NLO parton shower – dreamland or reality
The KRKMC Project

M. Skrzypek, S. Jadach

IFJ-PAN, Kraków and CERN, Geneva

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More in http://jadach.web.cern.ch/

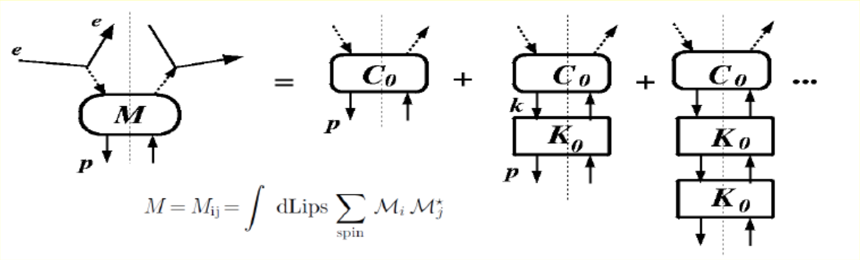
The aim of KRKMC project

Can we construct NLO Parton Shower Monte Carlo for QCD Initial State Radiation:

- based firmly on Feynman Diagrams (ME) and LIPS,
- based rigorously on the collinear factorization (EGMPR, CSS),
- implementing *exactly* NLO $\overline{MS}$ DGLAP evolution,
- implementing fully unintegrated PDFs (FunPDF); with NLO evolution done by MC itself, using EXClusive NLO kernels**???**

We are going to show that YES, we can do it**!**
And show first numerical implementation
– the proof of the concept.

"Raw" factorization of the IR collinear singularities



- Cut vertex M: spin sums and Lips integrations over all lines cut across
- C_0 and K_0 and are 2-particle irreducible (2PI)
- C_0 is IR finite, while K_0 encapsulates all IR collinear singularities
- Use of the axial gauge essential for the proof
- Formal proof given in EGMPR NP B152 (1979) 285
- Notation next slide

$$M = C_0\left(1 + K_0 + K_0^2 + \cdots\right) = C_0 \frac{1}{1 - K_0} \equiv C_0 \Gamma_0$$

Factorization of EGMPR improved by Furmanski and Petronzio (80):

$$F = C_0 \cdot \frac{1}{1-K_0} = C\left(\alpha, \frac{Q^2}{\mu^2}\right) \otimes \Gamma\left(\alpha, \frac{1}{\epsilon}\right),$$

$$= \left\{ C_0 \cdot \frac{1}{1-(1-\mathbf{P})\cdot K_0} \right\} \otimes \left\{ \frac{1}{1-\left(\mathbf{P}K_0 \cdot \frac{1}{1-(1-\mathbf{P})\cdot K_0}\right)} \right\}_{\otimes},$$

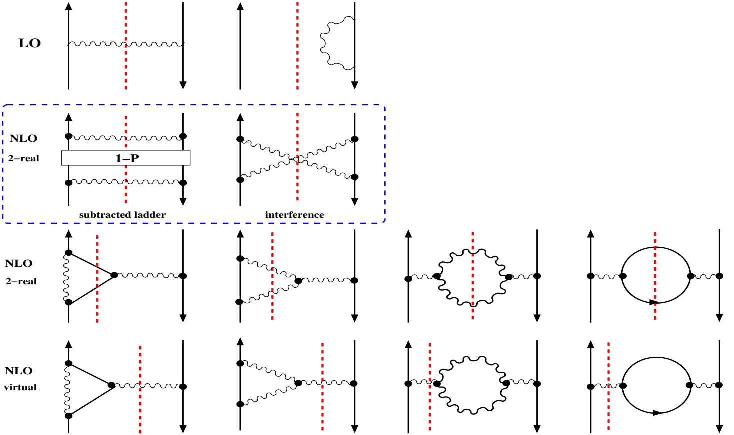
$$\Gamma\left(\alpha, \frac{1}{\epsilon}\right) \equiv \left(\frac{1}{1-K}\right)_{\otimes} = 1 + K + K \otimes K + K \otimes K \otimes K + \ldots,$$

$$K = \mathbf{P}K_0 \cdot \frac{1}{1-(1-\mathbf{P})\cdot K_0}, \quad C = C_0 \cdot \frac{1}{1-(1-\mathbf{P})\cdot K_0}.$$

Ladder part Γ corresponds to MC parton shower
C is the hard process part
$\mathbf{P}$ is the projection operator: $\mathbf{P} = P_{spin}\, P_{kin}\, PP$

Outline of KRKMC project

Step 1. Re-do the CFP calculation of the NLO kernels in the exclusive way, for various types of evolution time (virtuality, $k_\perp$, rapidity ...). Use $\overline{MS}$. For now only the C_F^2 part - blue frame



Step 2. Get rid of dimensional regularisation, go back to 4-dimensions. Include cut-off Δ instead

$$\frac{1}{\epsilon} \to \int_0^1 d\left(\frac{q^2}{Q^2}\right)\left(\frac{Q^2}{q^2}\right)^{1-\epsilon} \to \int_{\Delta^2/Q^2}^1 d\left(\frac{q^2}{Q^2}\right)\frac{Q^2}{q^2}$$

Calculate appropriate Sudakov form-factor to keep the momentum sum rule !

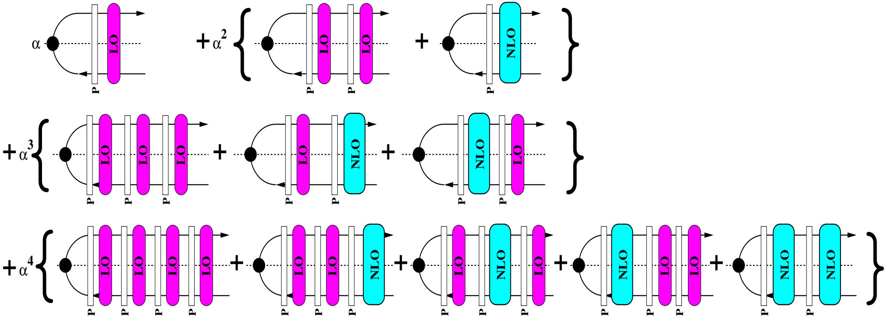
In full agreement with the $\overline{MS}$ scheme.

Step 3. Re-formulate the factorization formula:

⇒ DGLAP mixes orders of pert. expansion:

$$P = \alpha P^{LO} + \alpha^2 P^{NLO}$$

⇒we want order-by-order expansion (e.g. to avoid negative weights)



Step 4. Do the explicit Bose-Einstein symmetrisation:
⇒ In inclusive DGLAP one uses ordering: $|k_n| > |k_{n-1}| > \cdots > |k_0|$
⇒ In exclusive MC Bose-Einstein symmetric form is better because NLO contribution allows both $|k_i| > |k_{i-1}|$ and $|k_{i-1}| > |k_i|$

$$D_3^{L+N}(t,x) \sim \frac{1}{3!} \int_{k_{\min}}^{k_{\max}} \left(\prod_{i=1}^{3} \frac{d^3 k_i}{2k_i^0} \right) \delta_{x_0 - x = \alpha_1+\alpha_2+\alpha_3}\, \rho_3^{L+N}(k_3,k_2,k_1),$$

$$\rho_3^{L+N}(k_3,k_2,k_1) = \sum_{\pi} \left(\rho_3^L(k_{\pi_3},k_{\pi_2},k_{\pi_1}) + \rho_{3a}^N(k_{\pi_3},k_{\pi_2},k_{\pi_1}) + \rho_{3b}^N(k_{\pi_3},k_{\pi_2},k_{\pi_1}) \right),$$

$$\rho_3^L(k_3,k_2,k_1) = \rho^L(k_3|x_2)\, \rho^L(k_2|x_1)\, \rho^L(k_1|x_0)\, \theta_{|\mathbf{k}_3|>|\mathbf{k}_2|>|\mathbf{k}_1|},$$

$$\rho_{3a}^N(k_3,k_2,k_1) = \rho^L(k_3|x_2)\, b_2^{\theta N}(k_2,k_1|x_0)\, \theta_{|\mathbf{k}_3|>|\mathbf{k}_2|},$$

$$\rho_{3b}^N(k_3,k_2,k_1) = b_2^{\theta N}(k_3,k_2|x_1)\, \rho^L(k_1|x_0)\, \theta_{|\mathbf{k}_3|>|\mathbf{k}_1|}.$$

Step 5. Construct the NLO weight to be applied on top of the regular LO MC (Markovian or Constrained Markovian). For 3 emissions:

$$w = 1 + w^N_{3a} + w^N_{3b}$$

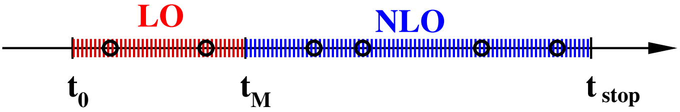
$$w^N_{3a} = \frac{b_2^{\theta N}(\tilde{k}_2, \tilde{k}_1|x_0)}{\rho^L(\tilde{k}_2|x_1)\ \rho^L(\tilde{k}_1|x_0)} \theta_{\tilde{t}_2 > t_M},$$

$$w^N_{3b} = \frac{b_2^{\theta N}(\tilde{k}_3, \tilde{k}_2|x_1)}{\rho^L(\tilde{k}_3|x_2)\ \rho^L(\tilde{k}_2|x_1)} \theta_{\tilde{t}_3 > t_M} + \frac{b_2^{\theta N}(\tilde{k}_3, \tilde{k}_1|x_1^{\pi_b^\bullet})}{\rho^L(\tilde{k}_3|x_2)\ \rho^L(\tilde{k}_1|x_0)}\ \frac{\rho^L(\tilde{k}_2|x_0)}{\rho^L(\tilde{k}_2|x_1)} \theta_{\tilde{t}_3 > t_M}.$$

Step 6. Solve problem of “lower limit of internal NLO phase space”.
NLO kernels contain “internal emission”:

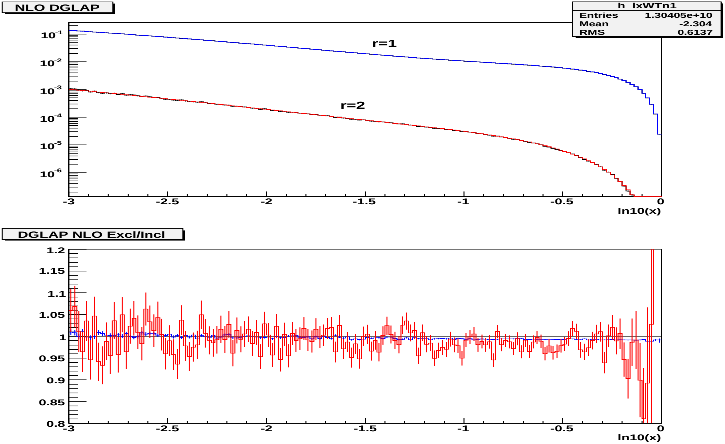
$$\text{Inclusive NLO: } \int_0^{Q^2} dq^2_{\text{internal}} \quad \Leftrightarrow \quad \text{Exclusive MC: } \int_{Q^2_{\text{min}}}^{Q^2} dq^2_{\text{internal}}$$

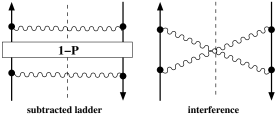
Two solutions:
⇒ Difference has to be calculated analytically
⇒ Use “pre-evolution” of pure LO type with VERY low t_0.
NLO corrections start at intermediate $t = t_M$.



Exclusive NLO MC agrees with $\overline{MS}$ NLO DGLAP

Comparison of new Exclusive NLO MC with $\overline{MS}$ NLO DGLAP





Both results agree!!

More on what is in these plots:

- Both evolutions on top of the same Markovian LO MC. (It can be put easily on top of non-Markovian CMC.)
- MC weights positive, weight distributions very reasonable,
- Evolution range from 10GeV to 1TeV.
- LO pre-evolution staring from $\delta(1-x)$ at 1GeV to 10GeV provides initial x-distribution for the LO+NLO continuation.
- Only C_F^2 part of gluonstrahlung.
- Non-ruining α_S.
- Term due to ε part of γ-traces omitted. done.
- NLO virtual corrections omitted. partly done.

Potential gains

While retaining exact NLO DGLAP evolution, excellent starting point for extensions:

- Possible extension towards CCFM, BFKL (low x limit)
- Correct soft limit and built-in colour coherence
- Realistic description of the quark thresholds
- The use of exact amplitudes for multigluon emission, the analog of Coherent Exclusive Exponentiation in QED (Jadach,Was,Ward)
- Easy connection between hard process ME and the shower parts
- In particular no negative weight events, no ambiguity of defining last emission before hard process, etc.
- Providing better tool for exploiting HERA DATA for LHC (fitting F_2 directly with MC)
- And more!!!

Summary and Prospects

- First serious feasibility study of the true NLO exclusive MC parton shower is under construction, well advanced...
- What next? Workplan well defined:
- Short range aim: Complete non-singlet. ($\sim C_F C_A$ at work.)
- Middle range aim: Complete singlet.
- Speed up the MC weight calculation.
- Better documentation needed on what was done.
- NLO MC for W/Z production for LHC, including SANC electroweak library.
- NLO MC for DIS@HERA and an example of BSM processes at LHC

1970 | 1980 | 1990 | 2000 | 2010

LO

- Moments OPE: (74) QCD: Georgi+Politzer
- Diagramatic: (72) QED: Gribov+Lipatov; (77) Altarelli+Parisi
- Monte Carlo: (85) Sjostrand; (88) Marchesini,Webber — 10 years

NLO

- Moments OPE: (78) Floratos+Ross+Sachrajda
- Diagramatic: (81) Curci+Furmanski+Petronzio
- Monte Carlo: 27 years later → (08) Jadach Skrzypek

WE ARE HERE!!!

NNLO

- Moments: (03) Moch+Verm.+Vogt
- Diagramatic: (03) Moch+Verm.+Vogt
- Monte Carlo: (15) ???

Excellent weight distribution!