NLL Jet Rates

- In k_T algorithm (only) NLL jet fractions computable to any order ($a = \alpha_S/\pi$, $L = \ln(1/y_{\rm cut})$):

$$
\begin{aligned}
R_2 &= 1 + a(R_{21}L + R_{22}L^2) + a^2(R_{23}L^3 + R_{24}L^4) + a^3(R_{25}L^5 + R_{26}L^6) + \cdots \\
R_{21} &= 3C_F/2 \\
R_{22} &= -C_F/2 \\
R_{23} &= -3C_F^2/4 - 11C_FC_A/36 + C_FN_f/18 \\
R_{24} &= C_F^2/8 \\
R_{25} &= 3C_F^3/16 + 11C_F^2C_A/72 - C_F^2N_f/36 \\
R_{26} &= -C_F^3/48 \\
R_3 &= a(R_{31}L + R_{32}L^2) + a^2(R_{33}L^3 + R_{34}L^4) + a^3(R_{35}L^5 + R_{36}L^6) + \cdots \\
R_{31} &= -3C_F/2 \\
R_{32} &= C_F/2 \\
R_{33} &= 3C_F^2/2 + 7C_FC_A/12 - C_FN_f/12 \\
R_{34} &= -C_F^2/4 - C_FC_A/48 \\
R_{35} &= -9C_F^3/16 - 137C_F^2C_A/288 - 7C_A^2C_F/160 + 5C_F^2N_f/72 + C_FC_AN_f/160 \\
R_{36} &= C_F^3/16 + C_F^2C_A/96 + C_FC_A^2/960 \\
R_4 &= a^2(R_{43}L^3 + R_{44}L^4) + a^3(R_{45}L^5 + R_{46}L^6) + \cdots \\
R_{43} &= -3C_F^2/4 - 5C_FC_A/18 + C_FN_f/36 \\
R_{44} &= C_F^2/8 + C_FC_A/48 \\
R_{45} &= 9C_F^3/16 + 71C_F^2C_A/144 + 217C_FC_A^2/2880 - 41C_F^2N_f/720 - C_FC_AN_f/120 \\
R_{46} &= -C_F^3/16 - C_F^2C_A/48 - 7C_FC_A^2/2880 \\
R_5 &= a^3(R_{55}L^5 + R_{56}L^6) + \cdots \\
R_{55} &= -3C_F^3/16 - 49C_F^2C_A/288 - 91C_FC_A^2/2880 + 11C_F^2N_f/720 + C_FC_AN_f/480 \\
R_{56} &= C_F^3/48 + C_F^2C_A/96 + C_FC_A^2/720
\end{aligned}
$$

- Resummed NLL jet fractions at c.m. energy Q and k_T-resolution $y_1 = Q_1^2/Q^2$:

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$$R_2(Q_1, Q) = [\Delta_q(Q_1, Q)]^2$$

$$R_3(Q_1, Q) = 2\,[\Delta_q(Q_1, Q)]^2 \int_{Q_1}^{Q} dq\, \Gamma_q(q, Q)\Delta_g(Q_1, q)$$

$$R_4(Q_1, Q) = 2\,[\Delta_q(Q_1, Q)]^2 \Bigg\{ \int_{Q_1}^{Q} dq\, \Gamma_q(q, Q)\Delta_g(Q_1, q) \int_{Q_1}^{Q} dq'\, \Gamma_q(q', Q)\Delta_g(Q_1, q')$$
$$+ \int_{Q_1}^{Q} dq\, \Gamma_q(q, Q)\Delta_g(Q_1, q) \int_{Q_1}^{q} dq'\, \Gamma_g(q', q)\Delta_g(Q_1, q')$$
$$+ \int_{Q_1}^{Q} dq\, \Gamma_q(q, Q)\Delta_g(Q_1, q) \int_{Q_1}^{q} dq'\, \Gamma_f(q')\Delta_f(Q_1, q') \Bigg\}$$

etc., where

$$\Gamma_q(q, Q) = \frac{2C_F}{\pi}\frac{\alpha_S(q)}{q}\left(\ln\frac{Q}{q} - \frac{3}{4}\right)$$

$$\Gamma_g(q, Q) = \frac{2C_A}{\pi}\frac{\alpha_S(q)}{q}\left(\ln\frac{Q}{q} - \frac{11}{12}\right)$$

$$\Gamma_f(q) = \frac{N_f}{3\pi}\frac{\alpha_S(q)}{q}$$

- $\Delta_{q,g}$ are quark and gluon Sudakov form factors:

$$\begin{aligned}\Delta_q(Q_1, Q) &= \exp\left(-\int_{Q_1}^{Q} dq\,\Gamma_q(q, Q)\right)\\ \Delta_g(Q_1, Q) &= \exp\left(-\int_{Q_1}^{Q} dq\,\left[\Gamma_q(q, Q) + \Gamma_f(q)\right]\right)\end{aligned}$$

  with

$$\Delta_f(Q_1, Q) = \left[\Delta_q(Q_1, Q)\right]^2 / \Delta_g(Q_1, Q)\ .$$

- Sudakov form factor $\Delta_i(Q_1, Q)$ represents probability for parton of type i to evolve from scale Q to scale Q_1 without any branching (resolvable at scale Q_1). Thus R_2 is probability that produced quark and antiquark both evolve without branching.

- More generally, probability to evolve from scale Q to scale q without branching (resolvable at scale Q_1) is

$$\frac{\Delta_i(Q_1, Q)}{\Delta_i(Q_1, q)}$$