Nuclear electronics

A simple introduction to some basic aspects of the analogue front end part of detector electronics

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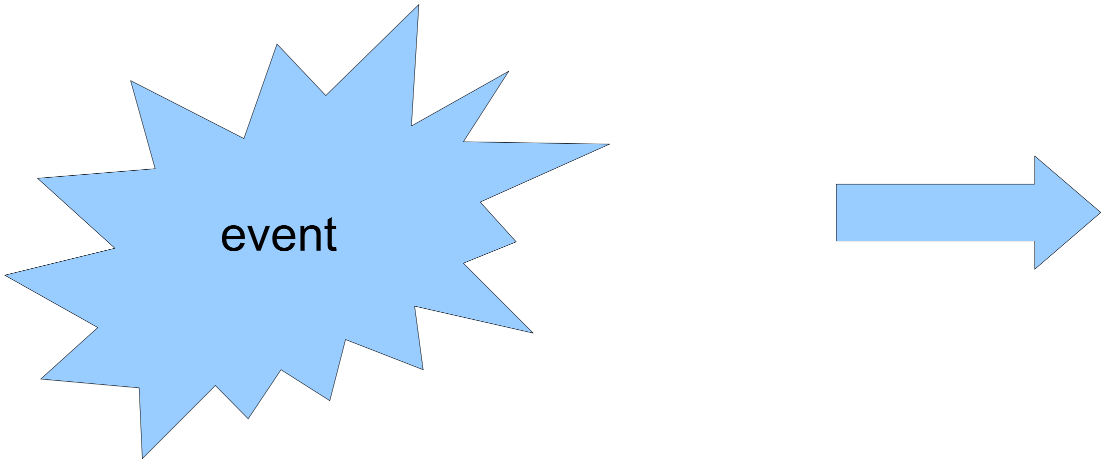
Detection, current, charge I



Number
Or
Decision

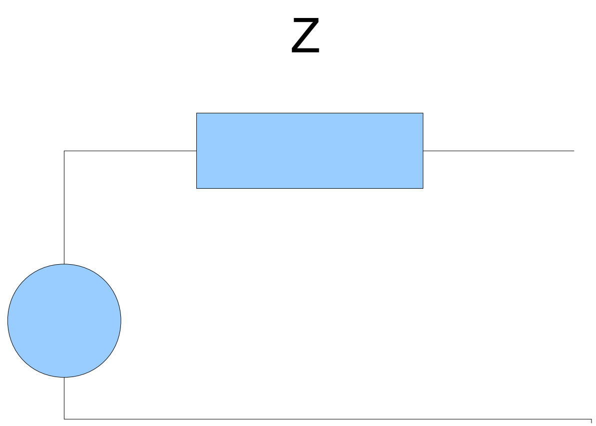
Physical detection → movement of charges in strong E field
- Induction of currents in conductors (electrodes)

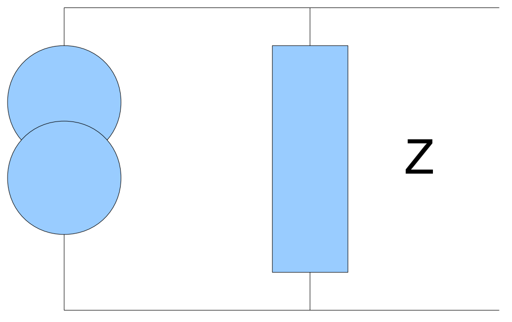
Most other instrumentation:
- Change in impedance
- Change in electro(chemical) potential

Thevenin and Norton

A « source » is represented by an electric circuit, which can be simplified as:

- An imperfect voltage source (Thevenin)
- An imperfect current source (Norton)





Amplifiers

Amplifiers are « matched » to sources.

- Ideal voltage sources with ideal voltage amplifiers
- Ideal current sources with ideal current amplifiers

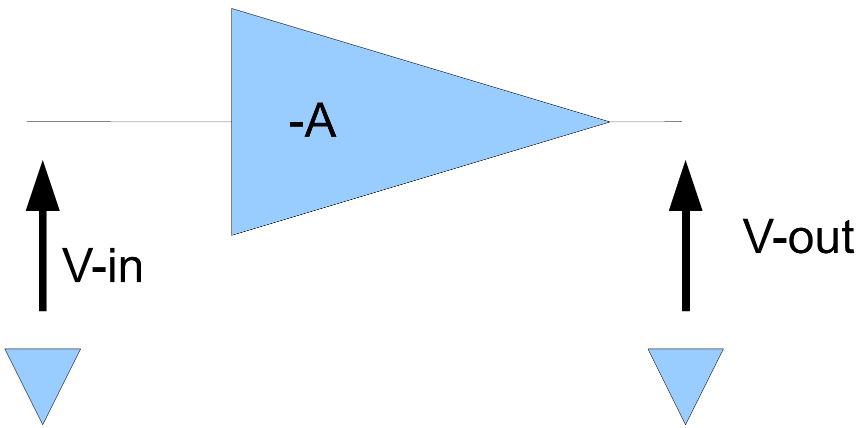
Voltage amplifier: high impedance

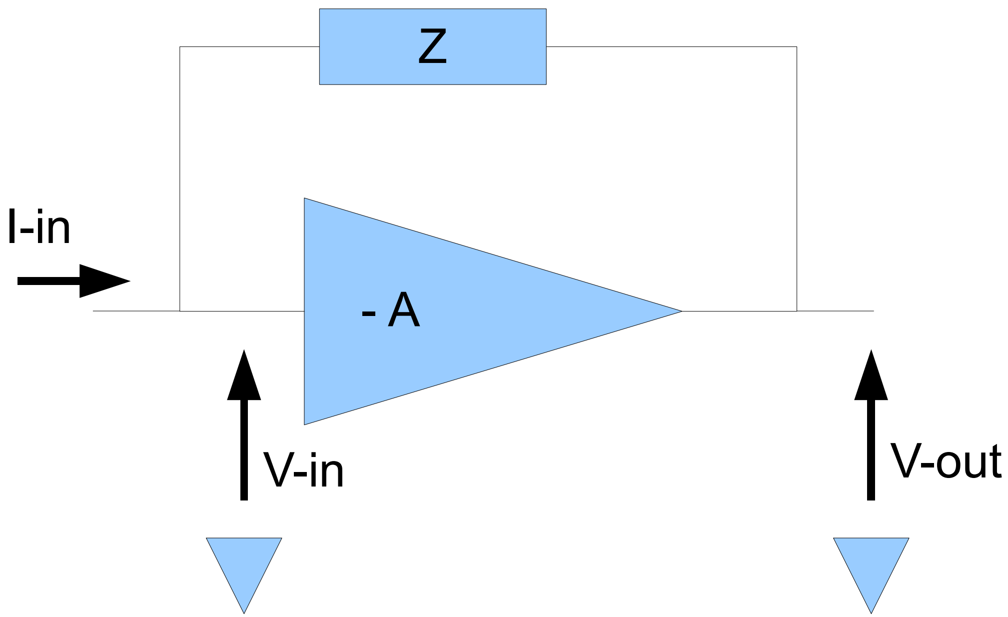
Current amplifier: low impedance

In fact, the output is always a voltage:

- Voltage – voltage amplifier (« normal » amplifier)
- Current – voltage amplifier (transimpedance amp.)

How to make a transimpedance amplifier ?



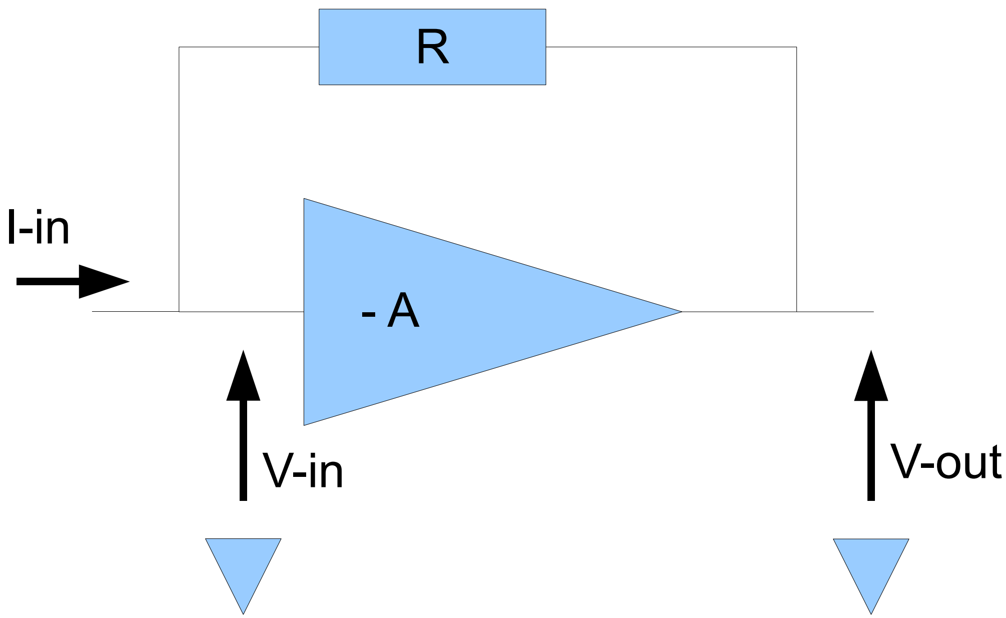


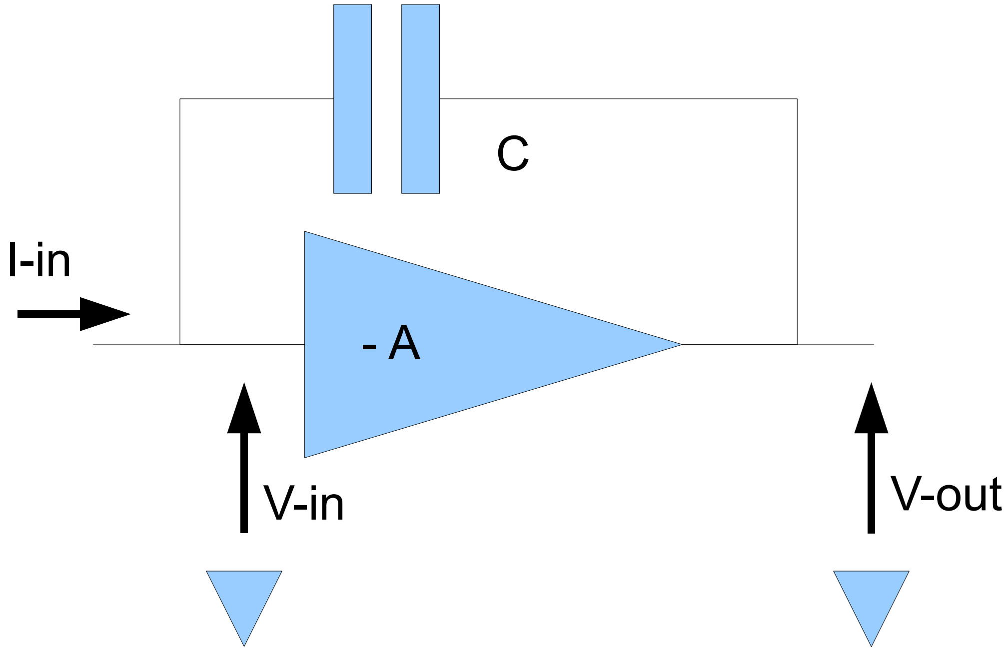
V-out = - A V-in

V-out = - A V-in ;
V-in – V-out = I-in Z ;
A very big, hence V-in much smaller than V-out, hence:

V-out = - Z I-in

« Current » vs. « Charge »





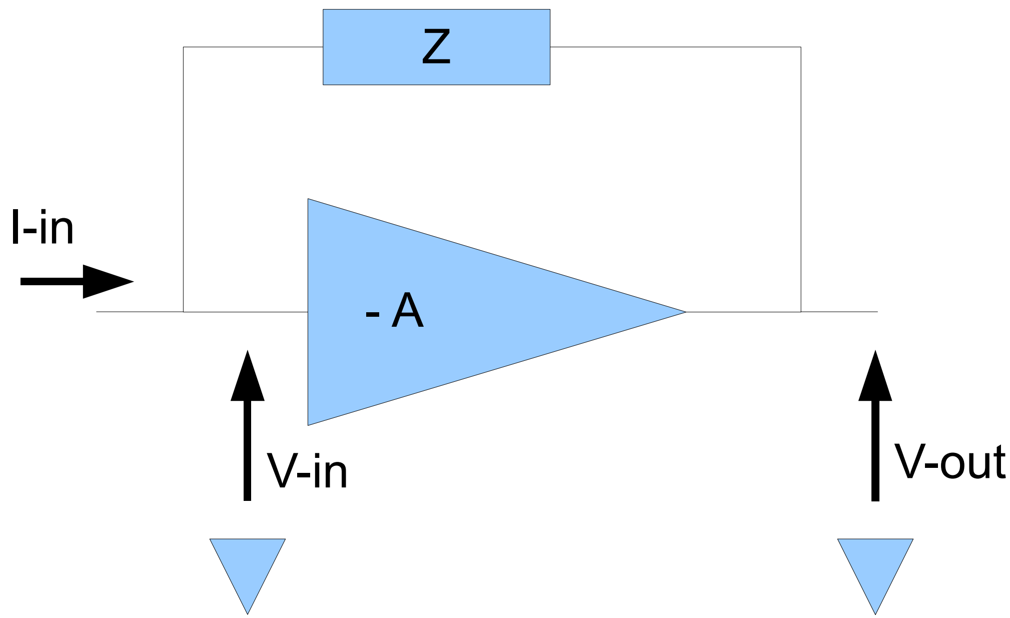
$$V_{out}(s) = -R\,I_i(s)$$

$$v_{out}(t) = -R\,i_i(t)$$

$$V_{out}(s) = \frac{-1}{s.C} I_i(s)$$

$$V_{out}(t) = \frac{-1}{C} \int i_i(t)\,dt = \frac{-Q(t)}{C}$$

Input impedance



$$V_i = V_{out} + i_i . Z$$

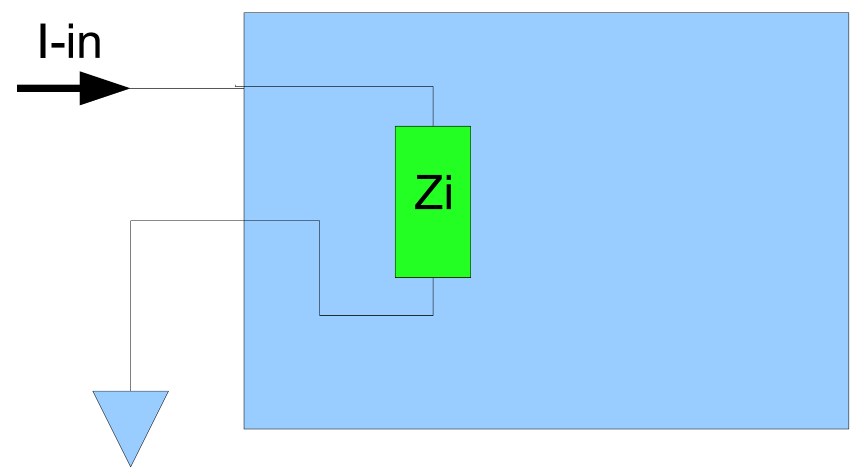
$$V_{out} = -A V_i$$

$$V_i = -A V_i + i_i Z$$

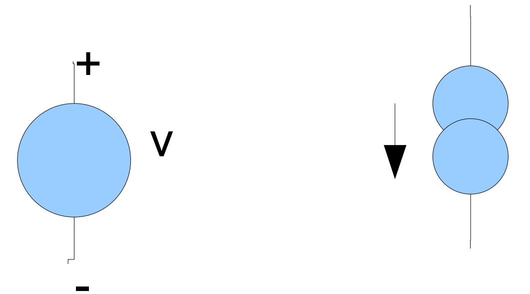
$$Z_i = -A . Z_i + Z$$

$$(1+A) Z_i = Z$$

$$Z_i = \frac{Z}{A+1}$$



Noise sources I



Represented by voltage or current sources

V or I is a random process with average 0 and given autocorrelation.

$$f(t_1)=X_1;\ f(t_2)=X_2$$

$$\langle X_1 \rangle = 0$$

$$R(\tau)=\langle X_1 X_2 \rangle;\ \tau = t_2 - t_1$$

$$R(0)=\langle X_1^2 \rangle = f_{rms}^2$$

Wiener-Khinchine: power spectral density (one-sided)

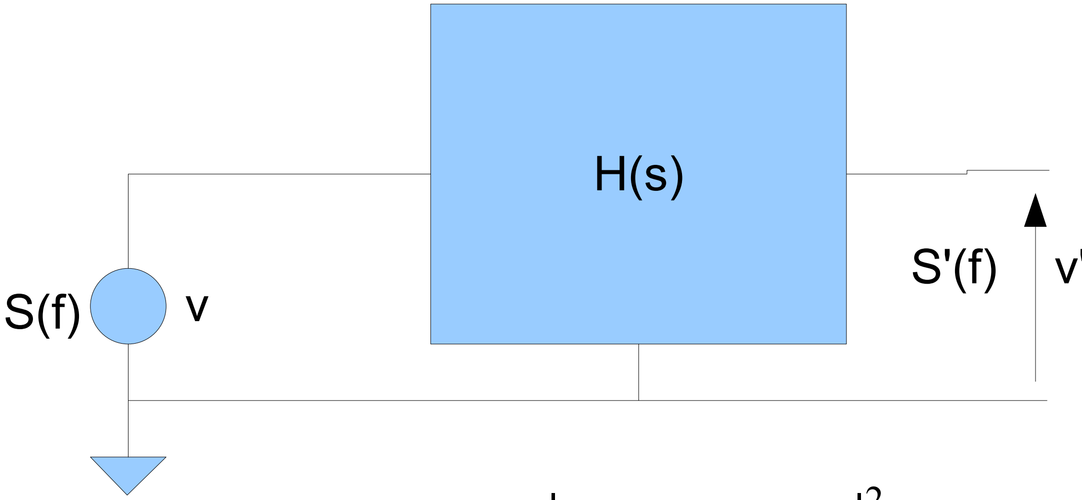
$$S(f)=4\int_{\tau=0}^{\infty} R(\tau)\cos(2\pi f \tau)\,d\tau$$

Noise sources II

- Power spectral density and RMS

$$v_{RMS} = \sqrt{R(0)} = \sqrt{\int_{f=0}^{\infty} S(f)\,df}$$

- Noise and linear filter



$$S'(f) = |H(j2\pi f)|^2 S(f)$$

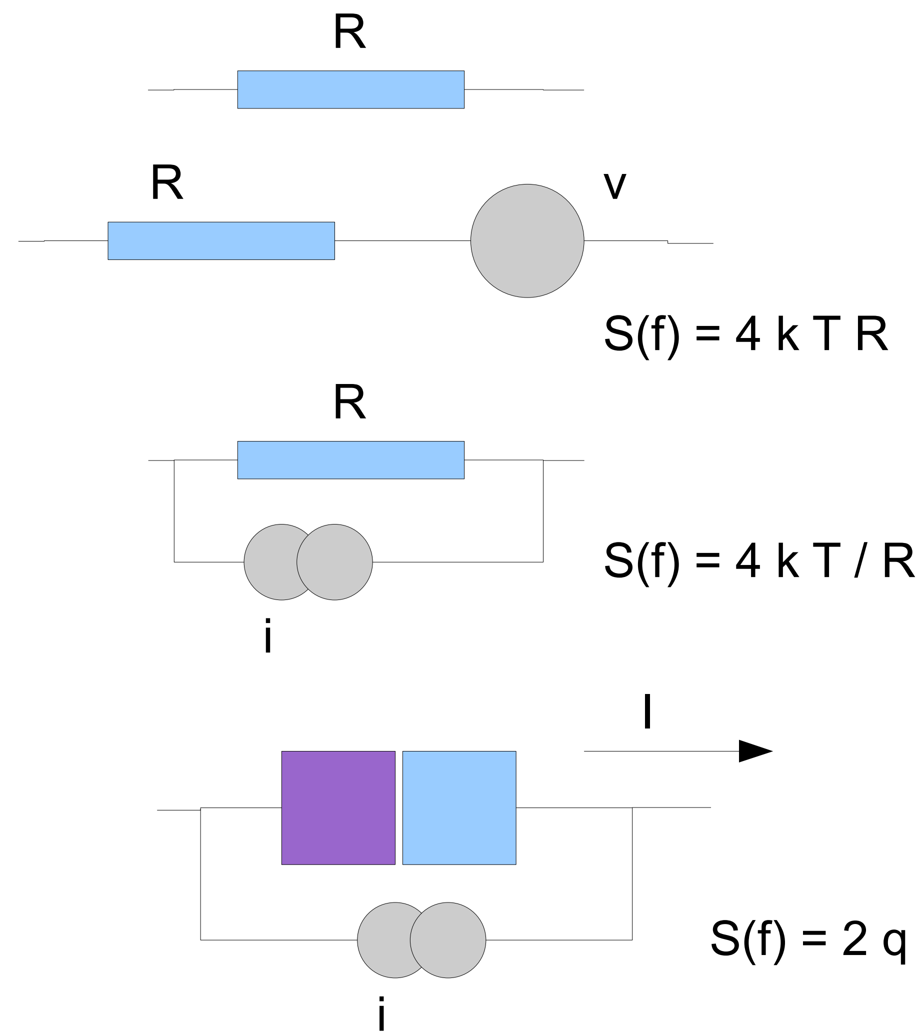
- White noise
  - S(f) = constant
  - R ~ Dirac function

Physical noise sources

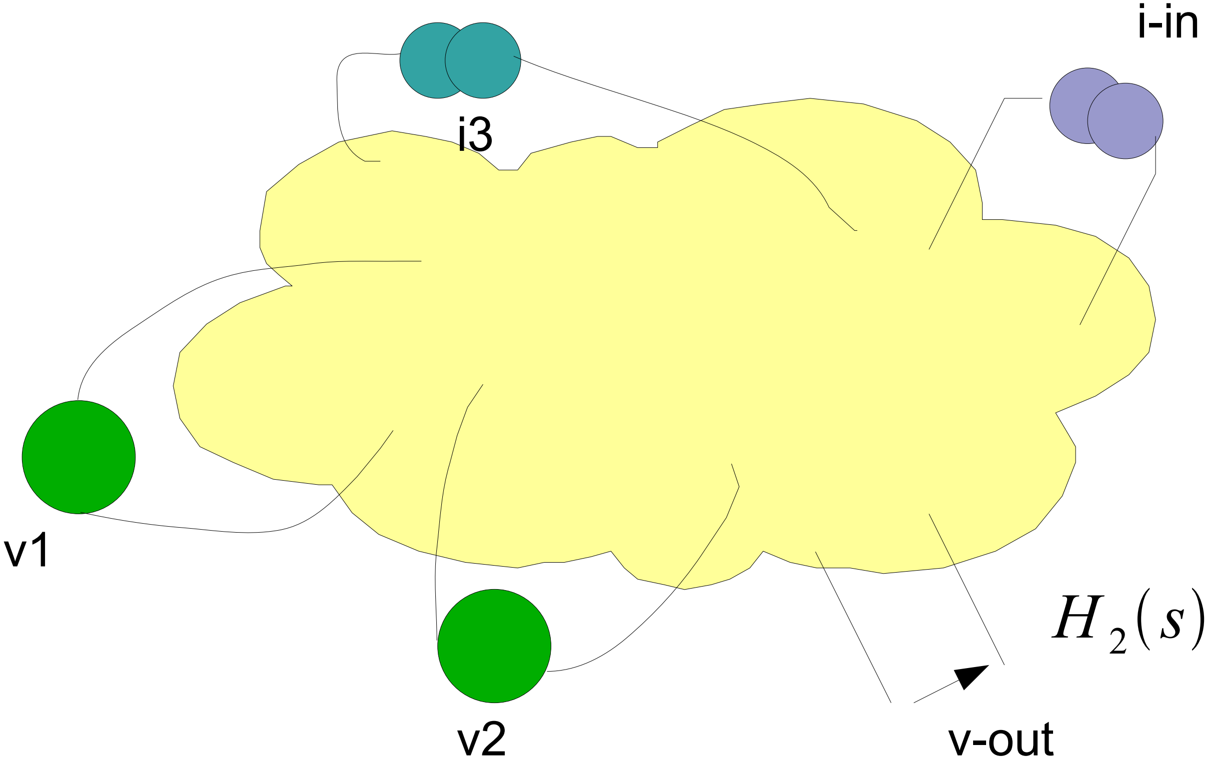
Thermal (Johnson or Nyquist) noise in resistors (white noise)

Shot noise in PN barriers (white noise)

Flicker noise, 1/f noise, pink noise



Networks with noise sources.

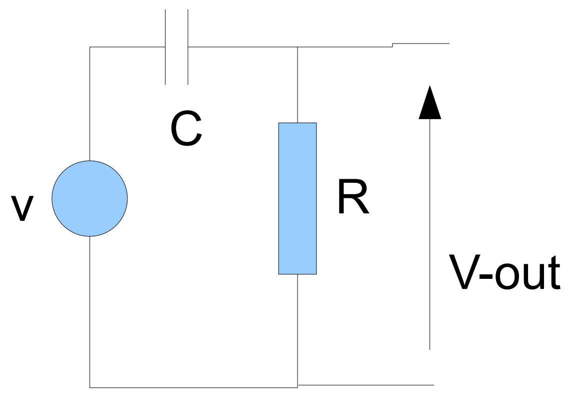


$$V_{out}(s) = H_1(s)V_1(s) + \ldots$$
$$H_2(s)V_2(s) + Z_3(s)I_3(s) + \underline{Z(s)I_i(s)}$$

$$S_{out}(f) = |H_1(j2\pi f)|^2 S_1(f) + |H_2(j2\pi f)|^2 S_2(f) + |Z_3(j2\pi f)|^2 S_3(f)$$

$$S_{equi}(f) = \frac{S_{out}(f)}{|Z(j2\pi f)|^2}$$

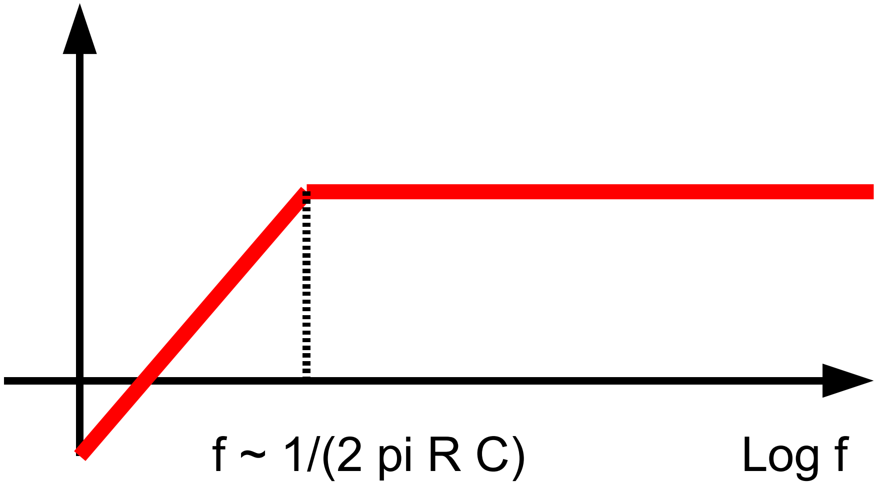
Example 1



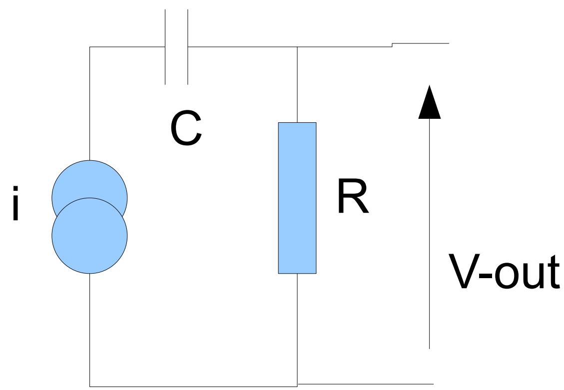
$$V_{out}(s) = \frac{R}{R + \frac{1}{sC}} V_i = \frac{RCs}{1 + RCs} V_i$$

$$|H(j2\pi f)|^2 = \left|\frac{j2\pi f RC}{1 + j2\pi f RC}\right|^2 = \frac{4\pi^2 f^2 R^2 C^2}{1 + 4\pi^2 f^2 R^2 C^2}$$

$$S'(f) = \frac{4\pi^2 f^2 R^2 C^2}{1 + 4\pi^2 f^2 R^2 C^2} S(f)$$



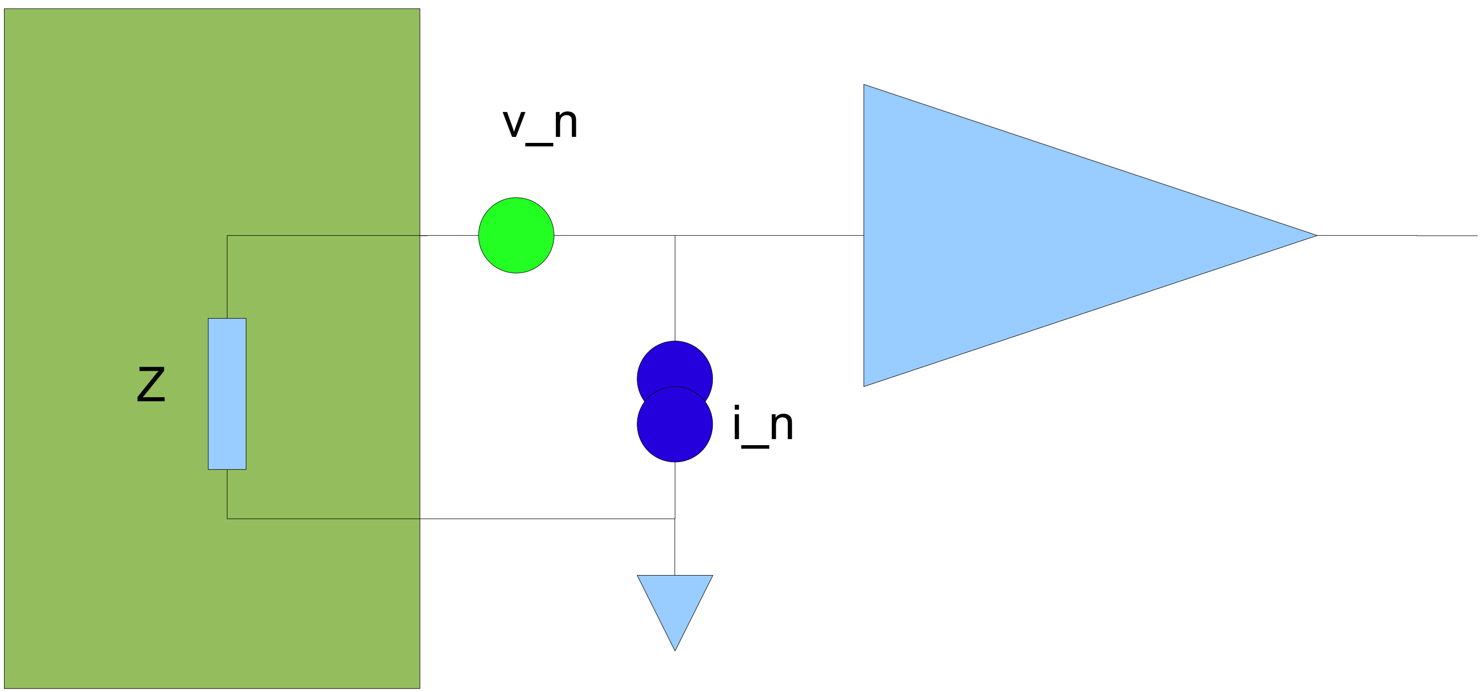
Example 2 (exercise)



$$V_{out}(s) = R\,I(s)$$

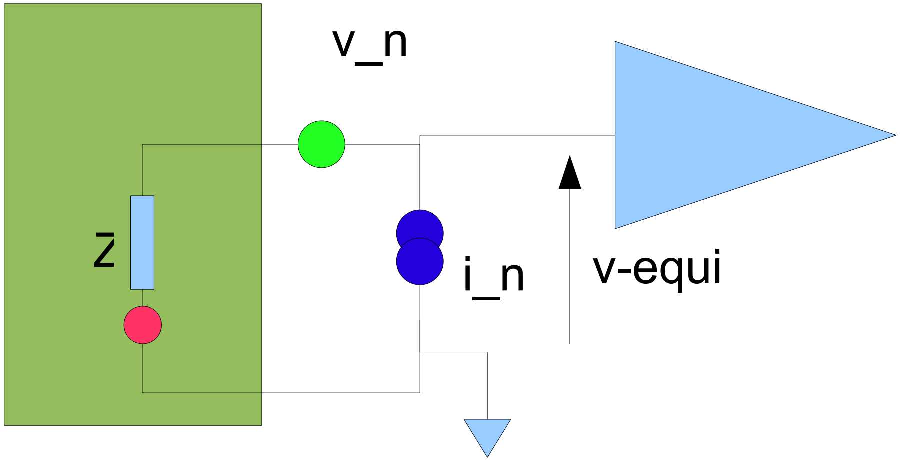
S'(f) = ?

Equivalent noise of amplifier



Equivalent noise of amplifier II

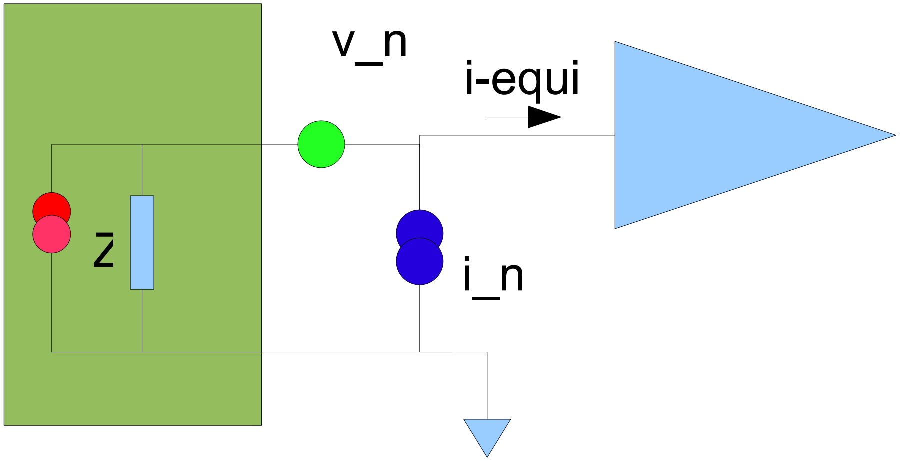
Case of voltage amplifier (Thevenin)



$$v_{equi}(s)=v_n(s)+Z(s)i_n(s)$$

$$S_{equi}(f)=S_{v_n}(f)+|Z(j2\pi f)|^2 S_{i_n}(f)$$

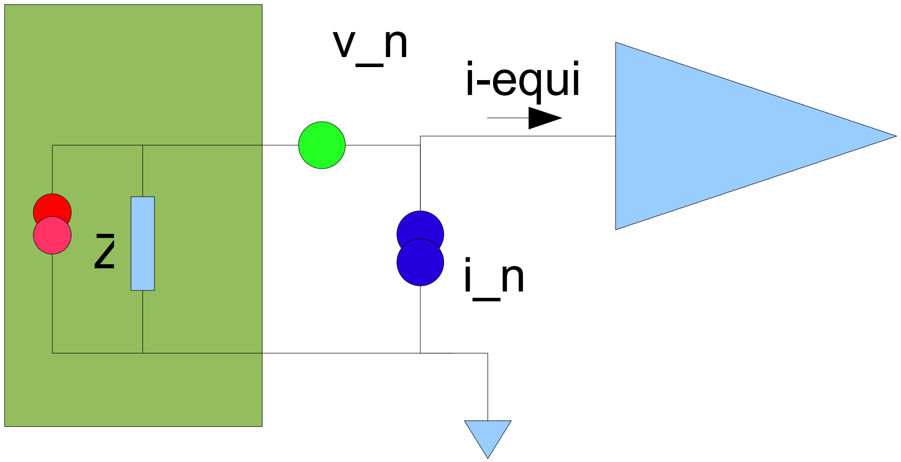
Case of current amplifier (Norton)



$$i_{equi}(s)=i_n(s)+\frac{v_n(s)}{Z(s)}$$

$$S_{equi}(f)=S_{i_n}(f)+\frac{S_{v_n}(f)}{|Z(j2\pi f)|^2}$$

Equivalent noise of amplifier III



$$S_{equi}(f) = S_{i_n}(f) + \frac{S_{v_n}(f)}{|Z(j2\pi f)|^2}$$

Load is capacitive: $Z = \frac{1}{sC}$

$$S_{equi}(f) = S_{i_n}(f) + 4\pi^2 f^2 S_{v_n}(f)$$

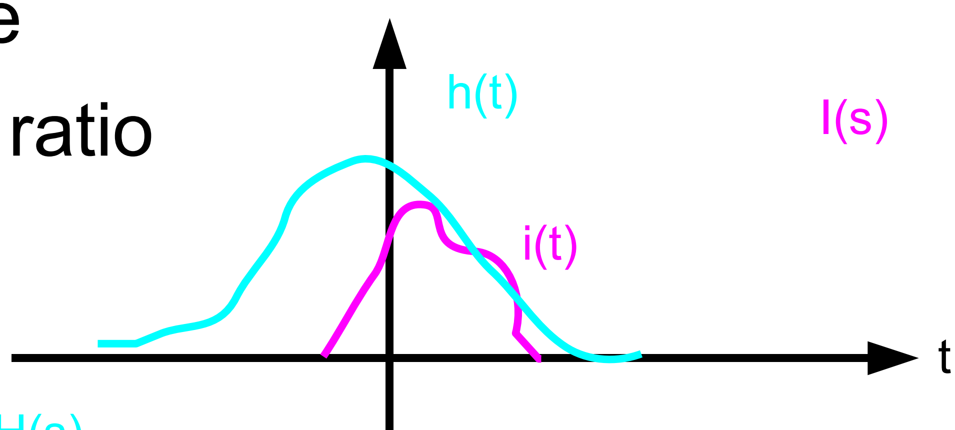
Load is resistive: Z = R

$$S_{equi}(f) = S_{i_n}(f) + \frac{S_{v_n}(f)}{R^2}$$

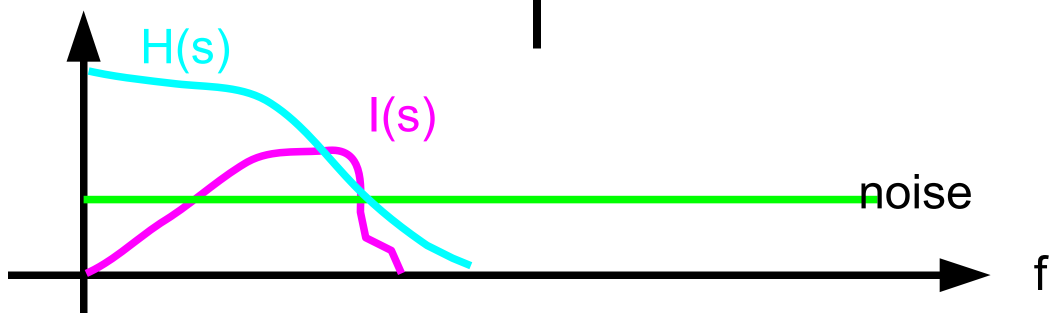
Shaping

Shaping for charge: impulse response of overall circuit

- long enough to « integrate » the charge
- short enough to limit « dead time »
- signal = maximum of pulse
- best possible signal/noise ratio



$$v_{out}(t) = \int_{\tau=-\infty}^{t} i(\tau)\, h(t-\tau)\, dt$$



$$V(s) = H(s)\, I(s)$$

Shaping and scaling

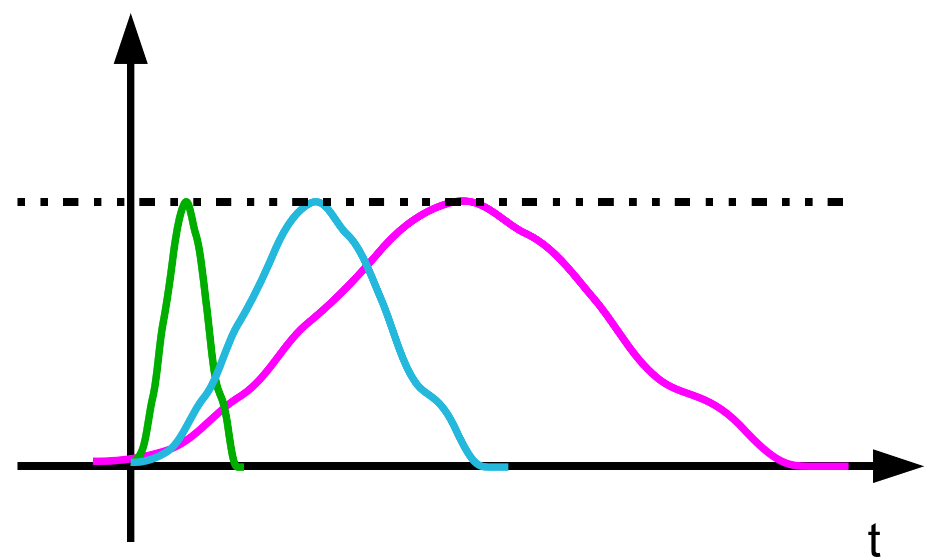
A « unit charge » will generate an impulse response of which the height is the « gain » of the amplifier chain.

Scaling the time axis (k times faster), but keeping the gain (maximum value):

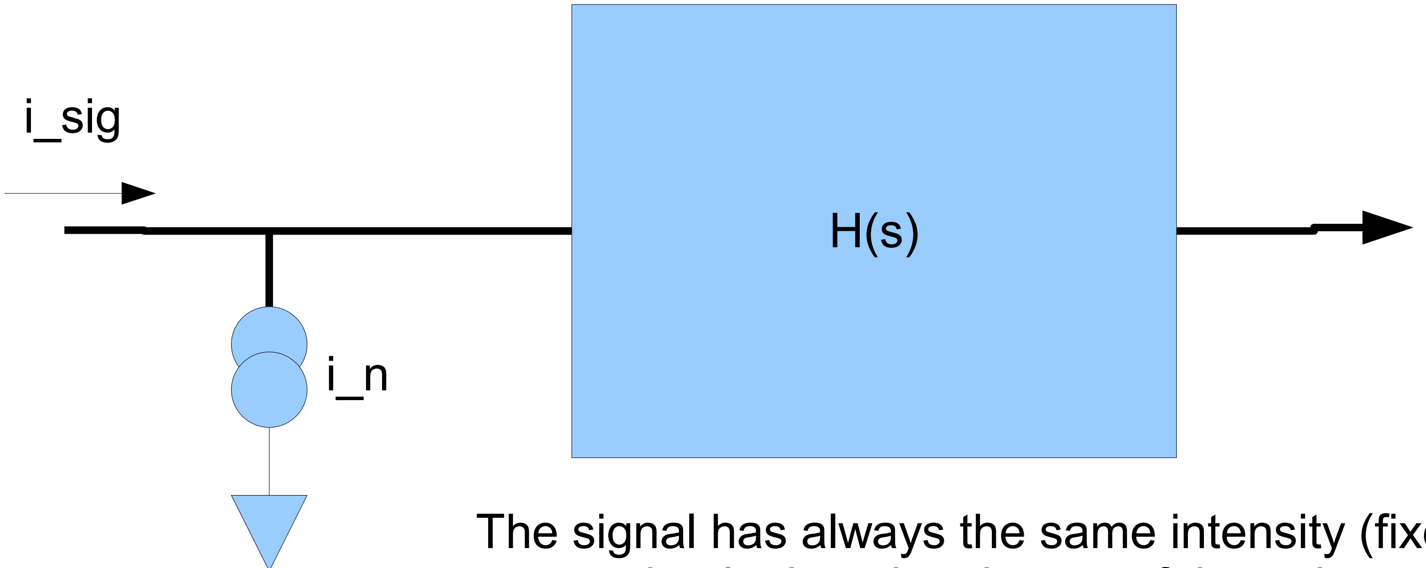
$$h(t) \rightarrow h'(t) = h(k\,t)$$

$$H(s) = \int_0^\infty h(t)\, e^{-st}\, dt$$

$$H(s) \rightarrow H'(s) = \frac{1}{|k|} H\left(\frac{s}{k}\right)$$



Application: scaling of S/N 1



The signal has always the same intensity (fixed gain)
we need to look at the change of the noise as a function of time scale.
Noise = RMS value of noise (to be compared with peak signal)

$$N_1 = \sqrt{\int_{f=0}^{\infty} |H(j2\pi f)|^2 S(f)\,df}$$

$$N_2 = \frac{1}{k}\sqrt{\int_{f=0}^{\infty} \left|H\left(j2\pi \frac{f}{k}\right)\right|^2 S(f)\,df}$$

$$= \frac{1}{\sqrt{k}}\sqrt{\int_{u=0}^{\infty} |H(j2\pi u)|^2 S(ku)\,du}$$

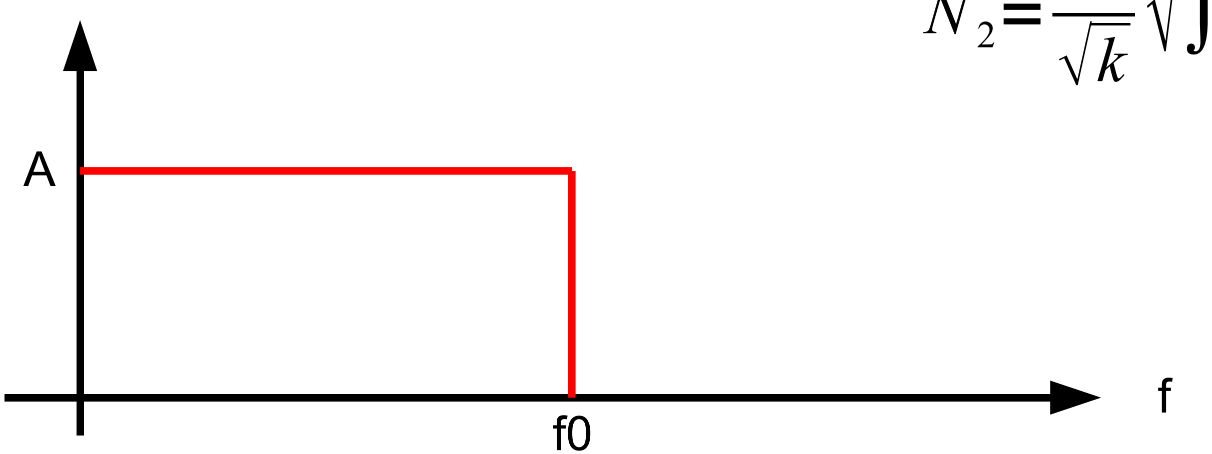
f/k = u

Scaling of S/N 2

Let us assume a very simple filter: perfect lowpass filter (with sinc response):

f < f0 then |H(f)| = A

f > f0 then H(f) = 0



$$N_1 = \sqrt{\int_{f=0}^{\infty} |H(j2\pi f)|^2 S(f)\,df}$$

$$N_1 = \sqrt{\int_{f=0}^{f_0} A^2 S(f)\,df}$$

$$N_2 = \frac{1}{\sqrt{k}} \sqrt{\int_{u=0}^{\infty} |H(j2\pi u)|^2 S(ku)\,du}$$

$$N_2 = \frac{1}{\sqrt{k}} \sqrt{\int_{u=0}^{f_0} A^2 S(ku)\,du}$$

Scaling of S/N 3

Case of resistive load

$$S_{equi}(f) = S_{i_n}(f) + \frac{S_{v_n}(f)}{R^2} = S_n$$

assume white equivalent noise sources

$$N_1 = \sqrt{\int_{f=0}^{f_0} A^2 S(f)\, df}$$

$$N_2 = \frac{1}{\sqrt{k}} \sqrt{\int_{u=0}^{f_0} A^2 S(k\,u)\, du}$$

$$N_1 = \sqrt{f_0 A^2 S_n}$$

$$N_2 = \frac{1}{\sqrt{k}} \sqrt{f_0 A^2 S_n} = \frac{N_1}{\sqrt{k}}$$

A k times FASTER amplifier has sqrt(k) times LESS noise ;
the signal-to-noise ratio IMPROVES for faster amplifiers!

Scaling of S/N 4

Capacitive load

$$S_{equi}(f) = S_{i_n}(f) + 4\pi^2 f^2 S_{v_n}(f)$$

Assume only voltage noise,

$$S_{equi}(f) = 4\pi^2 f^2 S_{v_n}$$

assume it to be white

$$N_1 = \sqrt{\int_{f=0}^{f_0} A^2 S(f)\, df}$$

current noise will behave as in the « resistive case »

$$N_2 = \frac{1}{\sqrt{k}} \sqrt{\int_{u=0}^{f_0} A^2 S(k u)\, du}$$

$$N_1 = \sqrt{\int_{f=0}^{f_0} A^2 4\pi^2 f^2 S_{v_n}\, df} = 2\pi A \sqrt{S_{v_n} \frac{f_0^3}{3}}$$

$$N_2 = \frac{1}{\sqrt{k}} \sqrt{\int_{u=0}^{f_0} A^2 4\pi^2 k^2 u^2 S_{v_n}\, du} = \sqrt{k} \sqrt{\int_{u=0}^{f_0} A^2 4\pi^2 u^2 S_{v_n}\, du} = \sqrt{(k)}\, N_1$$

This time, a k times FASTER amplifier has sqrt(k) times MORE noise ;
a faster amplifier DETERIORATES the S/N ratio.

Scaling of S/N 5

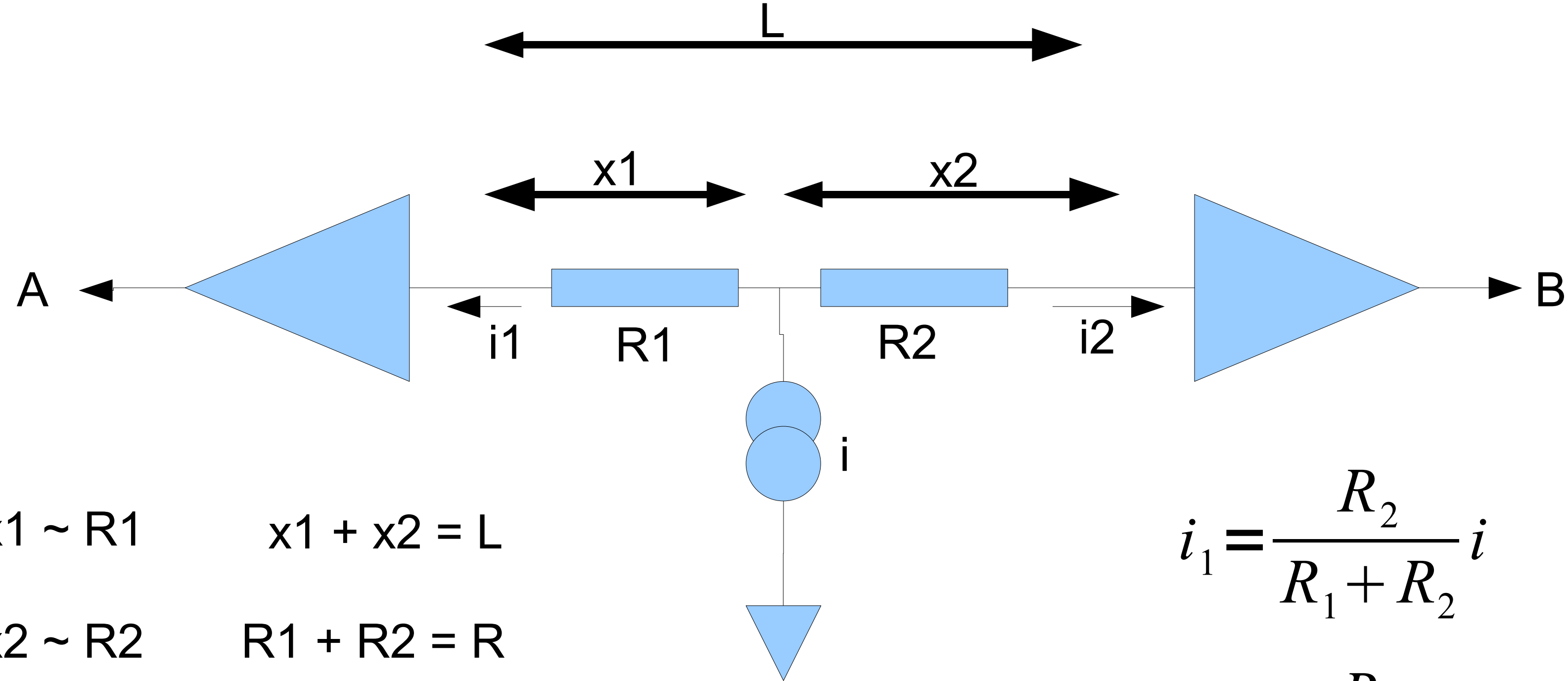
Current noise at the input: a faster amplifier IMPROVES S/N

Voltage noise with resistive load: a faster amplifier IMPROVES S/N

Voltage noise with capacitive load: a faster amplifier DETERIORATES S/N

Mixed case: it depends on the relative contributions: *there will be an optimum amplifier speed.*

Resistive charge division: principle



x1 ~ R1 x1 + x2 = L

x2 ~ R2 R1 + R2 = R

$$i_1 = \frac{R_2}{R_1 + R_2} i$$

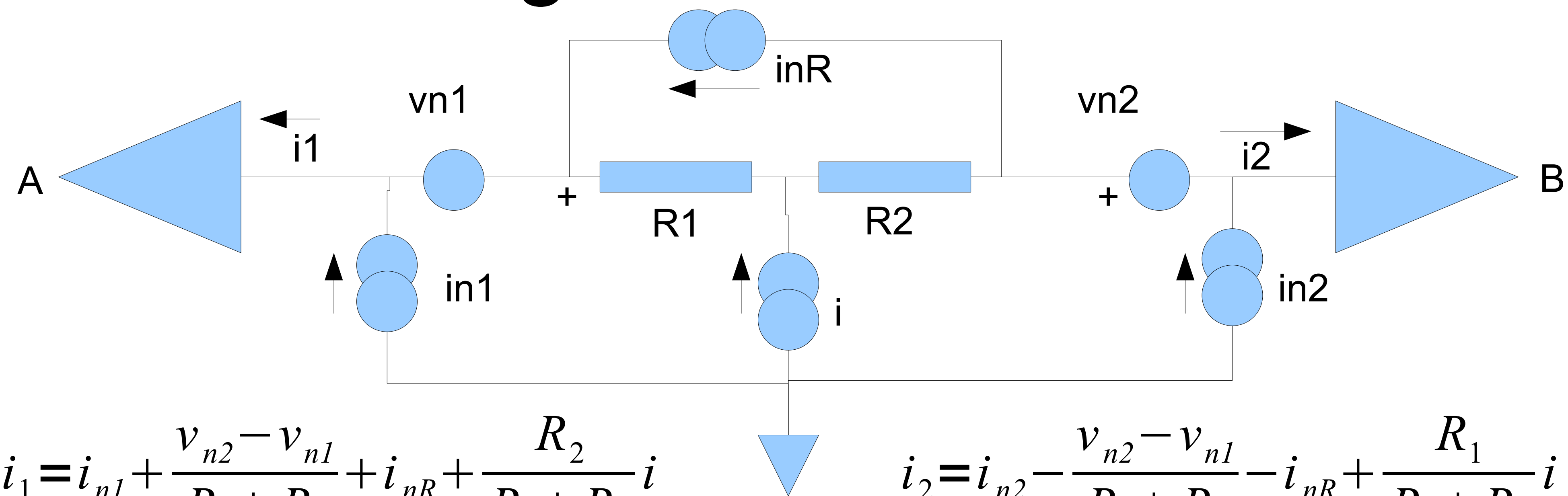
$$i_2 = \frac{R_1}{R_1 + R_2} i$$

A ~ i1

B ~ i2

$$\frac{x_2 - x_1}{L} = \frac{A - B}{A + B}$$

Charge division: noise.



$$i_1 = i_{n1} + \frac{v_{n2} - v_{n1}}{R_1 + R_2} + i_{nR} + \frac{R_2}{R_1 + R_2} i$$

$$i_2 = i_{n2} - \frac{v_{n2} - v_{n1}}{R_1 + R_2} - i_{nR} + \frac{R_1}{R_1 + R_2} i$$

$$i_1 - i_2 = i_{n1} - i_{n2} + 2\frac{v_{n2} - v_{n1}}{R_1 + R_2} + 2 i_{nR} + \frac{R_2 - R_1}{R_1 + R_2} i$$

$$i_1 + i_2 = i_{n1} + i_{n2} + i$$

$$S_{min} = S_i + S_i + \frac{4}{R^2}(S_v + S_v) + 4 S_R = 2 S_i + \frac{8 S_v}{R^2} + \frac{16 k T}{R}$$

$$S_{plus} = 2 S_i$$

Charge division: resolution

$$\delta_{rms} D = \sqrt{\int_{f=0}^{\infty} |H|^2 S_{min}\, df}$$

$$\delta_{rms} S = \sqrt{\int_{f=0}^{\infty} |H|^2 S_{plus}\, df}$$

$$p = \frac{D}{S}$$ Relative position from -1 to 1.

$$x = \frac{p}{2} L$$ Physical position

$$dp = \frac{dD}{S} - \frac{D}{S}\frac{dS}{S} = \frac{1}{S}(dD - p\, dS)$$

$$\delta_{rms}\, p = \frac{1}{S}\sqrt{(\delta_{rms} D)^2 + p^2(\delta_{rms} S)^2}$$

$$\delta_{rms}\, x = \frac{L}{2S}\sqrt{(\delta_{rms} D)^2 + p^2(\delta_{rms} S)^2}$$

Position resolution (1 sigma)