Numerical evaluation of high multiplicity one-loop amplitudes

“A Numerical Unitarity Formalism for Evaluating One-Loop Amplitudes”
R.K. Ellis, W.G. and Z. Kunszt
JHEP 0803:003,2008; arXiv:0708.2398 [hep-ph]

“Full one-loop amplitudes from tree amplitudes”
W.G., Z. Kunszt and K. Melnikov
JHEP 0804:049,2008; arXiv:0801.2237 [hep-ph]

“Masses, fermions and generalized *D*-dimensional unitarity”
R.K. Ellis, W.G.,K. Melnikov and Z. Kunszt
Nucl. Phys B; arXiv:0806.3467 [hep-ph]

W. Giele,
SM-BSM workshop
CERN, August 2009

- The long term goal of this project is to construct a NLO event generator (similar to e.g. ALPGEN at LO).

- The first step is the construction of an algorithm for the generation of the virtual corrections for processes with a high number of external particles.

- The algorithm should be able to generate the virtual corrections for processes such as $PP \rightarrow n$ jets, $PP \rightarrow V + n$ jets, $VV + n$ jets etc. where n is only limited by the computer resources.

- Recent developments bring such an algorithm within the realm of possibilities with a time dependence on n which is slower than factorial.

R. Britto, F. Catchatzo and B. Feng, hep-th/0412103
(see also Z. Bern, L. Dixon, D.C. Dunbar and D. Kosower, hep-ph/9403226)

- The first ingredient for the algorithm comes from the more formal side: the concept of "generalized unitarity"
    - Any one-loop integral can be expressed in box, triangle and bubble scalar integrals.
    - All we need to determine are the coefficients of the scalar integrals.
    - It was shown that the coefficient of the scalar box integral could be obtained by calculating the Feynman diagram residue of the 4 denominators appearing in the scalar box integral.
    - Because the residue sets the 4 internal lines on-shell the coefficient is simply the product of 4 tree-level amplitudes.
    - Problems remained with the triangle and bubble coefficients due to overlapping contributions.

G. Ossola, C. Papadopoulos and R. Pittau, hep-ph/0609007

- The second ingredient is the concept of parametric integration of one-loop expressions
  - The integrand of any one-loop expression can be parametrized by a limited set of loop expressions for which the integrals are known
  - The coefficients of these loop expressions can be determined numerically by calculating the residues of 2, 3 and 4 denominators for the one-loop expressions.

- The coefficients in the parametric form are connected to the coefficients of the scalar integrals and the subtraction terms (to remove the overlaps).

- By combining these two ingredients one can construct the one-loop amplitude from on-shell tree-level amplitudes.

- Combining generalized unitarity with parametric integration leads to a powerful numerical method.

W.G. and G. Zanderighi, arXiv:0805.2152; A. Lazopoulos, arXiv:0812.2998; J. Winter and W.G., arXiv:0902.0094

- For example, it is straightforward to construct an algorithm of polynomial complexity which evaluates the color ordered one-loop *n*-gluon amplitude for arbitrary *n*

$$\mathcal{M}^{(0,1)}(1,2\ldots,n) \sim \sum_{P(23\cdots n)} Tr\left(F^{a_1}F^{a_2}\cdots F^{a_n}\right)\mathcal{A}^{(0,1)}(1,2,\ldots,n)$$

- We see already a problem: there are *(n-1)!* ordered amplitudes, each with its own color factor:
    - How do we get rid of the factorial growth
    - How the perform the color sums when contracting in with leading order

- At tree-level both of these problems were solved by so-called color dressing

P. Draggiotis, R. Kleiss and C. Papadopoulos, hep-ph/9807207

  - Sample over the external colors by choosing explicit colors for each scattering

C. Duhr, S. Hoche and F. Maltoni, hep-ph/060757

  - Fold the explicit color weights in the recursively generated tree-level amplitudes

T. Gleisberg and S. Hoche, ArXiv:0808.3674 [hep-ph]

- Both these methods have been implemented in the leading order COMIX generator and shown to be working well. (The algorithm is of exponential complexity.)

- Because generalized unitarity use the on-shell tree-level amplitudes as building blocks, the one-loop amplitude inherits these properties of the tree-level amplitudes.

Conclusions

- By using color dressed tree-level amplitudes we can construct algorithms for the calculation of the full virtual corrections of a scattering process (summed over color).

- The algorithm grows exponentially (not factorial). See talk by Jan Winter for implementation of this algorithm.

- By using COMIX as the LO amplitude generator one will be able to calculate the virtual corrections to any process of relevance at the LHC.

- After that the phase space integration has to be added and/or shower matching to obtain a useful tool for LHC phenomenology.