All-order Corrections to Multi-jet Rates using t-channel Factorised Scattering Matrix Elements

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in collaboration with Jenni Smillie

SM and BSM physics at the LHC
August 11, 2009

What, Why, How?

What?

Develop a framework for reliably calculating many-parton rates inclusively (ensemble of $2, 3, 4, \ldots$ parton rates) and in a flexible way (jets, W+jets, Z+jets, Higgs+jets,$\ldots$)

Why?

$(n+1)$-jet rate not necessarily small compared to n-jet rate
Inclusive (hard) perturbative corrections important for e.g. hard end of W $p_\perp$-spectrum.

How?

Establish universal behaviour of radiative corrections (in the so-called High Energy Limit)
Supplement with constraint on sub-asymptotic behaviour (gauge-invariance and analyticity)

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Resummation and Matching

Consider the **perturbative expansion** of an observable

$$R = r_0 + r_1\alpha_s + r_2\alpha^2 + r_3\alpha^3 + r_4\alpha^4 + \cdots$$

Fixed order pert. QCD will calculate a fixed number of terms in this expansion. r_n may contain **logarithms** so that $\boldsymbol{\alpha_s \ln(\cdots)}$is large.

$$R = r_0 + \left(r_1^{LL}\ln(\cdots) + r_1^{NLL}\right)\alpha_s + \left(r_2^{LL}\ln^2(\cdots) + r_2^{NLL}\ln(\cdots) + r_2^{SL}\right)\alpha_s^2 + \cdots$$
$$= r_0 + \sum_n r_n^{LL}(\alpha_s \ln(\cdots))^n + \sum_n r_n^{NLL}\alpha_s(\alpha_s\ln(\cdots))^n + \text{sub-leading terms}$$

Need simplifying assumptions to get to all orders - useful **iff** the terms really do describe **the dominant part** of the **full pert. series.**
Matching combines **best of both worlds**:

$$R = r_0 + r_1\alpha_s + r_2\alpha^2 + \left(r_3^{LL}\ln^3(\cdots) + r_3^{NLL}\ln^2(\cdots) + r_3^{SL}\right)\alpha^3 + \cdots$$

Factorisation of QCD Matrix Elements

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It is **well known** that QCD matrix elements **factorise** in certain kinematical limits:
Soft limit $\longrightarrow$ **eikonal approximation** $\longrightarrow$ enters all parton shower (and much else) resummation.

Like all good limits, the eikonal approximation is applied **outside its strict region of validity**.

Will discuss the **less well-studied factorisation** of scattering amplitudes in a different kinematic limit, better suited for describing perturbative corrections from **hard parton emission**

Factorisation only **becomes exact** in a region **outside** the reach of any collider…

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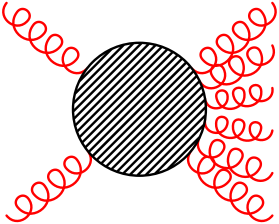
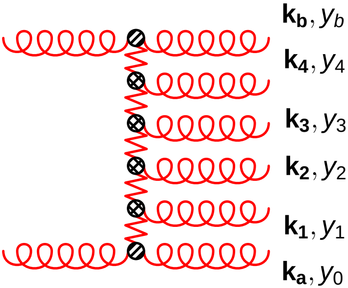
The Possibility for Predictions of n-jet Rates

The Power of Reggeisation



High Energy Limit

$$|\hat{t}| \quad \text{fixed}, \quad \hat{s} \to \infty$$



$$\mathcal{A}^R_{2\to 2+n} = \frac{\Gamma_{A'A}}{q_0^2} \left(\prod_{i=1}^{n} e^{\omega(q_i)(y_{i-1}-y_i)} \frac{V^{J_i}(q_i, q_{i+1})}{q_i^2 q_{i+1}^2} \right) e^{\omega(q_{n+1})(y_n - y_{n+1})} \frac{\Gamma_{B'B}}{q_{n+1}^2}$$

$q_i = \mathbf{k}_a + \sum_{l=1}^{i-1} \mathbf{k}_l$

LL: Fadin, Kuraev, Lipatov; NLL: Fadin, Fiore, Kozlov, Reznichenko

Maintain (at LL) terms of the form

$$\left(\alpha_s \ln \frac{\hat{s}_{ij}}{|\hat{t}_i|} \right)$$

to all orders in α_s.

At LL only gluon production; at NLL also quark–anti-quark pairs produced.
Approximation of any-jet rate possible.

Comparison of 3-jet scattering amplitudes

Universal behaviour of scattering amplitudes in the HE limit:

$$\left|\overline{\mathcal{M}}^{MRK}_{gg\to g\cdots g}\right|^2 = \frac{4\,s^2}{N_C^2-1}\,\frac{g^2\,C_A}{|p_{1\perp}|^2}\left(\prod_{i=2}^{n-1}\frac{4\,g^2 C_A}{|p_{i\perp}|^2}\right)\frac{g^2\,C_A}{|p_{n\perp}|^2}.$$

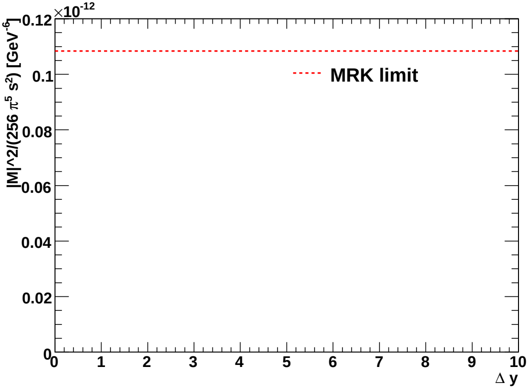
$$\left|\overline{\mathcal{M}}^{MRK}_{qg\to qg\cdots g}\right|^2 = \frac{4\,s^2}{N_C^2-1}\,\frac{g^2\,C_F}{|p_{1\perp}|^2}\left(\prod_{i=2}^{n-1}\frac{4\,g^2 C_A}{|p_{i\perp}|^2}\right)\frac{g^2\,C_A}{|p_{n\perp}|^2},$$

$$\left|\overline{\mathcal{M}}^{MRK}_{qQ\to qg\cdots Q}\right|^2 = \frac{4\,s^2}{N_C^2-1}\,\frac{g^2\,C_F}{|p_{1\perp}|^2}\left(\prod_{i=2}^{n-1}\frac{4\,g^2 C_A}{|p_{i\perp}|^2}\right)\frac{g^2\,C_F}{|p_{n\perp}|^2},$$

Allow for analytic resummation (BFKL equation).
However, how well does this actually approximate the amplitude?

Comparison of 3-jet scattering amplitudes

Study just a slice in phase space:



Correct limit is obtained - but outside LHC phase space. Limit alone irrelevant.

Universality obtained before limit is reached.

Will build frame-work which has the right MRK limit but also retains correct behaviour at smaller rapidities

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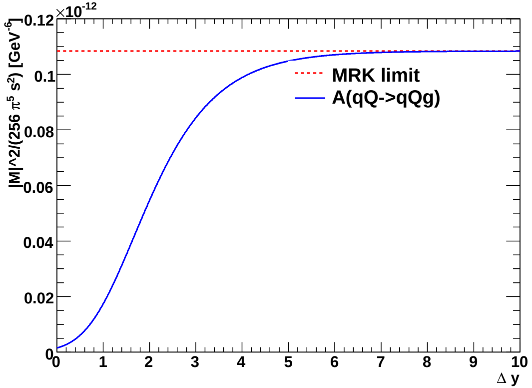

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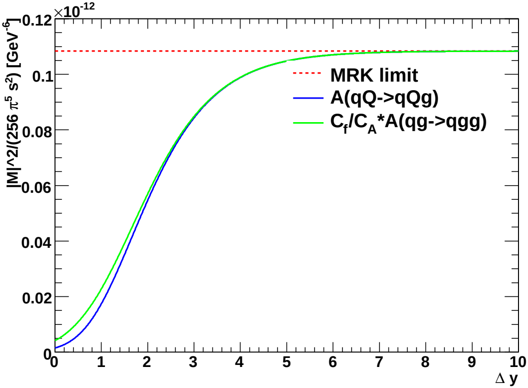

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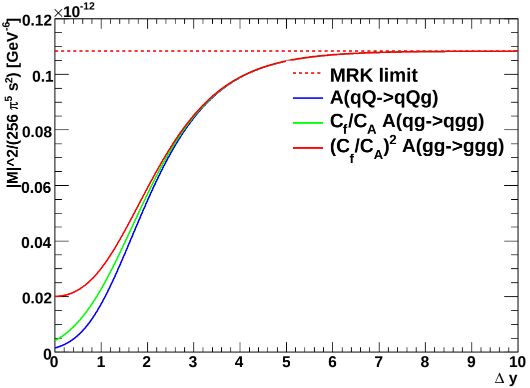

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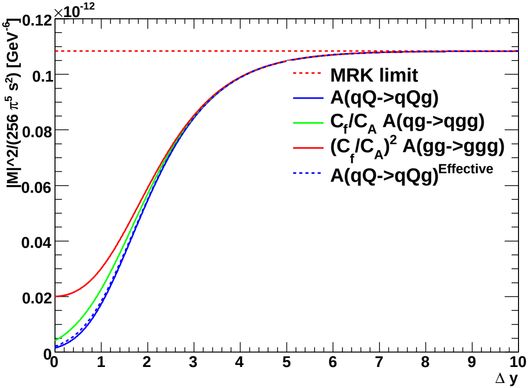

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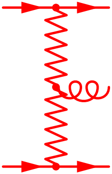
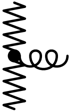
Building Blocks for an Amplitude

Identification of the **dominant contributions** to the **perturbative series** in the limit of well-separated particles

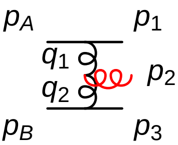


$$\frac{1}{q^2}\exp\left(\hat{\alpha}(q)\Delta y\right)$$

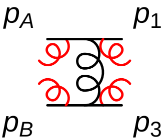


$$V^\mu(q_{i-1}, q_i)$$

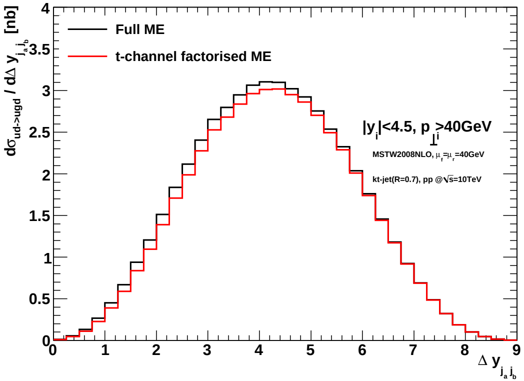
$$j^\nu = \overline{\psi}\gamma^\nu\psi$$



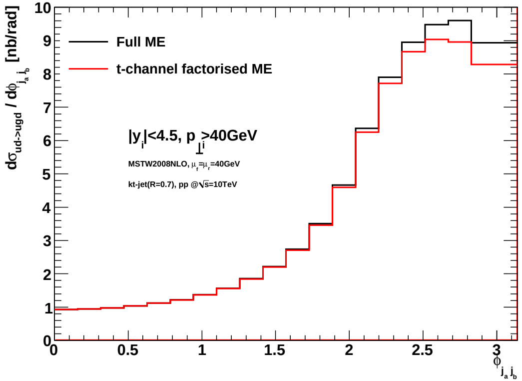
$$\begin{aligned} V^\rho(q_1, q_2) = & -(q_1+q_2)^\rho \\ & + \frac{p_A^\rho}{2}\left(\frac{q_1^2}{p_2\cdot p_A} + \frac{p_2\cdot p_B}{p_A\cdot p_B} + \frac{p_2\cdot p_n}{p_A\cdot p_n}\right) + p_A \leftrightarrow p_1 \\ & - \frac{p_B^\rho}{2}\left(\frac{q_2^2}{p_2\cdot p_B} + \frac{p_2\cdot p_A}{p_B\cdot p_A} + \frac{p_2\cdot p_1}{p_A\cdot p_1}\right) - p_B \leftrightarrow p_3. \end{aligned}$$



Jets @ LHC


Full ME
t-channel factorised ME
|y_i|<4.5, p_⊥i>40GeV
MSTW2008NLO, μ_f=μ_r=40GeV
kt-jet(R=0.7), pp @√s=10TeV
dσ_ud->ugd / dΔ y_jajb [nb]
Δ y_jajb

Jets @ LHC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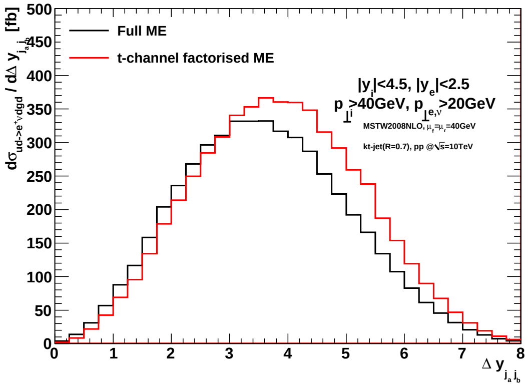
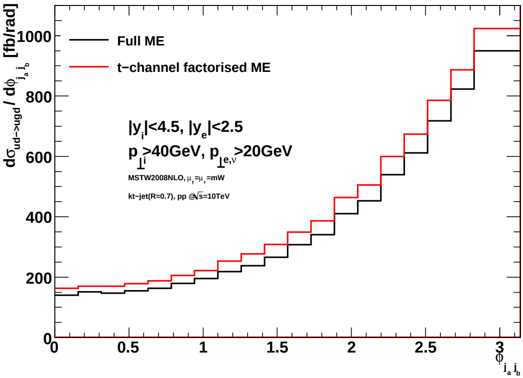
dσ_ud->ugd / dφ_jajb [nb/rad]
φ_jajb

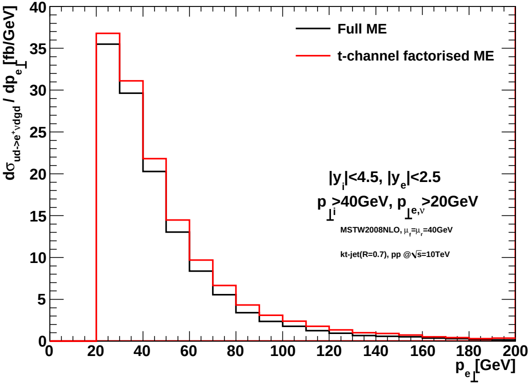
W+Jets @ LHC


Full ME
t-channel factorised ME
$|y_j|<4.5, |y_e|<2.5$
$p_{\perp j}>40$GeV, $p_{\perp e,\nu}>20$GeV
MSTW2008NLO, $\mu_f=\mu_r=40$GeV
kt-jet(R=0.7), pp @$\sqrt{s}$=10TeV
$d\sigma_{ud\to e^+\nu dgd} / d\Delta y_{j_a j_b}$ [fb]
$\Delta y_{j_a j_b}$

W+Jets @ LHC


Full ME
t−channel factorised ME
|y_j|<4.5, |y_e|<2.5
p_⊥j>40GeV, p_⊥e,ν>20GeV
MSTW2008NLO, μ_f=μ_r=mW
kt−jet(R=0.7), pp @√s=10TeV
dσ_ud->ugd / dφ_ja jb [fb/rad]
φ_ja jb
0
200
400
600
800
1000
0.5
1
1.5
2
2.5
3

W+Jets @ LHC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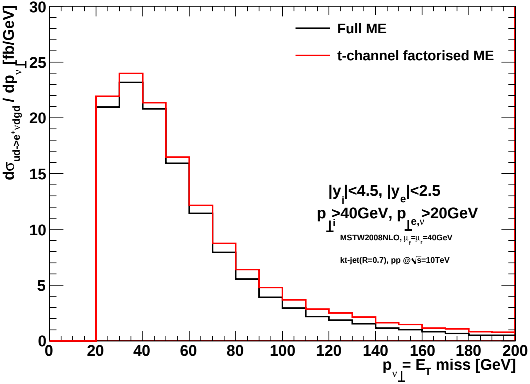
dσ_ud->e+νdgd / dp_e⊥ [fb/GeV]
p_e⊥[GeV]
0
20
40
60
80
100
120
140
160
180
200
5
10
15
20
25
30
35
40

W+Jets @ LHC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kt-jet(R=0.7), pp @$\sqrt{s}$=10TeV
$d\sigma_{ud\to e^+\nu dgd}$ / $dp_{\nu\perp}$ [fb/GeV]
$p_{\nu\perp}$ = E_T miss [GeV]
0
5
10
15
20
25
30
0
20
40
60
80
100
120
140
160
180
200

Outlook and Conclusions

Conclusions

- Emerging framework for the study of processes with multiple hard jets
- For each number of particles n, the approximation to the matrix element (real and virtual) is sufficiently simple to allow for the all-order summation to be constructed as an explicit sum over n-particle final states (exclusive studies possible)
- Resummation based on approximation which really does capture the behaviour of the scattering processes at the LHC
- Matching will correct the approximation where the full matrix element can be evaluated