.
Note that in the case of a capacitive load, the noise power spectral density has a term that has a factor f^2 for the voltage noise source, while in the case of the resistive load, there is no explicit frequency dependence other than that of the noise sources themselves.

WARNING: we didn't take into account the intrinsic noise of the resistor here.

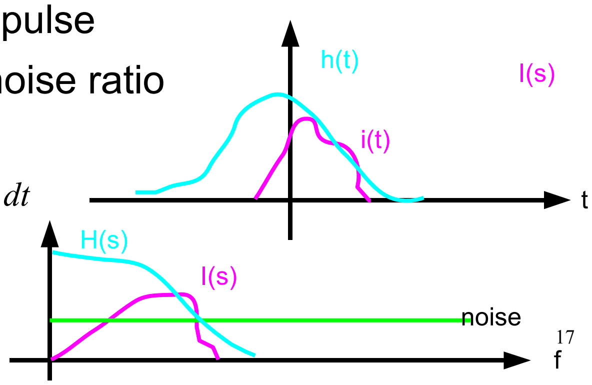
Shaping

Shaping for charge: impulse response of overall circuit

- long enough to « integrate » the charge
- short enough to limit « dead time »
- signal = maximum of pulse
- best possible signal/noise ratio

$$v_{out}(t)=\int_{\tau=-\infty}^{t} i(\tau)h(t-\tau)dt$$

$$V(s)=H(s)I(s)$$



Shaping for a charge amplifier is usually understood to have a circuit that gives an impulse response with a more or less bell-shaped form in the time domain. A good shaper, that allows a good charge measurement (as peak value of the output pulse) must be the compromise of several desires. As the impulse response is also the « weighting function » in the past of the current signal (convolution), it must be long enough to integrate most of the charge. As it is the width of the pulse, it must be short enough to have small dead time. It also needs to « cut away » most of the noise in the frequency domain, outside of the part where the signal is present.

Shaping and scaling

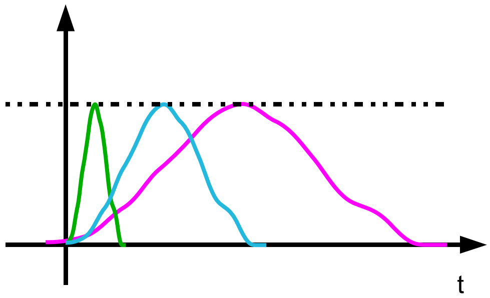
A « unit charge » will generate an impulse response of which the height is the « gain » of the amplifier chain.

Scaling the time axis (k times faster), but keeping the gain (maximum value):

$$h(t) \rightarrow h'(t) = h(kt)$$

$$H(s) = \int_0^\infty h(t) e^{-st} dt$$

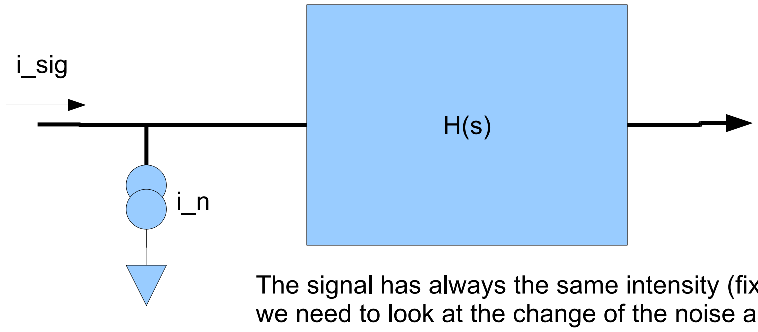
$$H(s) \rightarrow H'(s) = \frac{1}{|k|} H\left(\frac{s}{k}\right)$$



18

A charge amplifier produces, per event, an output pulse which ressembles more or less its impulse response. We consider the « charge value » as given by the peak value of that pulse. So what matters, for a charge amplifier, is the ratio of that peak value over the actual charge.
The gain is then that peak voltage per unit of charge (say, pico Coulomb).
If we « make the amplifier k times faster » while keeping the shape of the impulse response, and we want to keep the « gain » constant (the ratio of peak output voltage over the charge), then we have to scale the impulse response as shown.
The transfer function is the Laplace transform of the impulse response, and its scaling behaviour is well-known.

Application: scaling of S/N 1



The signal has always the same intensity (fixed gain)
we need to look at the change of the noise as a function of time scale.
Noise = RMS value of noise (to be compared with peak signal)

$$N_1 = \sqrt{\int_{f=0}^{\infty} |H(j2\pi f)|^2 S(f)\, df}$$

$$N_2 = \frac{1}{k}\sqrt{\int_{f=0}^{\infty} \left|H\left(j2\pi \frac{f}{k}\right)\right|^2 S(f)\, df}$$

$$= \frac{1}{\sqrt{k}}\sqrt{\int_{u=0}^{\infty} |H(j2\pi u)|^2 S(k u)\, du}$$

f/k = u

19

Assuming a given input signal (a short current pulse corresponding to a fixed charge), and assuming a fixed charge gain (maximum of the impulse response), the output signal level is constant (the height of the output pulse is constant). The signal-to-noise ratio is then determined by the RMS value of the noise.
Assuming an equivalent overall input noise current source with spectral power density S(f), we can calculate the RMS noise level at the output.
Applying the time scaling (k times faster), we find a new noise level. In order to go further, we need a specific f-dependence of the noise source spectral density and/or of the shaper.

Scaling of S/N 2

Let us assume a very simple filter: perfect lowpass filter (with sinc response):

f < f0 then |H(f)| = A

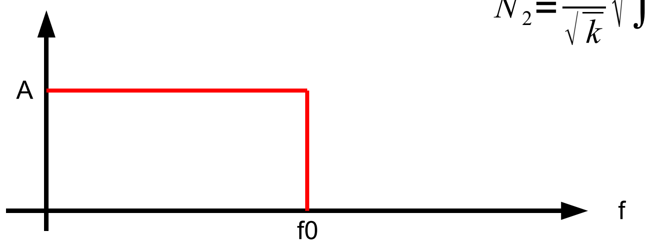
f > f0 then H(f) = 0

$$N_1 = \sqrt{\int_{f=0}^{\infty} |H(j2\pi f)|^2 S(f)\,df}$$

$$N_1 = \sqrt{\int_{f=0}^{f_0} A^2 S(f)\,df}$$

$$N_2 = \frac{1}{\sqrt{k}} \sqrt{\int_{u=0}^{\infty} |H(j2\pi u)|^2 S(k u)\,du}$$

$$N_2 = \frac{1}{\sqrt{k}} \sqrt{\int_{u=0}^{f_0} A^2 S(k u)\,du}$$



In order to get a clearer feeling of what is going on, we are going to use a very simple (although not totally realistic) transfer function: the « perfect low-pass filter » which lets through all frequencies equally and without phase shift below f0, and cuts them perfectly away beyond f0.
The impulse response of such a filter is a sinc function (sin(x)/x), and is not a very nice « pulse » because there are wobbles and so on. We take this filter because it will illustrate the essential behavior of the noise, and will make the calculations very easy.

Scaling of S/N 3

Case of resistive load

$$S_{equi}(f) = S_{i_n}(f) + \frac{S_{v_n}(f)}{R^2} = S_n$$

assume white equivalent noise sources

$$N_1 = \sqrt{\int_{f=0}^{f_0} A^2 S(f)\, df} \qquad N_2 = \frac{1}{\sqrt{k}} \sqrt{\int_{u=0}^{f_0} A^2 S(k u)\, du}$$

$$N_1 = \sqrt{f_0 A^2 S_n} \qquad N_2 = \frac{1}{\sqrt{k}} \sqrt{f_0 A^2 S_n} = \frac{N_1}{\sqrt{k}}$$

A k times FASTER amplifier has sqrt(k) times LESS noise ;
the signal-to-noise ratio IMPROVES for faster amplifiers!

Let us see what happens in this simple case when we have a purely resistive load, and where we assume that the equivalent current and voltage noise sources of the amplifier are white (as is most often the case). The total equivalent input noise is then also white as we have seen.
If we do the simple calculation, and compare the RMS noise of an amplifier with the RMS noise of the k times faster amplifier, we find (maybe to our surprise...) that the FASTER amplifier has a LOWER noise level (and hence a better S/N ratio)!

Scaling of S/N 4

Capacitive load

$$S_{equi}(f) = S_{i_n}(f) + 4\pi^2 f^2 S_{v_n}(f)$$

Assume only voltage noise,

$$S_{equi}(f) = 4\pi^2 f^2 S_{v_n}$$

assume it to be white

current noise will behave as in the « resistive case »

$$N_1 = \sqrt{\int_{f=0}^{f_0} A^2 S(f)\, df}$$

$$N_2 = \frac{1}{\sqrt{k}} \sqrt{\int_{u=0}^{f_0} A^2 S(k u)\, du}$$

$$N_1 = \sqrt{\int_{f=0}^{f_0} A^2 4\pi^2 f^2 S_{v_n}\, df} = 2\pi A \sqrt{S_{v_n} \frac{f_0^3}{3}}$$

$$N_2 = \frac{1}{\sqrt{k}} \sqrt{\int_{u=0}^{f_0} A^2 4\pi^2 k^2 u^2 S_{v_n}\, du} = \sqrt{k} \sqrt{\int_{u=0}^{f_0} A^2 4\pi^2 u^2 S_{v_n}\, du} = \sqrt{(k)}\, N_1$$

This time, a k times FASTER amplifier has sqrt(k) times MORE noise ;
a faster amplifier DETERIORATES the S/N ratio.

22

If we do the same exercise with a purely capacitive load (which is very often the case in particle detectors), and we assume again that the equivalent noise sources of the amplifier are white, then we have seen that there are two contributions to the equivalent input noise: a part that is « white » (and will behave as in the previous resistive case), which comes from the equivalent current noise, and a part that is « blue » and comes from the equivalent voltage noise. We limit ourselves to this last, « blue » noise.
A simple calculation then shows that this time, the faster amplifier has MORE rms noise, and hence a worse S/N ratio – as is in fact usually expected.

Scaling of S/N 5

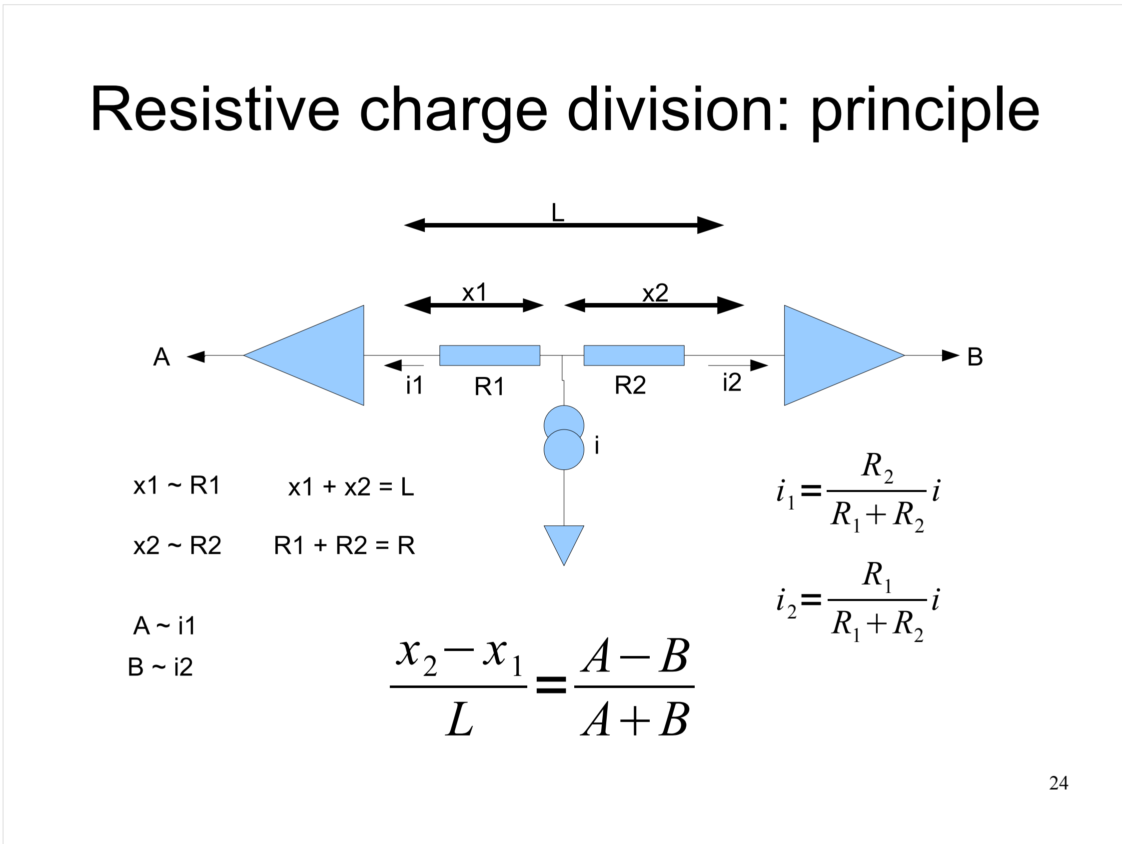
Current noise at the input: a faster amplifier IMPROVES S/N

Voltage noise with resistive load: a faster amplifier IMPROVES S/N

Voltage noise with capacitive load: a faster amplifier DETERIORATES S/N

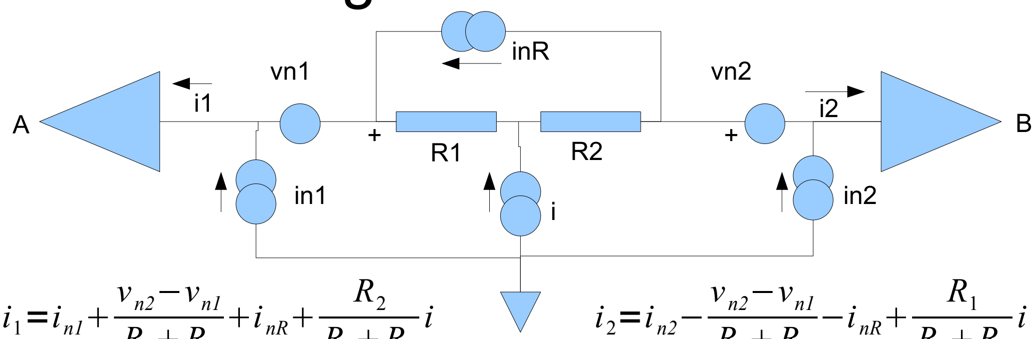
Mixed case: it depends on the relative contributions: *there will be an optimum amplifier speed.*

We have as such demonstrated some general properties which are good to summarize. Certain kinds of noise are suppressed with a faster amplifier (this can seem surprising!), and other kinds of noise become worse with a faster amplifier.
In the case both contributions are present, there will be an optimal « amplifier speed » for the best S/N ratio.
This compromise will depend strongly on the equivalent voltage noise and current noise sources of the amplifier.
Let us stress that all this is with amplifiers which have the same shape of impulse response. Changing the shape can improve also on the S/N ratio. In general, Gaussian-like shaping tends to give good results.



In resistive charge division, a resistive electrode of length L receives an injection of charge somewhere along its length due to a detection event. We assume that this injection happens at a distance x1 from the left end and at a distance x2 from the right end.
In the ideal case, as the amplifiers are perfect current amplifiers with 0 impedance, the injected current i will divide in i1 and i2 respectively. The amplifiers will amplify (and integrate) these respective currents. Their output will be proportional to the fraction of the current received.
Everything together gives us the relative position as a function of the ratio of the output signals.

Charge division: noise.



$$i_1 = i_{n1} + \frac{v_{n2} - v_{n1}}{R_1 + R_2} + i_{nR} + \frac{R_2}{R_1 + R_2} i \qquad i_2 = i_{n2} - \frac{v_{n2} - v_{n1}}{R_1 + R_2} - i_{nR} + \frac{R_1}{R_1 + R_2} i$$

$$i_1 - i_2 = i_{n1} - i_{n2} + 2\frac{v_{n2} - v_{n1}}{R_1 + R_2} + 2 i_{nR} + \frac{R_2 - R_1}{R_1 + R_2} i \qquad i_1 + i_2 = i_{n1} + i_{n2} + i$$

$$S_{min} = S_i + S_i + \frac{4}{R^2}(S_v + S_v) + 4 S_R = 2 S_i + \frac{8 S_v}{R^2} + \frac{16 k T}{R} \qquad S_{plus} = 2 S_i$$

We introduce all the noise sources in our setup: for each amplifier we have an equivalent current and voltage noise source, and the resistor itself has its Johnson noise (Norton equivalent).
Next we calculate the outputs i1 and i2 as a function of all these sources and the signal source.

But we will actually use the difference and the sum signals (A-B over A+B) and we can imagine constructing the signal i1-i2 and the signal i1+i2 which we consider now to be the « output signals ». We calculate those as a function of all the noise sources. Using the absolute square of the transfer functions, we can now calculate the noise power spectral density on the signals i1-i2 and i1+i2.

Charge division: resolution

$$\delta_{rms} D = \sqrt{\int_{f=0}^{\infty} |H|^2 S_{min} \, df} \qquad \delta_{rms} S = \sqrt{\int_{f=0}^{\infty} |H|^2 S_{plus} \, df}$$

$p = \dfrac{D}{S}$ Relative position from -1 to 1.

$$dp = \frac{dD}{S} - \frac{D}{S}\frac{dS}{S} = \frac{1}{S}(dD - p\, dS)$$

$x = \dfrac{p}{2} L$ Physical position

$$\delta_{rms} p = \frac{1}{S}\sqrt{(\delta_{rms} D)^2 + p^2 (\delta_{rms} S)^2}$$

$$\delta_{rms} x = \frac{L}{2S}\sqrt{(\delta_{rms} D)^2 + p^2 (\delta_{rms} S)^2}$$

Position resolution (1 sigma)

The difference and the sum of the outputs of the amplifiers can also be considered as the amplified outputs of the difference and the sum of the currents. H(i1) + H(i2) = H(i1+i2) for instance.
This means that the rms noise on the difference signal and the rms noise on the sum signal is given by the formula as if they passed through the amplifier.

The error propagation in the ratio: as the RMS noises are statistically independent on D and on S, we can combine them quadratically, to obtain the rms noise on p.