Colour-dressed one-loop amplitudes from generalized unitarity

[CERN, Geneva 2009]

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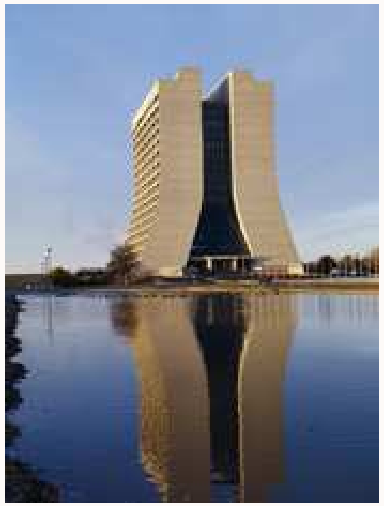
– Fermilab –



- *Results for colour-ordered amplitudes*
  - → *algorithm based on method by Ellis, Giele, Kunszt, Melnikov*
- *Extension of numerical method to colour-dressed amplitudes*
  - → *first preliminary results ... **work in progress***

[a] In collaboration with: W. Giele and Z. Kunszt

C++ code to calculate ordered amplitudes

➡ ***Implemented algorithm based on ...***

[Ellis, Giele, Kunszt, arXiv:0708.2398] 4dim method, cut-constructible part

[Giele, Kunszt, Melnikov, arXiv:0801.2237] Ddim method, rational part

[Giele, Zanderighi, arXiv:0805.2152] application of Ddim method to pure gluons

- independent implementation of EGKM method (from scratch, no translation of Fortran routines)
- good xcheck of generalized-unitarity method and its results

N external gluons & their polarizations ➡ ***(leading-)colour-ordered 1-loop amplitude (FDH)***

- accuracy – numerical stability of algorithm

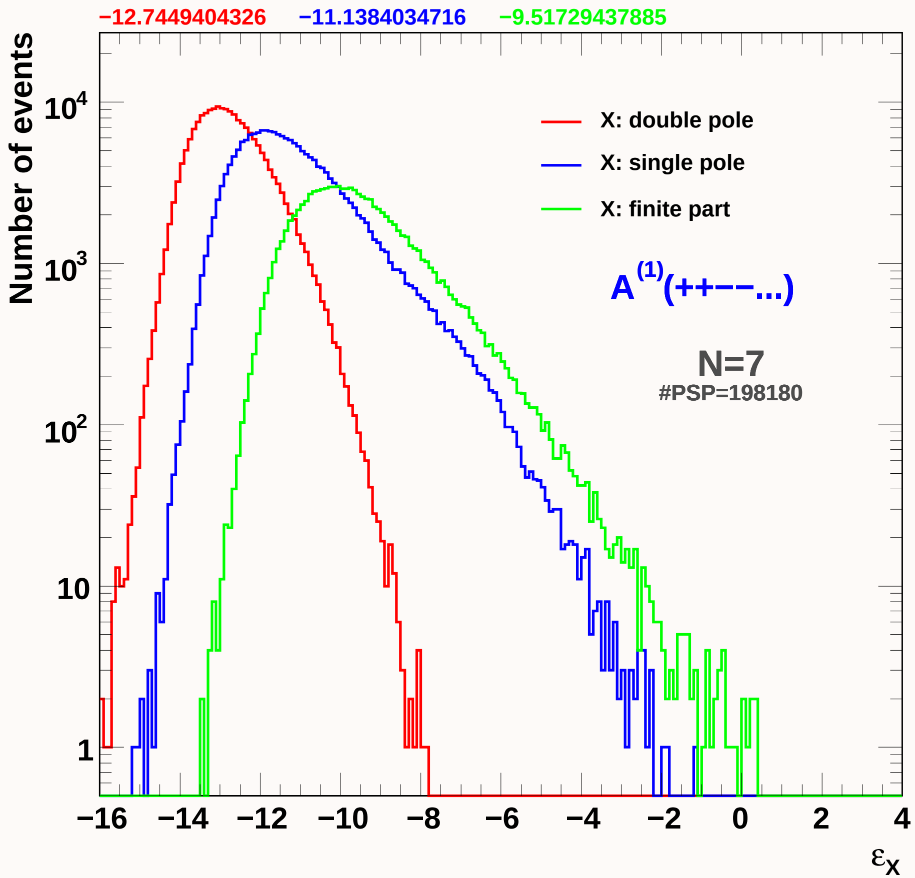
$$\varepsilon_{\rm dp,sp} = \log_{10} \frac{|\mathcal{A}^{(1)(\rm dp,sp)}_{N,\rm C++} - \mathcal{A}^{(1)(\rm dp,sp)}_{N,\rm anly}|}{|\mathcal{A}^{(1)(\rm dp,sp)}_{N,\rm anly}|}\,, \qquad \varepsilon_{\rm fp} = \log_{10} \frac{2\,|\mathcal{A}^{(1)(\rm fp)}_{N,\rm C++}[1] - \mathcal{A}^{(1)(\rm fp)}_{N,\rm C++}[2]|}{|\mathcal{A}^{(1)(\rm fp)}_{N,\rm C++}[1]| + |\mathcal{A}^{(1)(\rm fp)}_{N,\rm C++}[2]|}$$

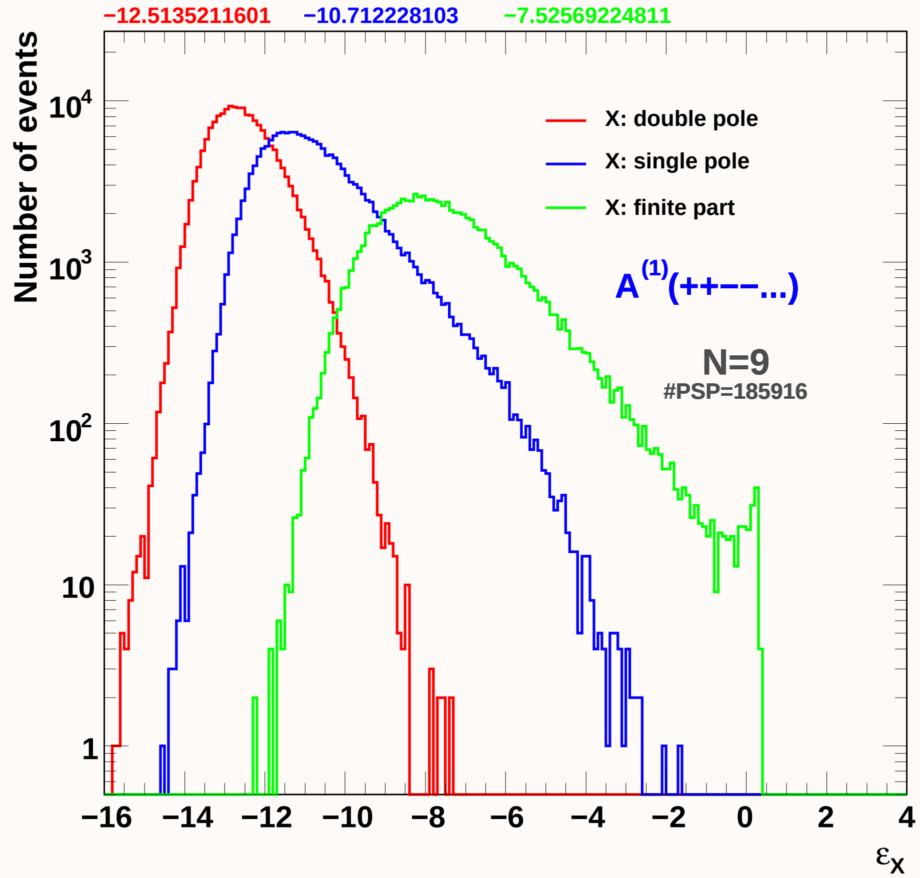
- efficiency – scaling of computing time with # of legs N $\rightarrow$ $\tau \sim N^9$

Accuracy

(calculations shown are in double precision only) [GIELE, WINTER, ARXIV:0902.0094]

- peak positions & tails are OK, comparable to Rocket (Rucola) results [GIELE, ZANDERIGHI]
- start losing finite-part precision about $N = 10, 11$, lost when $N = 15$
  (double precision not enough, too many large numbers involved)

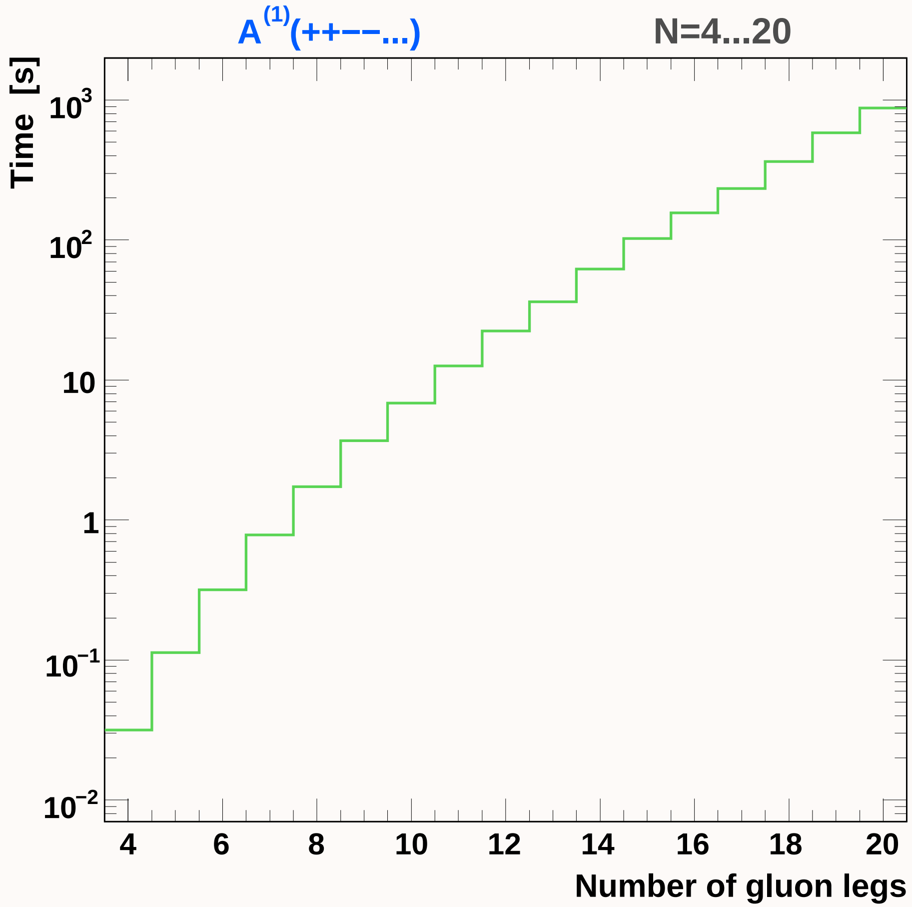


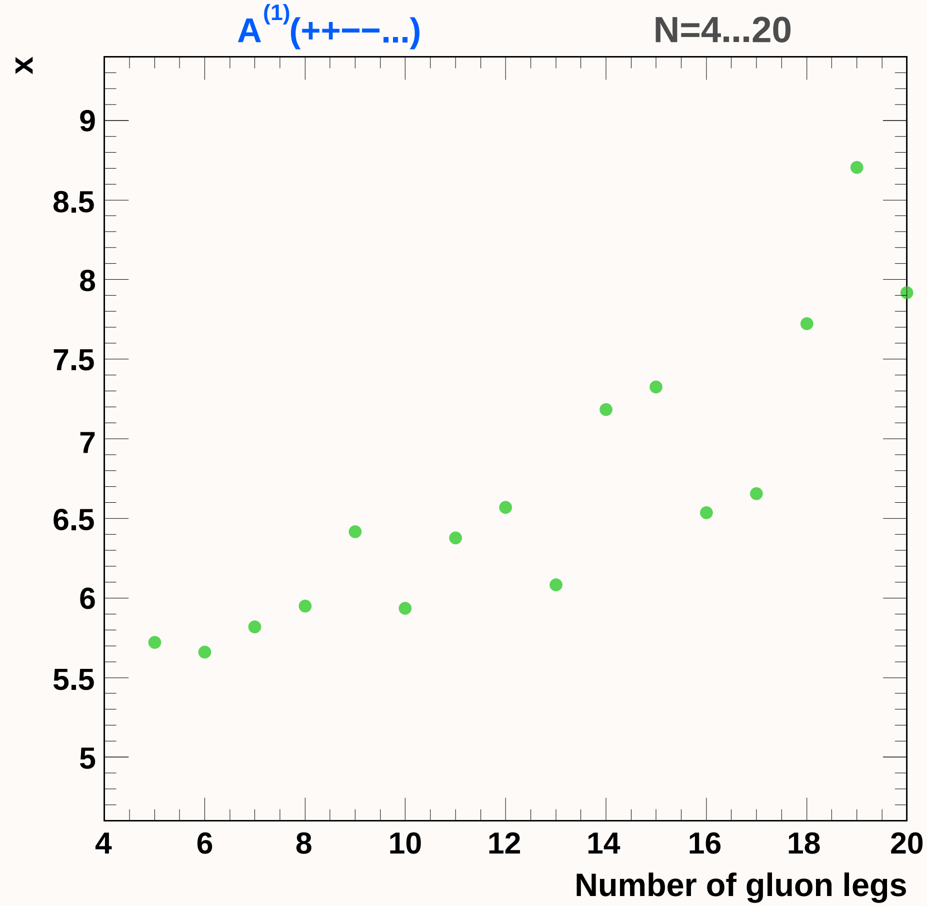


Speed of the calculation

(calculations shown are in double precision only) [GIELE, WINTER, ARXIV:0902.0094]

- we checked algorithm for polynomial complexity ($\tau \sim N^x$)
- we checked for asymptotic value of x using the fractions: $x = \ln \frac{\tau_{N+1}}{\tau_N} / \ln \frac{N+1}{N}$





Algorithm for full one-loop amplitudes

⇒ Construction of an algorithm of exponential complexity. Colours included. (see Giele's talk)

⇒ Our naive expectation of the asymptotic scaling is $(f \times 5)^N$ for N legs.

⇒ Colour-dressed recursions give factor $f > 1$, can be as large as 4.

⇒ Number of pentagons rise with 5^N ... asymptotic behaviour of $\mathcal{S}_2(N,5)$.

- input: external parton momenta & polarizations plus their explicit colours (colour-flow representation)

  output: amplitude $\mathcal{M}_1$ in form of complex number

➡ ***Based on EGKM algorithm for colour-ordered amplitudes. Extensions are necessary.***

- Decomposition of the integrand: sums over ordered cuts change into sums over partitions including non-cyclic, non-reflective permutations of the initial partitions.

$$\sum_{[i_1|i_k]} \quad \rightarrow \quad \sum_{RP_{\pi_1 \cdots \pi_k}(1,2,..,N)}$$

- Calculation of the integrand's residues: use colour-dressed recursions and sum over internal polarizations and internal colours.
- Symmetry factor of $1/2!$ needs to be applied to bubble coefficients.

Unordered gluons: partitions and subtractions

- *decomposition of integrand*

$$\mathcal{A}_N^{(D_s)}(\{p_i\},\ell) = \frac{\mathcal{N}_0(\{p_i\},\ell)+(D_s-4)\,\mathcal{N}_1(\{p_i\},\ell)}{d_1 d_2 \ldots d_N} = \sum_{k=2}^{5} \sum_{RP_{\pi_1\cdots\pi_k}(1,2,..,N)} \frac{\bar{C}^{(D_s)}_{\pi_1\cdots\pi_k}(\ell)}{d_{\pi_1}\ldots d_{\pi_k}}$$

- number of total partitions: $\max\{1,(k-1)!/2\}\times\mathcal{S}_2(N,k)$ $\Rightarrow$ increased number of terms

ord.) N	5-gons	boxes	triangles	bubbles	total
4	0	1	4	6	11
5	1	5	10	10	26
6	6	15	20	15	56
7	21	35	35	21	112
8	56	70	56	28	210
9	126	126	84	36	372

unord.) N	5-gons	boxes	triangles	bubbles	total
4	0	3	6	3	12
5	12	30	25	10	77
6	180	195	90	25	490
7	1680	1050	301	56	3087
8	12600	5103	966	119	18788
9	83412	23310	3025	246	109993

ord.) number of orderings however grows as $(N-1)!/2$, unord.) Stirling numbers grow as k^N

- *solving for numerator coefficients requires subtractions except for highest cut*

- subtraction terms identified by de-pinching, may cause shifts in loop momenta
- e.g. 4-gluon bubble 01|23 has 4 triangle subtraction terms:
  0|1|23 with $\hat{\ell}=\ell$ and $\hat{\ell}=-\ell+p_{23}$ and 2|3|01 with $\hat{\ell}=-\ell$ and $\hat{\ell}=\ell+p_{01}$

Colour-dressed recursion relations

- show exponential growth with N, cf. [DUHR, HÖCHE, MALTONI], implemented in ...

➡ ***COMIX ... SM tree-level ME generator based on generalized colour-dressed Berends–Giele recursions*** [GLEISBERG, HÖCHE]

- colour-flow decomposition for gluon currents used in our study

$$J_\mu^{IJ}(1,2,..,n) = \sum_{\sigma\in S_n} \delta^I_{j_{\sigma_1}} \delta^{i_{\sigma_1}}_{j_{\sigma_2}} \cdots \delta^{i_{\sigma_{n-1}}}_{j_{\sigma_n}} \delta^{i_{\sigma_n}}_J \; J_\mu(\sigma_1,\sigma_2,..,\sigma_n)$$

$$= \kappa^{-2}(1,2,..,n)\Big[\sum_{P_{\pi_1\pi_2}} \left(\delta^I_K\delta^L_M\delta^N_J - \delta^I_M\delta^N_K\delta^L_J\right)\left[J_\mu^{KL}(\pi_1), J_\mu^{MN}(\pi_2)\right] +$$

$$\sum_{P_{\pi_1\pi_2\pi_3}} \left(\delta^{ILNP}_{KMOJ} + \delta^{IPNL}_{OMKJ} - \delta^{ILPN}_{KOMJ} - \delta^{INPL}_{MOKJ}\right)\left(\{J_\mu^{KL}(\pi_1), J_\mu^{MN}(\pi_2), J_\mu^{OP}(\pi_3)\} + \pi_1 \leftrightarrow \pi_2\right)\Big]$$

- our tree-level amplitude calculations scale as 4^N
  (in COMIX, V_{gggg} is replaced by effective V_{ggg}, which yields 3^N scaling)
- needed to calculate the LHS of the parametric form when solving for the coefficients

$$\mathrm{Res}_{\kappa_1\cdots\kappa_n}\left(\mathcal{A}_N^{(D_s)}(\ell) - \sum_{k=n+1}^{5}\sum_{\text{parts}} \frac{\bar{C}^{(D_s)}_{\pi_1\cdots\pi_k}(\ell)}{d_{\pi_1}\dots d_{\pi_k}}\right) = \sum_{\substack{\{\lambda_j=1\}\\ \{(IJ)_j\}}}^{D_s-2} \prod_{i=1}^{n} \mathcal{M}_0\left(\ell_{\pi_{i-1}}^{(\lambda_{i-1}(IJ)_{i-1})}, \{p_{\pi_i}\}, -\ell_{\pi_i}^{(\lambda_i(JI)_i)}\right)$$

- internal colour sum is costly: reuse as many J_μ^{IJ} as possible, store & compute only non-zeros

First preliminary results

- Results can be tested: (not to mention internal consistency checks)

  (1) ⇒ double poles obey $\mathcal{M}_1^{(\mathrm{dp})} = -c_\Gamma\,\epsilon^{-2}\,N_\mathrm{C} N\,\mathcal{M}_0$

  (2) ⇒ vs colour decomposition into ordered amplitudes (using the "old" algorithm)

  schematically $\mathcal{M}_1 = \sum_{P(2,..,N)/Z_{N-1}} \left\{ \sum_r^{2^N} N_\mathrm{C}^{b(r)} \prod_s^N \delta_{j_s(r)}^{i_s(r)} \right\} m_1(1,\ldots,N)$

- Table of very first results: $2 \to N-2$ gluons, $(++--..)$ polarizations, $\binom{..1131..}{..1311..}$ colours & random PSPs obeying separation cuts ... computation times in secs (2.20GHz IntelCore2Duo)

ord.) N	cut-c,4D	factor	full,5D	factor	unord.) N	cut-c,4D	factor	full,5D	factor	OK?
4	0.025		0.045		4	0.05		0.105		✓
5	0.185	7.4	0.355	7.9	5	0.315	6.3	0.74	7.0	✓
6	0.83	4.5	2.7	7.6	6	1.37	4.3	4.59	6.2	✓
7	7.95	9.6	27.5	10.2	7	8.4	6.1	32.5	7.1	✓
8	86.5	10.9	439	16.0	8	52	6.2	234	7.2	✓
9	2220	25.7	**no**		9	380	7.3	3370	14.4	**no**

ord.) factors increase with larger N, unord.) growth seems to follow $(f \cdot 5)^N$, $1 < f < 2$

- Have to find the reason(s) for the odd $N = 9$ results.

Summary

- C++ code that implements Ellis–Giele–Kunszt–Melnikov method of calculating colour-ordered one-loop amplitudes using unitarity cuts.
  - ⇒ good double-precision results for gluon case.
  - ⇒ potential improvements: fitting coefficients, higher precision.
- Algorithm and implementation for full amplitudes using colour-dressed recursion relations.
  - ⇒ algorithm is of exponential complexity.
  - ⇒ asymptotic scaling of $\sim 7^N$ seen so far, does it persist for many legs?
  - ⇒ more to do: fully include quarks, squared amplitudes, OLE, xsecs (pure jets)
- First numerical results presented for colour-dressed one-loop amplitudes. Algorithm works.
  - ⇒ more stress tests needed: increase N, accuracy of results.
  - ⇒ colour-sampling convergence test when integrating $2\mathrm{Re}(\mathcal{M}_0\mathcal{M}_1^*)$.