Bayesian Statistics and Supersymmetry

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The University of Sheffield, England and

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with Roberto Ruiz de Austri and Roberto Trotta

and J. Silk, F. Feroz, M. Hobson, D. Lopez-Fogliani, S. Tsai and T. Varley (several papers)

new tool: SuperBayes package, available from www.superbayes.org

Main interests/activity

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- axino and gravitino dark matter (bounds from BBN and on T_R, and tests at the LHC)
- dark matter and Fermi
- low-energy SUSY models and signatures (DM, LHC), Bayesian approach
- SUSY parameter reconstruction from LHC data

MCMC + Bayesian Statistics

(MCMC=Markov Chain Monte Carlo)

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Powerful method of exploring multi–parameter models;

allows one to make global statements, expose correlations, etc.

huge improvement over “traditional” fixed-grid scans

Impact of varying SM parameters

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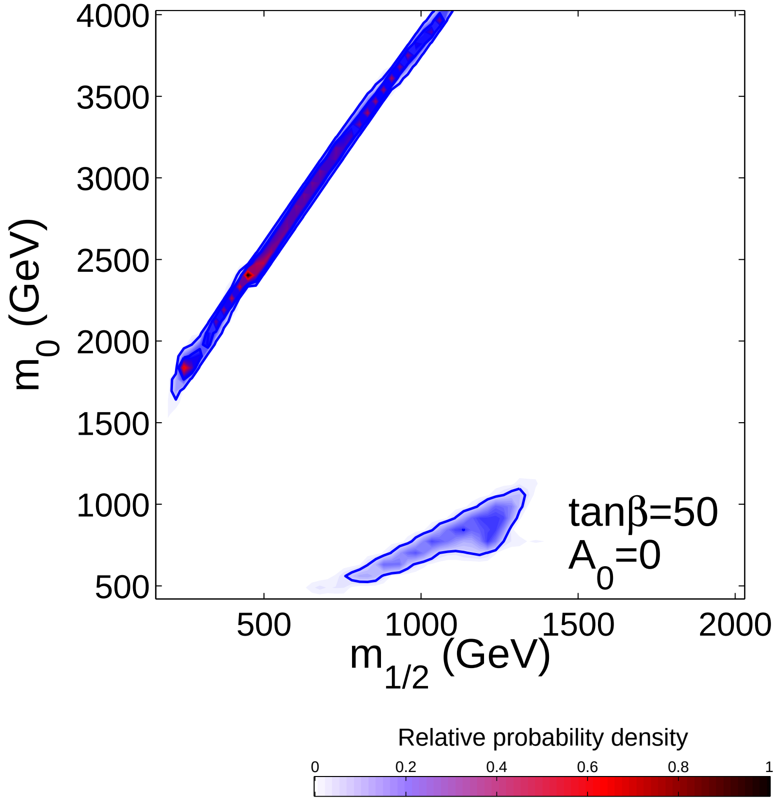
fix $\tan\beta$, A_0 + all SM param's



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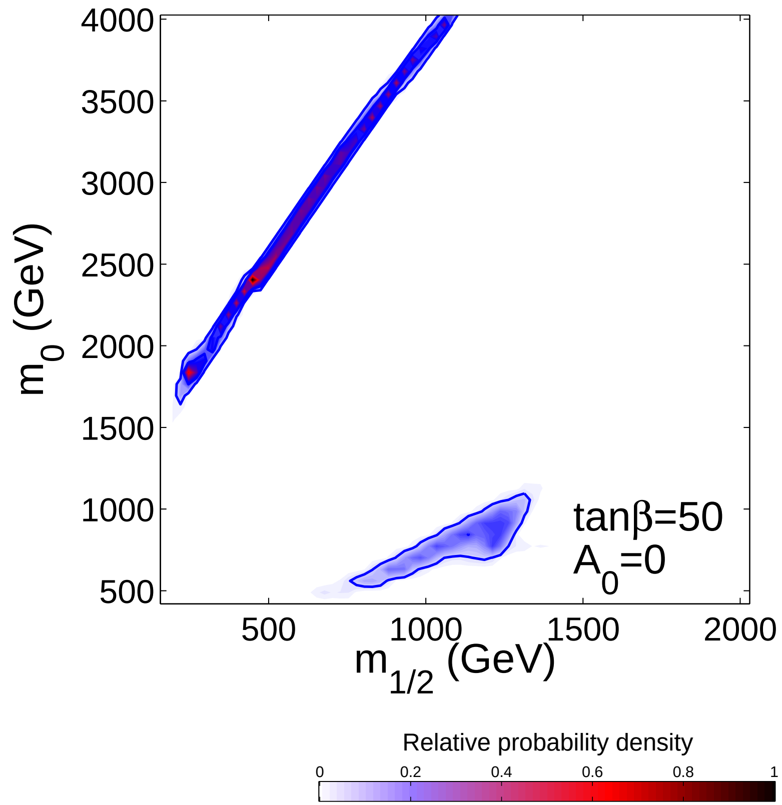
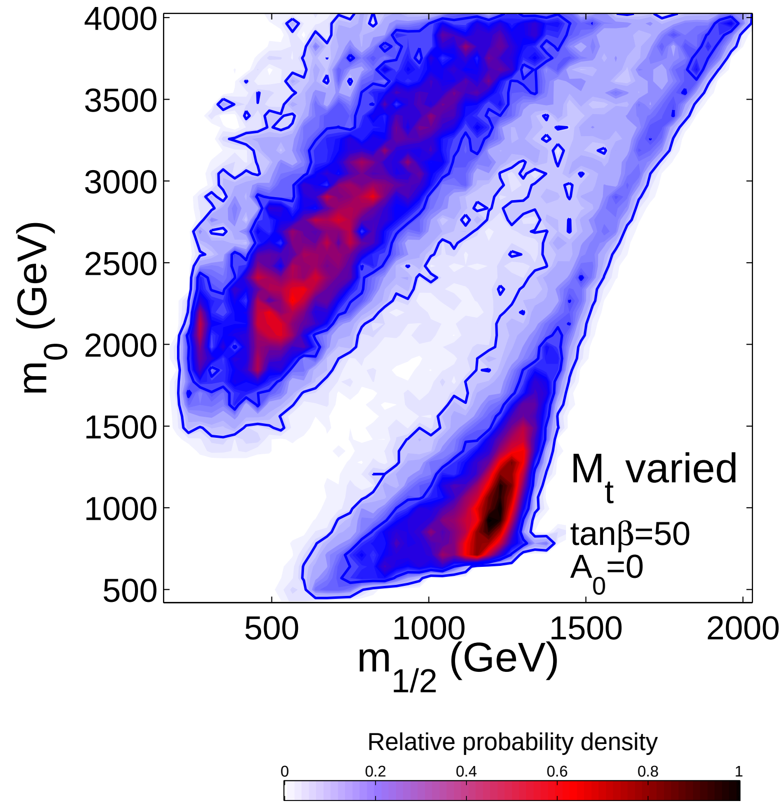


vary M_t



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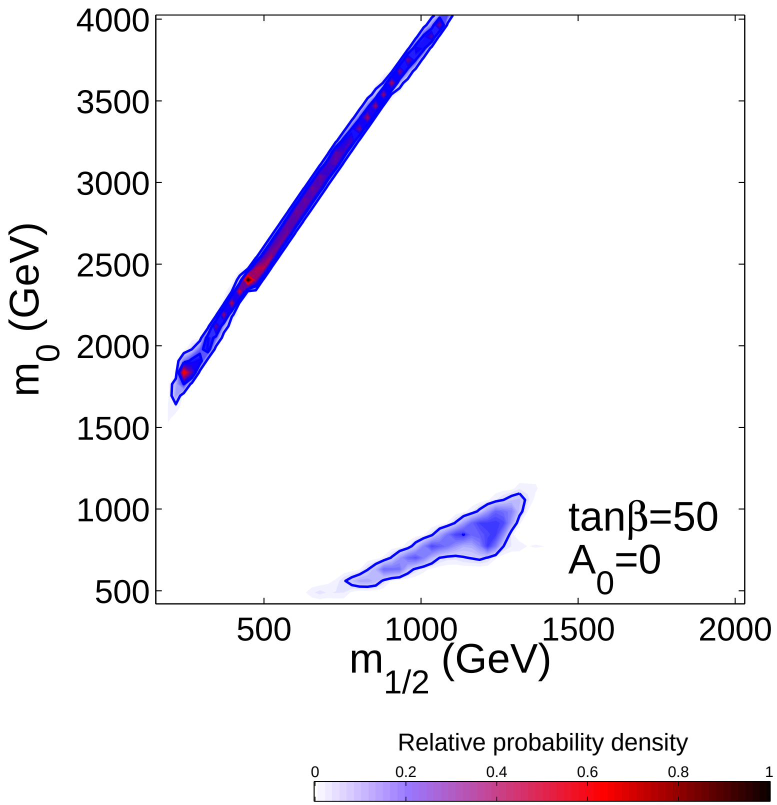

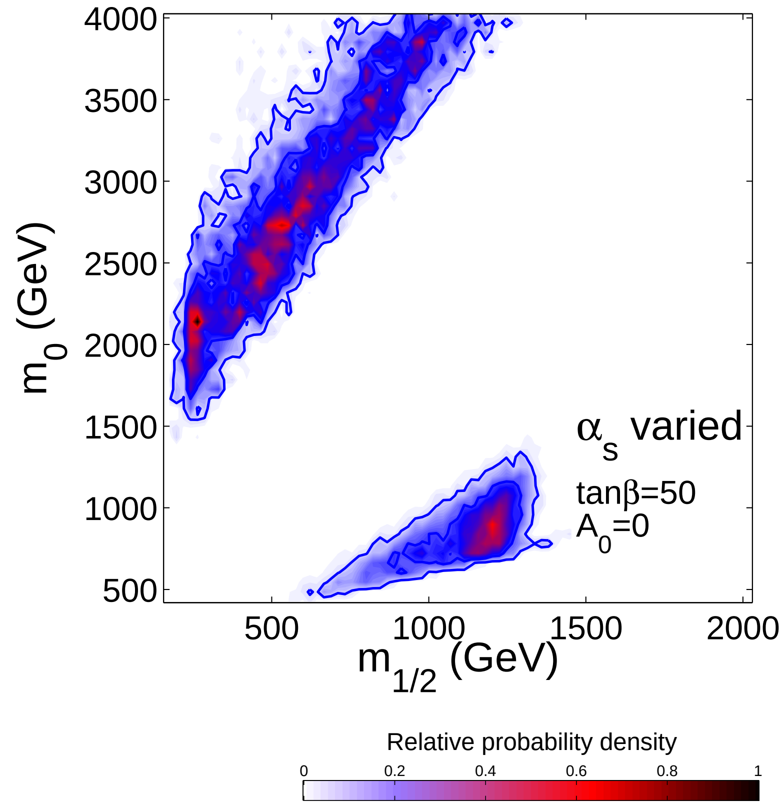

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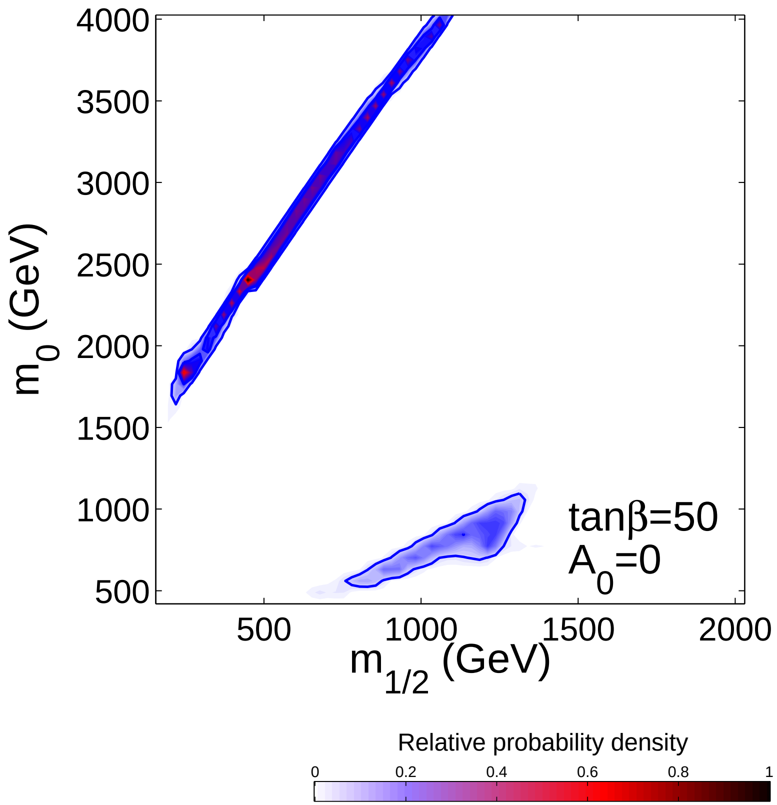

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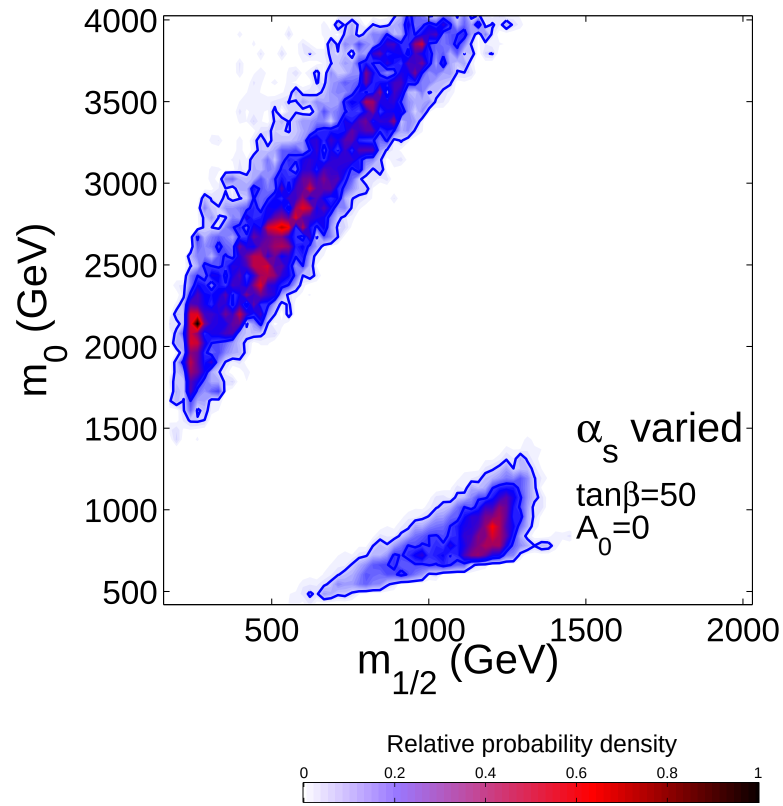

residual errors in SM parameters $\Rightarrow$ strong impact on favoured SUSY ranges

effect of varying A_0, $\tan\beta$ also substantial

Bayesian Analysis of the CMSSM

Apply to the CMSSM: new development, led by 2 groups

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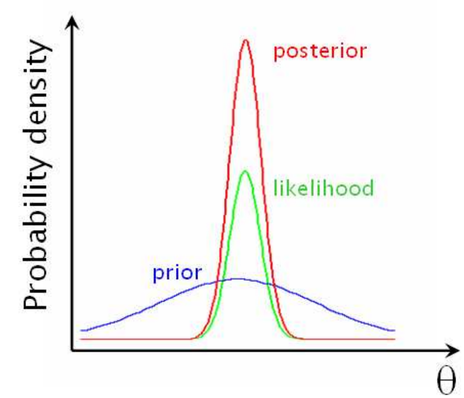

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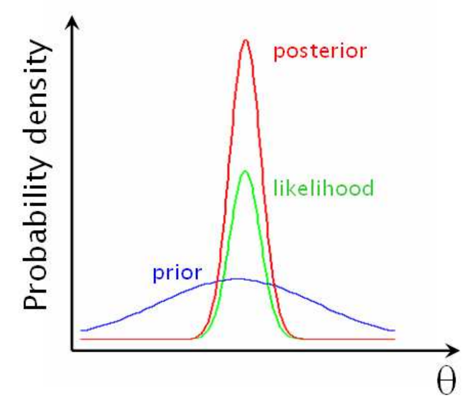
- Bayes' theorem: posterior pdf

$$\boxed{p(\theta, \psi | d) = \frac{p(d|\xi)\pi(\theta,\psi)}{p(d)}}$$

- $p(d|\xi) = \mathcal{L}$: likelihood
- $\pi(\theta, \psi)$: prior pdf
- $p(d)$: evidence (normalization factor)



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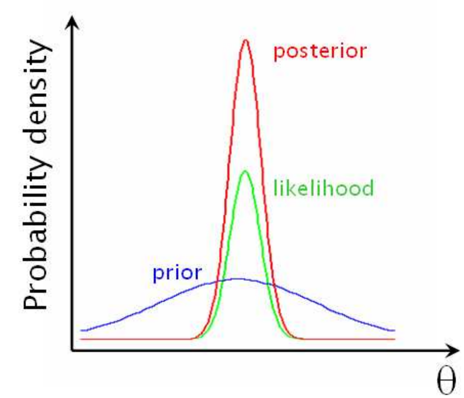

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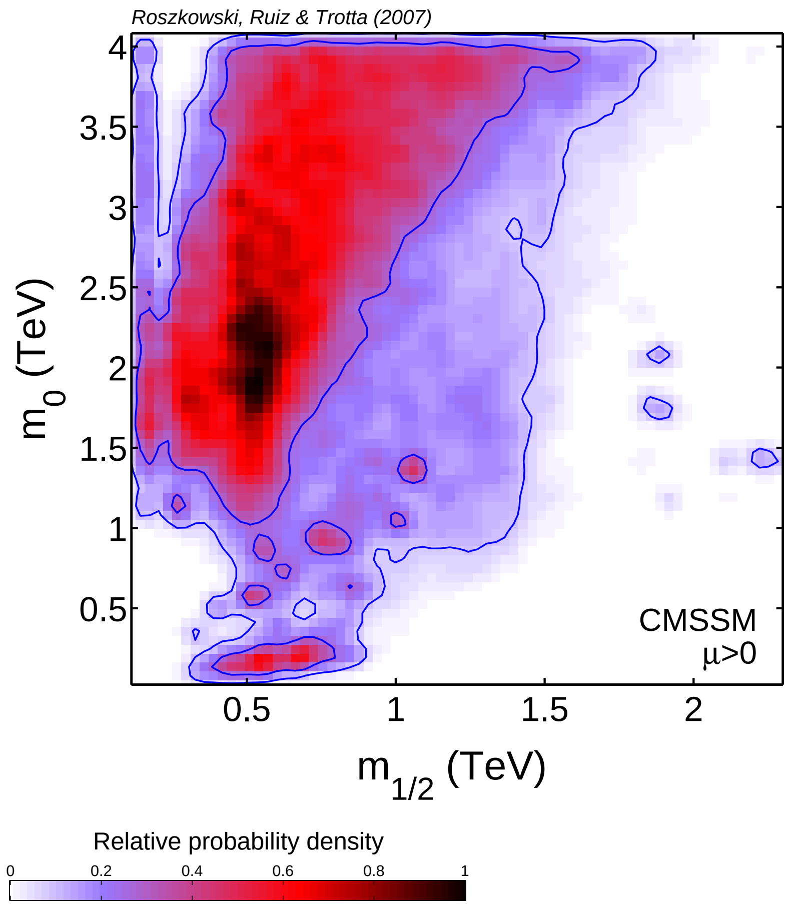
- usually marginalize over SM (nuisance) parameters ψ $\Rightarrow$ $\boxed{p(\theta|d)}$

Probability maps of the CMSSM

Bayesian posterior pdf

Probability maps of the CMSSM

arXiv:0705.2012



Bayesian posterior pdf

- MCMC scan (4 CMSSM + 4 SM param's)
- $50\,\mathrm{GeV} < m_{1/2}, m_0 < 4\,\mathrm{TeV}$, $|A_0| < 7\,\mathrm{TeV}$, $2 < \tan\beta < 62$
- relative probability density fn (pdf)
- flat priors
- 68% total prob. – inner contours
- 95% total prob. – outer contours
- 2-dim pdf $p(m_0, m_{1/2}|d)$
- favored: $m_0 \gg m_{1/2}$ (FP region)

Probability maps of the CMSSM

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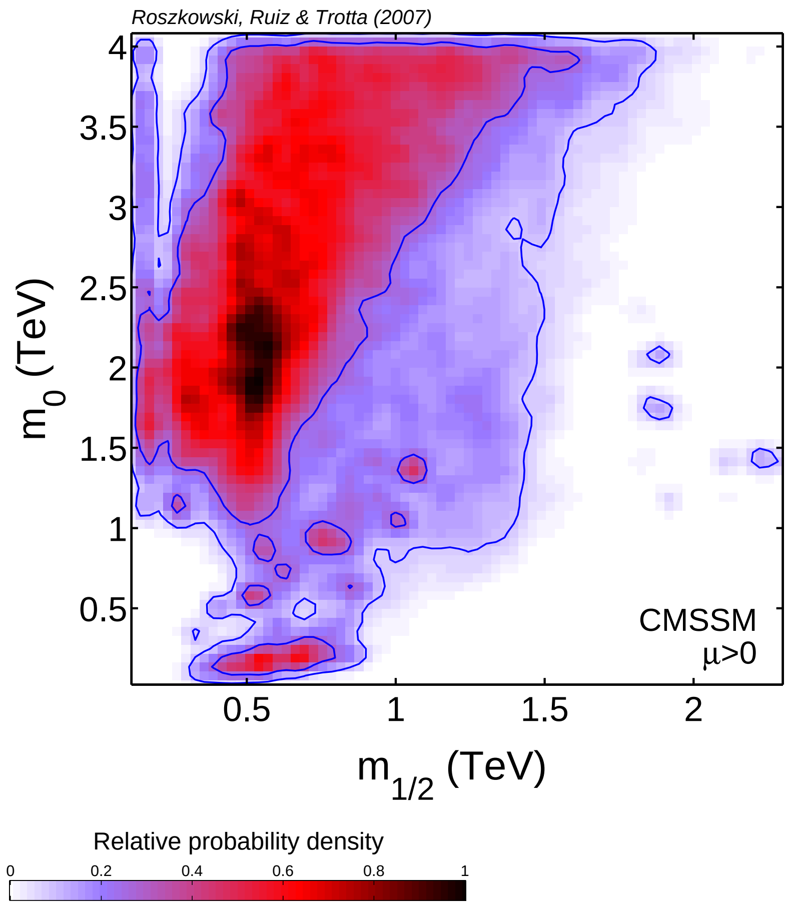

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similar study by Allanach+Lester et al.
see also, Ellis et al (EHOW, χ^2 approach)

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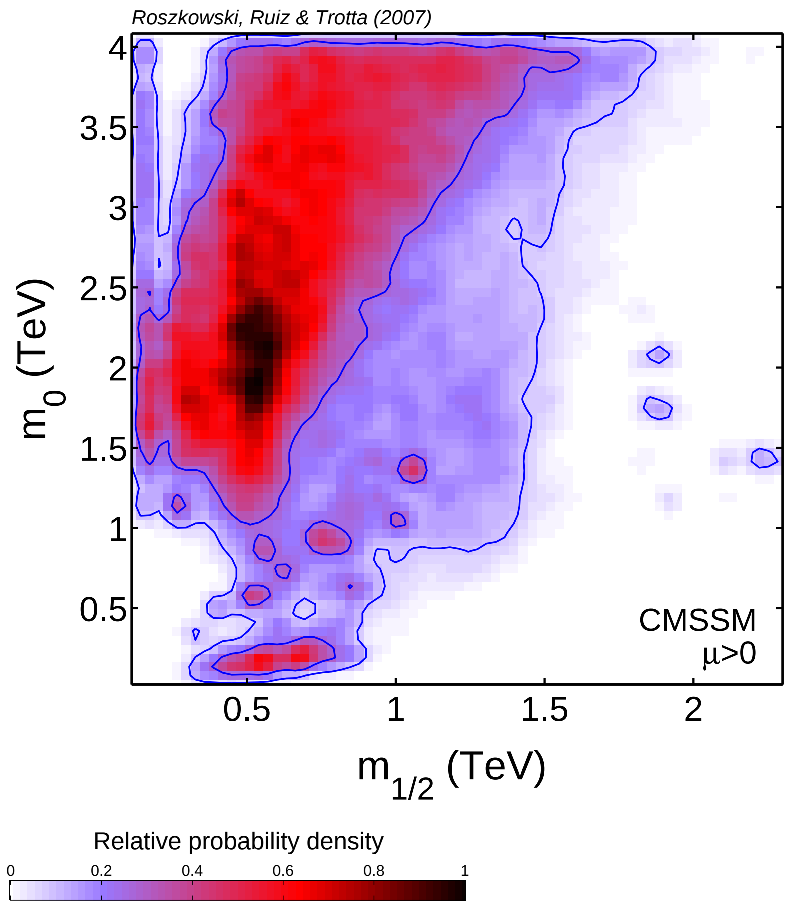

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important to vary also SM parameters

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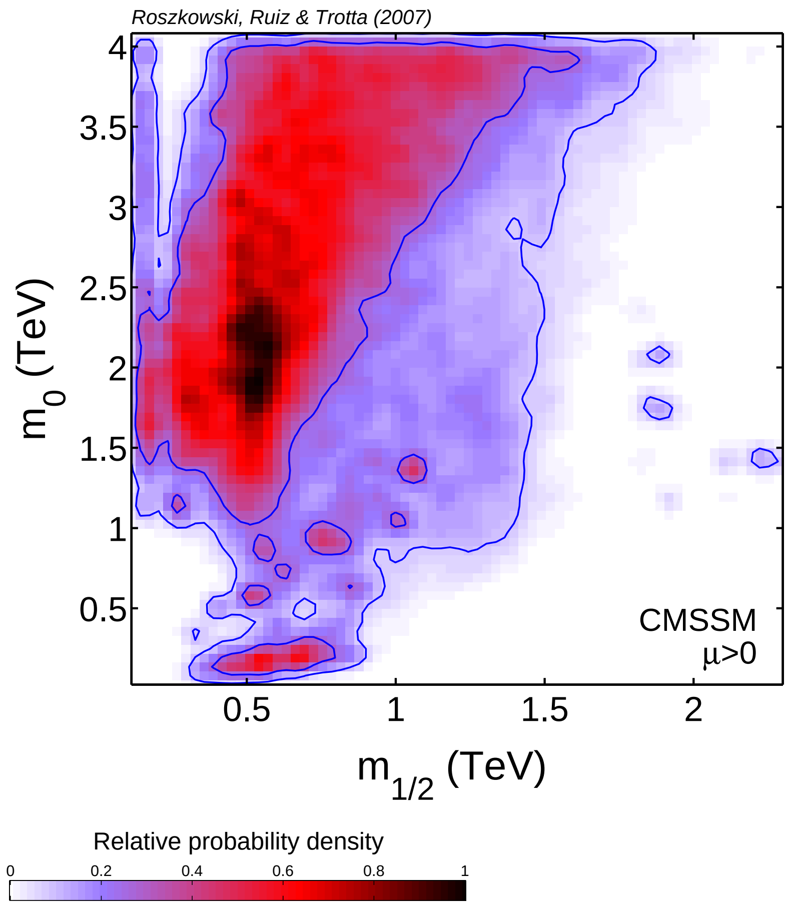

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$\Rightarrow$ Constrained MSSM is currently under-constrained

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some observables are rather robust

despite prior dependence

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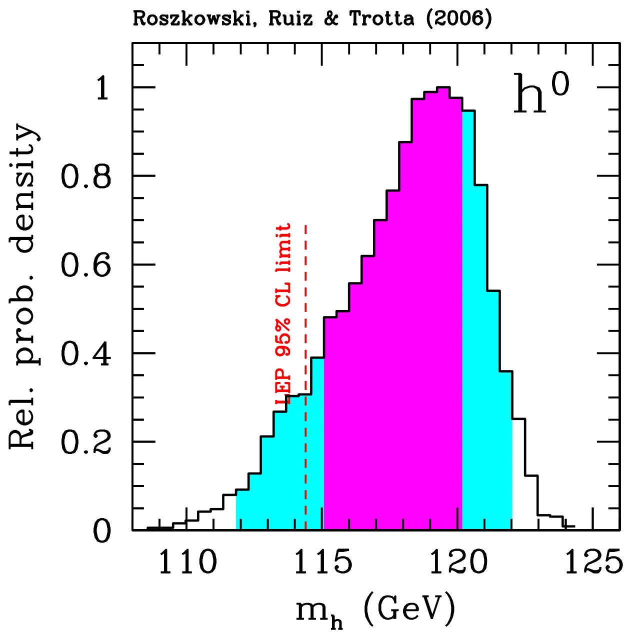

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- to be fully explored at the Tevatron

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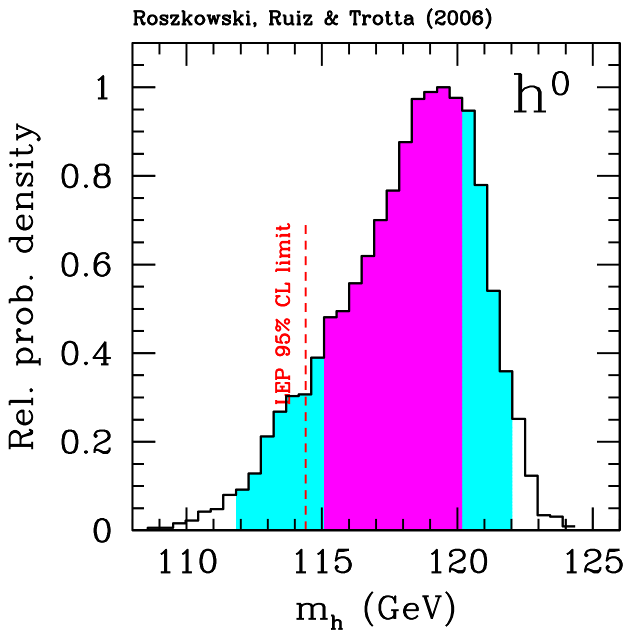

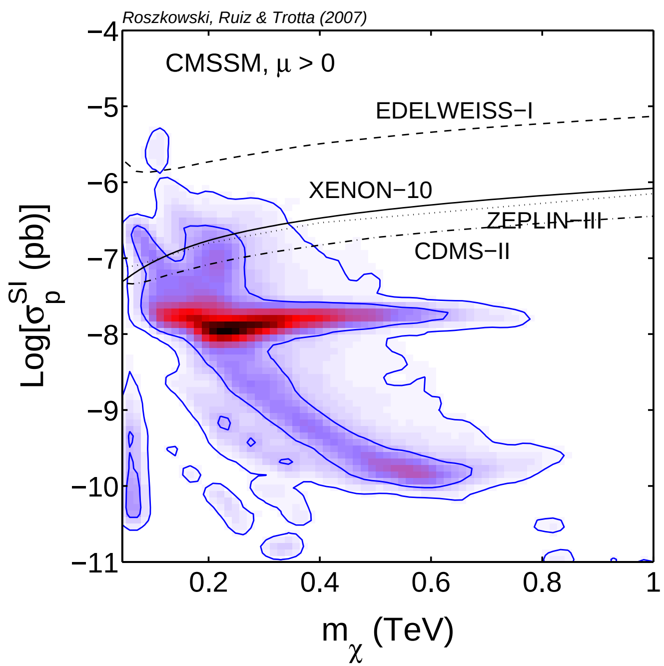

- SM-like light Higgs
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- already explored by current searches
- to be fully explored with one-tonne detectors

Future: CMSSM Point SU3

Atlas SU3 benchmark point, arXiv:0901.0512

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Parameter	SU3 benchmark value
m_0	100 GeV
$m_{1/2}$	300 GeV
$\tan\beta$	6.0
A_0	-300 GeV
$\Omega_\chi h^2$	0.23319 $\Leftarrow$
SUSY mass spectrum	
$\chi = \chi_1^0$	117.9 GeV
χ_2^0	223.4 GeV
$\widetilde{m}_{\widetilde{l}}$	152.2 GeV
$m_{\widetilde{q}}$	652.4 GeV

- $\widetilde{m}_{\widetilde{l}}$ - lightest slepton mass
- $m_{\widetilde{q}}$ - average light squark mass

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- study endpoint measurements
- dileptons + lepton+jets
- χ^2 minimization
- int. lum. 1 fb^{-1}

Table 8: Resulting SUSY particle masses and mass differences within SU3 and SU4 from the χ^2 minimization fit using the dilepton and lepton+jets edges. Shown are the measured masses $m_{\rm meas}$ and mass differences $\Delta m_{\rm meas}$ followed first by the parabolic errors as returned by MIGRAD and then by the jet energy scale errors. When the measured parameter is anticorrelated with the jet energy scale variation, this is indicated by a $\mp$ sign. The input Monte Carlo masses $m_{\rm MC}$ and mass differences $\Delta m_{\rm MC}$ are also shown. The integrated luminosity assumed is 1 fb^{-1} for SU3 and 0.5 fb^{-1} for SU4.

Observable	SU3 $m_{\rm meas}$ [GeV]	SU3 $m_{\rm MC}$ [GeV]	SU4 $m_{\rm meas}$ [GeV]	SU4 $m_{\rm MC}$ [GeV]
$m_{\tilde\chi_1^0}$	$88 \pm 60 \mp 2$	118	$62 \pm 126 \mp 0.4$	60
$m_{\tilde\chi_2^0}$	$189 \pm 60 \mp 2$	219	$115 \pm 126 \mp 0.4$	114
$m_{\tilde q}$	$614 \pm 91 \pm 11$	634	$406 \pm 180 \pm 9$	416
$m_{\tilde l}$	$122 \pm 61 \mp 2$	155		
Observable	SU3 $\Delta m_{\rm meas}$ [GeV]	SU3 $\Delta m_{\rm MC}$ [GeV]	SU4 $\Delta m_{\rm meas}$ [GeV]	SU4 $\Delta m_{\rm MC}$ [GeV]
$m_{\tilde\chi_2^0} - m_{\tilde\chi_1^0}$	$100.6 \pm 1.9 \mp 0.0$	100.7	$52.7 \pm 2.4 \mp 0.0$	53.6
$m_{\tilde q} - m_{\tilde\chi_1^0}$	$526 \pm 34 \pm 13$	516.0	$344 \pm 53 \pm 9$	356
$m_{\tilde l} - m_{\tilde\chi_1^0}$	$34.2 \pm 3.8 \mp 0.1$	37.6		

- $\widetilde{m}_{\tilde{l}}$ - lightest slepton mass
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- 1st errors: parabolic
- 2nd errors: jet energy scale

Atlas: reconstruction of SU3 point

Aad, *et al.*, arXiv:0901.0512

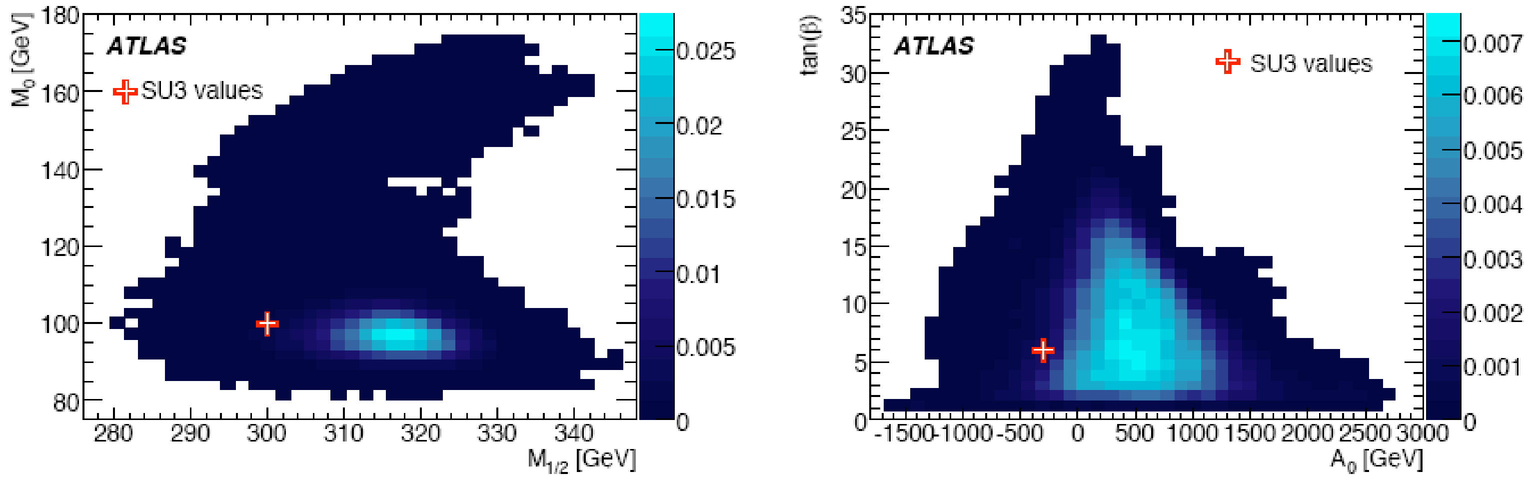

Figure 12: Two-dimensional Markov chain likelihood maps for mSUGRA parameters M_0 and $M_{1/2}$ (left) as well as $\tan\beta$ and A_0 (right) for sign $\mu = +1$, for benchmark point SU3, with integrated luminosity of 1 fb^{-1}. The crosses indicate the actual values of the parameters for that benchmark point.

- 2D likelihood maps (int. lum. 1 fb^{-1})
- theory errors neglected
- neglect effect of SM parameters
- ranges around the true value found

Reconstructing $m_{1/2}, m_0$ with SB

Case study: Atlas SU3 benchmark point

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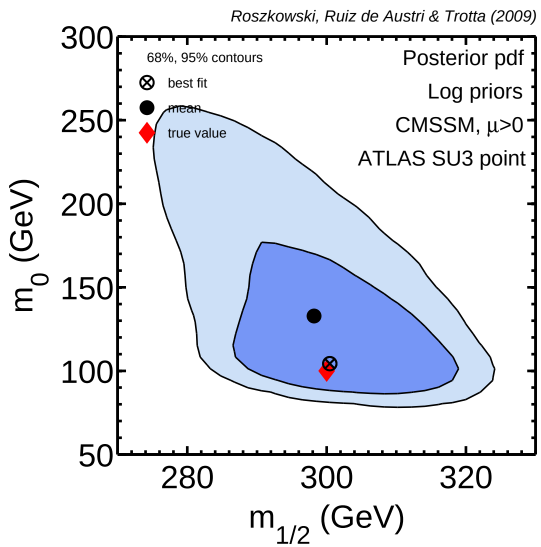
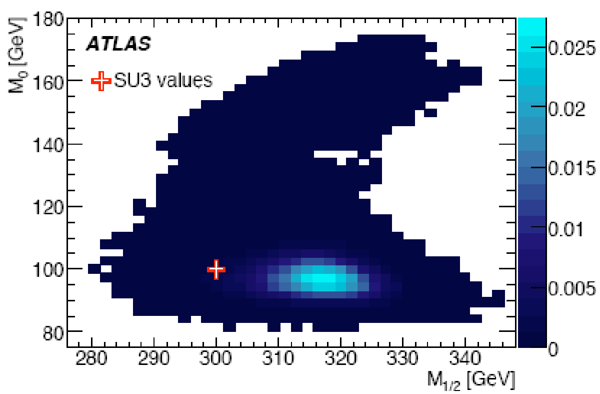


Atlas analysis



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- cross in circle: best-fit value
- black dot: posterior mean
- dark blue: 68% total prob. region
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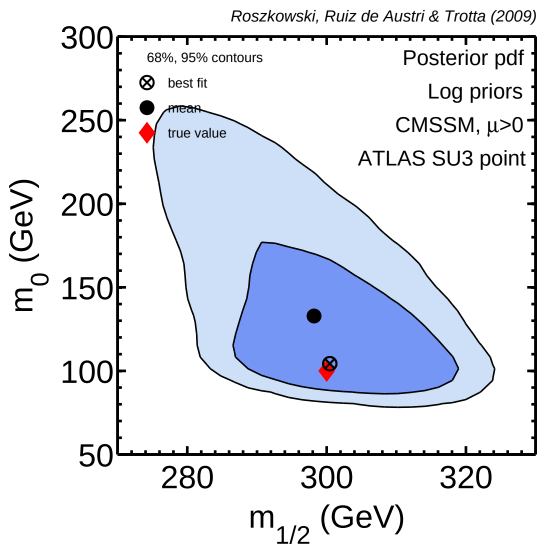

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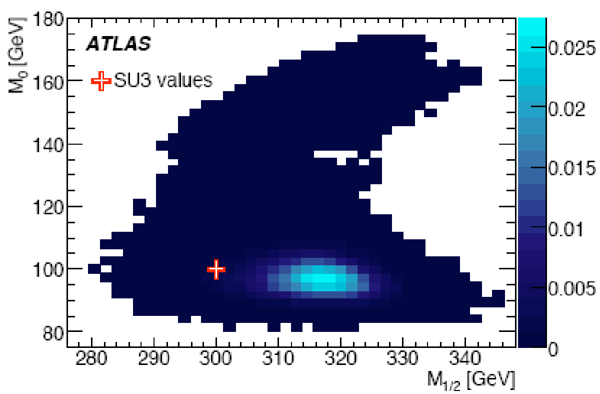

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- follow Atlas input + covariance matrix
- fix SM (nuisance) parameters
- NO exptal constraints applied ($b \longrightarrow s\gamma$, $\Omega_\chi h^2$, etc)
- similar for flat prior and profile like (akin to χ^2)
- determination similar to that claimed by Atlas

Reconstructing $A_0, \tan\beta$ with SB

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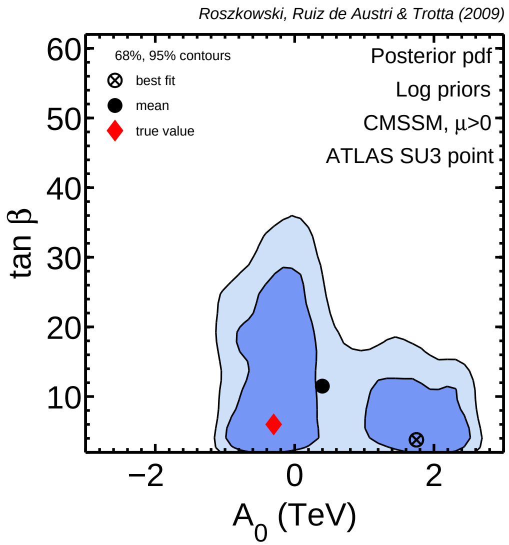

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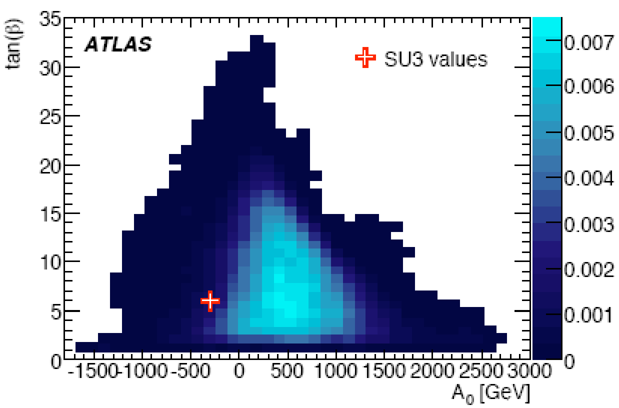

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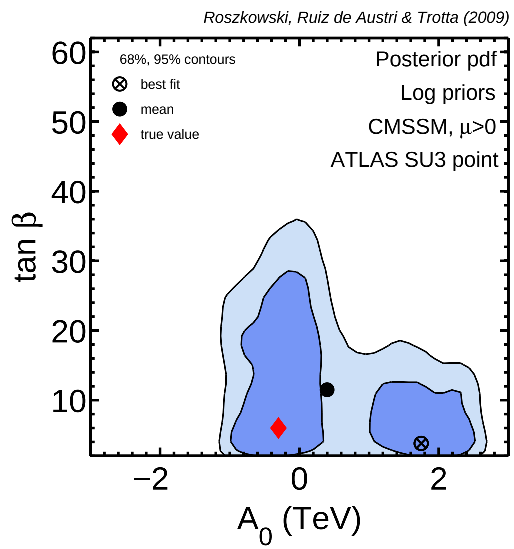

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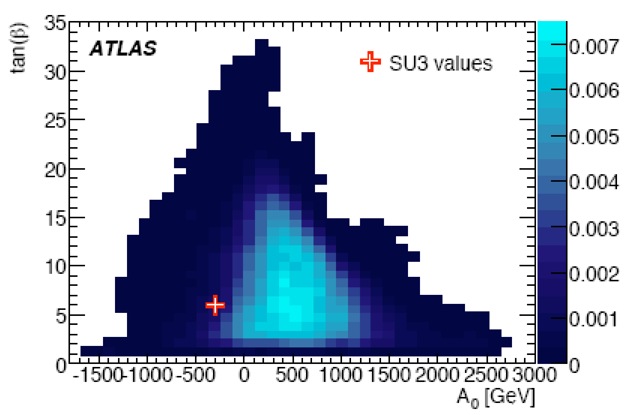

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- similar result for flat prior and profile like (akin to χ^2)
- determination a bit poorer than claimed by Atlas?
- cannot resolve sign of A_0

Add info on dark matter

WMAP error 0.0062

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- similar result for log prior and profile likelihood

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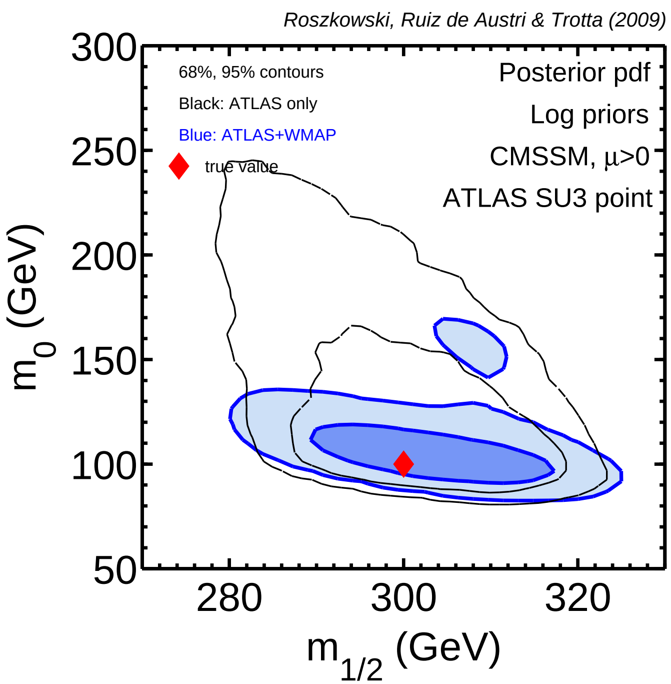

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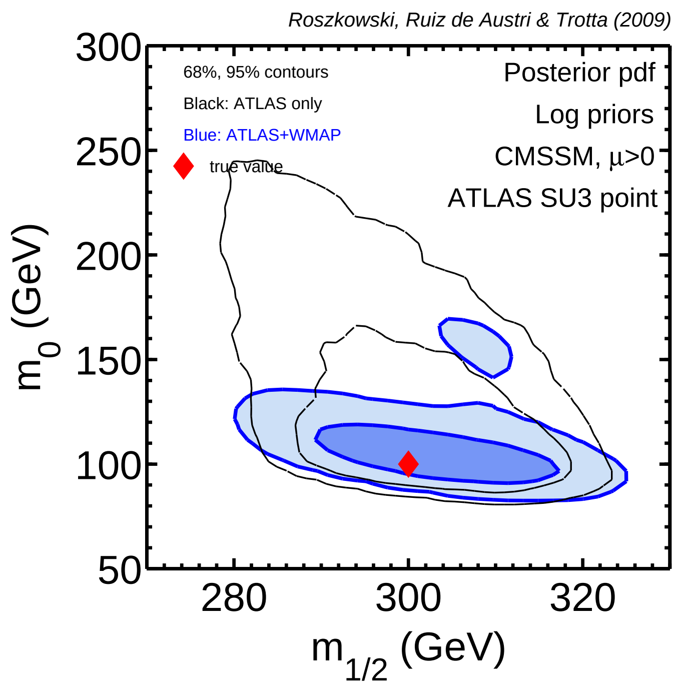

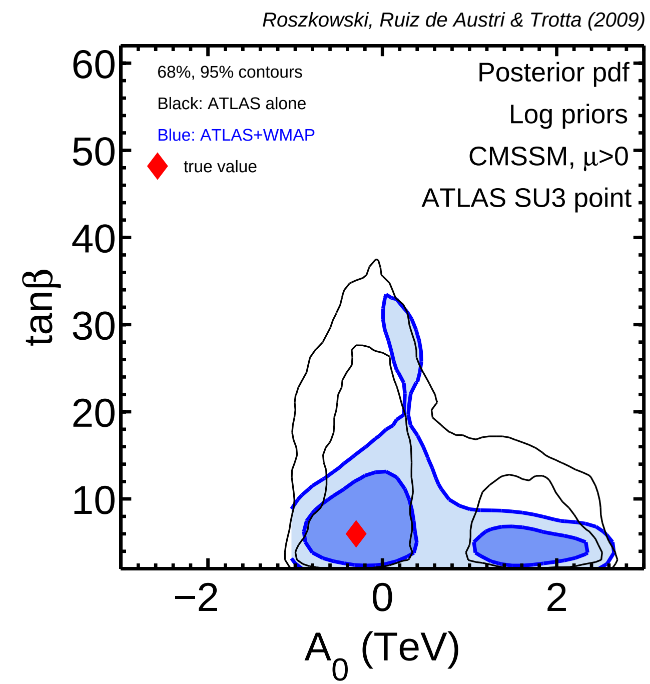

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- red diamond: SU3 point
- similar result for log prior and profile likelihood

Add info on dark matter

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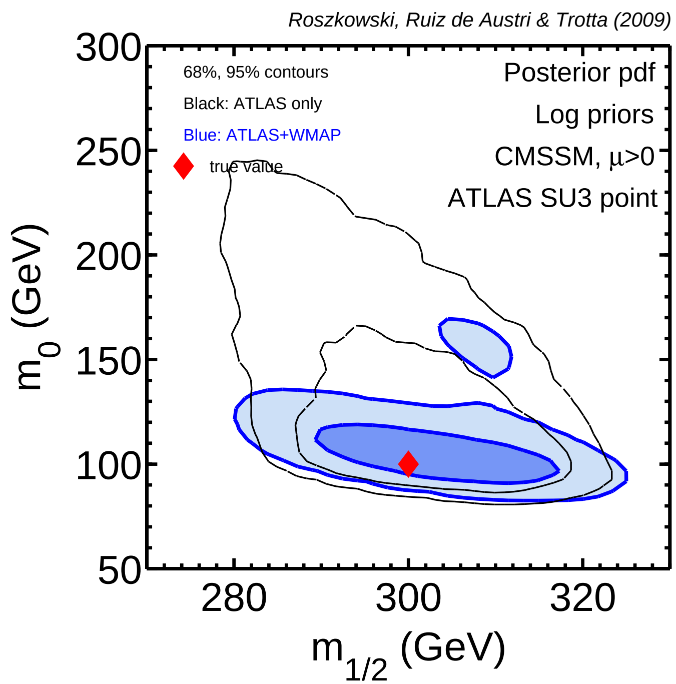

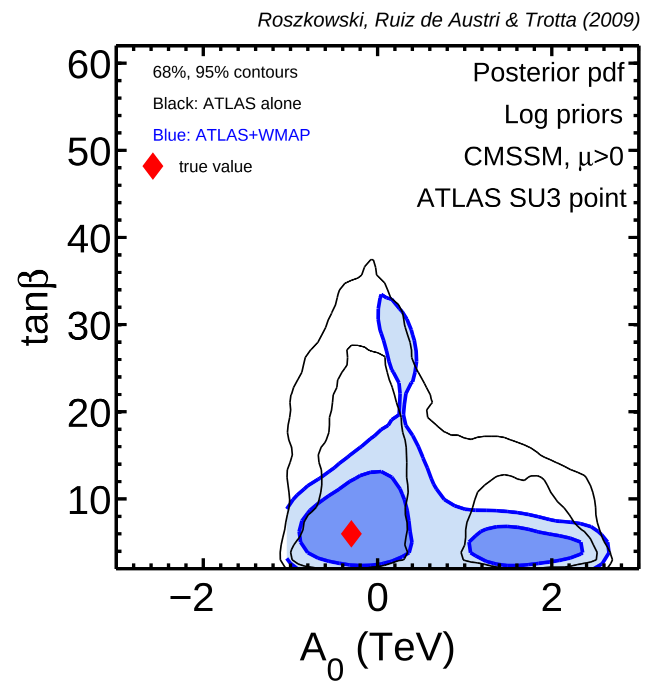

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$\Rightarrow$ easy to add relevant extra constraints

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SuperBayes tool: easily applicable to other multi-parameter models (NUHM, NMSSM, add your own...)