Matrix Elements and Shower CKKW and MLM

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Cern, 11-8-2009

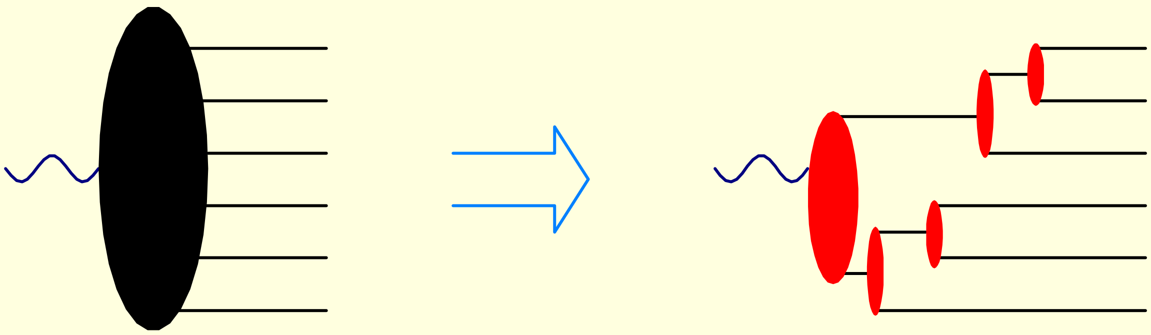
Historical approach: CKKW

Catani, Krauss, Kuhn, Webber (2001), (in e^+e^- annihilation).

In a nutshell:

Clusterize ME partons to reconstruct a shower skeleton
(by pairing up particles that yield smallest hardness t recursively)

Red blobs have decreasing t



- Do not allow t below a given cut $t_{\rm cut}$.
- Re-evaluate ME couplings at scales t of vertices in shower skeleton
- Assign Sudakov form factors to the skeleton (as in Shower MC)
- Continue the shower for $t < t_{\rm cut}$ with the Shower MC

CKKW: details

CKKW based upon the theory of soft-collinear radiation in QCD, through the following steps:

A) Theory of multiple emissions in the soft collinear regions (Mueller, 1981; Ermolaev and Fadin, 1981; Bassetto, Ciafaloni, Marchesini, etc.)

B) k_T-cluster multiplicity calculable at the NLL level in framework A) (Catani, Dokshitzer, Olsson, Turnock and Webber, 1991)

C) exact k_T-cluster ME cross section can be improved with Sudakov form factors and running α_s (i.e. dominant virtual corrections) from B)

D) Completion of the algorithm with subsequent angular ordered shower

Why k_T-clusters?

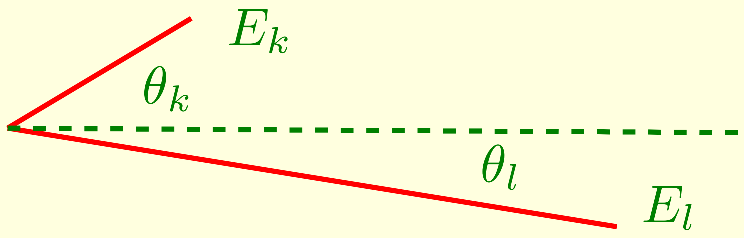
- Because we want to improve the cross section of the most energetic jets
- Because it can be computed at the double log and NLL level

k_T-clusters

Given a set of n particles in an e^+e^- final state, reconstruct jets by pairing up recursively pairs of particles with minimum

$$y_{kl} = 2(1 - \cos\theta_{kl}) \min(E_k^2, E_l^2)/Q^2.$$

In essence: $Q\, y_{kl} \approx \theta^2_{\text{softest}}\, E^2_{\text{softest}}$



The pair of particles with minimum y_{kl} are combined into a single pseudo-particle, with momentum $p_{kl} = p_k + p_l$ (or any variant of this)

k_T-cluster multiplicities can be computed at NLL level using the theory of multiple soft gluon emission.

In CKKW it is shown how to compute cluster multiplicities at NLL using angular ordering (i.e. reproduce the results of Catani, Dokshitzer, Olsson, Turnock and Webber)

k_T-clusters multiplicity

Sudakov form factor as in angular ordered shower, but veto radiation that yields $y > y_{\min}$. Introducing: $Q_{\min} = \sqrt{y_{\min}}\, Q$, $t = \theta E$, $q = k_T = \sqrt{t}\, z(1-z)$

$$\Delta(Q) = \exp\left[-\int_0^{Q^2} \frac{dt}{t} \int dz\, \frac{\alpha_s(q)}{2\pi} P(z)\theta(q - Q_{\min}) \right]$$

$$= \exp\left[-2 \int \frac{dq}{q}\, dz\, \frac{\alpha_s(q)}{2\pi} P(z)\theta(q - Q_{\min})\theta(Q\, z(1-z) - q) \right].$$

For example, for P_{qq}:

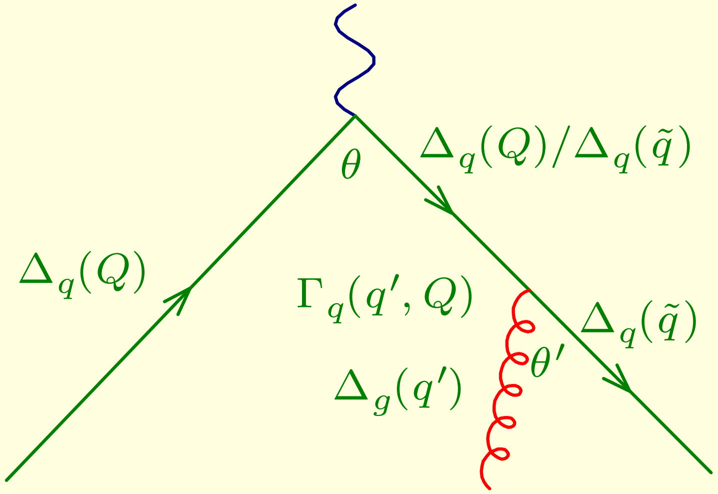
$$\Delta_q(Q) = \exp\left[-\int_{Q_{\min}}^{Q} \Gamma_q(q, Q)\, dq \right], \qquad \Gamma_q(q, Q) = \frac{2C_F}{\pi} \frac{\alpha_s(q)}{q} \left(\log\frac{Q}{q} - \frac{3}{4} \right)$$

For P_{gg}, P_{gq}: $\Gamma_g(q, Q) = \frac{2C_F}{\pi} \frac{\alpha_s(q)}{q} \left(\log\frac{Q}{q} - \frac{11}{12} \right)$, $\Gamma_f(q, Q) = \frac{N_F}{3\pi} \frac{\alpha_s(q)}{q}$

and $\Delta_g(Q) = \exp\left[-\int_{Q_{\min}}^{Q} [\Gamma_g(q, Q) + \Gamma_f(q, Q)]\, dq \right]$

Thus, the 2-clusters multiplicity is: $\frac{\sigma_2}{\sigma_{\text{tot}}} = \Delta_q^2(Q)$.

3-clusters: The antiquark line gets a factor $\Delta_q(Q)$ as before.

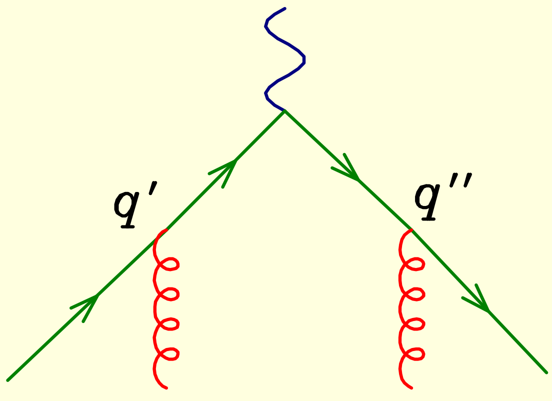


The gluon line from the gluon vertex gets a factor $\Delta_g(q')$, where $q' = \theta' E_g$.
(This uses angular ordering! no gluon radiation with angles $> \theta'$)
The quark line from the gluon vertex gets a factor $\Delta_q(\tilde{q})$, where $\tilde{q} = \theta' E_q$.
The quark line from the photon to the gluon vertex gets a factor:

$$\exp\left[-\int_{\theta'}^{\theta} \frac{d\theta^2}{\theta^2} \int dz \, \frac{\alpha_s(q)}{2\pi} P(z) \theta(q - Q_{\min}) \right] \approx \frac{\Delta_q(Q)}{\Delta_q(\tilde{q})} \text{ (in the soft approximation!)}$$

The gluon vertex gets a factor $\Gamma_q(Q')$.

Thus, the 3-clusters multiplicity is $2\Delta_q^2(Q) \int_{Q_{\min}}^{Q} \Delta_g(q')\Gamma_g(q', Q)\, dq'$



Same (angular ordering) arguments lead to the following values for the 4-cluster multiplicity diagrams:

$$\Gamma_q(q', Q)\Gamma_q(q'', Q)\,\Delta_{q\bar{q}\,gg}, \qquad \Gamma_q(q', Q)\Gamma_q(q'', q')\,\Delta_{q\bar{q}\,gg},$$

with $\Delta_{q\bar{q}\,gg} = \Delta_q^2(Q)\,\Delta_g(q')\,\Delta_g(q'')$.
The following observation holds to all orders: the Sudakov factors depend only upon the nodal values of the k_T scales q', q'', ... at which branching occours, and on the parton type.

Notice: in an angular ordered shower, the starting evolution scale of a branched soft parton, $E_{\rm soft}\,\theta$, is equal to the k_T.

In CKKW: replace approximate Γ factors by exact matrix element.

Detailed prescription:

- Consider the cross section $d\sigma_n$ to produce n partons ($n \leqslant N$), all separated by a minimum distance parameter $y_{\rm min}$, computed with a fixed value of α_s. Generate n body kinematics with probability $d\sigma_n$.
- From the given kinematics reconstruct the scheleton, by pairing up recursively partons with smallest y. Only pair up partons that can come from the same splitting process (i.e. $g\,g$, $q\,g$, $q\bar{q}$; no $q\,q$, $q'\bar{q}$, etc.). Assign to each vertex i of the skeleton the corresponding $q_i = Q\sqrt{y_i}$.
- Associate factors $\Delta(q_i)/\Delta(q_j)$ ($q_i > q_j$) with each intermediate line of the skeleton, a factor $\Delta(q_i)$ with each final line of the skeleton, and $\alpha_s(q_i)/\alpha_s(Q)$ with each node of the skeleton. Compute the product of all this factors and accept the event with a probability equal to this product.

Originally, N (and/or $Q_{\rm min}$) was assumed to be large enough, so that the result was insensitive to N (i.e., most events had less than N clusters)

CKKW with finite N

In the original CKKW scheme, N is assumed to be large enough (i.e., almost negiglible amount of final states with N clusters). Since N is finite, this means that $Q_{\min}$ should be kept large enough. A practical alternative to this (Mrenna and Richardson, 2003; Schaelicke and Krauss, 2005) is the following:

In the matrix element for N clusters, replace the $Q_{\min}$ scale used to compute the Sudakov form factors and the vetoed showers with q_n, (the $\sqrt{y}Q$ value of the smallest cluster.)

This was, starting from the N clusters event, the parton shower will be able to generate $N+1$, $N+2$, etc. clusters with merging scales larger than $Q_{\min}$. The N hardest clusters will be accurate at the matrix element level, while the subsequent ones will be only MC accurate.

Notice: with this prescription $Q_{\min}$ can be chosen as low as one likes (i.e., even near the shower cutoff). In this limiting case, no subsequent showers will be generated by the Monte Carlo for events with less than N clusters.

The scale $Q_{\min}$ and N appear here as the delimiter between the exact matrix element calculation and the shower approach: production of more than N clusters will rely upon the SMC, as well as production of clusters below $Q_{\min}$.

Interfacing to a Shower

We must complete the calculation with a full shower.
We only included splittings with k_T above $Q_{\min}$. In the shower we should:

A) Avoid to generate splittings with $k_T > Q_{\min}$; those were already generated by the matrix elements

B) Include all missing radiation with $k_T < Q_{\min}$

(A) is achieved by introducing a $\theta(Q_{\min} - k_T)$ in the splitting vertices and Sudakov form factors of the Shower Monte Carlo. In practice, this is achieved by the veto algorithm:

$\rightarrow$ At any stage of the generation of a branching starting from a scale t' in the SMC, generate the branching at a scale $t'' < t'$ and generate the z value with the usual method.

$\rightarrow$ If $k_T = \sqrt{t} z(1-z) > Q_{\min}$, discard the current branching, set t' to the value t'', and go back to the previous step. Otherwise, continue.

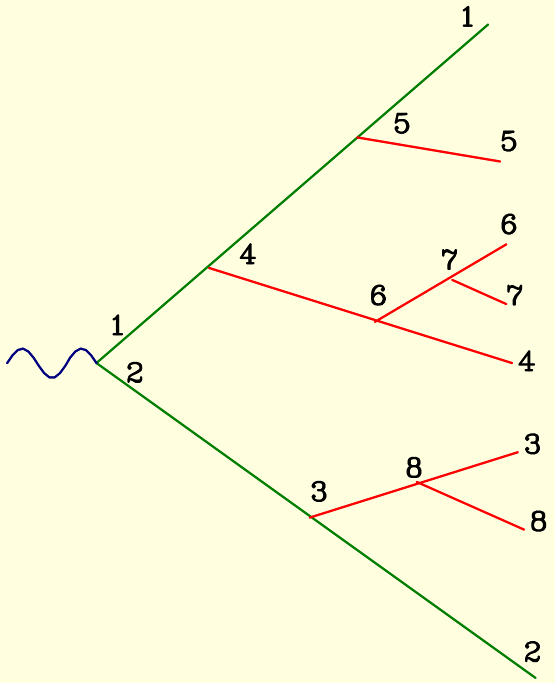
(B) is more subtle: one should allow branchings from each intermediate and final line of the skeleton that were not included in the ME calculation.

CKKW proposed the following:

The final state particles are fed into an angular ordered Monte Carlo, their initial showering angle is set equal to the angle at the vertex where the parton was initially produced.

The vertex where the parton is initially produced is found by walking up from the given final state parton in the shower skeleton, skipping vertices where the parton in question is merged with a softer parton, and stopping at the first vertex where this is not the case.

Introduce the following notation: when drawing a vertex, draw the final state hardest line parallel to the incoming line.



green: fermions, red: gluons; straight lines: hardest

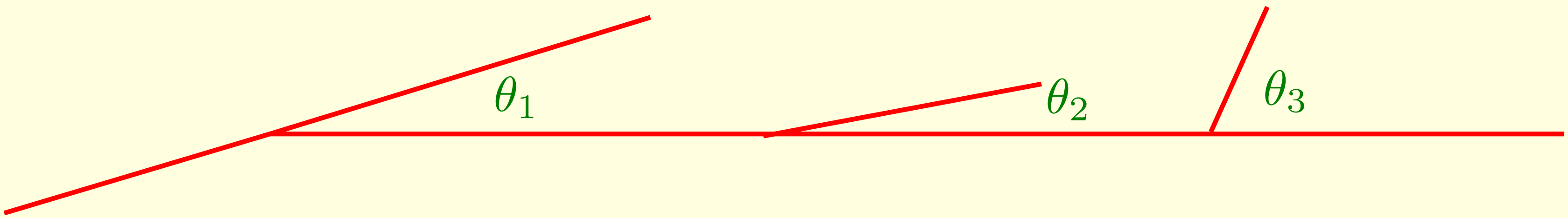
The production vertex of each parton (1 to 8) is indicated with the corresponding number.

The product of Sudakov form factors in CKKW is equivalent to introduce a singe $\Delta(q)$ for each final state line, with q computed at its initial production vertex

Equivalently, we define the hardest lines in the graph as the lines going from a final state parton to its initial production vertex.
The system of hardest lines covers the whole graph.

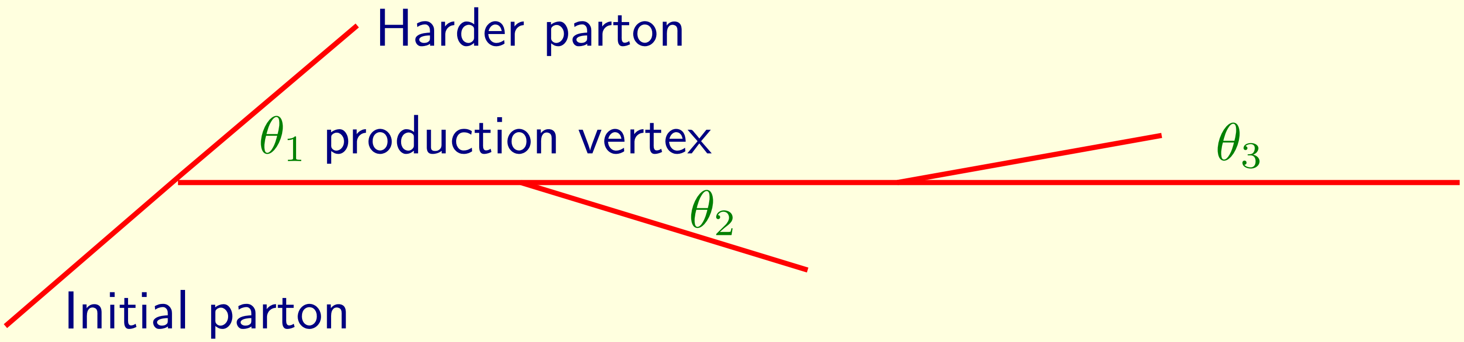
Reordering in angle

The k_T-ordered skeleton can be reordered in angle without changing the q value associated with each final line.



If $\theta_3 > \theta_2$ we can permute the two lines (they are both soft) without changing k_T (which is $q_2 = \theta_2 E_2$ and $q_3 = \theta_3 E_3$).
If $\theta_3 > \theta_1$ we can move vertex 3 to the initial line; θ_3 does not change, and neither does q_3.

After angular ordering:



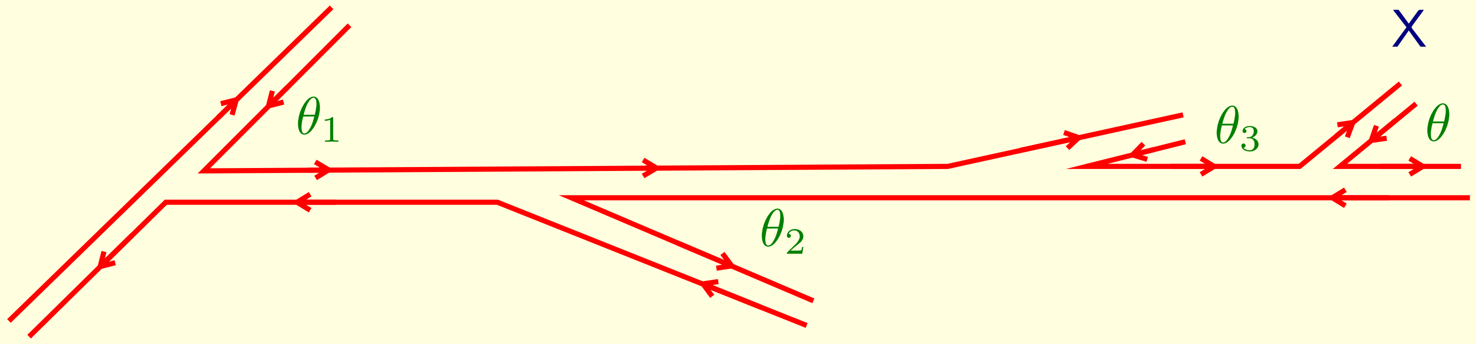
The CKKW prescription provides a single shower from θ_1. The horizontal line from θ_1 has almost constant energy, since radiation from 2, 3 is soft. So, a shower from θ_1 to the minimum is like a shower from θ_1 to θ_2, plus a shower from θ_2 to θ_3 plus a shower from θ_3 to the minimum.

It is then obvious that the CKKW prescription is kinematically equivalent to add truncated showers (P.N. 2004) to the skeleton.

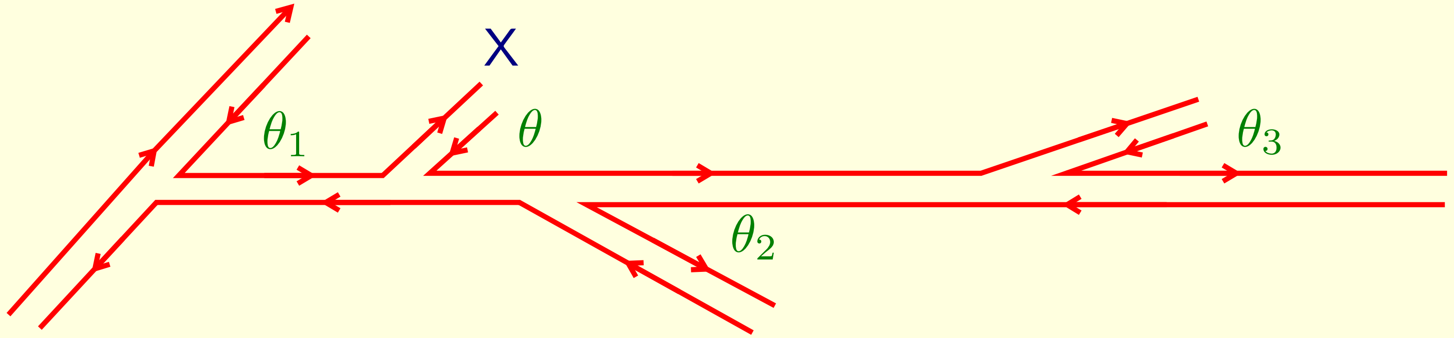
HOWEVER: incorrect colour pattern ...

If the MC generates parton X at angle θ, with $\theta_1 > \theta > \theta_2$:

Colour connection in CKKW:



Colour connection with truncated shower:



So: Larger colour gaps with CKKW

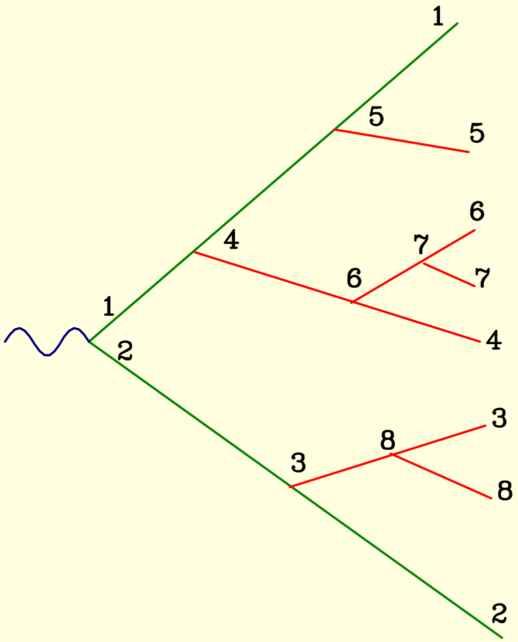
“Theorist” view of MLM matching

MLM: Generate the matrix element kinematics, build the skeleton, and reweight with $\alpha_s(q)$ as in CKKW. Start the shower from the given final state, but kill the event if the jets reconstructed after the shower do not match the parton jets.

Is it the same as CKKW?

In the following I will show that the answer is YES,
if the initial condition for the shower is according to the CKKW prescription
(i.e. equal to the angle where the parton was initially produced).

Assume that the showering scales are set according to the CKKW method,



Looking (for example) at parton 1, the MC builds a shower starting from the wide angle (1,2).

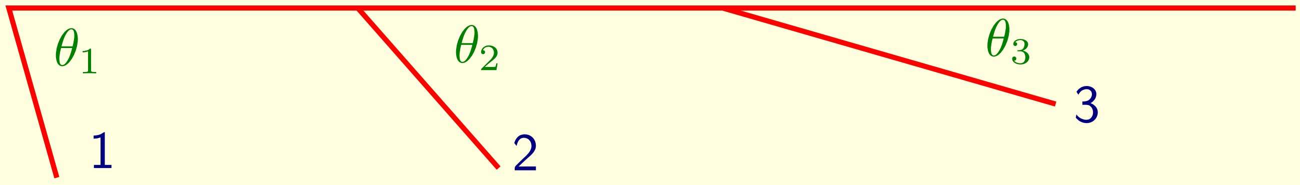
In CKKW: a probability factor $\Delta_q(Q)$ is provided. There is one such factor for each hardest line.

In MLM: the event is killed if if the MC generates a $k_T > Q_{\rm min}$. The survival probability is $\Delta_q(Q)$, as in CKKW. There is one such factor for each hardest line.

(Recall: $\Delta_q(Q) = \exp\left[-\int_0^{Q^2} \frac{dt}{t} \int dz \frac{\alpha_s(q)}{2\pi} P_{qq}(z) \theta(k_T - Q_{\rm min}) \right]$)

Same weight for the event; is the radiation below $Q_{\rm min}$ also the same?

The probability to undergo a few radiations in MLM is



$$\Delta(\theta_1,\theta_2)\frac{d\theta_2}{\theta_2}\frac{\alpha_s(k_T^{(2)})}{2\pi}P(z_1)\Delta(\theta_2,\theta_3)\frac{d\theta_3}{\theta_3}\frac{\alpha_s(k_T^{(3)})}{2\pi}\,\theta(Q_{\min}>k_T^{(2)},k_T^{(3)})\Delta(\theta_3)$$

If we normalise the above to the survival probability (we divide the above expression to $\Delta(Q)=\exp\Big[-\int_0^{Q^2}\frac{dt}{t}\int dz\,\frac{\alpha_s(q)}{2\pi}P_{qq}(z)\theta(k_T-Q_{\min})\Big]$) we take away from the exponent in each Sudakov form factor, the region with $k_T>Q_{\min}$. So, we end up with the vetoed shower required in CKKW.

So:

same radiation below $Q_{\min}$

Colour ordering

What about colour ordering? CKKW (and MLM) are incorrect;
One should reorder the skeleton in angle, and attach properly the MC emissions originating inside the skeleton (may be messy)

Simpler alternative:
At the end of the shower, build recursively an angular ordered skeleton, and assign colours according to it.

Conclusions

- If we can use for initial showering angle the angle at the vertex where the parton was initially produced, MLM and CKKW are equivalent
- Colour problem in CKKW can be easily fixed

Prospects

What is the use of this? (I don't know, may be useless)

Why use MLM matching in a first place? (I don't know, ask Michelangelo)

This result has emerged as part of a (personal) effort to understand the relationship between different matching schemes:

- traditional CKKW (with angular ordered showers)
- MLM matching
- matching with dipole showers

and how coherence is preserved with the various methods.