Heavy Flavours and PDF's

P. Nason
INFN
Milano-Bicocca

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History

How do we treat a heavy flavour (mass $m_h \gg \Lambda_{\rm QCD}$) in QCD processes?

In standard $\overline{\rm MS}$ scheme, heavy flavour effects persist even for $Q \ll m_h$
(n_f, not $n_l = n_f - 1$, appear in the running coupling ...).
It would be cumbersome to worry about top when doing DIS at 10 GeV^2.

Use Decoupling renormalization scheme with n_l light flavours (all but h)
(Collins, Wilczek, Zee 1978;). CWZ prescription:

- the $\overline{\rm MS}$ scheme for light flavours
- a zero momentum subtraction for heavy flavour graphs

heavy flavour graphs: graphs that include heavy flavour line, or
counterterms to heavy flavour graphs.

Advantages: $m_h \to \infty$ limit easily treated (forget the heavy quark!)
(i.e., if the scale of the process is $\ll m_h$, forget the heavy quark)

How decoupling works

If the external momenta $p \ll m_h$:

- Convergent graphs with heavy lines are dominated by internal momenta $k \approx p$, so the heavy lines yield $k/m_h \approx p/m_h$ suppression factors.
- Divergent graphs with heavy lines are dominated by momenta $k \gg m_h$; subtracting them at zero momentum:

$$\Sigma(k, p, m_k) - \Sigma(k, 0, m_k) \approx \mathcal{O}(p/m_k)$$

Evolution of α_s and parton densities

- Decoupling scheme: as in $\overline{\text{MS}}$ scheme with n_l flavours
- Standard $\overline{\text{MS}}$: evolution with $n_f = n_l + 1$ flavours

In both cases m_h does not enter in the evolution.

Relations among the schemes (suffix d fro decoupling scheme)

$$
\begin{aligned}
\alpha(\mu) &= \alpha_d(\mu) + c_1(\mu/m_h)\,\alpha_d^2(\mu) + c_2(\mu/m_h)\,\alpha_d^3(\mu) + \ldots \\
f_i(\mu) &= \sum_j A_{ij}(\mu/m_h) \otimes f_j^d(\mu) \\
A_{ij}(\mu/m_h) &= \delta_{ij} + A_{ij}^{(1)}(\mu/m_h)\alpha_d(\mu) + \underbrace{A_{ij}^{(2)}(\mu/m_h)\alpha_d^2(\mu)}_{\text{(Buza etal, 1996)}} + \ldots
\end{aligned}
$$

VFNS (Collins, Tung, 1986;)
Use CWZ scheme treating as heavy all quarks heavier than μ.
VFNS has variable flavour number depending upon the scale.

Easy applications: μ not much larger than m_h

Use the decoupling scheme! (jargon: Massive scheme)

Accuracy: (if Born term is $\mathcal{O}(\alpha_s^b)$) an $\mathcal{O}(\alpha_s^{b+n})$ calculation has reminder of $\mathcal{O}(\alpha_s^{b+n+1})$; however, for $\mu \gg m_h$, terms of order $(\alpha_s L)^n$ (with $L = \log\frac{\mu}{m_h}$) arise at all orders, and the remainder is $\mathcal{O}(\alpha_s^{b+n+1} L^n)$ (for $\alpha_s L \approx 1$, $\mathcal{O}(\alpha_s^b)$)

In some cases (F_2, examples in Maltoni's monday talk) powers of L also arise in the Born term.

Easy applications: $\mu \gg m_h$

$\overline{\text{MS}}$ bar, neglecting m_h (jargon: Massless scheme)

If we do not ask explicitly for the presence or absence of h in the final state (i.e. for INCLUSIVE cross sections) we can treat all $n_f = n_l + 1$ partons as massless, throwing away effects suppressed by powers of m_h/μ.

Cross section formulae as in massless n_f flavour theory.

Accuracy: $\mathcal{O}(\alpha_s^{b+n})$ calculation has reminder of $\mathcal{O}(\alpha_s^{b+n+1})$

all terms of order $(\alpha_s \log\frac{\mu}{m_h})^k$ are resummed to all orders in k, for any n

However: powers suppressed effects (by powers of m_h/μ) are not included

Accuracy: (k and l stand for ANY integer from 0 to ∞)

$$\sigma = \sum_{j=1}^{n_l} f_j^{(n_l)}(x,\mu)\, \hat{\sigma}_j^{n_l}(px,\mu,m_h,....) \text{ (Massive scheme)}$$

Born	NLO	NNLO	...
$\alpha_s^b \times (\alpha_s \log \mu/\Lambda)^k$	$\alpha_s^{b+1} \times (\alpha_s \log \mu/\Lambda)^k$	$\alpha_s^{b+2} \times (\alpha_s \log \mu/\Lambda)^k$	

$$\sigma = \sum_{j=1}^{n_f} f_j^{(n_f)}(x,\mu)\, \hat{\sigma}_j^{(n_f)}(px,\mu,...) \text{ (Massless scheme)}$$

Born	NLO	NNLO	...
$\alpha_s^b \times (\alpha_s \log \mu/\Lambda)^k \times (\alpha_s \log \mu/m_h)^l + \mathcal{O}(m_h/\mu)$	$\alpha_s^{b+1} \times (\alpha_s \log \mu/\Lambda)^k \times (\alpha_s \log \mu/m_h)^l + \mathcal{O}(m_h/\mu)$	$\alpha_s^{b+2} \times (\alpha_s \log \mu/\Lambda)^k \times (\alpha_s \log \mu/m_h)^l + \mathcal{O}(m_h/\mu)$	

Everybody agrees on massive and massless schemes; no controversies there.

Phenomenological applications

The decoupling scheme has been used in all calculations of heavy flavour production processes involving incoming hadrons:

- Hadroproduction (Dawson, Ellis, P.N., 1988; Beenakker etal, 1991)
- Photoproduction (Ellis, P.N. 1989; Smith, Van Neerven, 1992)
- Electroproduction(Laenen, Riemersma, Smith, Van Neerven, 1993)

All these calculations include consistently mass effects.
The massless scheme has been used in high p_T hadro and photoproduction of charm and bottom (Cacciari and Greco, 1994)

Gluck,Reya,Vogt,(1992): straightforward application of the decoupling scheme in DIS fits. They work within a 3-flavour scheme, and compute heavy flavour effect from the $\gamma^\star g \to h\bar{h}$ process.

Matched calculations

Can we get the best of both worlds?
Mass effects present in the decoupling scheme,
plus log resummation present in massless scheme?

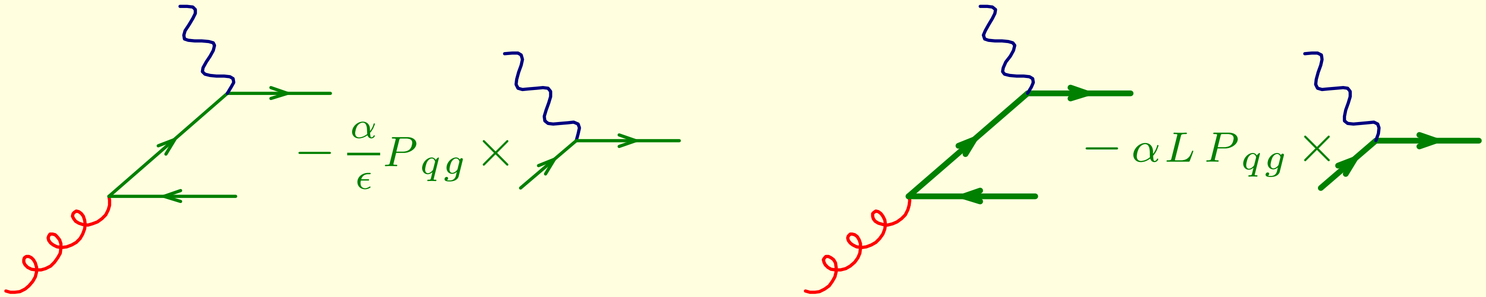
Several proposals have appeared;

- ACOT scheme (Aivazis, Collins, Olness, Tung, 1988,1994):
  use the $\overline{\rm MS}$ scheme above m_h without setting m_h to zero.
- (Thorne and Roberts, 1998,) Modify massless scheme coeff. function to achieve continuity with structure functions from massive calculation
- FONLL scheme (Cacciari, Greco, P.N., 1998):
  use the massless scheme, replace terms that are known in the massive scheme with the exact massive result.

ACOT (Aivazis, Collins, Olness, Tung, 1988,1994)

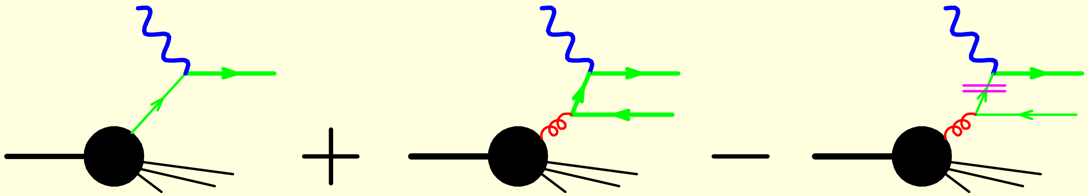
Use the $\overline{\text{MS}}$ scheme above m_h without setting m_h to zero.

If $m_h = 0$: $1/\epsilon$ poles to subtract;   if $m_h > 0$, $L = \log Q/m_h$ terms to subtract



Formal basis: factorization with massive quarks (Collins,1998)

ACOT At NLO: PDF subtraction (3rd graph) depends upon 1st graph.



How to include mass in the 1st graph is not fully specified ...

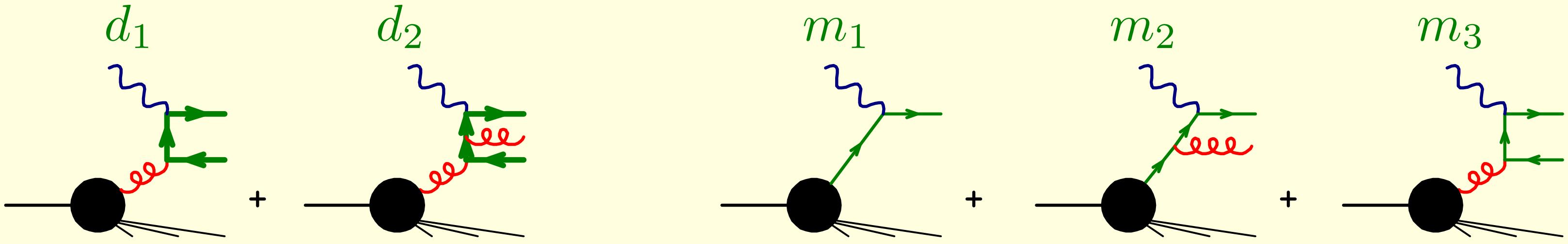
3rd graph takes away from 2nd graph what was already included in 1st.

In spite of ACOT formulation in 1994, up to CTEQ 6.1, the massless approximation has been used in the computation of DIS structure functions.

From CTEQ6.5 (end of 2006) the ACOT scheme has been implemented.

TR SCHEME (Thorne and Roberts, 1998)

Basic idea: a structure function computed in the decoupling scheme does not match a structure function computed in the massless scheme when $Q \approx m_h$.
Correct the massless scheme so that they match.

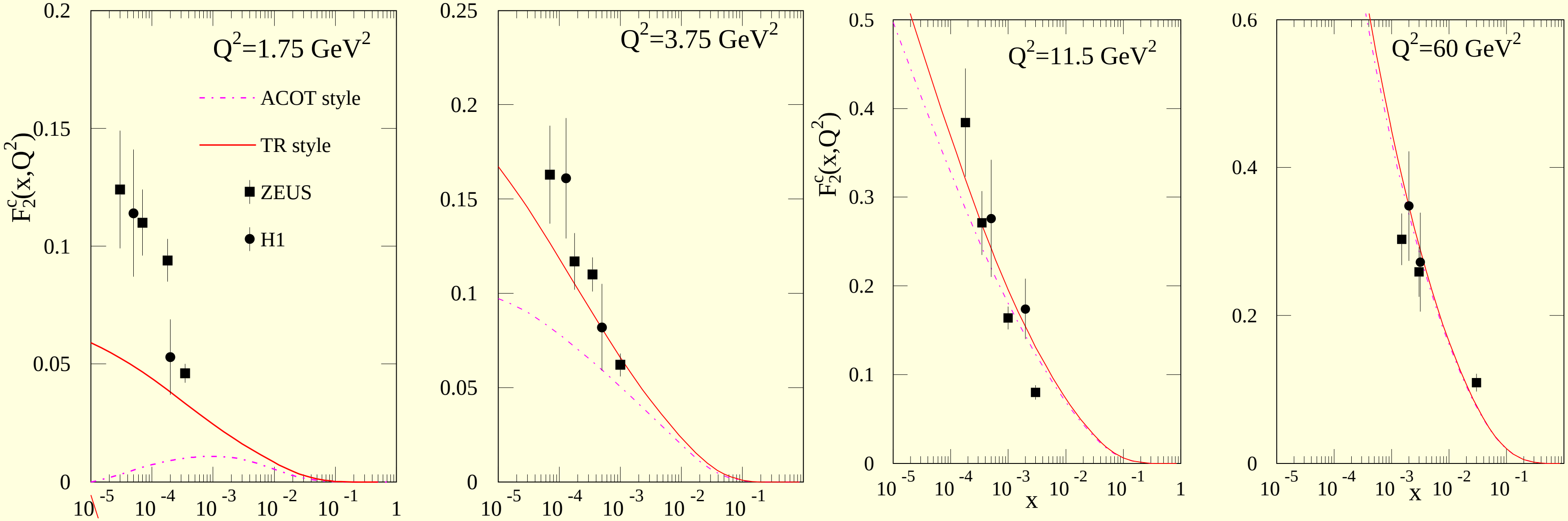


$Q \approx m_h$: $\mathcal{O}(\alpha_s)$ $\mathcal{O}(\alpha_s^2)$ $\mathcal{O}(\alpha_s)$ $\mathcal{O}(\alpha_s^2)$ $\mathcal{O}(\alpha_s)$

(When counting the order for $Q \approx m_h$, remember that $f_h \approx \alpha_s$)
Since F_2 is $\mathcal{O}(1)$, matching at NLO can be interpreted as: $\mathcal{O}(\alpha_s)$ terms only.
This approach essentially recovers ACOT

TR try to match at NLO, including $\mathcal{O}(\alpha_s^2)$ terms by imposing continuity at $Q = M$ at $\mathcal{O}(\alpha_s^2)$, up to the derivative with respect to Q.
Since m_1 and m_2 vanish at $Q = m$, this implies that d_2 is added to their result. They add $d_2(Q = m_h)$, to avoid $(\alpha_s L)^2$ terms arising for $Q \gg m_h$.

Thorne and Tung (2009) now seem to agree that this constant term summarizes the difference in their approaches (is it ALL the difference?)
It is beyond the declared accuracy of ACOT (Since it is $\mathcal{O}(\alpha_s^2)$).
Notice: it is frozen at $Q = m_h$, so $(\alpha L)^2 \approx 1$ terms do not arise at large Q^2.
Thorne (2006) shows that it is relevant at low x and Q^2.



Thorne and Tung (2009) paper has represented a considerable step forward in understanding similarities and differences between the two approaches.

From Collins (1998) factorization paper:

> Roberts and Thorne [10, 11] appear to have a scheme similar to the one in the present paper. But they do not present complete proofs, and they make a number of incorrect or misleading statements. For example, they state that "the detailed construction of the coefficient functions ...is extremely difficult if not impossible." As regards the general formalism, the construction is exactly as difficult as in the light-quark case. The only computational complication is that in a calculation of the coefficient functions, heavy quark masses must be retained. All the necessary Feynman-graph calculations for computing the coefficient functions at order α_s^2 have been done in Refs. [8], and all that remains is to organize them to form the coefficient function by use of the recursion relation Eq. (65). This recursion relation is of the same form as the one used to obtain the coefficient functions in the massless case.

so, some similarities were recognized, but differences were not fully understood.

The core of the difference:

In ACOT, the massive result is included up to $\mathcal{O}(\alpha_s)$
In TR, it is included up to $\mathcal{O}(\alpha_s^2)$.

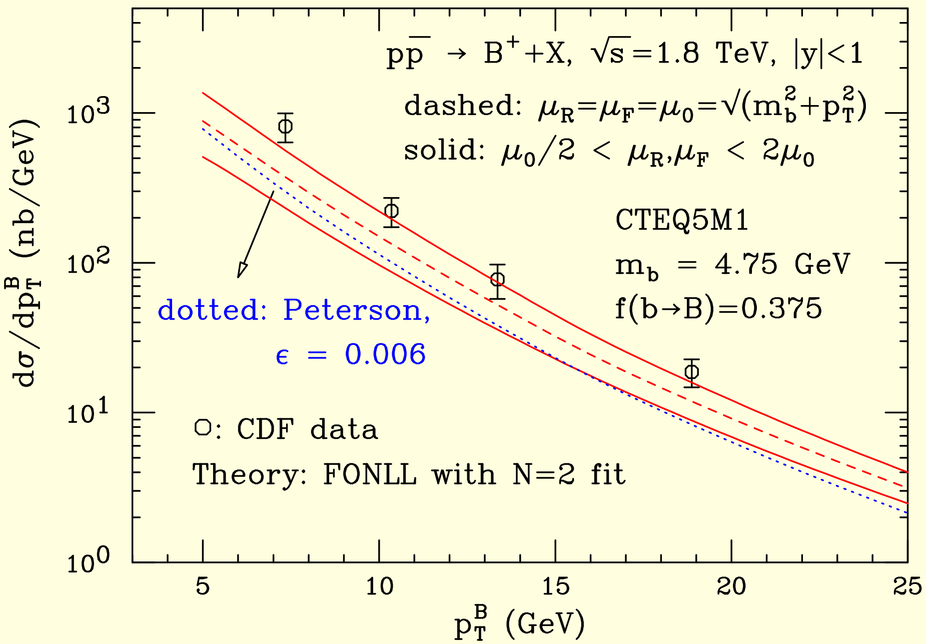
The same mismatch is present at NNLO, where an estimate of the $\mathcal{O}(\alpha_s^3)$ massive result is needed in TR (Thorne, 2006).

Within ACOT, at NLO, only the $\mathcal{O}(\alpha_s)$ massive result enters naturally within the heavy flavour factorization scheme propose by Collins.

The question however remains:
The $\mathcal{O}(\alpha_s^2)$ massive result is available, it requires NLO α_s and pdf's.
Why should it not be used in an NLO fit?

FONLL (Cacciari, Greco, P.N., 1998)

Totally independent approach introduced in the context of heavy flavour production at high p_T, in order to address the discrepancy between theory and Tevatron data in b production. Besides the pdf, it also deals with b fragmentation.



It was used to match the NLO heavy flavour production calculation of Ellis, Dawson and P.N. (massive scheme) with that of Cacciari and Greco (massless scheme). The method is totally general. It has been applied to heavy flavour production in e^+e^- annihilation, but never to DIS.

FONLL in few words

A cross section in the decoupling scheme can be seen as a fixed order power expansion in α_s with mass dependent coefficients. The coefficients have logarithmic behaviour at large scale.
A cross section in the massless scheme can be seen as a power series in α_s with coefficients that are polynomials in L. All large logs are resummed.

So: add them up, deleting from the second the terms of the same order in α_s present in the first.

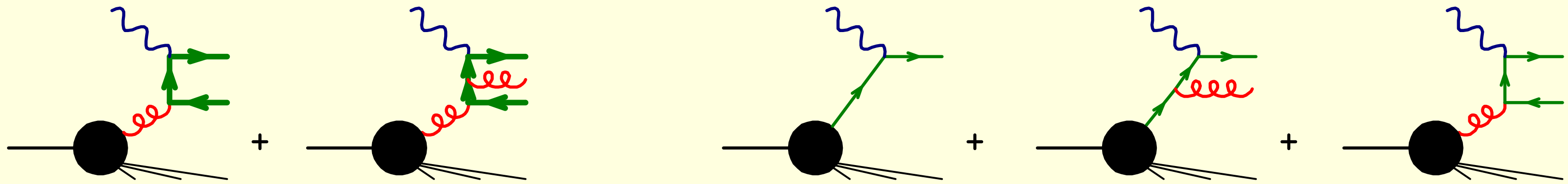
This way, the coefficients of powers of α_s that are only approximate in the massless expression, are replaced with the coefficients that include the exact mass dependence.

Advantages: it is simple, the proof takes one page. It works at all orders (in spite of the name...). It does not need new calculations. The heavy flavour calculation was done by putting together NDE and CG programs.

FONLL in details

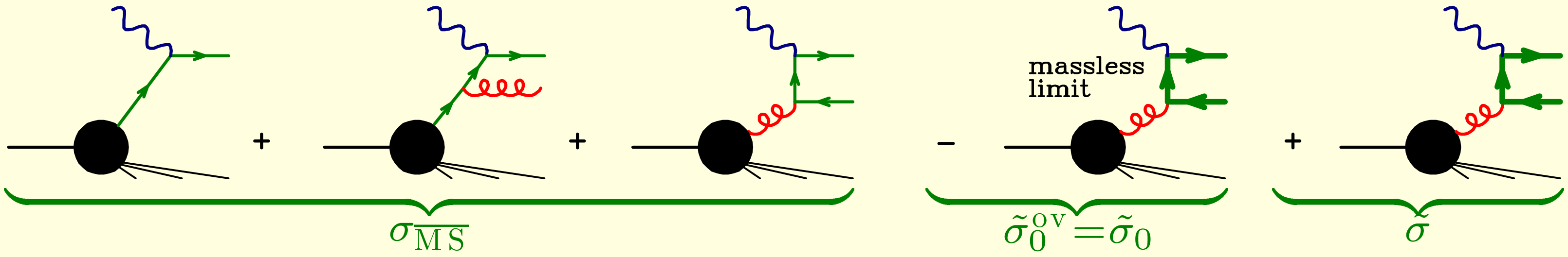
- Call σ a heavy flavour cross section in the decoupling scheme.
  σ is given in terms of $\alpha_s^{(n_l)}(\mu)$ and $f_i^{(n_l)}(\mu)$.
  Express σ in terms of $\alpha_s^{(n_f)}(\mu)$ and $f_{i\neq h}^{(n_f)}(\mu)$, using the matching equations. We call $\tilde{\sigma}$ the new expression (this procedure is elementary at NLO)
- In the limit $m\to 0$, $\tilde{\sigma}\to\tilde{\sigma}_0$, where $\tilde{\sigma}_0$ is a polynomial in α_s and L with mass independent coefficients. It is the massless limit of $\tilde{\sigma}$, in the sense $\lim_{m\to 0}(\tilde{\sigma}-\tilde{\sigma}_0)=0$.
- The massless scheme cross section, $\sigma_{\overline{\rm MS}}$, is given in terms of $\alpha_s^{(n_f)}(\mu)$ and $f_i^{(n_f)}(\mu)$. If we express $f_h^{(n_f)}(\mu)$ as a functional of the $f_{i\neq h}^{(n_f)}(\mu)$ using the evolution equations and the matching conditions, $f_h^{(n_f)}(\mu)$ is a power series in α_s and L with mass independent coefficients. In this way $\sigma_{\overline{\rm MS}}$ can be viewed as a power series in α_s and L with mass independent coefficients.
  Let us call $\tilde{\sigma}_0^{\rm ov}$ (ov for overlap) what we get from $\tilde{\sigma}_0$ deleting all terms that are not in $\sigma_{\overline{\rm MS}}$.
- The FONLL expression is $\sigma_{\rm FONLL}=\sigma_{\overline{\rm MS}}-\tilde{\sigma}_0^{\rm ov}+\tilde{\sigma}$

FONLL at NLO

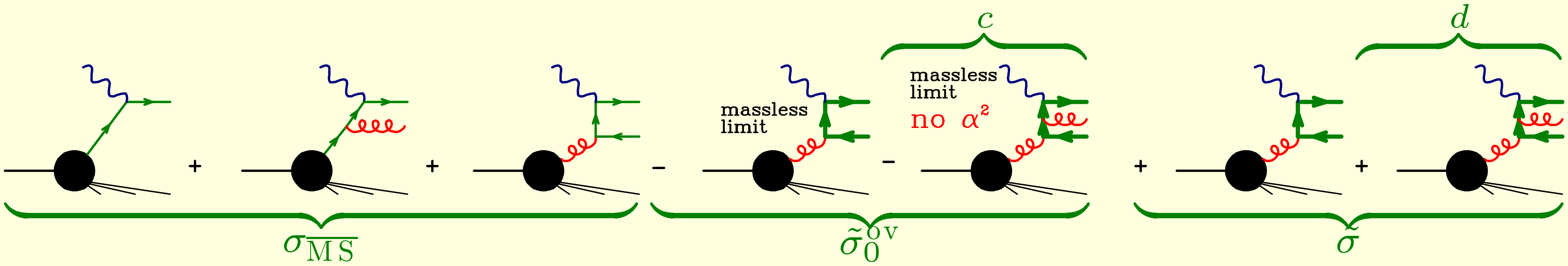


$\alpha_s + \alpha_s L$ $\quad$ $\alpha_s^2 + \alpha_s^2 L + \alpha_s^2 L^2$ $\quad$ $\alpha_s L + (\alpha_s^2 L + \dots)$ $\quad$ $\alpha_s^2 L + (\alpha_s^2 L^2 + \dots)$ $\quad$ α_s

If we uso LO for massive: (Same as S-ACOT!)



If we use NLO for massive: (reminiscent of TR!) $d - c \approx \mathcal{O}(\alpha_s^2), \text{no } L \text{ powers}$



Mass ambiguities in FONLL

In $\sigma_{\rm FONLL} = (\sigma_{\overline{\rm MS}} - \tilde{\sigma}_0^o) + \tilde{\sigma}$, we always have the freedom to introduce mass dependent modifications of the round bracket, that are suppressed by powers of the mass. For example, in DIS:

$$(\sigma_{\overline{\rm MS}} - \tilde{\sigma}_0^o) + \tilde{\sigma} \Rightarrow (\sigma_{\overline{\rm MS}} - \tilde{\sigma}_0^o)\, f_{\rm thr}(Q) + \tilde{\sigma}\,,$$

as long as $f_{\rm thr}(Q) \to 1$ for $Q \gg m_h$. Or the value of x in $\sigma_{\overline{\rm MS}} - \tilde{\sigma}_0^o$ can be rescaled: $x \to \chi_h = x(1 + 4m_h^2/Q^2)$. This freedom follows from the facts:

- $\sigma_{\overline{\rm MS}} - \tilde{\sigma}_0^o$ does not contain terms of the same order of those in $\tilde{\sigma}$
- $\sigma_{\overline{\rm MS}} - \tilde{\sigma}_0^o$ is valid only for $Q \gg m_h$

The treatment of terms computed in the massless approximation can be modified by a mass suppressed correction.

FONLL now being applied to DIS (Forte, Piccione, Rojo, P.N.; in preparation)

Advantages:

- Extreme simplicity (does not rely upon factorization with masses, etc.)
- No new calculation needed (σ_0 easily derived numerically from σ; also available in DIS from (Buza etal, 1996)
- More general: it allows inclusion of α_s^2 term in NLO implementation

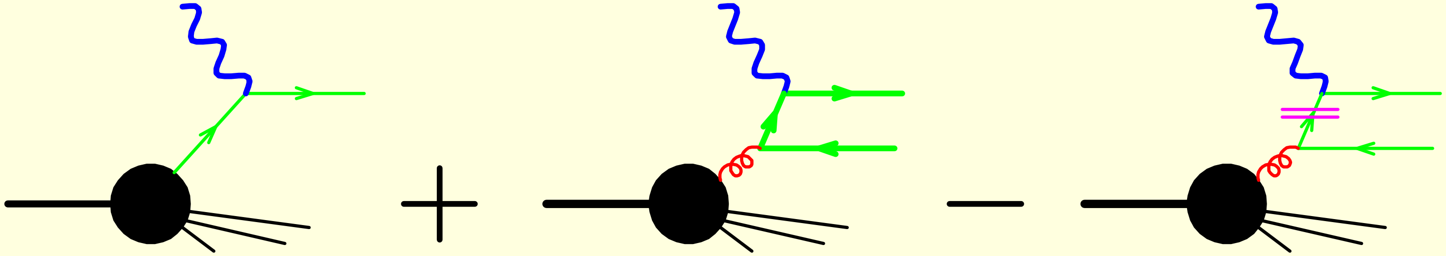
Without the α_s^2 term it is identical to SACOT at NLO; if χ-scaling is included, identical to ACOT-χ.

With α_s^2 term it constitutes a more transparent implementation of TR scheme.

NNLO straightforward to implement; should be identical to SACOT, if the $\mathcal{O}(\alpha_s^2)$ massive result is used. Or it can be improved invluding an estimate of the $\mathcal{O}(\alpha_s^3)$ massive result, as done by Thorne (2006).

Note:

S-ACOT (for Simplified ACOT, Kräemer, Olness, Soper, 2000).
Variant over ACOT, exploiting freedom in the choice of mass effects.
Use mass only in heavy quark lines not coming from the hadron



Modern CTEQ and MRST implementations use S-ACOT
(plus: χ-scaling, S-ACOT-χ).

Mass ambiguities

If mass effects are included, mass ambiguities remain only in the terms computed in the massless approximation.
These terms are correct only if $Q \gg m_h$. For $Q \approx m_h$, they are of higher order.

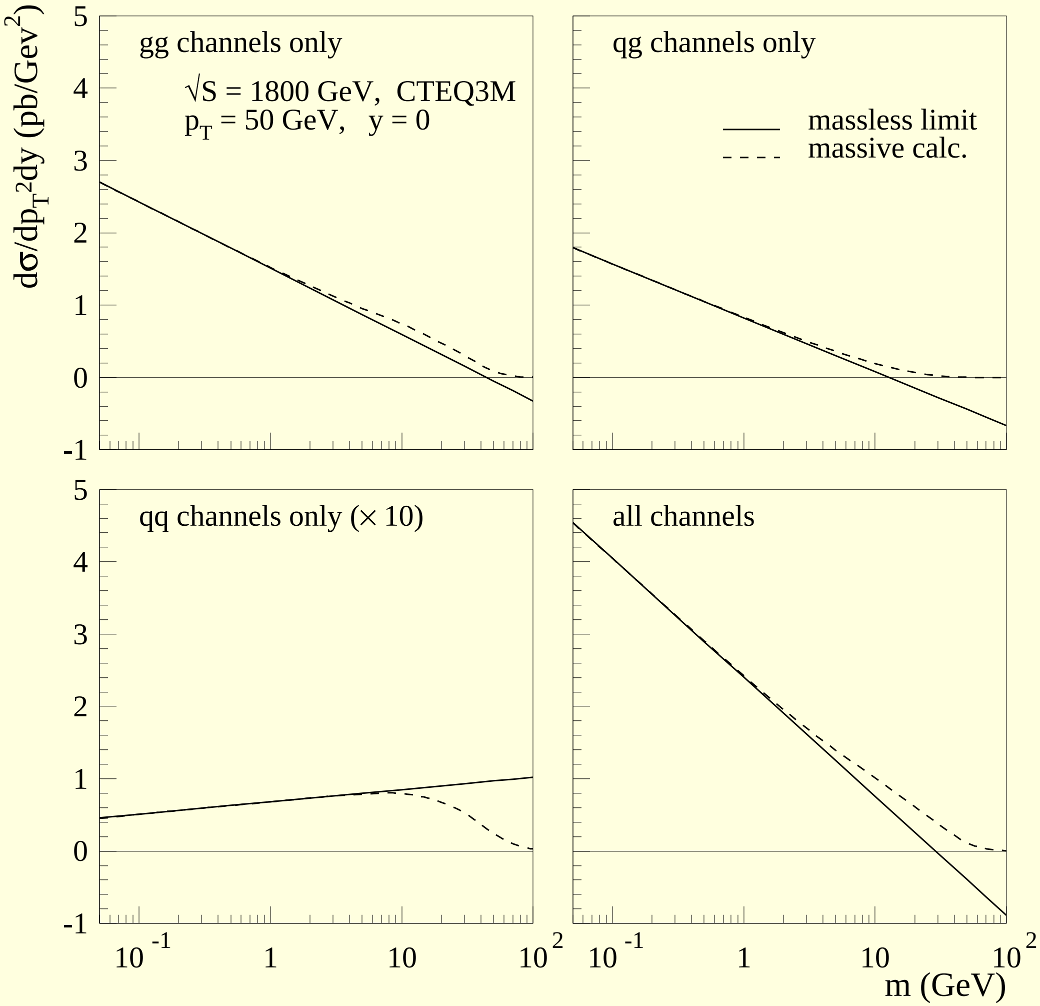
In practice, however, they may be important if for $Q \approx m_h$ they differ drastically from the full massive result.

So: the approach to the asymptotic form should be studied explicitly;

This problem is related to the one raised by F. Maltoni in his short presentation on Monday. Two questions:

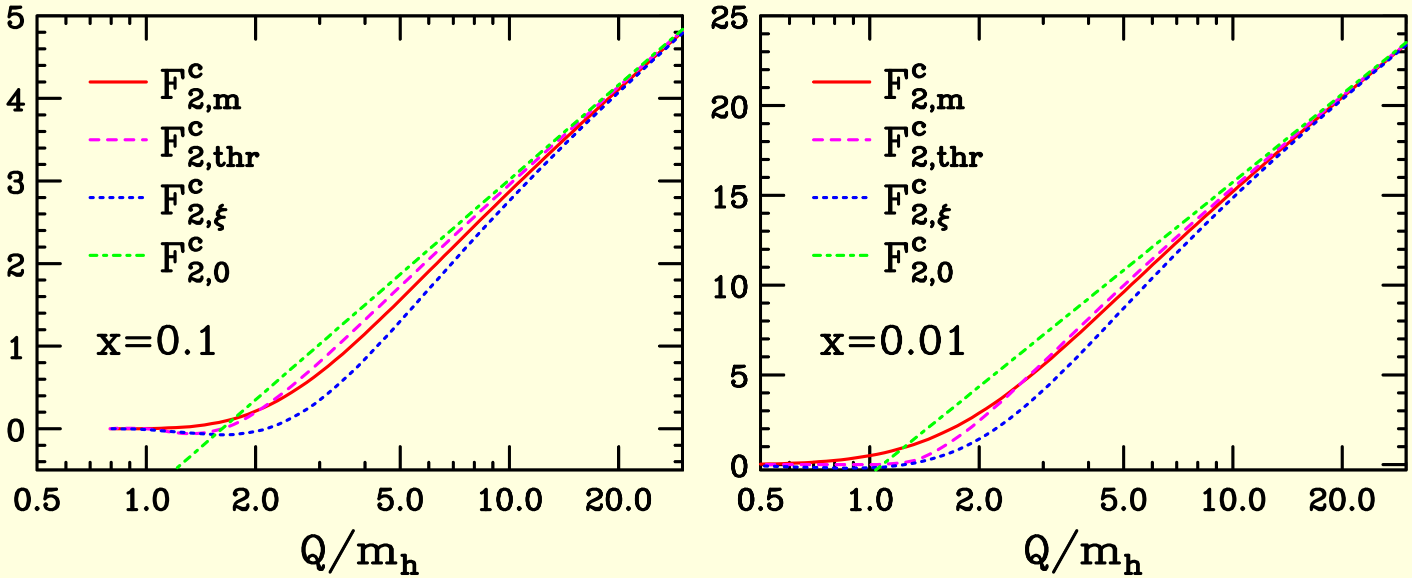
- When are mass effects truly negligible?
- When are large logs truly important?

Heavy Flavour hadroproduction
(Cacciari, Greco, P.N., 97)
In single inclusive heavy flavour production, at fixed p_T, $\mu = p_T$, sending the mass to zero, the asymptotic is a linear function of $\log m_h$.
For $p_T = 50$, for $m_h \lesssim 10$ the asymptotic form and the exact one start to differ sensibly.
Effect more pronounced
if flavour excitation is present?

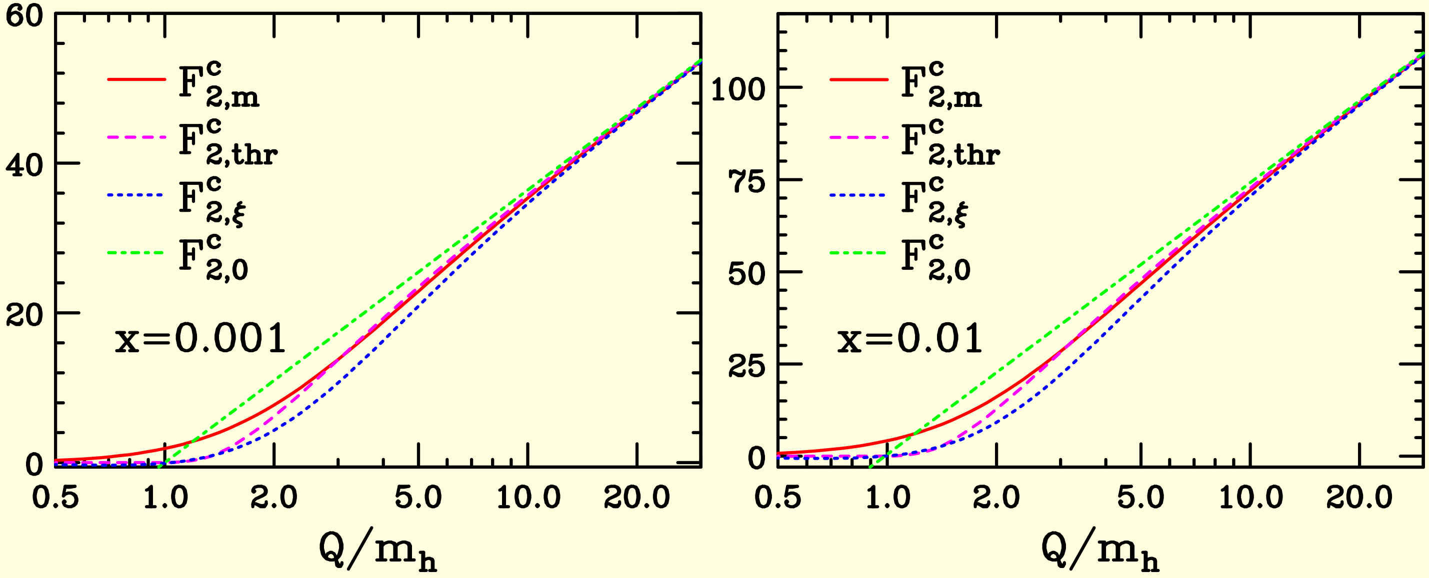


In F_2: compare photon-gluon fusion result in:

- Full massive
- massless approximation
- massless + χ-scaling
- massless times threshold factor $\theta(Q - m_h)(1 - m_h^2/Q^2)^2$



Use fake, Q^2 independent pdf, and fix α_s.



Again, $Q/m_h \gtrsim 3$ before any massless approximation works well ...
Same exercise should be applied to α_s^2 contribution ... What we learn:

- No prescription makes miracles
- Even the simplest $\theta(Q - m_h)$ is not bad...

Final considerations

- Matched calculations are easy to implement; no escuse to leave out mass effects or large logs from DIS fits.
- Proposals to use kinematic procedure to fudge mass effects can be made to work (Tung, Nadolski, 2009). Can be useful, but should not replace exact methods. Also: not universal? (i.e. confined to DIS?).
- Much to understand on the real impact of mass effects and large logarithms in hadron collisions (Maltoni, monday talk).
Matched calculations can help to clarify the problem.

Topics not discussed

- Fixed flavour pdf's (as in Stirling's talk)
- Infrared problems with F_c^2
- Which mass (pole, $\overline{\text{MS}}$, etc.)
- Intrinsic charm (non perturbative effects in charm initial condition)