Uncertainties about Uncertainties

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CMS and CDF

PDF4LHC

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Motivation

PDF's needed for LHC predictions

Could be dominant uncertainty for some searches for New Physics (e.g.quark substructure)

i.e. Could mistakenly claim **discovery** if estimated uncertainty too **small**

Could unfortunately **miss** discovery if estimated uncertainty too **large**

So want uncertainties reliably estimated

Standard method for params and uncertainties

Fit theory to data:

Construct $\chi^2(\mu_1, \mu_2, \mu_3\ldots)$ and minimise wrt μ_i

Check that χ^2_{min} is reasonable

Expected $\chi^2 = \nu \pm \sqrt{2\nu}$

ν = NDF ~ d – f

d = # data points

f = # free params

σ_i from $\partial^2\chi^2/\partial\mu_i\partial\mu_j = 2$ * inverse error matrix

If μ_i uncertainties uncorrelated, $\partial^2\chi^2/\partial\mu_i^2 = 2/\sigma_i^2$

OR: For parabolic $\chi^2(\mu_i)$, $\chi^2(\mu_i \pm \sigma_i) = \chi^2(\mu_i) + 1$

{This works for correlated μ, provided χ^2 is profiled wrt other μ}

Rule gives {error on mean} = {individual error} / $\sqrt{n}$

What about 10±1 and 15±1 ?

N.B. NO MENTION OF $\chi^2(\mu_i \pm \sigma_i) = \chi^2(\mu_i) + \sqrt{2\nu}$

Parton fits, and toy

Fits to lots of data (~37 sets) with pdf's for u d s u-bar d-bar (s-bar) and g, each parametrised as $f(x,Q_0^2)$, and then evolved to $f(x,Q^2)$. ~25 params, and 3000 data points.

[Is parametrisation reasonable? Maybe non-param method?]

Overall fit to data has χ^2/ν reasonable, but errors from $\Delta\chi^2 = 1$ are "too small" {MSTW, CTEQ}

Hard to contemplate, so consider simple toy

Fit theory ($y = a + b*x$) to sets of data

a and b correlated: cov(a,b) ~<x>

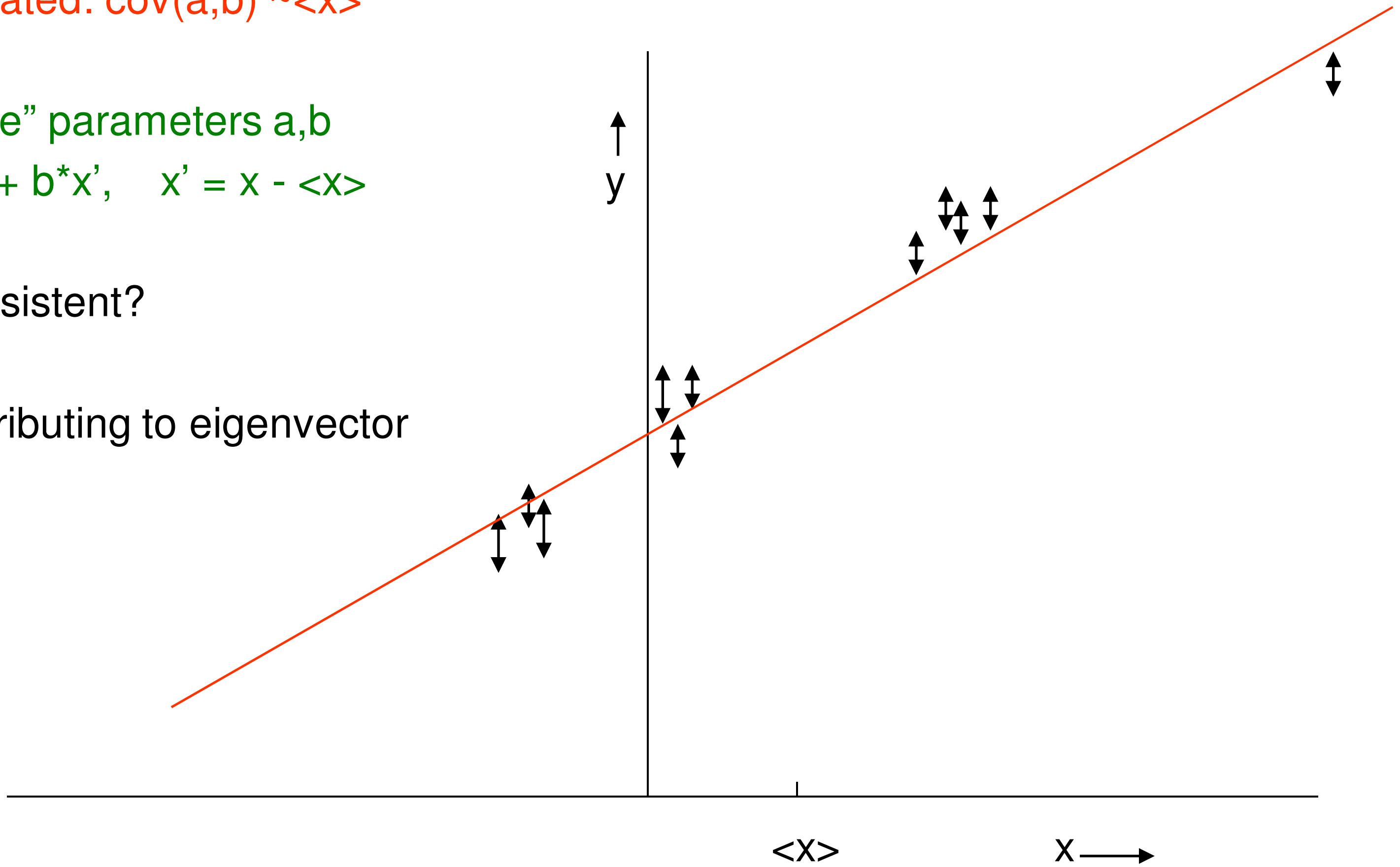
Decorrelate:

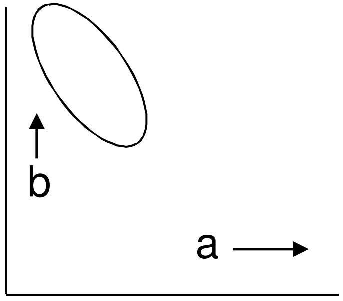
i) "Rotate" parameters a,b

ii) $y = a' + b*x'$, $x' = x - \langle x \rangle$

New data consistent?

Data not contributing to eigenvector





PDF groups use eigenvectors

Trying to reconcile χ^2/ν and $\Delta\chi^2$

Overall rescaling of errors doesn't work

Too much data reduces stat errors, hit some systematic

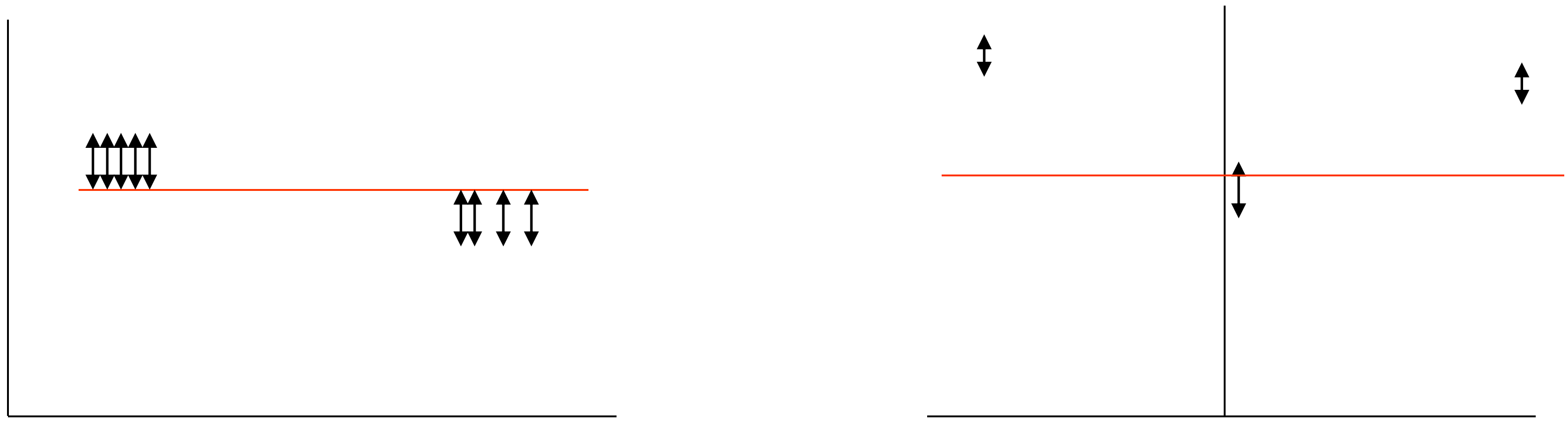
All data points off by one sigma (and correlations incorrectly dealt with)

Most of data OK, but irrelevant for params.

Few relevant data inconsistent

e.g. $\chi^2/\nu = 2850/2900$ and $200/100$ → $3050/3000$

OK and inconsistent → OK



"Errors are too small"

Comes from:

i) Using different data sets, cuts, parton parametrisations

ii) Ideas about systematics, including theory

iii) CTEQ/MSTW differences

iv) Gut feeling

$\Delta\chi^2 = 50$ (MRST), 100 (CTEQ) for **90%** intervals. Tolerance

Groups fitting less data use $\Delta\chi^2 = 1$ for **68%** intervals

Probably inflated parton errors are of ~ suitable magnitude.

But can we do better?

Some Sources of Uncertainty

It is vital to consider theoretical/assumption-dependent uncertainties:

* Methods of determining "best fit" and uncertainties.
* Underlying assumptions in procedure, e.g. parameterisations and data used.
* Treatment of heavy flavours.
* PDF and α_s correlations.
* QED and Weak (comparable to NNLO ?) Sometime enhancements.
* Standard higher orders (NNLO)
* Resummations, e.g. small x or large x
* Low Q^2 (higher twist), saturation

Lead to differences in current partons, and to corrections in predicted cross-sections.

R.T, Wednesday

MSTW

Have gone away from tolerance on $\Delta\chi^2$ (varied with eigenvector)

1) Fix a few params to avoid degeneracy (~20 left → eigenfunctions)
2) χ^2 calculated for each data set (renormalised to median), as eigenvalue is varied → range of acceptable values (68% or 90% χ^2 quantile)
3) Overall range: Smallest upper limit, and largest lower limit.
4) Quartic term in χ^2
5) Combination of asymmetric errors (combine in quadrature) for predictions

MSTW

Have gone away from tolerance on $\Delta\chi^2$ (varied with eigenvector)

1) Fix a few params to avoid degeneracy[a] (~20 left → eigenfunctions)
2) χ^2 calculated for each data set (renormalised to median[b]), as eigenvalue is varied → range of acceptable values (68% or 90% χ^2 quantile[c])
3) Overall range: Smallest upper limit, and largest lower limit[d].
4) Quartic term in χ^2
5) Combination of asymmetric errors (combine in quadrature) for predictions

a) Explanation of some discrepancies with CTEQ?
b) How many degrees of freedom?
c) Not standard rule for parameter uncertainty
   Different tolerances per eigenvector (understandable?)
d) Curious way of combining data – new data may not give improvement

CTEQ

$\sqrt{2\nu}$ for χ^2

Weight some data sets in χ^2

Quartic term in χ^2

Assess whether data is useful by whether it changes central value. (Does 10 ± 1 improve on 10 ± 10?)

Envelope of fits to assess errors

Combination of asymmetric errors (combine in quadrature) for predictions

Jon Pumplin recently looking at consistency of individual data set with rest of data.

(Uses further diagonalisation of error matrix. Loses ability to compare different data sets contribution to eigenvalues)

Finds old muon experiments discrepant.

Neural Nets

Varied input data:

Not specific to NN approach

Bootstrap?

Sets of PDF's → Central values (mean or median?)

Error on mean

Add new data: Error can increase

Negative PDF's

Future

Use combined HERA data, when available.

More firmly based method for uncertainties.

(Use jackknife to determine uncertainties?)

Is agreement among different groups (and experiments) OK?

Possible 5σ range for uncertainties.

PARADOX

Histogram with 100 bins

Fit 1 parameter

S_{min}: χ^2 with NDF = 99 (Expected $\chi^2 = 99 \pm 14$)

For our data, $S_{min}(p_0) = 90$

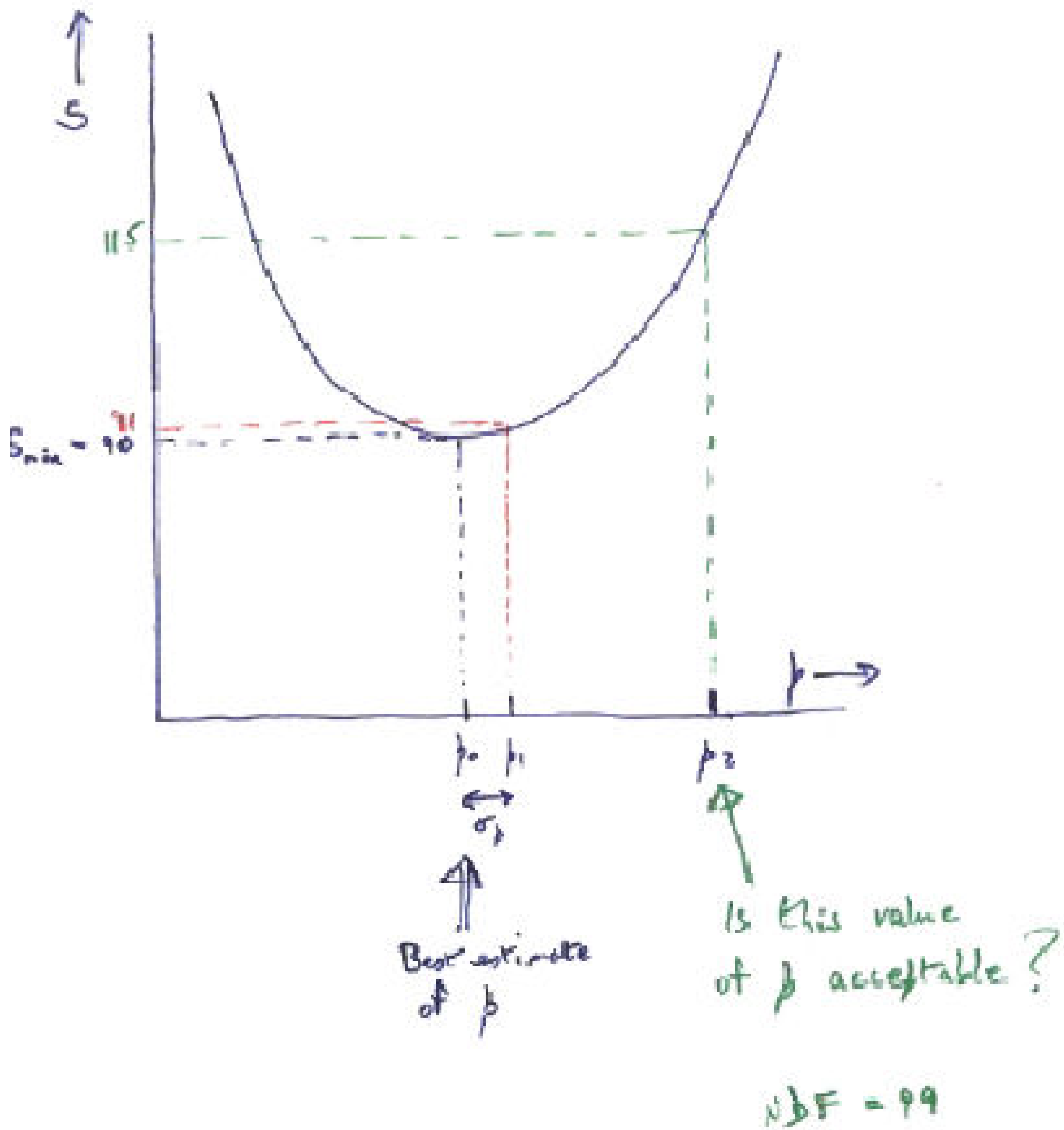
Is p_1 acceptable if $S(p_1) = 115$?

1) YES. Very acceptable χ^2 probability

2) NO. σ_p from $S(p_0 + \sigma_p) = S_{min} + 1 = 91$

But $S(p_1) - S(p_0) = 25$

So p_1 is 5σ away from best value


S
115
91
Smin = 90
p
p0 p1
p2
σp
Best estimate of p
Is this value of p acceptable?
NDF = 99