CERN Academic Training Program, 2007

Quantum Chromo-Dynamics and high energy nuclear collisions

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Quarks and the Standard Model

QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

1/2 of all “elementary” particles of the Standard Model are not observable;

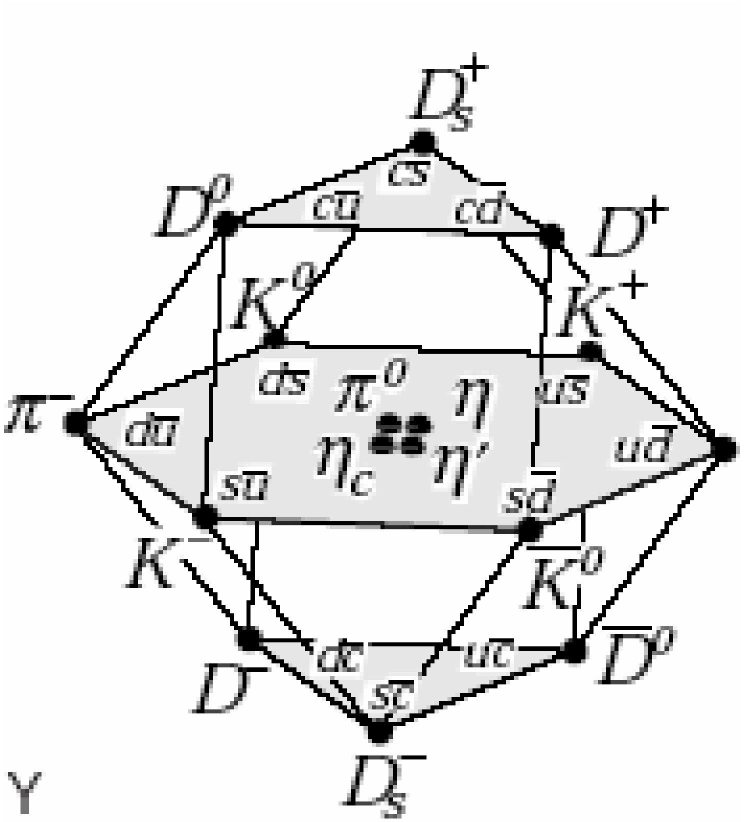
they are confined within hadrons by “color” interactions

Outline

- Quantum Chromo-Dynamics -
  the theory of strong interactions
- QCD at high energies
- QCD at high temperatures
- A glimpse of RHIC results
- New facets of QCD at RHIC and LHC:
  - Color Glass Condensate & thermalization
  - (strongly coupled) Quark-Gluon Plasma
  - topological effects at finite T

What is QCD?

QCD = Quark Model + Gauge Invariance



local gauge transformation:

$$q(x) \to \exp\left(i\omega_a(x)T^a\right)\; q(x),$$

$$\left[T^a, T^b\right] = i f^{abc} T_c$$

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QCD and the origin of mass

$$\mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + \sum_f \bar{q}^a_f (i\gamma_\mu D_\mu - m_f) q^a_f;$$

$$D_\mu = \partial_\mu - igA^a_\mu t^a$$

gluons

quarks

Invariant under scale ($x \rightarrow \lambda x$) and chiral Left $\longleftrightarrow$ Right

transformations in the limit of massless quarks

Experiment: u,d quarks are almost massless…

… but then… all hadrons must be massless as well!

Where does the “dark mass” of the proton come from?

QCD and quantum anomalies

$$\mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + \sum_f \bar{q}^a_f (i\gamma_\mu D_\mu - m_f) q^a_f;$$

Classical scale invariance is broken by quantum effects:

scale anomaly

$$\theta^\mu_\mu = \frac{\beta(g)}{2g} G^{\alpha\beta a} G^a_{\alpha\beta} + \sum_q m_q \bar{q} q$$

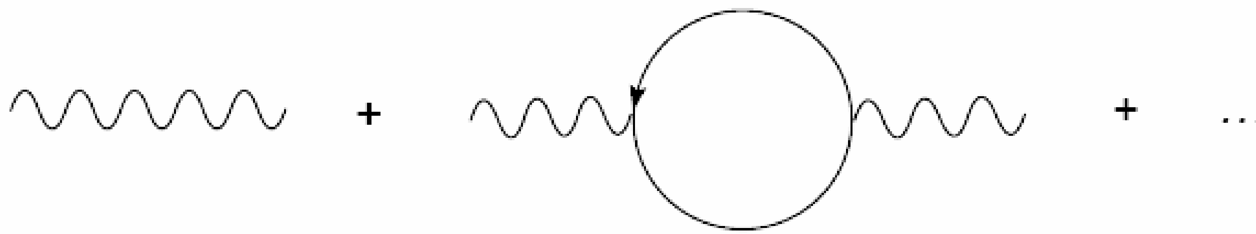
trace of the energy-momentum tensor

“beta-function”; describes the dependence of coupling on momentum

Hadrons get masses ⟺ coupling runs with the distance

Running coupling and renormalization group

QED:



Running coupling:
(screening)

$$e^2(r) = \frac{e^2(r_0)}{1 + \frac{2e^2(r_0)}{3\pi} \ln \frac{r}{r_0}}$$

Two paradoxes:

1. At large distances $r \gg r_0$ the "bare" coupling is unobservable! (only a universal, ~1/N coupling is)
2. In the local limit $r_0 \to 0$ if the bare coupling is finite, the interaction vanishes! ("Moscow zero charge")

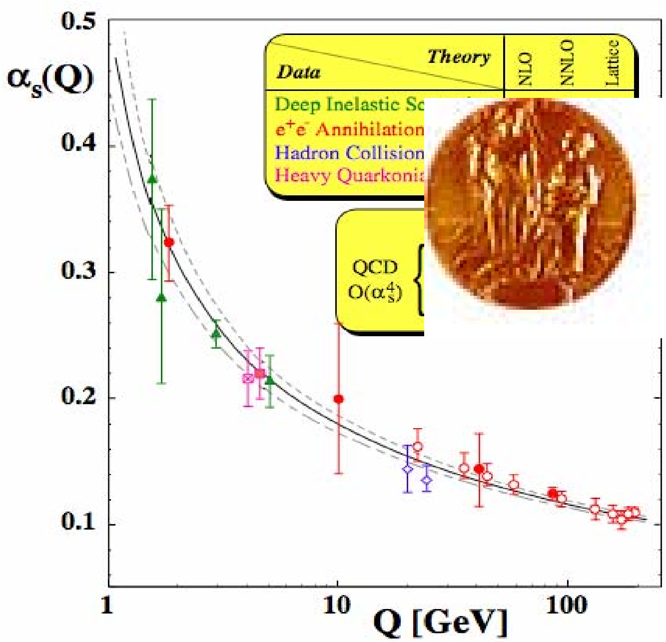
The difficulties of QED

“Zero charge” problem: the presence of screening in QED does not allow to reconcile the presence of interactions with the local limit of the theory
(Landau: strong interactions cannot be described by a field theory)

Ways out?

1. QED is not a fundamental theory, has to be modified at short distances; or
2. At strong coupling (short distances) pQED is misleading, and a true answer is entirely non-perturbative

Asymptotic Freedom



At short distances,
the strong force becomes weak
(anti-screening) -
one can access the "asymptotically free" regime in hard processes

and in super-dense matter
(inter-particle distances ~ 1/T)

$$\alpha_s(Q) \simeq \frac{4\pi}{b\ln(Q^2/\Lambda^2)}$$

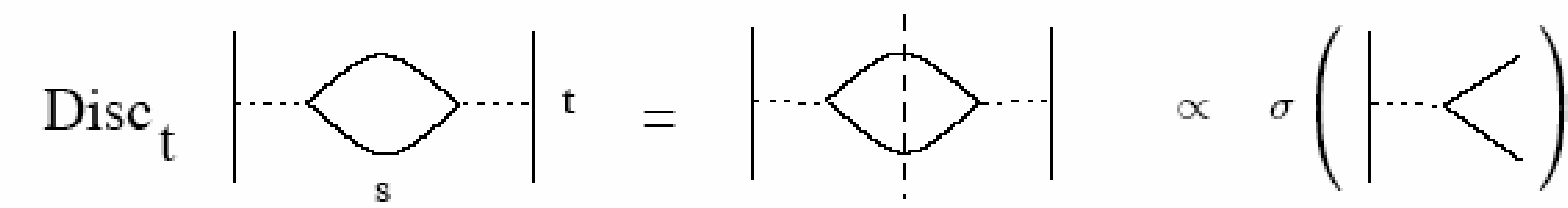
number of colors

number of flavors

$$b = (11N_c - 2N_f)/3$$

Running coupling in QCD (Coulomb potential)

Spectral representation in the t-channel: $V(R) = \sum_m \sigma(m^2)\,\frac{\exp(-mR)}{R}$



If physical particles can be produced (positive spectral density), then unitarity implies <u>screening</u>

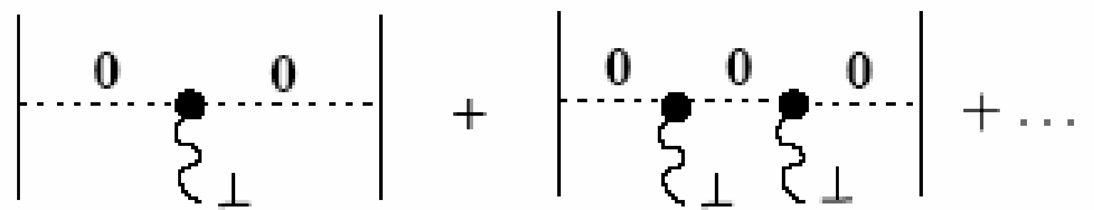
$$\left\{ \frac{d\,\alpha_s^{-1}(R)}{d \ln R} \right\}^{\text{phys}} \propto \frac{1}{3} N + \frac{2}{3} n_f$$

Gluons (transverse) Quarks

Coulomb potential in QCD - II

Missing non-Abelian effect: instantaneous Coulomb exchange dressed by (zero modes of) transverse gluons

$$\sum_n \left[0 + \perp \rightarrow 0\right]^n =$$



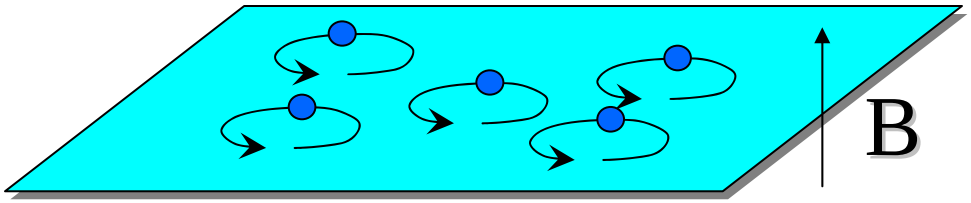
Negative sign
(the shift of the ground level
due to perturbations - unstable vacuum!):

$$\delta E \equiv E - E_0 = \sum_n \frac{|\langle 0|\delta V|n\rangle|^2}{E_0 - E_n} < 0$$

Anti-screening

$$\left\{ \frac{d\,\alpha_s^{-1}(R)}{d\ln R} \right\}^{\text{stat}} \propto -4N$$

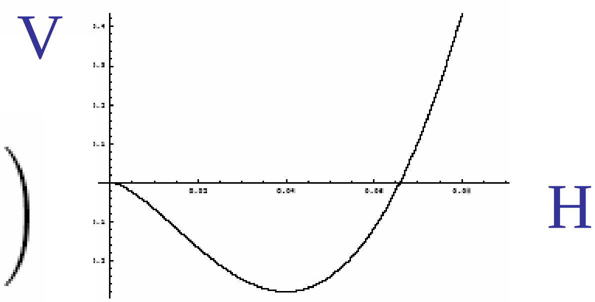
Asymptotic freedom and Landau levels of 2D parton gas



The effective potential: sum over 2D Landau levels

$$V_{\rm pert}(H) = \frac{g\,H}{4\,\pi^2} \int dp_z \sum_{n=0}^{\infty} \sum_{s_z=\pm 1} \sqrt{2\,g\,H\,(n+1/2-s_z)+p_z^2}.$$

Paramagnetic response of the vacuum:



$${\rm Re}\,V_{\rm pert}(H) = \frac{1}{2}\,H^2 + (g\,H)^2\,\frac{b}{32\,\pi^2}\left(\ln\frac{g\,H}{\mu^2} - \frac{1}{2}\right)$$

1. The lowest level n=0 of radius $\sim (gH)^{-1/2}$ is unstable!

2. Strong fields ⟷ Short distances

QCD and the classical limit

Classical dynamics applies when the action $S = \int d^4x \ \ \mathcal{L}(x)$ is large in units of the Planck constant (Bohr-Sommerfeld quantization)

$$\frac{S_{QCD}}{\hbar} \sim \frac{1}{g^2\hbar} \int d^4x \ \mathrm{tr}\ G^{\mu\nu}(x) G_{\mu\nu}(x) \gg 1$$

(equivalent to setting $\hbar \longrightarrow 0$)

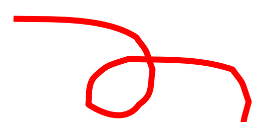
=> Need weak coupling and strong fields

$$A^2 \ll \frac{p^2}{g^2}$$

weak field

$$D_\mu = \partial_\mu - igA_\mu^a t^a$$

$$A^2 \sim \frac{p^2}{g^2}$$



strong field

Renormalization group and the effective action

RG constraints the form of the effective action:

$$\mathcal{L}_{\mathrm{eff}} = -\frac{1}{4\bar{g}^2(t)}\, G^2, \quad t \equiv \ln\left(\frac{G^2}{\Lambda^4}\right)$$

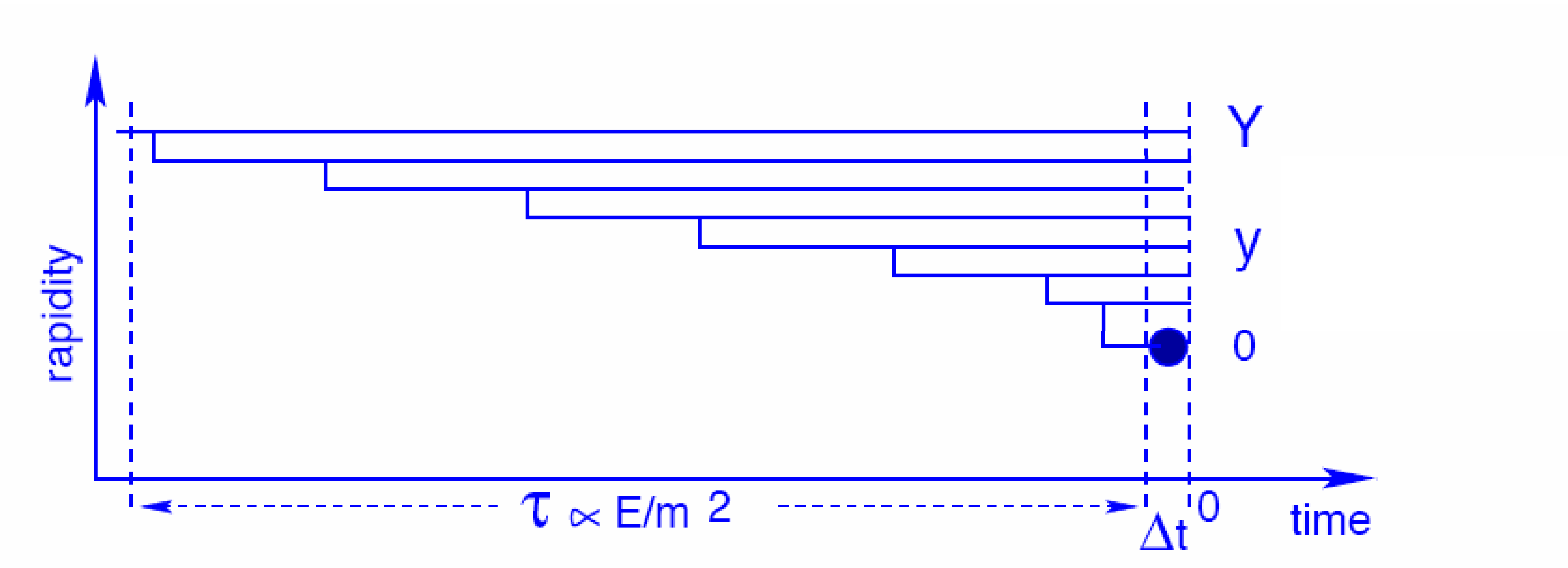
the coupling is defined through

$$t = \int_g^{\bar{g}(t)} \frac{dg}{\beta(g)};$$

At large t (strong color field),

$$\frac{1}{\bar{g}^2(t)} \sim t + \ldots \quad \text{and} \quad \mathcal{L}_{\mathrm{eff}} \sim G^2\, \ln\left(\frac{G^2}{\Lambda^4}\right)$$

The space-time picture of high-energy interactions in QCD

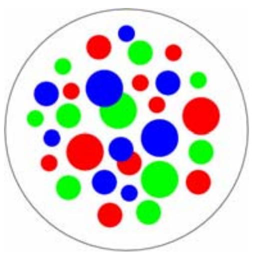


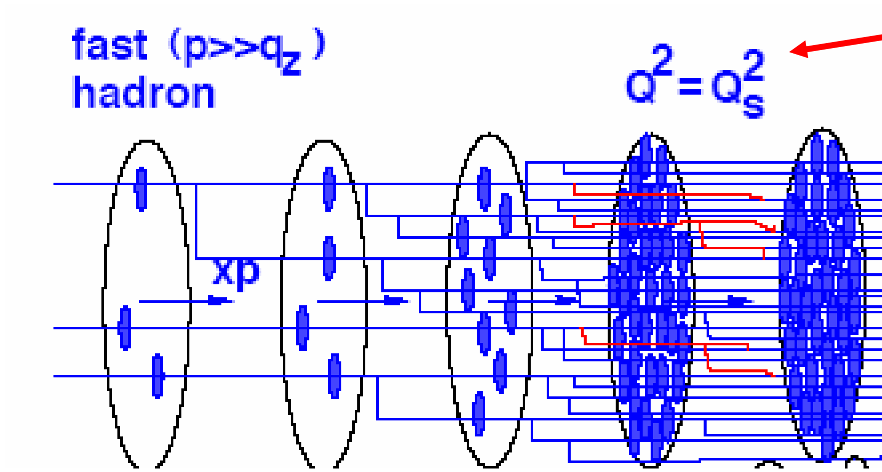
1. Fast (large y) partons live for a long time;
2. Parton splitting probability is ~ α_s y - not small!

Building up strong color fields: small x (high energy) and large A (heavy nuclei)

Bjorken x : the fraction of hadron's momentum carried by a parton; high energies s open access to small $x = Q^2/s$

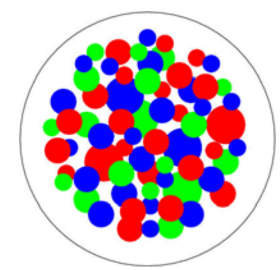
Large x





the boundary of non-linear regime: partons of size $1/Q > 1/Q_s$ overlap

small x

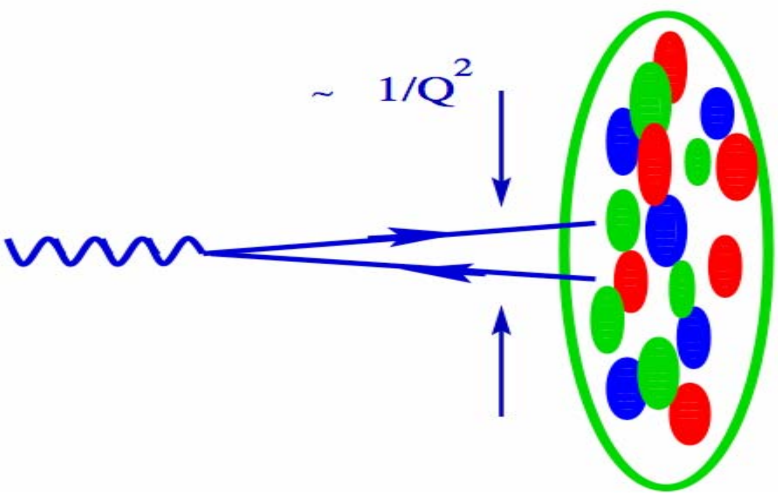


Because the probability to emit an extra gluon is ~ $\alpha_s \ln(1/x) \sim 1$, the number of gluons at small x grows; the transverse area is limited

→ transverse density becomes large

Strong color fields in heavy nuclei

At small Bjorken x, hard processes develop over large longitudinal distances $l_c \sim \frac{2\nu}{Q^2} = \frac{1}{mx}$



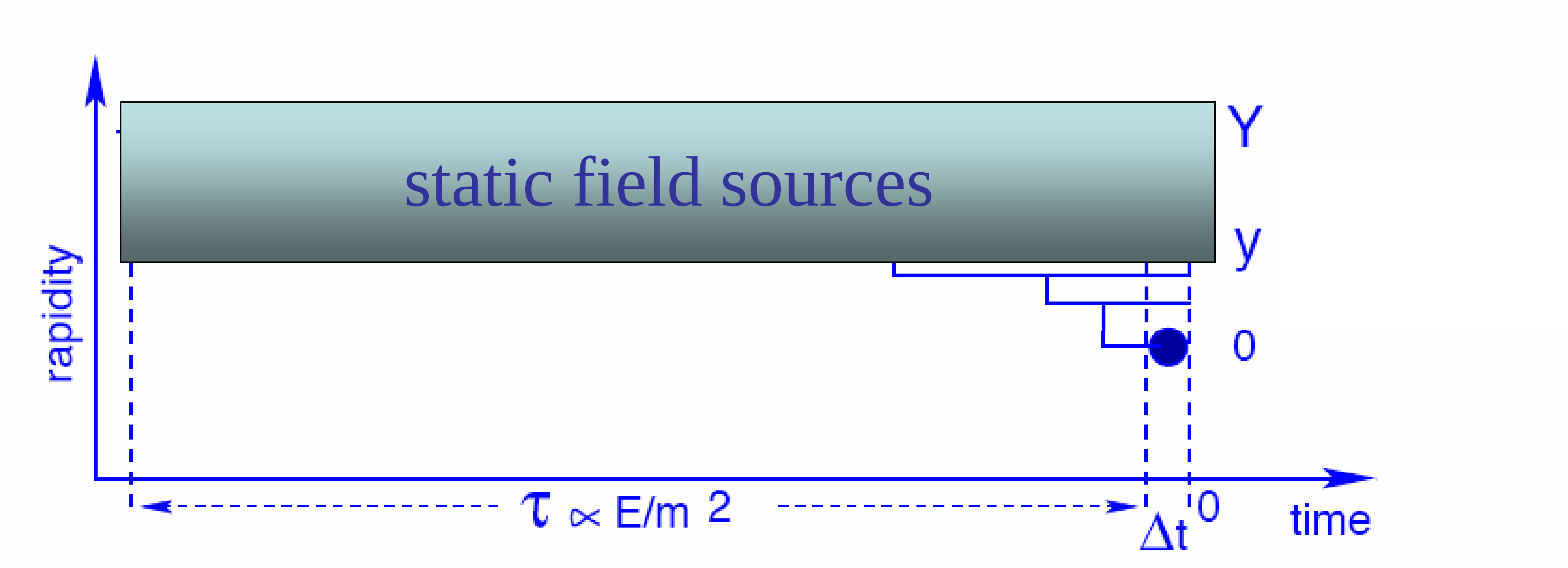
Density of partons in the transverse plane as a new dimensionful parameter Q_s ("saturation scale")

Gribov, Levin, Ryskin

All partons contribute coherently => at sufficiently small x and/or large A strong fields, weak coupling!

McLerran, Venugopalan

The origin of classical background field



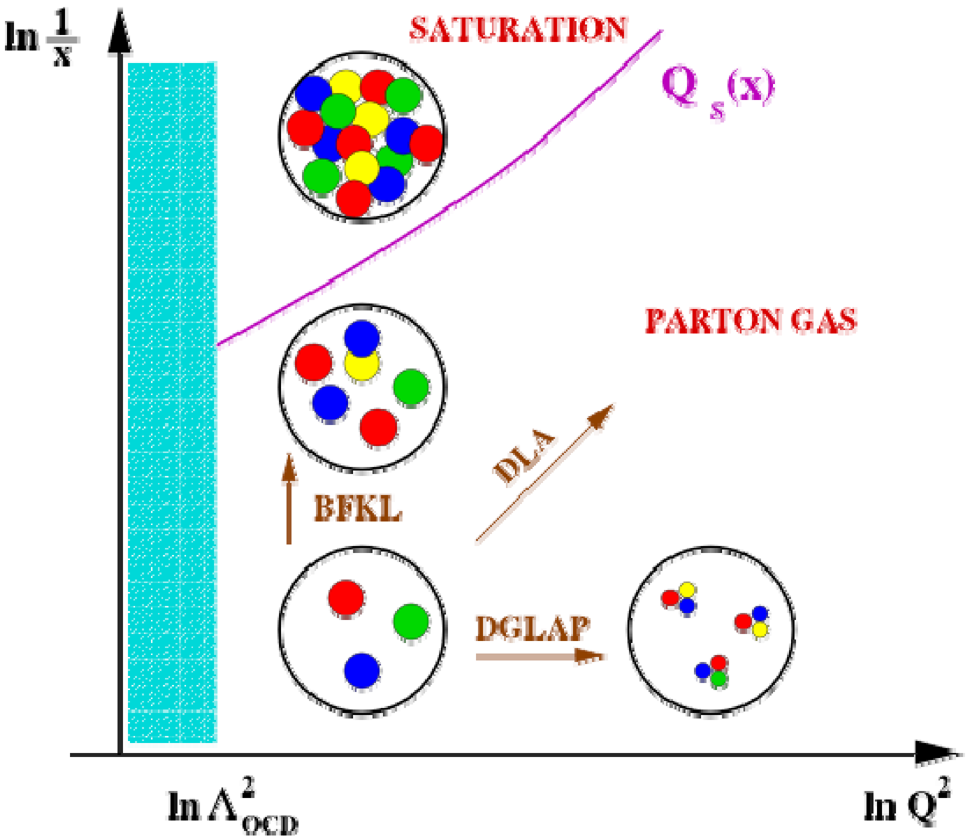
Gluons with large rapidity and large occupation number act as a background field for the production of slower gluons

"Color Glass Condensate"

Non-linear QCD evolution and population growth

time $t \to \ln \frac{1}{x} \equiv y$ rapidity

Color glass condensate



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Linear evolution: T. Malthus (1798)

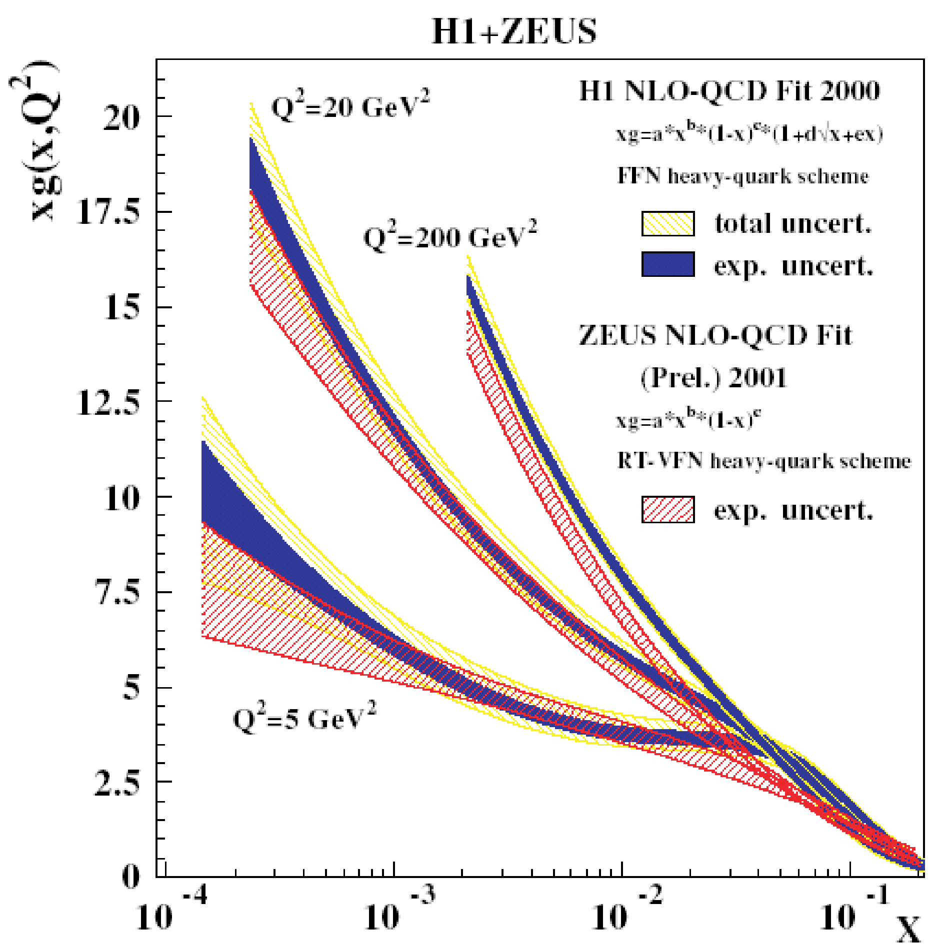
$$\frac{d}{dt} N(t) = r\ N(t)$$

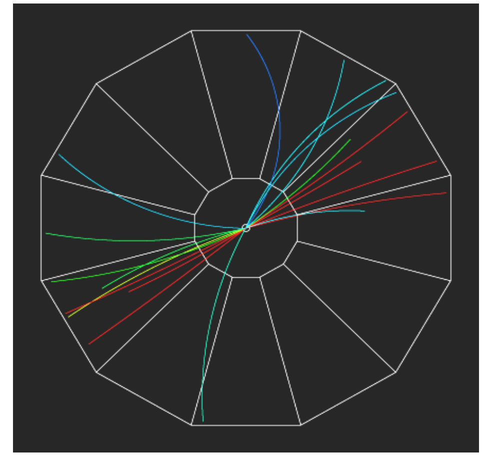
r - rate of maximum population growth

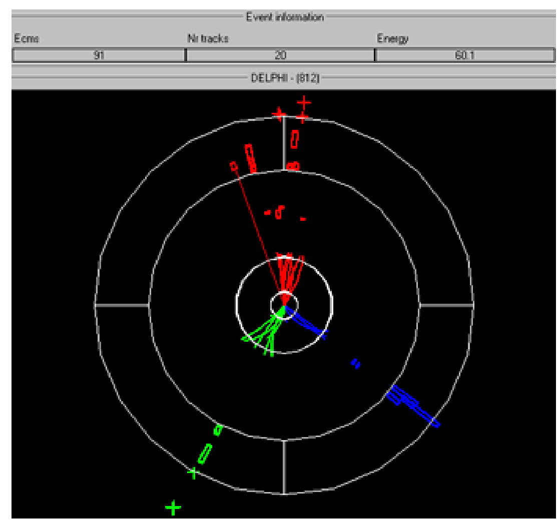
$$N(t) = N_0\ \exp(r\ t)$$

Unlimited growth!

Resolving the gluon cloud
at small x and short distances ~ $1/Q^2$







number of gluons

“jets”: high momentum partons