CKKW merging at NLO

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Introduction

- Starting point is CKKW(-L)
- We want to add events generated with NLO ME's
- The corresponding terms must be subtracted from the standard CKKW events.

Standard CKKW(-L) merging

Start out with events generated according to (inclusive) tree-level ME's

$$d\sigma_{+n}^{\text{tree}} = C_n(\Omega_n)\alpha_s^n(\mu_R)d\Omega_n$$

where $\Omega_n = (q_1, \ldots, q_m; p_1, \ldots, p_n)$ is the phase space for an m-particle Born process with n extra jets ($0 \leq n \leq N$).

The divergencies are regularized by a jet-like phase space cut, $k_{\perp MS}$.

Here we will assume that the parton shower is ordered in ρ, which is the same variable as $k_{\perp MS}$.

In this way we don't have to worry about vetoed/trunkated showers. We can simply add a shower below $k_{\perp MS}$ (except for the highest jet multiplicity).

CKKW-L is designed to work with mixed ordering/merging scales, but the notation becomes cumbersome.

(If you're interested, we can return to that in the discussion)

The basic idea

- Above $k_{\perp MS}$, the phase space should be populated by jets/partons given by the tree-level ME.
- Below $k_{\perp MS}$, we have the parton shower
- For the highest multiplicity ($n = N$), PS jets are allowed above $k_{\perp MS}$, as long as they are below the ME-jets.
- The ME states must be made exclusive by adding appropriate Sudakov Form factors.

First we do a mapping to the patron shower phase space

$$\Omega_n \mapsto \Omega_n^{\mathrm{PS}} = (q_1, \ldots, q_m; \rho_1, x_1 \ldots, \rho_n, x_n)$$

I.e. a shower history is constructed, with emissions (ρ_i, x_i).

Then we reweight

$$d\sigma_{+n}^{\mathrm{CKKW}} = C_n(\Omega_n)\alpha_{\mathrm{s}}^n(\mu_R)\prod_{i=1}^{n}\frac{\alpha_{\mathrm{s}}^{\mathrm{PS}}(\rho_i)}{\alpha_{\mathrm{s}}(\mu_R)}\prod_{i=0}^{n}\Delta_{S_i}(\rho_i, \rho_{i+1})d\Omega_n$$

with $\rho_{n+1} = k_{\perp MS}$ and ρ_0 is the maximum scale for the shower if started from the reconstructed Born-state.

$\alpha_s^{PS}(\rho_i)$ is the coupling the shower would have used in the corresponding emissions.

$\Delta_{S_i}(\rho_i, \rho_{i+1})$ is the no-emission probability in the shower from the reconstructed state S_i between the scales ρ_i and ρ_{i+1}. This is by definition the Sudakov form factor used in the shower.

If $n = N$ the last Sudakov,$\Delta_{S_N}(\rho_N, \rho_{N+1})$, is omitted and the shower is added below ρ_N, rather than $k_{\perp MS}$.

CKKW vs. CKKW-L

- Sudakov form factors are calculated analytically in CKKW. In -L they are calculated by the shower itself (including all funny kinematic effects).
- CKKW only reconstructs scales with a jet algorithm. In -L a full parton shower history is reconstructed.
- CKKW has trouble when the PS ordering is not the same as the jet measure used for $k_{\perp MS}$.
- CKKW-L needs a PS with on-shell explicit intermediate states.

Adding one-loop ME's

Now we want to look at n-jet events generatet to one-loop order

$$d\sigma_{+n}^{\text{loop}} = C_n(\Omega_n)\alpha_s^n(\mu_R)\left[1 + C_{n,1}(\Omega_n)\alpha_s(\mu_R)\right] d\Omega_n$$

Where $C_{n,1}$ is obtained from the virtual and real corrections integrated up to the merging scale $k_{\perp MS}$.

$\sigma_{+n}^{\text{loop}}$ should give the NLO-approximation to the exclusive cross section for n extra jets above $k_{\perp MS}$.

(physical quantity - no subtraction-scheme dependence)

- $\sigma_{+n}^{\mathrm{CKKW}}$ gives exclusive n-jet states approximately correct (as far as the PS is correct) to all orders in α_{s}.
- $\sigma_{+n}^{\mathrm{loop}}$ gives exclusive n-jet states exactly correct to the leading two orders in α_{s}.

In both cases we can add a shower below $k_{\perp MS}$.

The strategy will be to add events from both, but remove the LO and NLO terms from the CKKW.

We want to use $\sigma_{+n}^{\mathrm{CKKW}}$ with the first two orders in α_{s} subtracted. So we expand the CKKW weight (including a K-factor):

$$K \prod_{i=1}^{n} \frac{\alpha_{\mathrm{s}}^{\mathrm{PS}}(\rho_i)}{\alpha_{\mathrm{s}}(\mu_R)} \prod_{i=0}^{n} \Delta_{S_i}(\rho_i, \rho_{i+1}) = 1 + \alpha_{\mathrm{s}}(\mu_R) B^{\mathrm{PS}} + \mathcal{O}\left(\alpha_{\mathrm{s}}^2(\mu_R)\right)$$

So we reweight the tree-level events by a modified CKKW weight:

$$\begin{aligned} d\sigma_{+n}^{\mathrm{PScorr}} &= C_n(\Omega_n)\alpha_{\mathrm{s}}^n(\mu_R) d\Omega_n \\ &\times \left[K \prod_{i=1}^{n} \frac{\alpha_{\mathrm{s}}^{\mathrm{PS}}(\rho_i)}{\alpha_{\mathrm{s}}(\mu_R)} \prod_{i=0}^{n} \Delta_{S_i}(\rho_i, \rho_{i+1}) - 1 - \alpha_{\mathrm{s}}(\mu_R) B^{\mathrm{PS}} \right] \end{aligned}$$

$$K = 1 + k_1 \alpha_s(\mu_R)$$

$$\frac{\alpha_s^{PS}(\rho)}{\alpha_s(\mu_R)} = 1 - \frac{\log \frac{b\rho}{\mu_R}}{\alpha_0} \alpha_s(\mu_R) + \mathcal{O}\left(\alpha_s^2(\mu_R)\right)$$

$$\begin{aligned} \Delta_{S_i}(\rho_i, \rho_{i+1}) &= \exp\left(-\int_{\rho_{i+1}}^{\rho_i} d\rho \alpha_s(\rho) \Gamma_{S_i}(\rho)\right) \\ &= 1 - \alpha_s(\mu_R) \int_{\rho_{i+1}}^{\rho_i} d\rho \Gamma_{S_i}(\rho) + \mathcal{O}\left(\alpha_s^2(\mu_R)\right) \end{aligned}$$

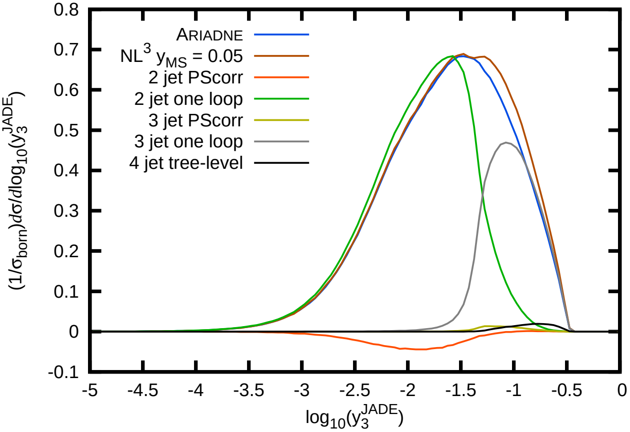
- $\sigma_{+n}^{\text{loop}} + \sigma_{+n}^{\text{PScorr}}$ gives exclusive n-jet states exactly correct to the first two orders in α_{s} and approximately correct to all other orders in α_{s}.

All weights are positive as long as

- $k_{\perp MS}$ is large enough for the NLO ME to be positive
- $\mu_R < b\rho_i$

The net result is events generated so that all n-jet observables (above the merging scale and $n < N$) will be correct to NLO with a PS-simulated resummation. And N-jet observables will correct to LO+PSresum.

This works for e^+e^-:



(Note, this is without the extra α_s-scale, otherwise ARIADNE is almost identical to NLO.)

Outlook

- CKKW-L-like NLO+PS merging works.
- So far only for e^+e^-.
- Should be trivial to apply to standard CKKW as well.
- Works for high jet multiplicities
  (cf. MC@NLO and POWHEG).
- NNLO matching is (in principle) possible.

- Extending to *pp* collisions (eg. W+jets) should be possible, but not necessarily trivial.

  We need to worry about factorization scheme dependencies. $\sigma_{+n}^{\mathrm{loop}}$ contains PDFs which means that it is not just $\alpha_{\mathrm{s}}(\mu_R)^n$ and $\alpha_{\mathrm{s}}(\mu_R)^{n+1}$ terms, but a full resummation.

The CKKW-reweighting also changes.

The no-emission probabilities are no longer simple Sudakov form factors, but contain PDF ratios. Difficult to disentangle how these overlap with the PDFs in the NLO ME.

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The CKKW-L reweighting becomes

$$K \prod_{i=1}^{n} \frac{\alpha_s^{\mathrm{PS}}(\rho_i)}{\alpha_s(\mu_R)} \prod_{i=0}^{n} \frac{f_i(x_i, \rho_i)}{f_i(x_i, \rho_{i+1})} \Pi_{S_i}(x, \rho_i, \rho_{i+1})$$

(assuming $\mu_F = k_{\perp MS} = \rho_{n+1}$)

$$\begin{aligned}\Pi_{S_i}(x, \rho_i, \rho_{i+1}) &= \Delta_{S_i}(\rho_i, \rho_{i+1}) \\ &\times \exp\left(- \int_{\rho_i}^{\rho_{i+1}} \frac{d\rho}{\rho} \int \frac{dz}{z} \frac{\alpha_s(\rho)}{2\pi} \sum_a P_{ai}(z) \frac{f_a(\frac{x}{z}, \rho)}{f_i(x, \rho)} \right)\end{aligned}$$

From the pink bible we have

$$\frac{f_b(x,t_0)}{f_b(x,t_1)} \exp\left(-\int_{t_1}^{t_0} dt \sum_a \int_{S(t)} \frac{dz}{z} \frac{f_a(\frac{x}{z},t)}{f_b(x,t)} P_{a\to bc}(z)\right) =$$

$$= \exp\left(-\int_{t_1}^{t_0} dt \frac{\alpha_S(t)}{2\pi} \sum_d \int_{S'(t)} dz P_{b\to de}(z)\right)$$

(which is used by Frank in CKKW)