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QCD: are we ready for the LHC?

Academic training – Lecture 4

CERN, 4 – 7/12/2006

Summary of lecture 3

- Fixed order: lots at NLO, a few at NNLO
- Highly-automated generation of tree-level diagrams
- High-accuracy resummed computations available for a few key observables
- Resummed and fixed-order results are complementary
- PDFs with errors must be considered for serious assessment of systematics. Computing intensive
- Progress being made in (semi)-numerical approaches to loop computations, resummations

Event Generators

Remind that an Event Generator aims at giving a *complete* description of collision processes

The core of Event Generators is the *Parton Shower* mechanism, which serves two main purposes:

- To provide estimates of higher-order corrections that are enhanced by large kinematic logarithms
- To generate high-multiplicity partonic states which can readily be converted into the observed hadrons

The Parton Shower is built on the same concept as resummations: logarithmically dominant contributions to the cross section are "universal". Power-suppressed and finite terms are neglected

Parton Showers are more flexible than (analytical or numerical) resummation results. This comes at a price, since more approximations need be made

The problem

- A lot of physics at the LHC will involve many-jet events, and processes with large K factors
- Monte Carlos cannot give sensible descriptions of many-jet events, and cannot compute K factors
- Although Monte Carlos should not be seen as discovery tools, these issues must be addressed for a good understanding of LHC physics

Event Generators in a nutshell

- Infinite number of dominant Feynman diagrams

  Generate high-multiplicity parton final state: shower

- Models for hadronization, underlying event

  Convert partons into incoming and outgoing hadrons

- PDG information embedded

  Used to decay particles with correct branching ratios

Let's discuss the Parton Shower

Before going into that, let me stress that the problem of the sensible generation of the underlying event is a serious one, owing to

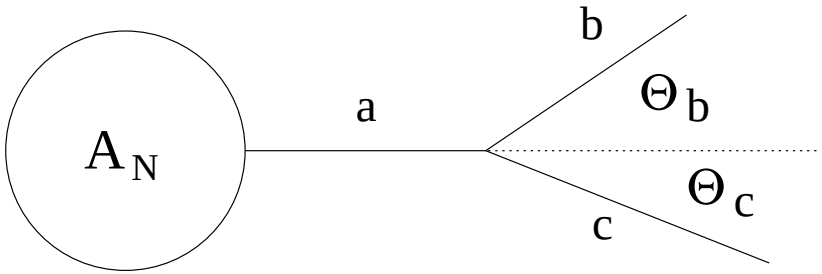
- its importance for all kind of physics simulations
- the still-poor theoretical understanding of its mechanisms

The process of checking the predictions of and of improving the models for the underlying event will start immediately after the LHC turn on

There is a lot of ongoing activity on this issue, which I won't report

Let's start by ignoring the problem of soft singularities

Collinear kinematics



$$\begin{aligned} z &= E_b/E_a \quad t = k_a^2 \\ \Theta &= \Theta_b + \Theta_c \\ &= \frac{\Theta_b}{1-z} = \frac{\Theta_c}{z} \\ &= \frac{1}{E_a}\sqrt{\frac{t}{z(1-z)}} \end{aligned}$$

Work in axial gauges

$$\begin{aligned} d\sigma_{N+1} &= d\sigma_N \frac{dt}{t}\frac{d\phi}{2\pi}dz\frac{\alpha_S}{2\pi}\left|K_{ba}(z)\right|^2 \\ d\bar{\sigma}_{N+1} &= d\bar{\sigma}_N \frac{dt}{t}dz\frac{\alpha_S}{2\pi}P_{ba}(z) \end{aligned}$$

as we already know from fixed-order and resummed computations

In the phase space, ϕ can be conveniently identified with the azimuthal angle between the plane of branching and the polarization of a

It is easy to iterate the branching process (splittings are called branchings in this context)

$$a(t) \longrightarrow b(z) + c\,, \quad b(t') \longrightarrow d(z') + e$$

$$d\bar{\sigma}_{N+2} = d\bar{\sigma}_N \frac{dt}{t} dz \frac{dt'}{t'} dz' \left(\frac{\alpha_S}{2\pi}\right)^2 P_{ba}(z) P_{db}(z')$$

This is a *Markov process*, ie a random process in which the probability of the next step only depends on the present values of the random variables. In formulae

$$\tau_1 < \ldots < \tau_n \implies$$
$$P\Big(x(\tau_n) < x_n | x(\tau_{n-1}), \ldots, x(\tau_1)\Big) = P(x(\tau_n) < x_n | x(\tau_{n-1}))$$

In our case, the probability of each branching depends on the type of splitting ($g \to gg$, ...), the virtuality t, and the energy fraction z

Following a given line in a branching tree, it is clear that enhanced contributions will be due to the strongly-ordered region

$$Q^2 \gg t_1 \gg t_2 \gg \ldots t_N \gg Q_0^2$$

$$\sigma_N \propto \sigma_0 \alpha_S^N \int_{Q_0^2}^{Q^2} \frac{dt_1}{t_1} \int_{Q_0^2}^{t_1} \frac{dt_2}{t_2} \ldots \int_{Q_0^2}^{t_{N-1}} \frac{dt_N}{t_N} = \sigma_0 \frac{\alpha_S^N}{N!} \left(\log \frac{Q^2}{Q_0^2} \right)^N$$

Denote by

$$\Phi_a[E, Q^2]$$

the ensemble of parton cascades initiated by a parton a of energy E emerging from a hard process with scale Q^2. Also, denote by

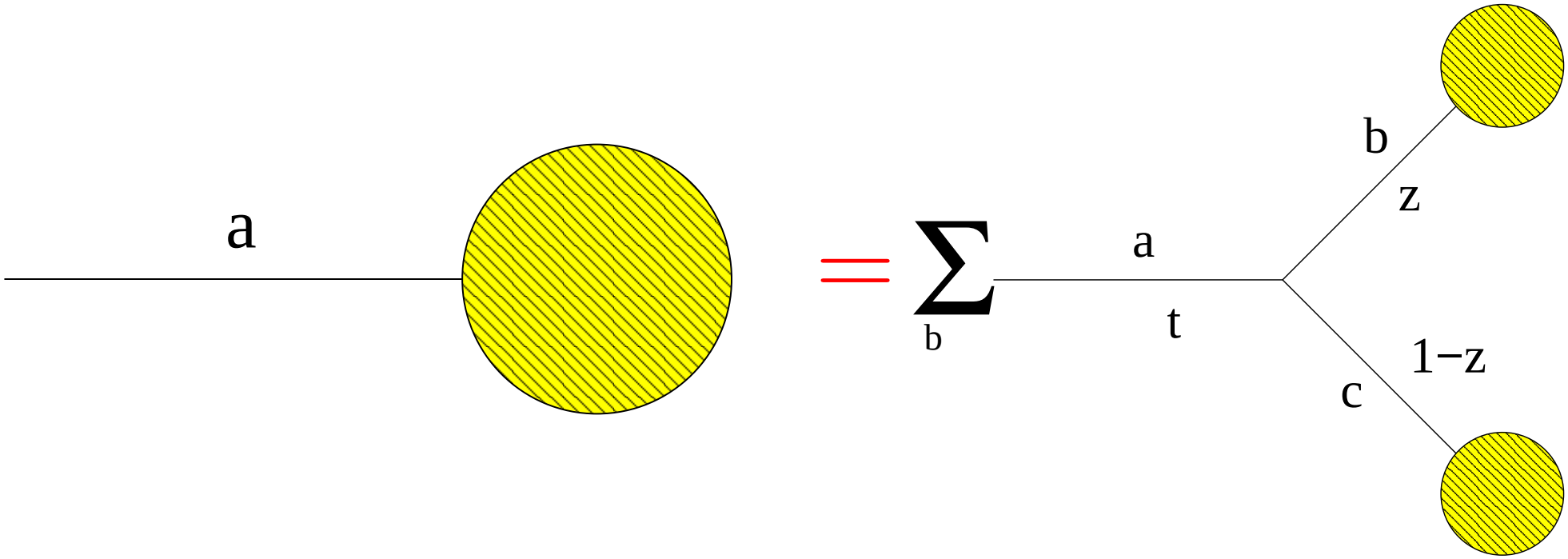
$$\Delta_a(Q_1^2, Q_2^2)$$

the probability that a does not branch for virtualities $Q_2^2 < t < Q_1^2$

With this, it is easy to write a formula that takes into account all the branches in a branching tree:

$$\begin{aligned}\Phi_a[E,Q^2] &= \Delta_a(Q^2,Q_0^2)\Phi_a[E,Q_0^2] \\ &+ \int_{Q_0^2}^{Q^2}\frac{dt}{t}\Delta_a(Q^2,t)\sum_b\int dz\frac{\alpha_S}{2\pi}P_{ba}(z)\Phi_b[zE,t]\Phi_c[(1-z)E,t]\end{aligned}$$

which has an immediate pictorial representation



Now simply impose that no information is lost during the parton shower: the sum of all the probabilities associated with the branchings of partons must be one. Therefore

$$1 = \Delta_a(Q^2, Q_0^2) + \int_{Q_0^2}^{Q^2} \frac{dt}{t} \Delta_a(Q^2, t) \sum_b \int dz \frac{\alpha_S}{2\pi} P_{ba}(z)$$

which can be solved:

$$\Delta_a(Q^2, Q_0^2) = \exp\left(- \int_{Q_0^2}^{Q^2} \frac{dt}{t} \sum_b \int dz \frac{\alpha_S}{2\pi} P_{ba}(z) \right)$$

Note

- This Sudakov form factor looks familiar $\longrightarrow$ resummation
- *Some* virtual corrections must be included, otherwise unitarity couldn't be imposed!

It's clear that a Sudakov must appear: resummation and parton shower described the same physics

Double logs

Keep in mind: this treatment is valid only in the collinear limit. Choices which affect the behaviour away from this limit are equivalent

For example, the choice of the shower variable t affects the double-log structure

$$
\begin{aligned}
t &= z(1-z)\theta^2 E^2 \quad \text{(virtuality)} \qquad \Longrightarrow \qquad \frac{1}{2}\log^2\frac{t}{E^2} \\
t &= z^2(1-z)^2\theta^2 E^2 \quad (p_T^2) \qquad \Longrightarrow \qquad \log^2\frac{t}{E^2} \\
t &= \theta^2 E^2 \quad \text{(angle)} \qquad \Longrightarrow \qquad \log\frac{t}{\Lambda}\log\frac{E}{\Lambda}
\end{aligned}
$$

owing to soft divergences. In MC's they are easy to locate:

$$
z \to 1 \qquad \Longrightarrow \qquad P_{qq},\ P_{gg} \sim \frac{1}{1-z}
$$

So the study of soft emission may give extra information on the proper choice for t

Soft emissions

Using soft-gluon techniques (Bassetto, Ciafaloni, Marchesini)

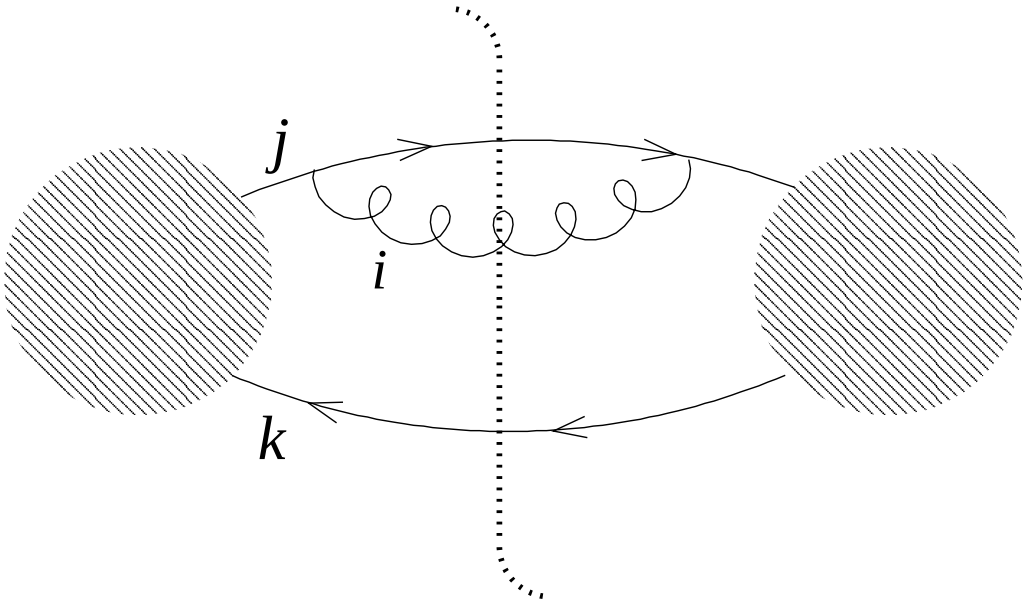
$$d\bar{\sigma}_{N+1} = -d\bar{\sigma}_N \frac{dE_i}{E_i} \frac{d\Omega_i}{2\pi} \frac{\alpha_S}{2\pi} \sum_{jk} \mathbf{T}_j \cdot \mathbf{T}_k \frac{\zeta_{jk}}{\zeta_{ij}\zeta_{ik}}$$

Gluon i has collinear singularities to j and k

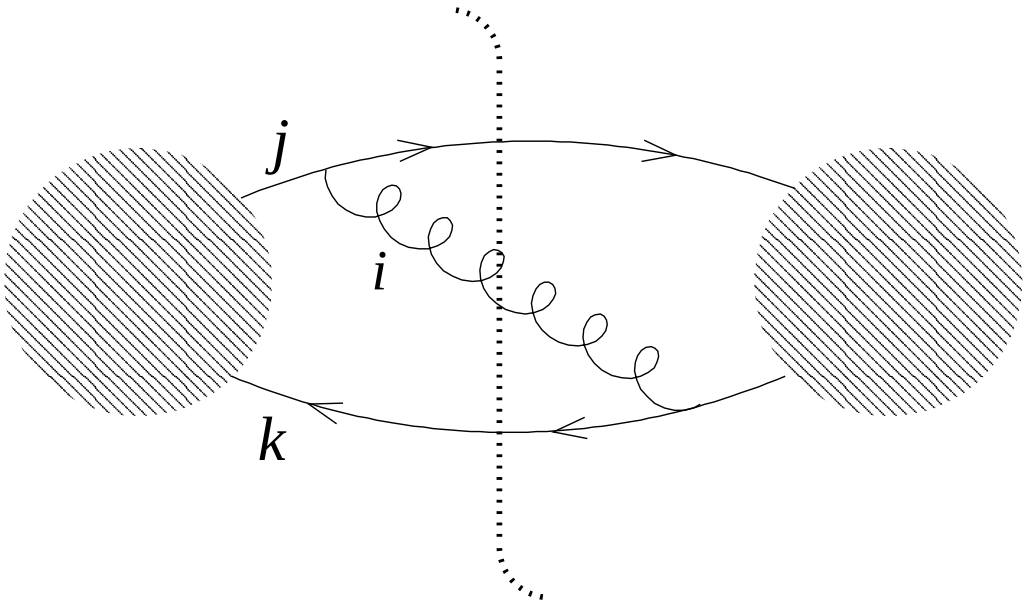
$$\begin{aligned} \zeta_{ab} &= \frac{k_a \cdot k_b}{E_a E_b} = 1 - \cos\theta_{ab} \\ \mathbf{T}_a &= \langle c_a | T^a \qquad \text{colour} - \text{charge operator} \\ \mathbf{T}_g^2 &= C_A\,, \quad \mathbf{T}_q^2 = C_F \end{aligned}$$

When iterating this formula to the next emission, one gets

- A non-positive definite expression (owing to interference)
- A non-Markovian structure (step 2 depends on step 1 *and* 0)



Collinear



Soft

Manipulate the radiation function

$$
\begin{aligned}
W_{jk} &= \frac{\zeta_{jk}}{\zeta_{ij}\zeta_{ik}} = W_{jk}^{[j]} + W_{jk}^{[k]} \\
W_{jk}^{[j]} &= \frac{1}{2}\left(\frac{\zeta_{jk}}{\zeta_{ij}\zeta_{ik}} + \frac{1}{\zeta_{ij}} - \frac{1}{\zeta_{ik}}\right)
\end{aligned}
$$

This decomposition has two remarkable properties

- It disentangles the collinear singularities
- It has *angular ordering*

$$
\int_0^{2\pi} d\phi_{ij} W_{jk}^{[j]} = \begin{cases} 1/\zeta_{ij} & \zeta_{ij} < \zeta_{jk} \\ 0 & \zeta_{ij} > \zeta_{jk} \end{cases}
$$

Angular ordering is a manifestation of (destructive) interference effects present in gauge theories – eg in QED

Angular ordering implies that after azimuthal average we have

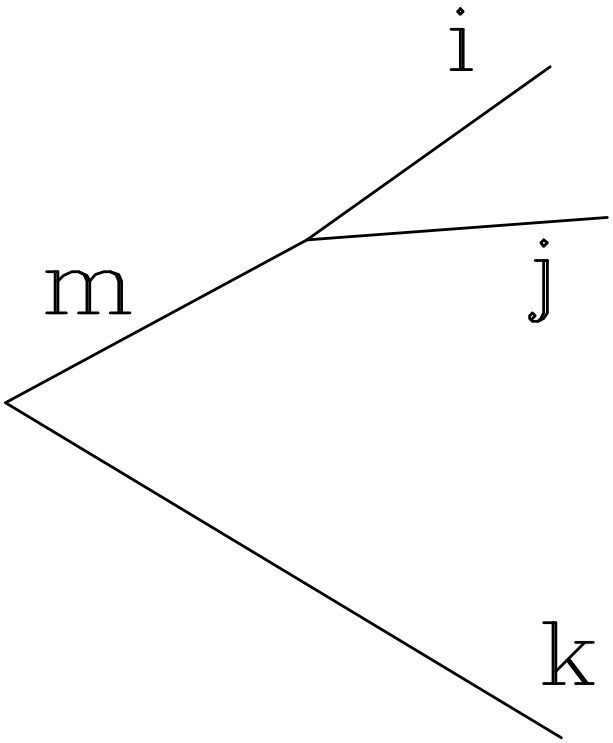
$$d\bar{\sigma}_{N+1} \quad = \quad -d\bar{\sigma}_N \frac{dE_i}{E_i} \frac{\alpha_S}{2\pi} \sum_{jk} 2\mathbf{T}_j \cdot \mathbf{T}_k \int_0^{\zeta_{jk}} \frac{d\zeta_{ij}}{\zeta_{ij}}$$

This looks promising: may be interpreted as

$$\ldots \longrightarrow j + k\,; \quad j \longrightarrow i + j' \ \ldots$$

The process is fully symmetric in $j \longleftrightarrow k$

In order to study the emission pattern in more details, we must at least consider the next branching



Consider the emission of a soft gluon from the *colour singlet* formed by the three partons i, j and k

The radiation pattern will be obtained by attaching a soft gluon to the three external legs i, j, k

$$W_{ijk} = -\mathbf{T}_i \cdot \mathbf{T}_j W_{ij} - \mathbf{T}_j \cdot \mathbf{T}_k W_{jk} - \mathbf{T}_i \cdot \mathbf{T}_k W_{ik}$$

Assuming that $\theta_{mk} \gg \theta_{ij}$ one gets

$$W_{ijk} = \mathbf{T}_i^2 W_{ij}^{[i]} + \mathbf{T}_j^2 W_{ij}^{[j]} + \mathbf{T}_k^2 W_{km}^{[k]} + \mathbf{T}_m^2 W_{km}^{[m]} \Theta(\theta_{mg} > \theta_{ij})$$

- Inside the cone (ij), the gluon is emitted by two independent charges $\mathbf{T}_i^2$ and $\mathbf{T}_j^2$
- Outside of this code, the gluon cannot resolve i and j, and only "sees" $\mathbf{T}_m^2 = (\mathbf{T}_i + \mathbf{T}_j)^2$

$\Longrightarrow$ A Markov structure has emerged: $(ijk) \equiv ((i+j)k) + (ij)$

Indeed, we can obtain

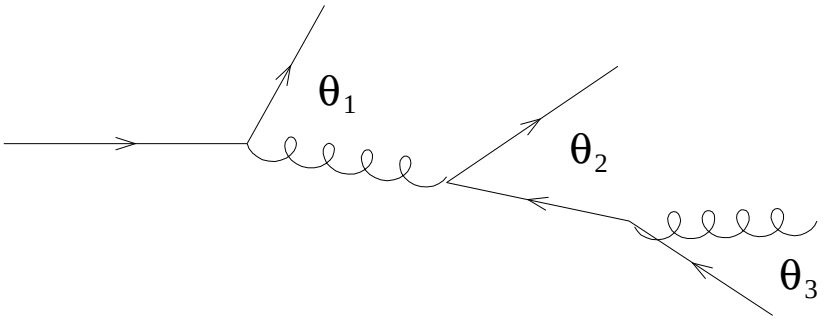
$$W_{ijk} = \mathbf{T}_i^2 W_{ij}^{[i]} + \mathbf{T}_j^2 W_{ij}^{[j]} + \mathbf{T}_k^2 W_{km}^{[k]} + \mathbf{T}_m^2 W_{km}^{[m]} \Theta(\theta_{mg} > \theta_{ij})$$

as a two-step branching process. First, attach the soft gluon to the pair (mk), ie

$$\mathbf{T}_k^2 W_{km}^{[k]} + \mathbf{T}_m^2 W_{km}^{[m]}$$

Note that m is on shell!. Next, after the branching $m \longrightarrow ij$ with $\theta_{ij} < \theta_{mg}$, attach the soft gluon to the pair (ij), ie

$$\mathbf{T}_i^2 W_{ij}^{[i]} + \mathbf{T}_j^2 W_{ij}^{[j]}$$



Angular ordering

$$\theta_1 > \theta_2 > \theta_3$$

Coherent branching

What done above can be combined with the collinear branching stuff. One arrives at a coherent branching formalism, which correctly incorporates collinear *and* soft enhancements to all orders

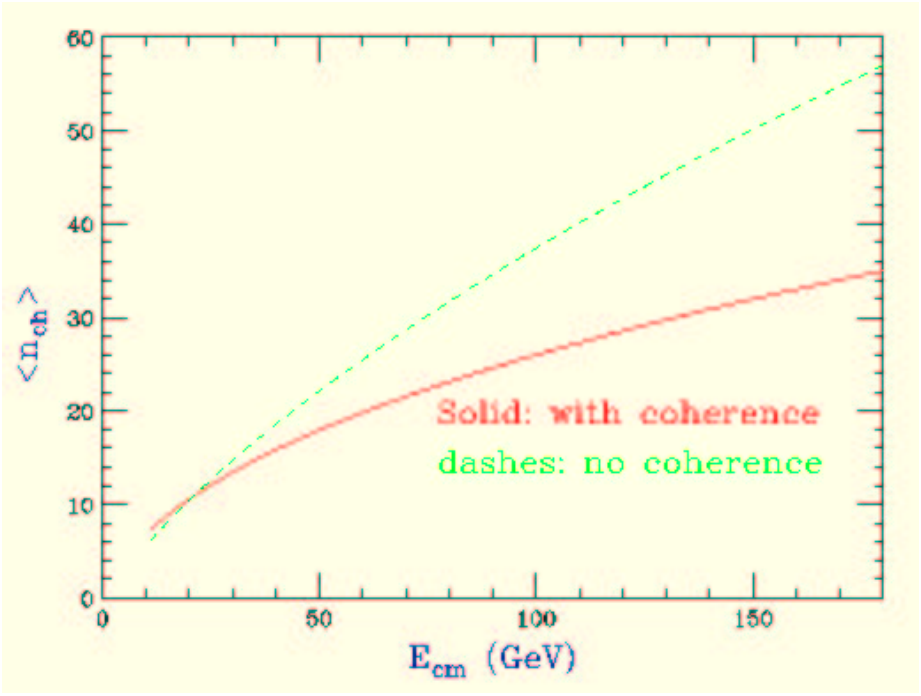
The most straightforward approach it that of replacing the shower variable t with $\zeta = 1 - \cos\theta$, and impose $\zeta_{n+1} < \zeta_n$. Iterated cross section formulae now read

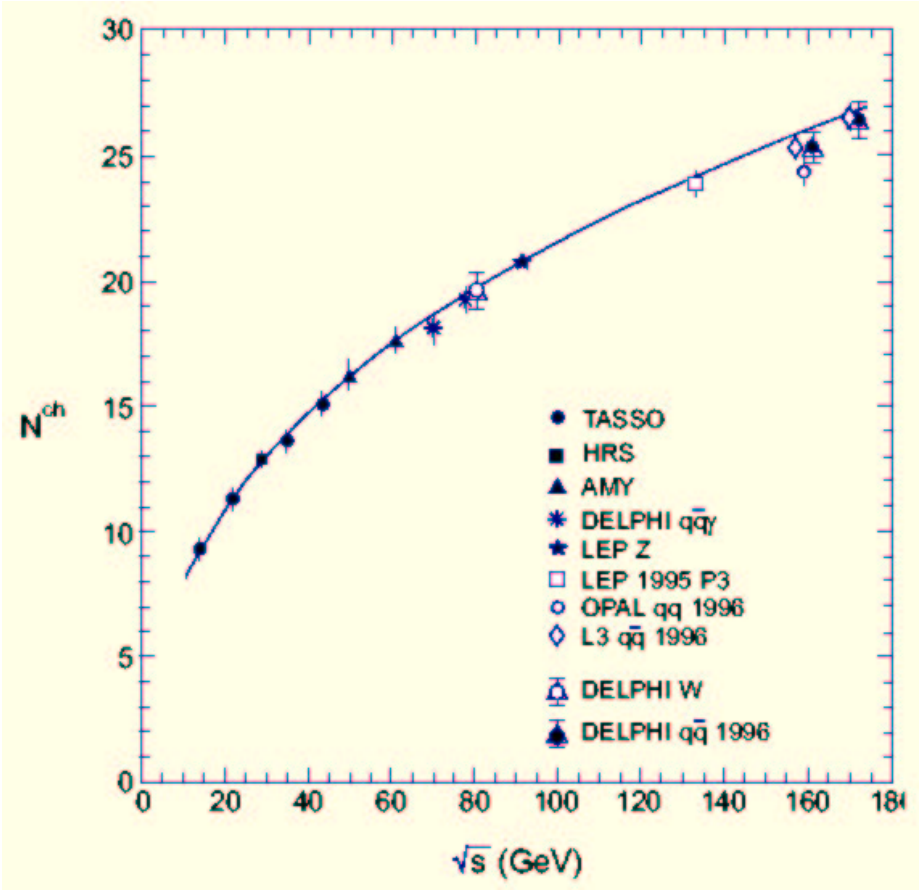
$$d\bar{\sigma}_{N+1} \quad = \quad d\bar{\sigma}_N \frac{d\zeta}{\zeta} dz \frac{\alpha_S}{2\pi} P_{ba}(z)$$

In practice, to take into account emission from non-zero-mass lines, it's more convenient to use as shower variable for $a \to bc$ (HERWIG)

$$Q_a^2 = E_a^2 \zeta_a \, ; \qquad \zeta_a > \zeta_b \implies Q_b^2 < z_b^2 Q_a^2$$

There are non-accessible kinematic regions (dead zones)





Coherence can be seen in data

3 very successful implementations

HERWIG	PYTHIA	ARIADNE
$t = E^2\zeta$	$t = M^2$	$t = p_T^2$
hardest not first	hardest first	hardest first
coherent	coherence forced	coherent
dead zones	no dead zones	no dead zones
ISR easy	ISR easy	ISR difficult
$g \to q\bar{q}$ OK	$g \to q\bar{q}$ OK	$g \to q\bar{q}$ difficult

Each has pros and cons: don't be lazy, try to use more than one

Summary on Event Generators

0) Start from a *leading order* hard subprocess

1) Let initial- and final-state partons branch

2) Iterate 1) (ie shower) till reaching a small scale Q_0

3) For final-state partons, use a model to convert partons into hadrons; for initial-state partons, force further branchings till valence flavours are generated, and fold with $f(x, Q_0)$

4) Add low-p_T stuff (underlying events, ...)

Troubles

As in the case of resummed computations, the perturbative part of parton shower is based on collinear/soft approximations

It is easier, however, to misuse an MC than a CSI implementing resummation

Example: W hadroproduction. One may want to study not only W properties, but also consider the accompanying jets

- Can't do this with a CSI: it's inclusive in W, jets are simply not there
- Can do it with an MC: the partons against which the W recoils are available in the event record, and jets can be reconstructed

This is OK, if the jets are not too hard, and not too far from each other

But hard and well-separated jets can be generated by the MC – and no warning is given that the corresponding cross section is *totally wrong*

It is left to you to determine whether you are using an MC outside the range of validity of its approximation. It is a very common mistake to abuse of this freedom

We have seen how to improve CSI with resummation, by matching them with fixed-order results. In order to do this, one needs

- To identify the large logs in the cross section
- To remove from the fixed-order result those logs already present in the resummed result

Unfortunately, this strategy will not work in the case of MC's, since it is observable-specific

However, the basic idea is the same: more information on fixed-order results need be passed to the Event Generator

How to improve Monte Carlos?

The key issue is to go beyond the collinear approximation

$\Longrightarrow$ use exact matrix elements of order higher than leading

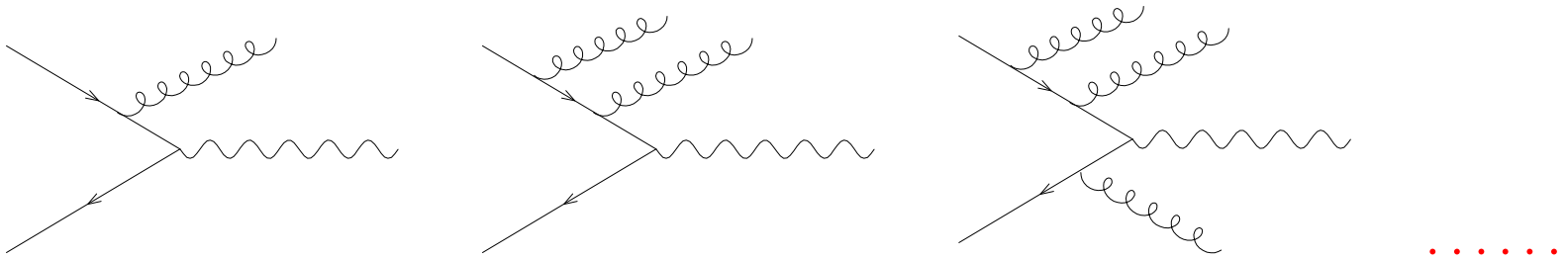
Which ones?

There are two possible choices, that lead to two vastly different strategies:

- Matrix Element Corrections $\longrightarrow$ tree level
- NLOwPS $\longrightarrow$ tree level and loop

Matrix Element Corrections

Compute (exactly) as many as possible real emission diagrams before starting the shower. Example: W production



Then use the kinematics configurations generated in this way as initial conditions for the shower

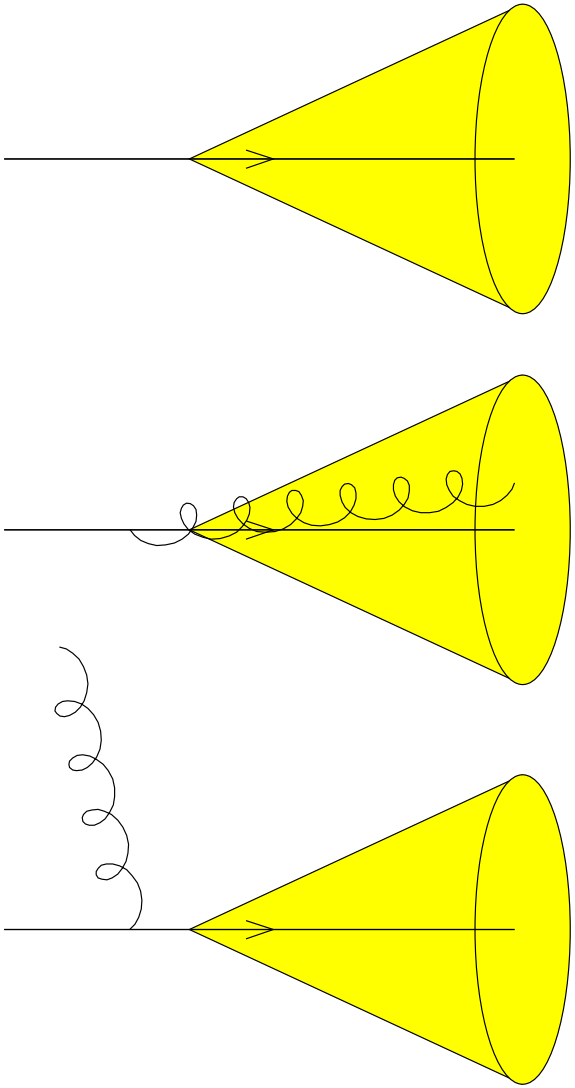
- The idea: large-p_T, well-separated partons will evolve into large-p_T, well-separated jets

Problems

- Double counting (the shower can generate the same diagrams)
- The diagrams are divergent

The two problems are connected: the matrix elements diverge in the soft/collinear regions, which are those "preferred" by showers

This suggests a (naive) solution: use some observable to decide if two partons are close to each other or not. If not, use the matrix elements, otherwise use the parton shower



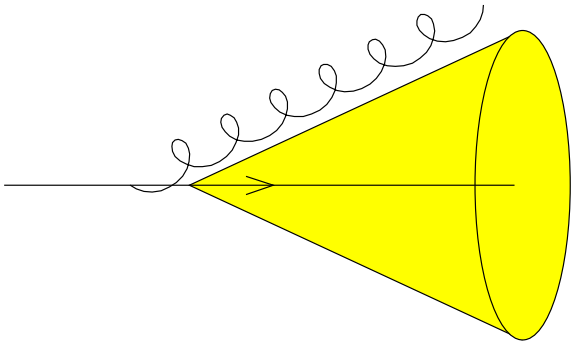
Simplest example: jet with cone algorithm

Close = in the cone

Far = outside the cone

Close $\Longrightarrow$ use PS

Far $\Longrightarrow$ use ME's



How about this one?

There are other obvious problems

- Partons emitted from far away parton which re-enter the cone
- Relative weight of ME's with different multiplicities unspecified
- What happens when changing cone size, or jet-finding algorithm?

The basic idea, however, is correct, and needs only be refined

I start discussing the approach of CKKW (Catani, Krauss, Kuhn, Webber), originally formulated for e^+e^- and then extended to hadronic collisions

Definition of interparton distance

It is conveniently suggested by a jet-finding algorithm, which *has nothing to do* with that used in the analysis. CKKW choose the k_T algorithm, where

$$\begin{aligned} d_i &= p_{Ti}^2 && \text{parton} - \text{beam distance} \\ d_{ij} &= \min\left(p_{Ti}^2, p_{Tj}^2\right) R_{ij}^2 && \text{parton} - \text{parton distance} \\ R_{ij}^2 &= (\varphi_i - \varphi_j)^2 + (\eta_i - \eta_j)^2 \end{aligned}$$

One then introduces a stopping value d_{ini} (typically, of $\mathcal{O}(10\text{ GeV})$)

Two partons i, j are close to each other, or one parton i is close to the beam (ie to an incoming parton), if

$$d_{ij} < d_{ini}\,, \qquad d_i < d_{ini}$$

If two partons are close, they can be recombined into one pseudo-parton. By iteration, one arrives at a set of partons and pseudo-partons all far away from each other

The prescription of CKKW

We are interested in $p_1 + p_2 \longrightarrow X + many\ jets$

1) Compute the probabilities

$$P_n^{(0)} = \sigma_n^{(0)} \Big/ \sum_{i=1}^{N} \sigma_i^{(0)}$$

with $\sigma_n^{(0)}$ the tree-level n-jet cross section for k_T-jets with resolution scale d_{ini}; use $\alpha_S = \alpha_S(d_{ini})$

$\sigma_n^{(0)} \longleftarrow$ n-parton matrix elements, with partons separated by d_{ini}

2) Choose a multiplicity $0 \leq \bar{n} \leq N$ with probability $P_{\bar{n}}^{(0)}$

3) Use the matrix elements $\mathcal{M}(p_1 + p_2 \rightarrow X + \bar{n}\ \mathrm{partons})$ to generate an $X + \bar{n}\ \mathrm{partons}$ kinematic configuration

We have now an $\bar{n}\ \mathrm{partons}$ unweighted hard event

4) Cluster the $\bar{n}$ partons using the k_T-algorithm, and find the *nodal values*

$$d_1 > d_2 > \ldots d_{\bar{n}} > d_{ini}$$

at which 1, 2, $\ldots$ $\bar{n}$ jets are resolved

The $\bar{n}$-parton configuration can now be depicted as a branching tree, with successive branchings at scales d_i

5) Apply a coupling reweighting factor

$$\alpha_S(d_1)\alpha_S(d_2)\ldots\alpha_S(d_{\bar{n}})\Big/\Big(\alpha_S(d_{ini})\Big)^{\bar{n}} \leq 1$$

Had we known the branching tree, we should have computed the ME's with these couplings

6) Apply a Sudakov reweighting factor

$$\Delta(d_{ini}, d_i)\Big/\Delta(d_{ini}, d_j)$$

to each line from a node with scale d_i to the next node with scale $d_j < d_i$. If the line is external, $d_j = d_{ini}$

7) Unweight again the hard configuration, ie accept it if the product of coupling and Sudakov reweighting factors is larger than a random number. Otherwise, start again from 2)

8) The accepted configuration is the initial condition for the parton shower. Branchings $a \longrightarrow bc$ in the shower *must be vetoed* if $d_{bc} > d_{ini}$

When an emission is vetoed it does not take place, but the shower scale for the next branching is recomputed as if the branching had occurred

In words: what happens in CKKW

- A jet clustering algorithm is used to separate the ME-dominated from the PS-dominated regions
- In the ME-dominated regions the ME's are corrected, as if they were generated (kinematically) by the PS. The Sudakov factors make sure that a PS would not emit extra partons wrt those entering the ME's
- In the PS-dominated regions the PS does its job, but it's prevented, owing to the veto, from emitting large-p_T, well-separated partons

If one goes through this considerable mess, he/she would like to be sure that in the end the predicted IR-safe observables will *independent* of the choice of the *jet-clustering algorithm*, and of d_{ini}

Accuracy in CKKW

A formal statement has been given only for jet observables in e^+e^- collisions, but is believed to be correct also for hadronic observables

- The separation of the ME- and PS-dominated regions introduces a dependence

$$\sigma_n \sim \alpha_S^{n-2} \sum_k a_k \alpha_S^k \log^{2k} \frac{d_{ini}}{s}$$

  in the n-jet cross section

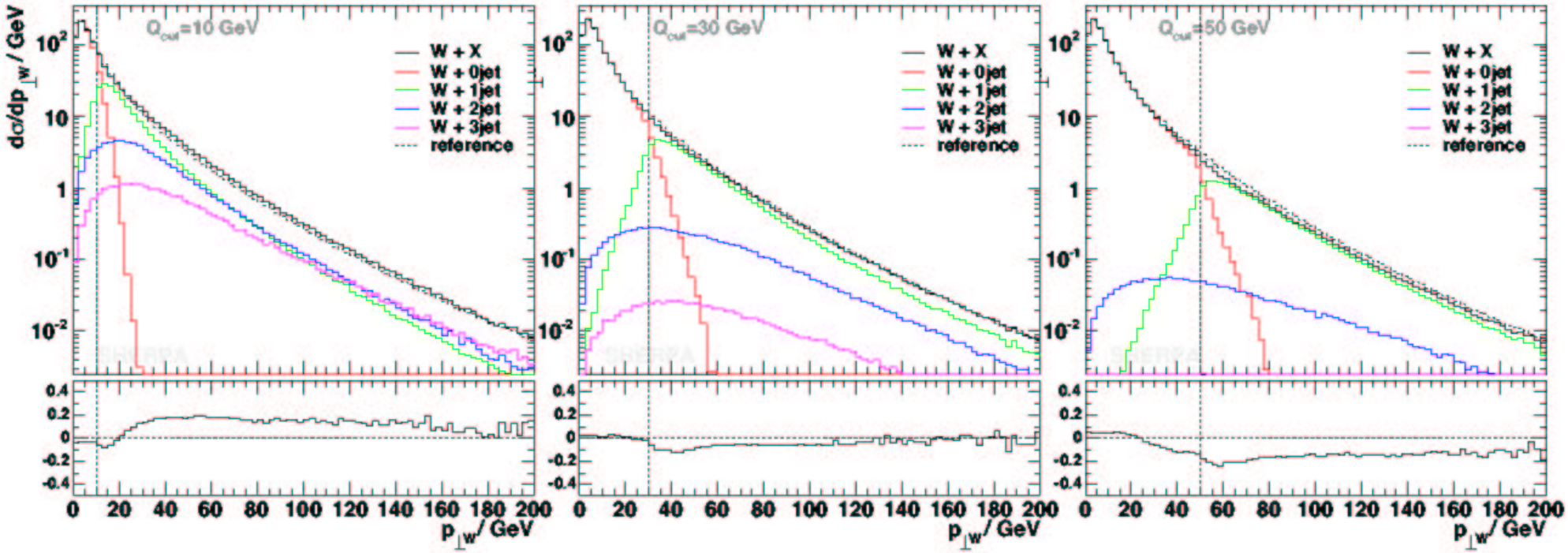
- At the end of the CKKW procedure, this is reduced to

$$\sigma_n \sim \alpha_S^{n-2} \left(\left(\frac{d_{ini}}{s} \right)^a + \sum_k b_k \alpha_S^k \log^{2k-2} \frac{d_{ini}}{s} \right)$$

  ie it is cancelled to NLL accuracy

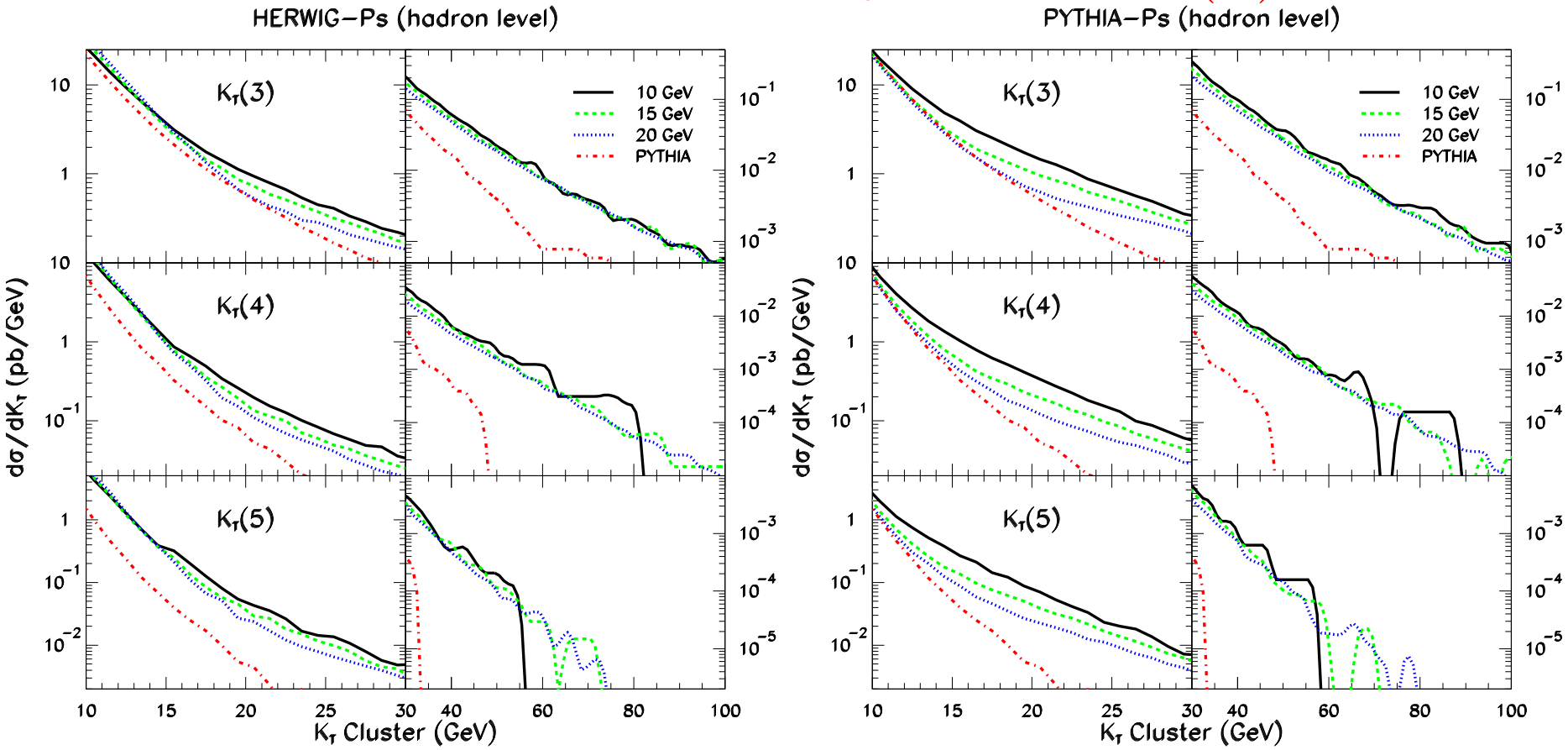
Is this good enough?

Test case: W+jets $\longrightarrow$ $p_T(W)$



- Here $Q_{cut} \equiv d_{ini}$
- The larger d_{ini}, the smaller the impact of high-multiplicity ME's
- A 20% bias is acceptable, and can be used to tune to the data

Test case: W+jets $\longrightarrow$ $k_T(n)$



- $k_T(n)$ is the value of the resolution scale at which an n-jet configuration becomes an $(n-1)$-jet one
- The dependence on d_{ini} is of the same order as that for $p_T(W)$
- Clear improvements wrt standard parton showers (black vs red lines)

CKKW is an *interpolation* procedure between a PS and the ME's. It defines a fremework, but there is a lot of freedom left, which can be used to reduce unphysical biases on observables

- Clustering algorithm and momentum-recombination scheme
- Sudakov definitions
- Scale choices
- Corrections due to $N < \infty$ (highest-multiplicity ME)

Never forget that the d_{ini} dependence can be reduced but not eliminated. So make sure, before embarking in extensive physics studies, that d_{ini} is properly chosen, and the biases are small

An alternative approach: MLM matching

Proposed by Mangano. This is now used in ALPGEN

1) Generate hard unweighted events with the ME's, imposing

$$E_T > E_T^{min}\,, \qquad R_{ij} > R_{min}$$

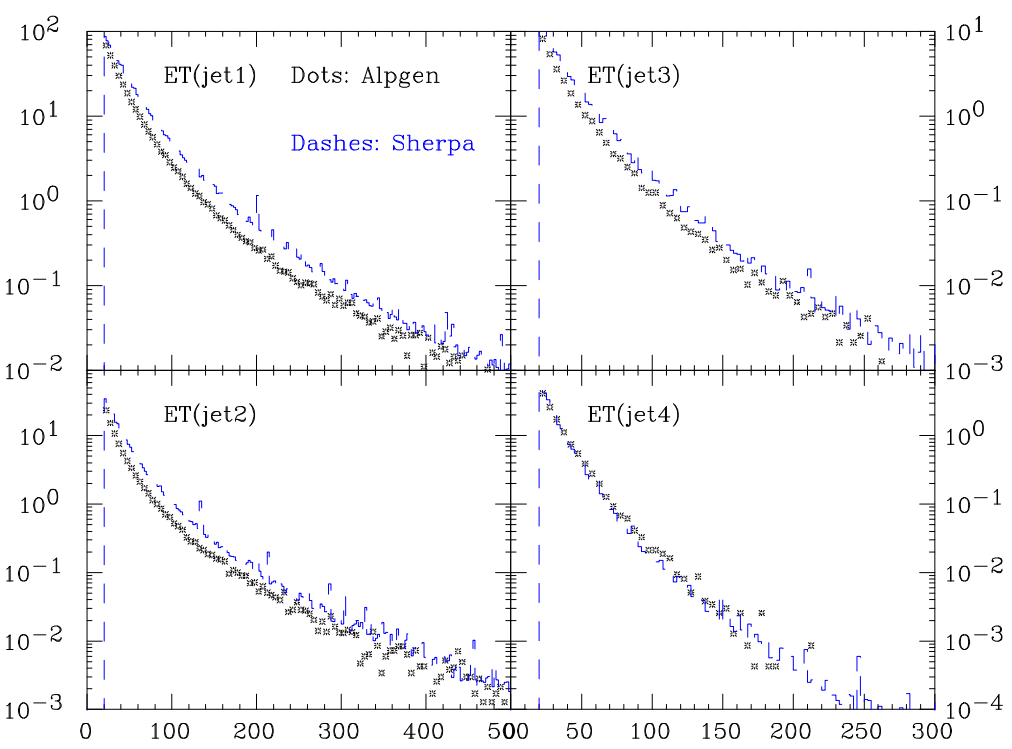
2) Define a branching-tree structure as done in CKKW, but using colour flows extracted from the ME calculations

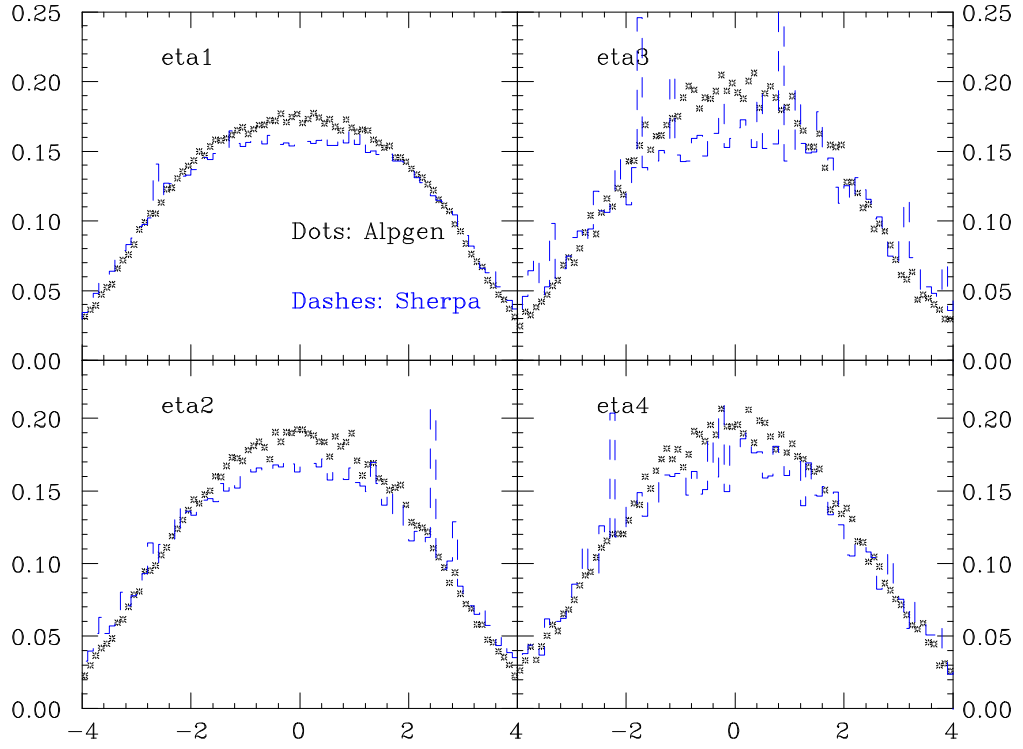
3) Compute α_S at the nodal values found in 2), but do not apply any Sudakov reweight factors

4) Shower the hard event, without applying any veto; when done, find jets using a cone algorithm with (E_T^{min}, R_{min})

5) Require jets be matched to hard partons. Events with more jets than hard partons are rejected, except for the highest-multiplicity ME's

CKKW vs MLM for W+jets





- Differences in leading-jets distributions slightly larger than for $p_T(W)$
- Differences may be due to the matching algorithm, the shower (SHERPA for CKKW, HERWIG for MLM), or a combination of the two

Summary on Matrix Element Corrections

- Various approaches and implementations on the market; the use of standard PSMC for multi-jet studies cannot be justified any longer
- Overall, existing approaches are robust, and lead to tolerably small dependence on unphysical parameters, if these are cleverly chosen
- There are discrepancies among the different approaches; there is a lot of flexibility in implementation details
- Tuning to data is strongly recommended, and anyhow necessary to figure out the correct normalization: these are LO QCD computation!

Matching parameter systematics must be assessed

Try different codes and implementations

How to improve Monte Carlos?

The key issue is to go beyond the collinear approximation

$\Longrightarrow$ use exact matrix elements of order higher than leading

Which ones?

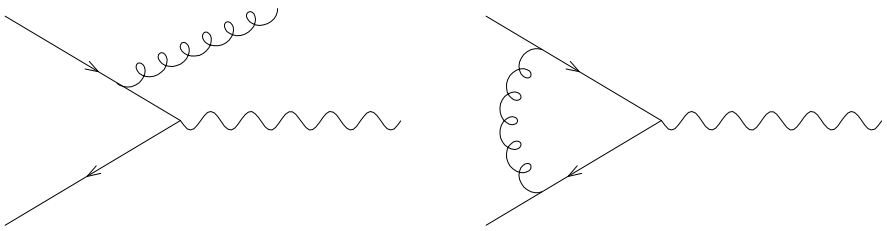
There are two possible choices, that lead to two vastly different strategies:

- Matrix Element Corrections
- NLOwPS

I now discuss the second option: NLOwPS

NLOwPS

Compute all the NLO diagrams (and only those) before starting the shower.
Example: W production



.....

Then use the kinematics configurations generated in this way as initial conditions for the shower

Problems

- Double counting (the shower can generate *some of* the same diagrams)
- The diagrams are divergent

- The problems are *almost* identical to those encountered in MEC. The solution, however, is completely different

Why NLO corrections?

Among the many good reasons, let me mention those that are likely to have the largest impact on phenomenology

- *K factors*: the only way to compute them consistently (i.e., no bin-by-bin reweighting), and to use this information in detector simulations
- *Shapes* of observables do have NLO corrections $\Longrightarrow$ impact on acceptances and physics studies in general
- *Theoretical systematics*: scale dependence can be computed – this procedure is either meaningless or very difficult to perform with standard Monte Carlos
- *Predictive power*: the MC can become a tool for "precision" physics

NLOwPS versus MEC

- Why is it difficult to incorporate matrix elements into MC's?

  The problem is a serious one: KLN cancellation is achieved in standard MC's through unitarity, and embedded in Sudakovs. This is no longer possible: IR singularities do appear in hard ME's

So the trick used in MEC is: avoid IR singularities cutting them off by hand, and correct for the bias so introduced. This is unavoidable, since only virtual diagrams can cancel the divergences of real matrix elements

But we do want to include virtual diagrams into NLOwPS. This makes all the difference wrt MEC. We expect that:

- There is no need to introduce cutoffs
- An increased complexity in the computations

NLOwPS is a relatively new field

Although somewhat undermanned, there is a lot of ongoing activity

- First hadronic code: Φ-veto (Dobbs, 2001)
- First general solution: MC@NLO (SF, Webber, 2002)
- Automated computations of ME's: grcNLO (GRACE group, 2003)
- Absence of negative weights: POWHEG (Nason, 2004)
- Showers with high log accuracy in ϕ_6^3 (Collins, Zu, 2002–2004)
- Proposals for $e^+e^- \longrightarrow jets$ (Soper, Krämer, Nagy, 2003–2005)
- SCET (Bauer, Schwartz, 2006)
- Use of dipoles (Giele, Kosower, Skands, in progress)

Only MC@NLO and, to a more limited extent so far, POWHEG, have resulted in ready-to-use computer programs

Matching NLO with MC: NLOwPS

What do we want? Let's *define* it:

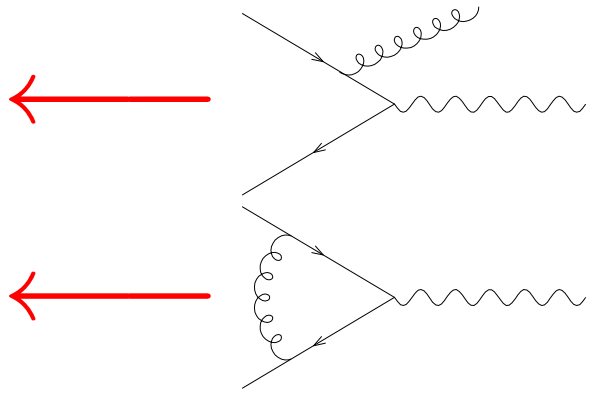
- Total rates are accurate to NLO
- Hard emissions are treated as in NLO computations
- Soft/collinear emissions are treated as in MC
- NLO results are recovered upon expansion of NLOwPS results in α_S. In other words: there is no double counting in NLOwPS
- The matching between hard- and soft/collinear-emission regions is smooth
- The output is a set of events, which are fully exclusive
- MC hadronization models are adopted

Note: in general, negative-weight events can be generated

NLO and MC computations

NLO cross section (based on subtraction)

$$\left(\frac{d\sigma}{dO}\right)_{subt} = \sum_{ab} \int dx_1\, dx_2\, d\phi_{n+1}\, f_a(x_1) f_b(x_2) \times$$
$$\Big[\delta(O - O(2 \to n+1))\mathcal{M}_{ab}^{(r)} +$$
$$\delta(O - O(2 \to n))\left(\mathcal{M}_{ab}^{(b,v,c)} - \mathcal{M}_{ab}^{(c.t.)}\right)\Big]$$



MC

$$\mathcal{F}_{\text{MC}} = \sum_{ab} \int dx_1\, dx_2\, d\phi_n\, f_a(x_1) f_b(x_2)\, \mathcal{F}_{\text{MC}}^{(2\to n)} \mathcal{M}_{ab}^{(b)}$$

- Matrix elements $\longrightarrow$ normalization, hard kinematic configurations
- δ-functions, $\mathcal{F}_{\text{MC}}^{(2\to n)} \equiv$ showers $\longrightarrow$ observable final states

NLO + MC $\longrightarrow$ NLOwPS?

Naive first try: use the NLO kinematic configurations as initial conditions for showers, rather than for directly computing the observables

- $\delta(O - O(2 \to n)) \longrightarrow$ start the MC with n "hard" emissions: $\mathcal{F}_{\text{MC}}^{(2\to n)}$
- $\delta(O - O(2 \to n+1)) \longrightarrow$ start the MC with $n+1$ "hard" emission: $\mathcal{F}_{\text{MC}}^{(2\to n+1)}$

$$\mathcal{F}_{\text{naive}} = \sum_{ab} \int dx_1\, dx_2\, d\phi_{n+1}\, f_a(x_1) f_b(x_2) \times \left[\mathcal{F}_{\text{MC}}^{(2\to n+1)} \mathcal{M}_{ab}^{(r)} + \mathcal{F}_{\text{MC}}^{(2\to n)} \left(\mathcal{M}_{ab}^{(b,v,c)} - \mathcal{M}_{ab}^{(c.t.)} \right) \right]$$

It doesn't work:

- Cancellations between $2 \to n+1$ and $2 \to n$ contributions occur after the shower: hopeless from the practical point of view; and, unweighting is impossible
- $(d\sigma/dO)_{naive} - (d\sigma/dO)_{NLO} = \mathcal{O}(\alpha_S)$. In words: double counting

MC@NLO: formalism (SF, Webber (2002))

The naive prescription doesn't work: MC evolution results in spurious NLO terms

$\longrightarrow$ *Eliminate the spurious NLO terms "by hand": MC counterterms*

- The generating functional is

$$\mathcal{F}_{\text{MC@NLO}} = \sum_{ab} \int dx_1\, dx_2\, d\phi_{n+1}\, f_a(x_1) f_b(x_2) \times \left[\mathcal{F}_{\text{MC}}^{(2\to n+1)} \left(\mathcal{M}_{ab}^{(r)} - \mathcal{M}_{ab}^{(\text{MC})} \right) + \mathcal{F}_{\text{MC}}^{(2\to n)} \left(\mathcal{M}_{ab}^{(b,v,c)} - \mathcal{M}_{ab}^{(c.t.)} + \mathcal{M}_{ab}^{(\text{MC})} \right) \right]$$

$$\mathcal{M}_{\mathcal{F}(ab)}^{(\text{MC})} = \mathcal{F}_{\text{MC}}^{(2\to n)} \mathcal{M}_{ab}^{(b)} + \mathcal{O}(\alpha_S^2 \alpha_S^b)$$

There are *two* MC counterterms: they eliminate the spurious NLO terms due to the branching of a final-state parton, and to the non-branching probability

Let's have a look at the weight functions

$$w_{\mathbb{H}} = \mathcal{M}^{(r)}_{ab} - \mathcal{M}^{(\mathrm{MC})}_{ab}$$

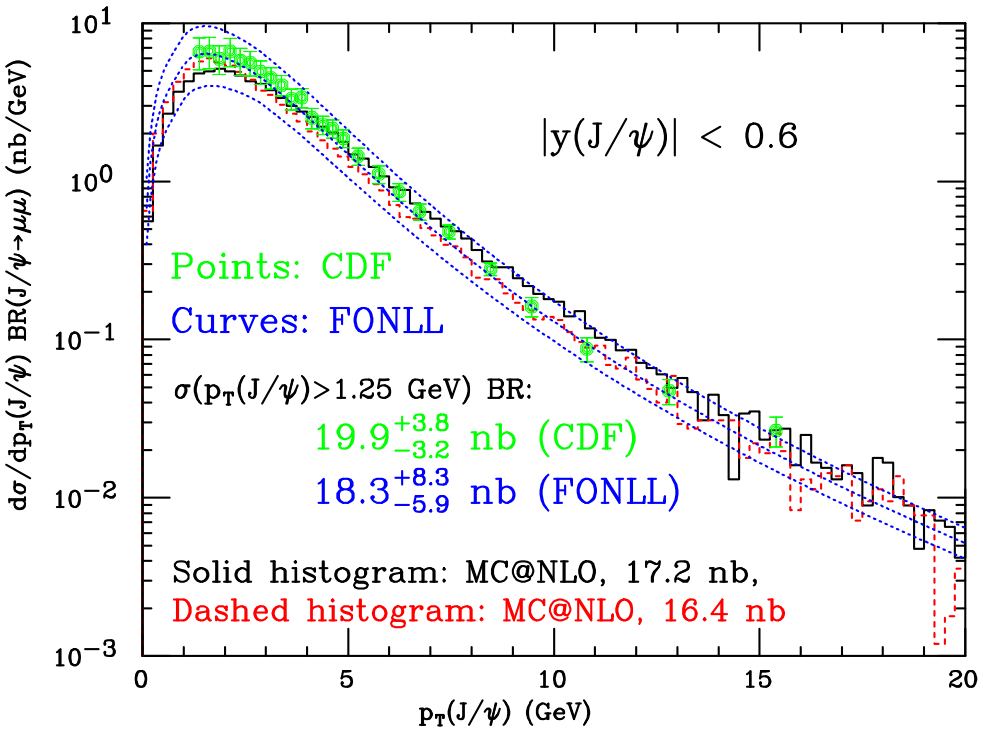
$$w_{\mathbb{S}} = \mathcal{M}^{(b,v,c)}_{ab} - \mathcal{M}^{(c.t.)}_{ab} + \mathcal{M}^{(\mathrm{MC})}_{ab}$$

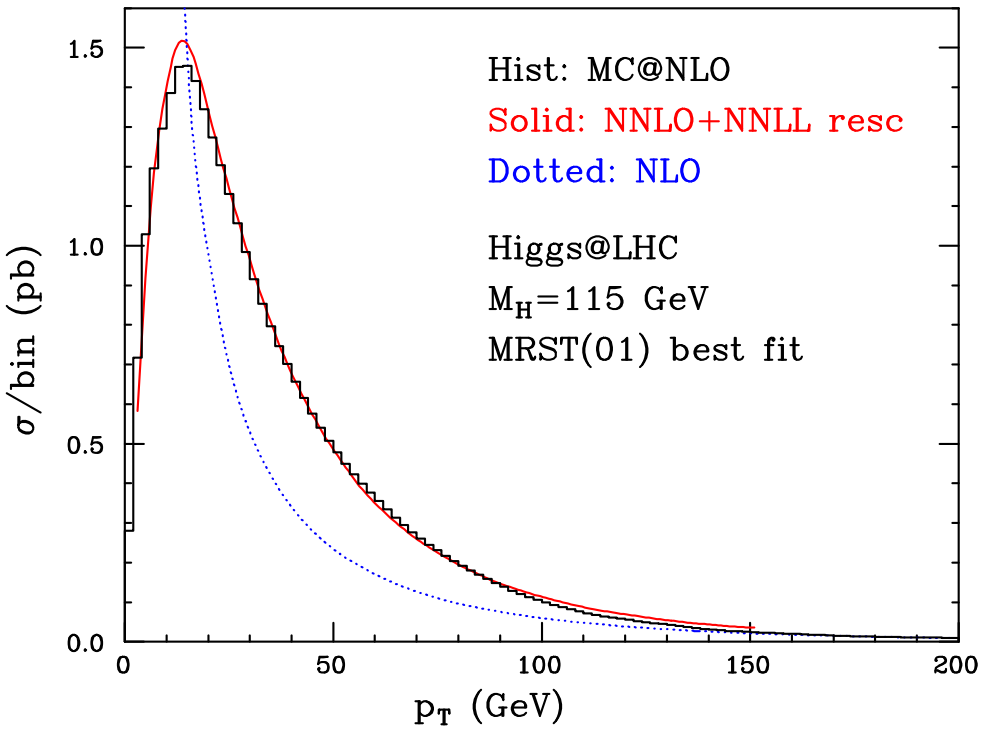
These are finite (i.e. don't diverge) for any phase-space point!

The MC provides local, observable-independent, counterterms $\Longrightarrow$ greater numerical stability, unweighting possible

By solving the problem of double counting, one also cancels the singularities at the level of *hard matrix elements* (i.e., with no reference to a specific observable). Configurations with different final states can then be showered independently

MC@NLO results





- Implements several hadroproduction processes; used by experimenters
- Left plot: we have another *predictive* way to show that b production is under control
- Excellent agreement with matched analytic computation of formally higher log accuracy

POWHEG (Nason (2004))

The proposal stems from the following *theorem*

A shower can be defined which has the largest-p_T emission at the first branching, and is equivalent (to LL accuracy) to the angular-ordered shower

Such a shower goes through the following steps

- Do the first branching as usual. It will define branching variables z and $t_0 < t < t_{ini}$, and a p_T
- Do a shower from each of the two legs from the first branching, with upper scales $z^2 t$ and $(1-z)^2 t$, and veto all emissions with a relative transverse momentum larger than p_T (vetoed showers)
- Do a further p_T-vetoed shower, with upper scale t_{ini} and lower scale t (vetoed truncated shower) $\longrightarrow$ restores coherence

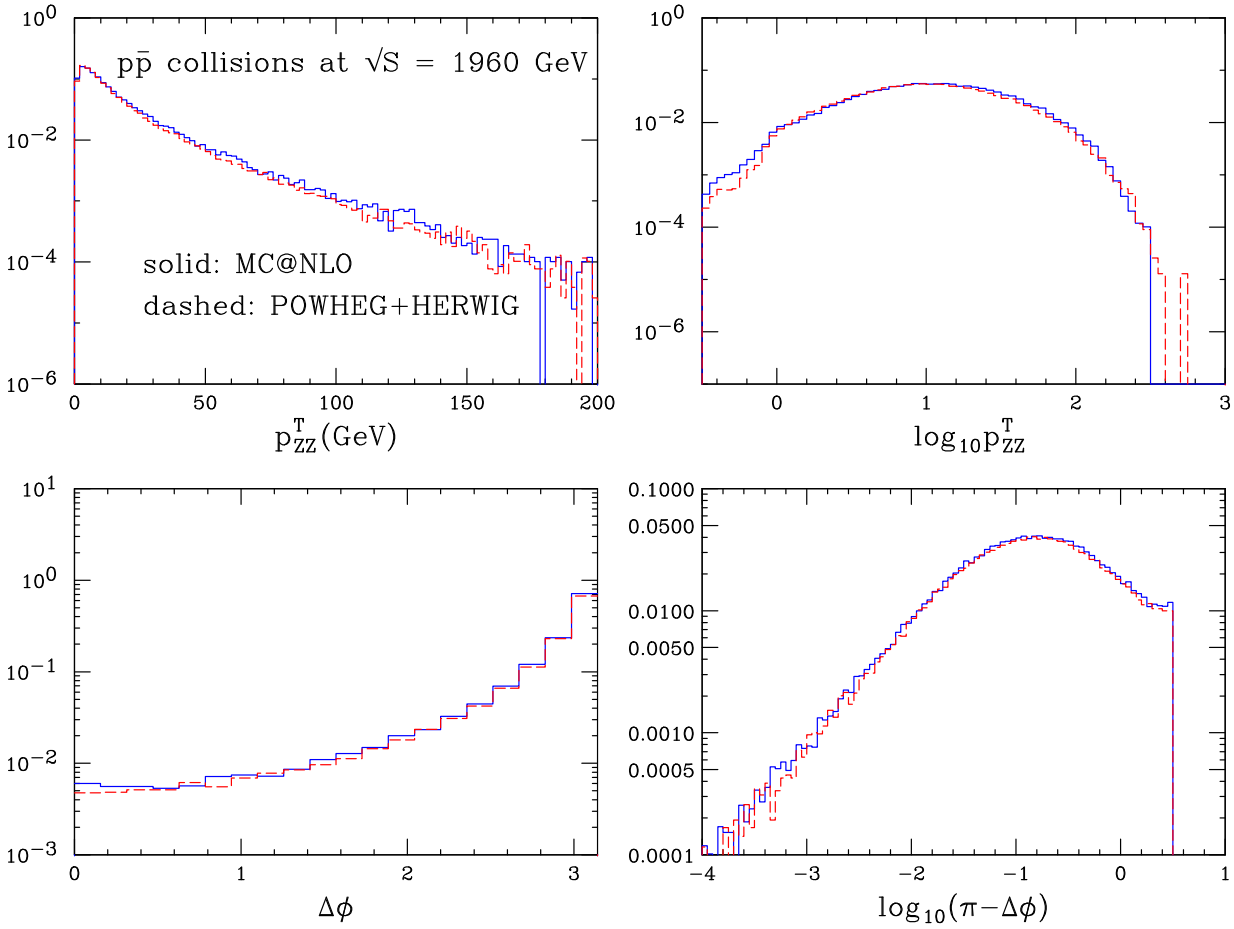
Proposal for POWHEG

The basic idea builds upon the previous theorem

Exponentiate the full real corrections into a Sudakov, and use that for the first branching. Then proceed as before, with vetoed and vetoed truncated showers

- MC must be capable of handling vetoed truncated showers;
  not the case at the moment
- May use a separate package for the vetoed truncated showers
- Beyond-LL structure changed: need for re-tuning?
- |MC@NLO-pMC@NLO|=$\mathcal{O}(\alpha_S^2)$ $\longleftarrow$ how large α_S^2 terms?

Proof-of-concept for $pp \to ZZ$ (Nason, Ridolfi), without vetoed truncated showers.
General formulation and vetoed truncated showers under way (SF, Nason, Oleari)



- ZZ hadroproduction now implemented in POWHEG
- Vetoed-truncated showers not yet available
- Should not matter much for inclusive observable in ZZ: excellent agreement with MC@NLO

General formulation and other implementations are under way

Summary on NLOwPS

- Event Generators including the typical benefits of NLO computations now exist
- Sensible predictions for total rates and large-p_T tails; it is meaningful to study scale dependence in a realistic experimental environment
- Absence of matching parameters, matching systematics (which may be introduced if needed). Increased predictive power wrt MEC
- Multileg NLO results are difficult to obtain. At present, MEC and NLOwPS are therefore complementary
- Next steps: more NLOwPS formalisms, extension of CKKW-like procedures at the NLO

Conclusions

This is the decade of hadron colliders – and the most exciting time in high-energy particle physics after 1984. We can't tell what lies ahead, and thus we must have reliable predictions for what we believe we know

QCD theorists have responded remarkably well, with major breakthroughs in the past few years *(many topics seemed unrealistic 5 years ago)*

I've tried to give you an overview on selected topics which will presumably have a strong impact on the LHC programme

- Fixed-order computations at tree-level, NLO and NNLO
- Resummed and matched computations
- PDFs
- Monte Carlos of the new generation

There are of course so many things I did not even mention

- Twistors
- Soft and semihard physics
- Small-x physics
- Quarkonia (NRQCD)
- Power corrections
- Diffraction

... therefore: are we ready?

There is a lot of work to be completed, which is supposed to be relevant for LHC physics, both in the perturbative and non-perturbative domains

But a lot has already been achieved and, more importantly, I am confident that, thanks to what we have understood, we'll be able to solve the problems which is reasonable to expect from the LHC

Thanks for your attention

Good luck with your searches...