DRIVER BEAM PROPAGATION

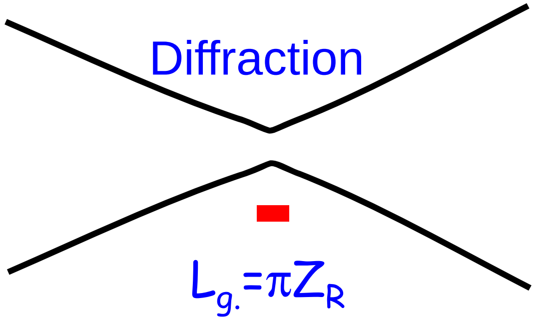
GUIDING
NONLINEAR EFFECTS
INSTABILITIES

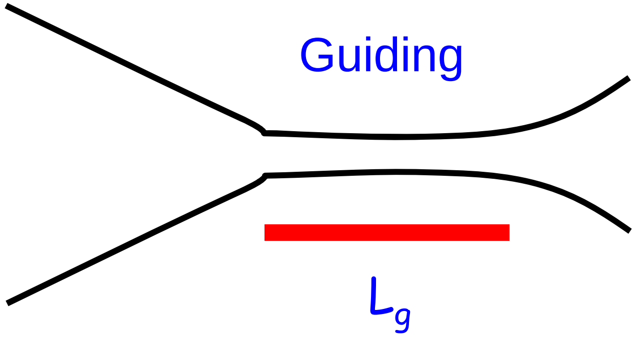
Summary

LASER DRIVER		BEAM DRIVER
$L_{deph} = \frac{\lambda_p^3}{\lambda_0^2}$	Dephasing length	$L_{deph} \simeq \gamma_b^2 \lambda_p$
$L_{dp} \simeq \frac{\gamma_p^2 \lambda_p}{a_0^2} = \frac{L_{deph}}{a_0^2}$	Depletion length	$L_{dp} = 2 m_e c^2 \gamma_b \frac{\epsilon_0 \pi r_1^2}{e^2 N_1}$
Dephasing	Limitation	Depletion
$\Delta\gamma = 4 \frac{\omega_0^2}{\omega_p^2} \phi_0$	Energy gain	$\Delta\gamma \simeq \gamma_b$

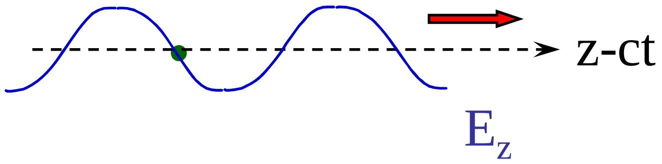
3 limits to acceleration

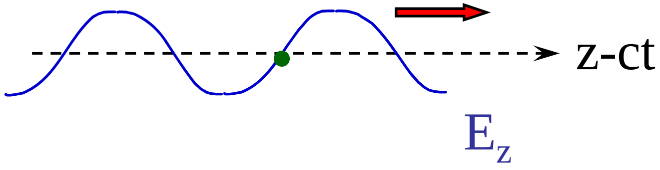
- L_g: the guiding length.





- L_{deph}: the dephasing length

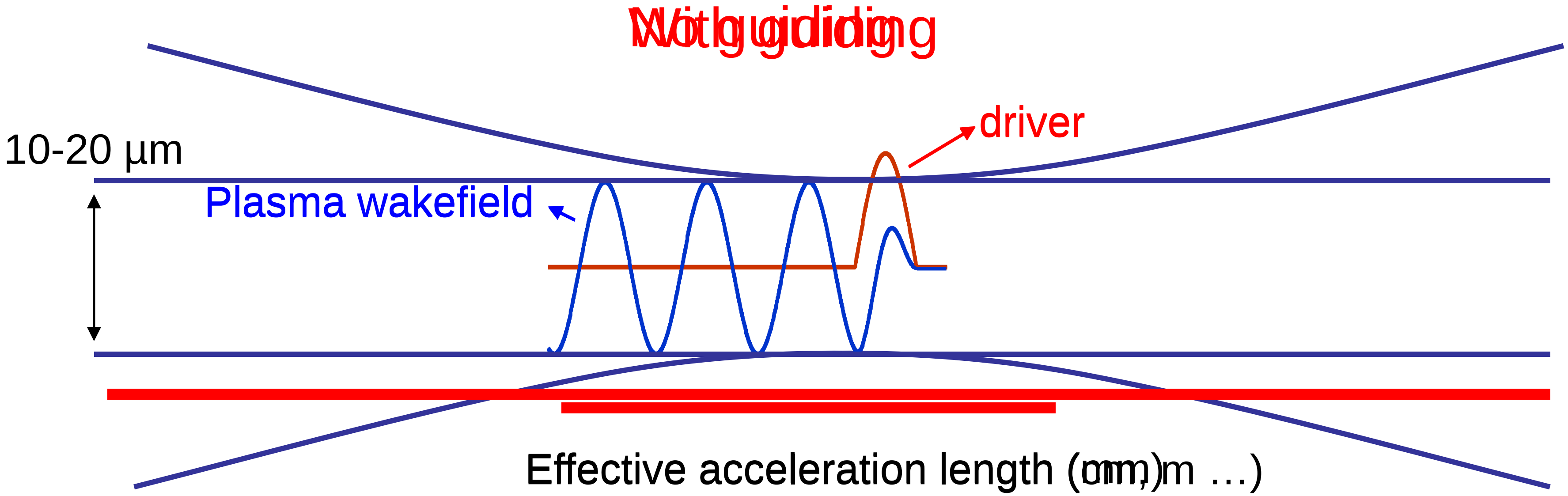




- L_{dp}: the depletion length
  - The laser looses energy as it propagates

Conceptual design of an accelerator: importance of guiding

- Injector
- Accelerating field: RF (plasma wakefield)
- Guiding struture provides acceleration over longer distances



No guiding

With guiding

driver

10-20 µm

Plasma wakefield

Effective acceleration length (mm)

Effective acceleration length (cm, m …)

CASE OF A LASER DRIVER

Wave equation and index of refraction

- Wave equation

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) a = \frac{n k_p^2}{n_0 \gamma} a$$

- Linearization $n=n_0+\delta n$ and γy$(1+a^2/2)^{1/2}$

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) a = k_p^2 \left(1 + \frac{\delta n}{n_0} - \frac{\langle a^2 \rangle}{2}\right) a$$

- Take Fourier transform → dispersion equation

$$\frac{\omega^2}{c^2} - k_0^2 = k_p^2 \left(1 + \frac{\delta n}{n_0} - \frac{\langle a^2 \rangle}{2}\right)$$

- Index of refraction: $\eta = ck/\omega$

$$\eta \simeq \left[1 - \frac{1}{2}\frac{\omega_p^2}{\omega_0^2}\left(1 + \frac{\delta n}{n_0} - \frac{\langle a^2 \rangle}{2}\right)\right]$$

Nonlinear effects in the plasma

- Index of refraction: $\eta = ck/\omega$

$$\eta \simeq \left[1 - \frac{1}{2}\frac{\omega_p^2}{\omega_0^2}\left(1 + \frac{\delta n}{n_0} - \frac{\langle a^2 \rangle}{2}\right)\right]$$

Linear index of refraction

Nonlinearity due to density perturbation (plasma wakefield)

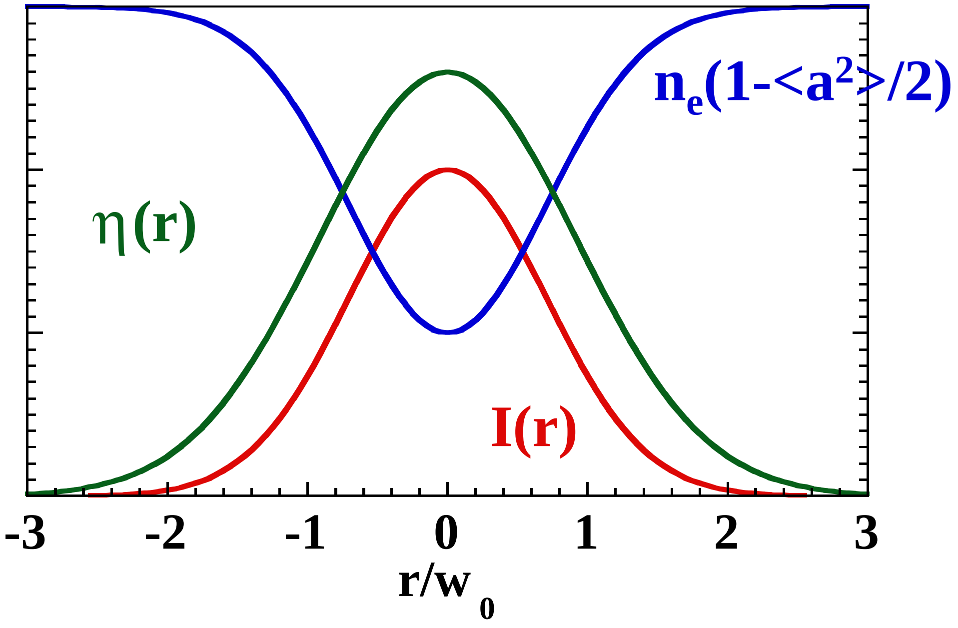
Nonlinearity due to relativistic motion of electrons in laser field
→Relativistic nonlinear optics

- Strong laser field perturbs plasma density ($\delta n/n_0$)
- These perturbations modifies the plasma index of refraction…
- … which in turns modifies the laser pulse propagation

Relativistic self-focusing

- Considering only the effect of the laser (neglect plasma term δn)

$$\eta \simeq \left[1 - \frac{1}{2}\frac{\omega_p^2}{\omega_0^2}\left(1 - \frac{\langle a^2 \rangle}{2}\right)\right]$$



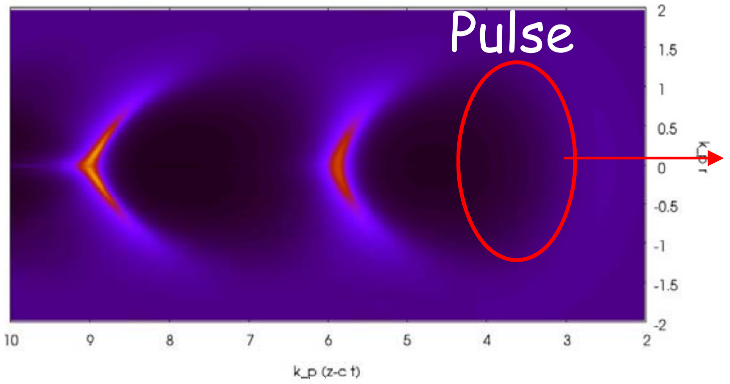
- Relativistic self-focusing occurs when

$$P > P_c[GW] = 16.2\,\frac{\omega_0^2}{\omega_p^2}$$

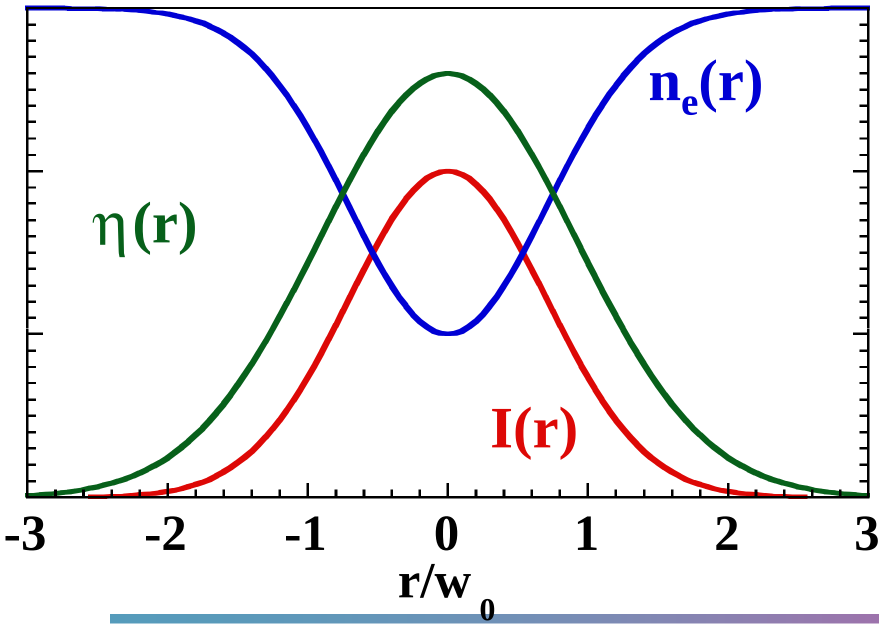
- In typical experiments
  P_C=1-10 TW (1TW=10^{12} W)
  state of the art lasers: 1 J, 30 fs
  P=30 TW > P_C

Ponderomotive self-focusing (bubble regime)

- Considering only the effect of the plasma (neglect laser term <a^2>)



$$\eta \simeq \left[1 - \frac{1}{2}\frac{\omega_p^2}{\omega_0^2}\left(1 + \frac{\delta n}{n_0}\right)\right]$$



- Condition for self-guiding is on a_0 not on P

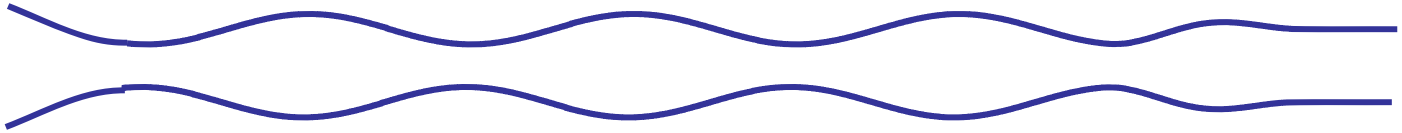
$$\boxed{a_0^c \approx \left(\frac{\omega_0}{\omega_p}\right)^{\frac{2}{5}}}$$

- Dominant effect in short pulse laser propagation (a_0 > 3)

Matched / unmatched beam (relativistic SF)

- For perfect beam propagation, we want $P=P_C$
  - Diffraction is exactly balanced by self-focusing
  - Matched beam

- $P>P_C$: the beam is not matched
  - Oscillation of beam envelope

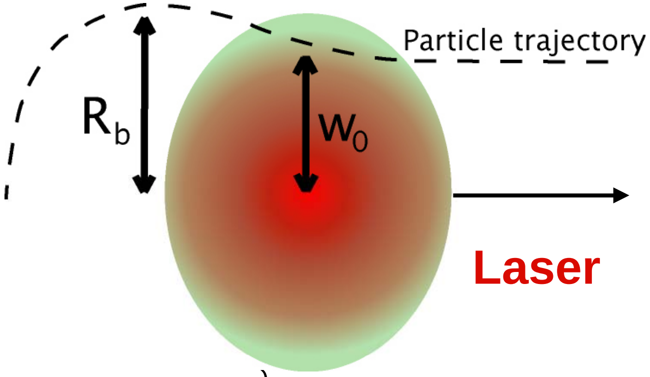


- $P<P_C$ no self-guiding

Matched beam (ponderomotive SF)

- Matched laser spot size

For given laser power P, there is a matched laser spot size W_0, which is approximately equal the blowout radius R_b



$$k_p w_0 \approx k_p R_b$$

Balance of forces :

$$\left\{\begin{array}{ll}\text{Ponderomotive :} & \dfrac{\nabla a_0^2}{\gamma} \sim \dfrac{a_0^2}{k_p R_b \gamma} \sim \dfrac{a_0}{k_p R_b} \\ \text{Ion channel :} & \overline{E}_r \sim \dfrac{\partial \overline{\phi}}{\partial \overline{r}} \sim k_p R_b \end{array}\right\} \Rightarrow k_p R_b \sim \sqrt{a_0}$$

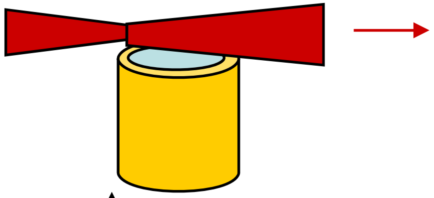
Approximately :

$$W_0^{matched} \approx \lambda_p \frac{\sqrt{a_0}}{\pi}$$

Wei Lu, UCLA, 2005

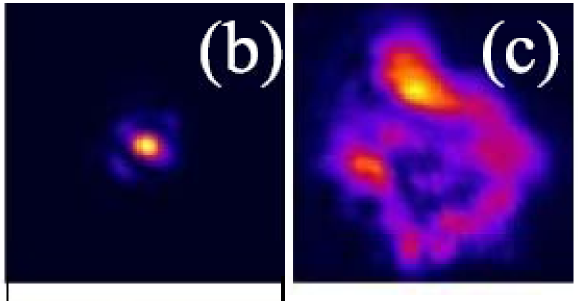
Experimental observations of self-focussing

Vacuum propagation

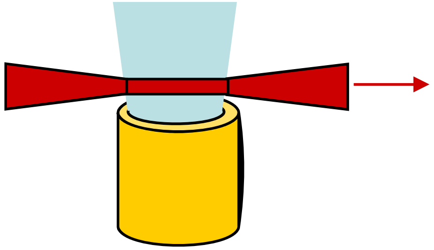


Entrance plane

Exit plane

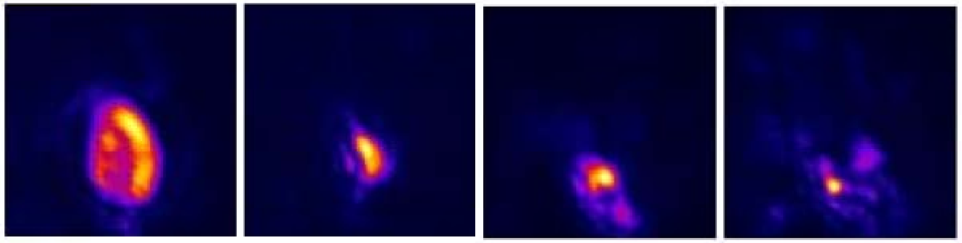


Plasma propagation



n_e (cm^{-3})

$3.3\mathrm{x}10^{18}$ $6.7\mathrm{x}10^{18}$ $1.0\mathrm{x}10^{19}$ $1.3\mathrm{x}10^{19}$

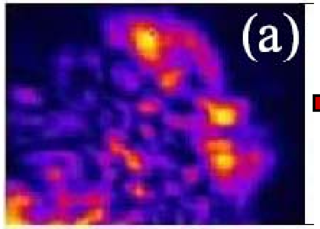


Below threshold

Matched beam

Filamentation

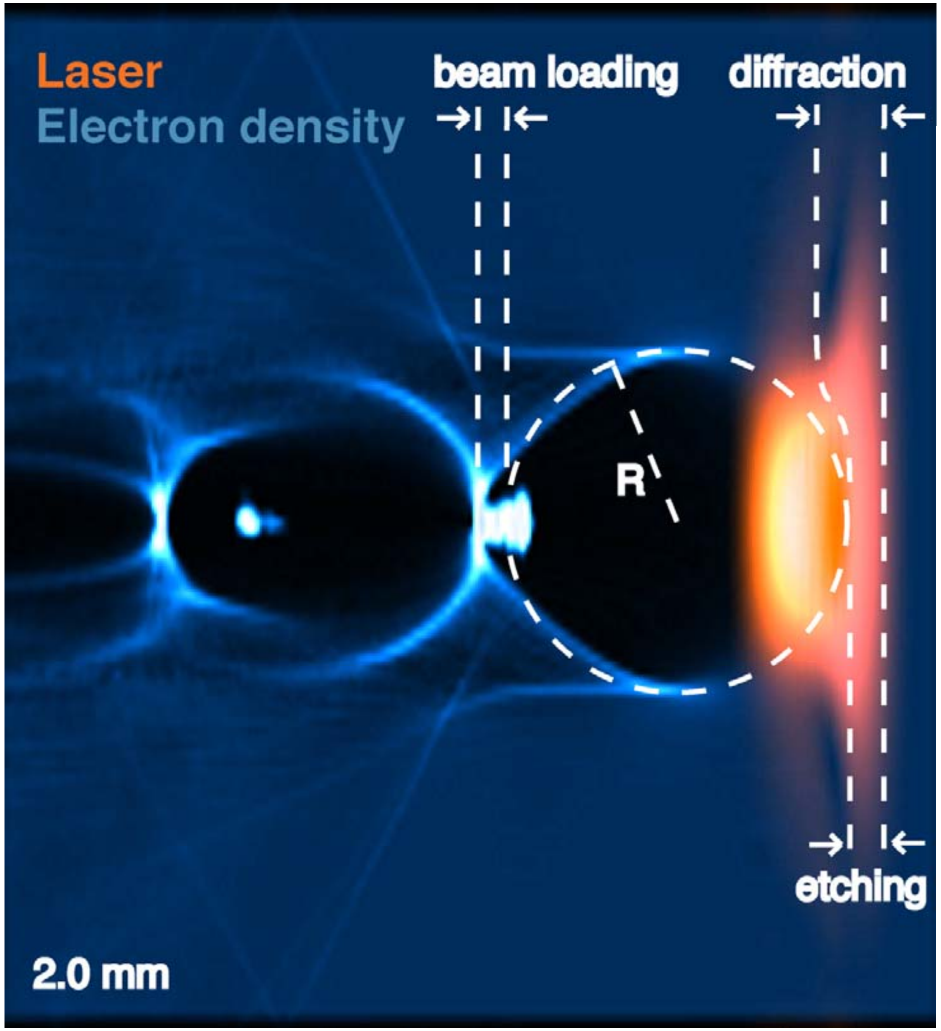
Using an unmatched beam (focused too tightly)



Filamentation

A. Thomas et al, Imperial College group

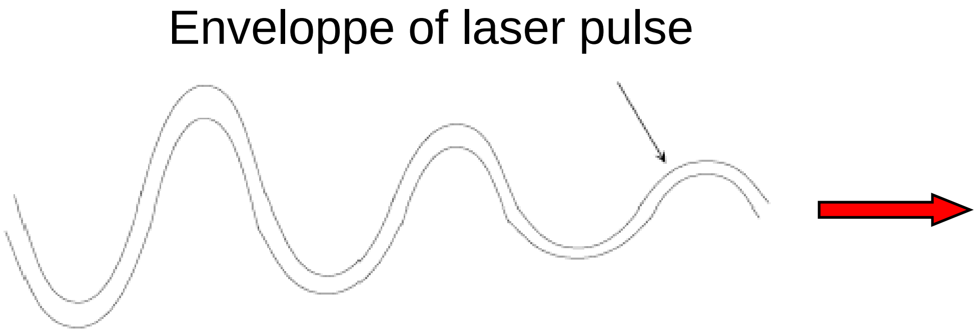
Nonlinear pulse evolution: erosion, shortening



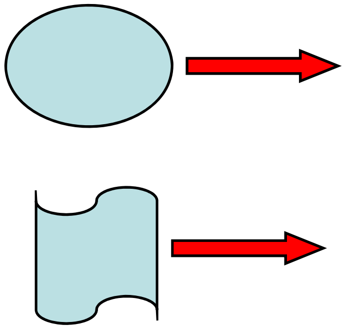
- The front of the laser pulse interacts with the plasma. As a result it loses energy (**Local pump depletion)** and etches back.
- The shape and size of the accelerating structure slightly change.
- Electrons are self-injected in the ion channel at the tail of the ion channel due to the accelerating and focusing fields.
- The trapped electrons slightly elongate the back of the spheroid.

Other instabilities: hosing …

- Hosing: radial motion of beam centroid



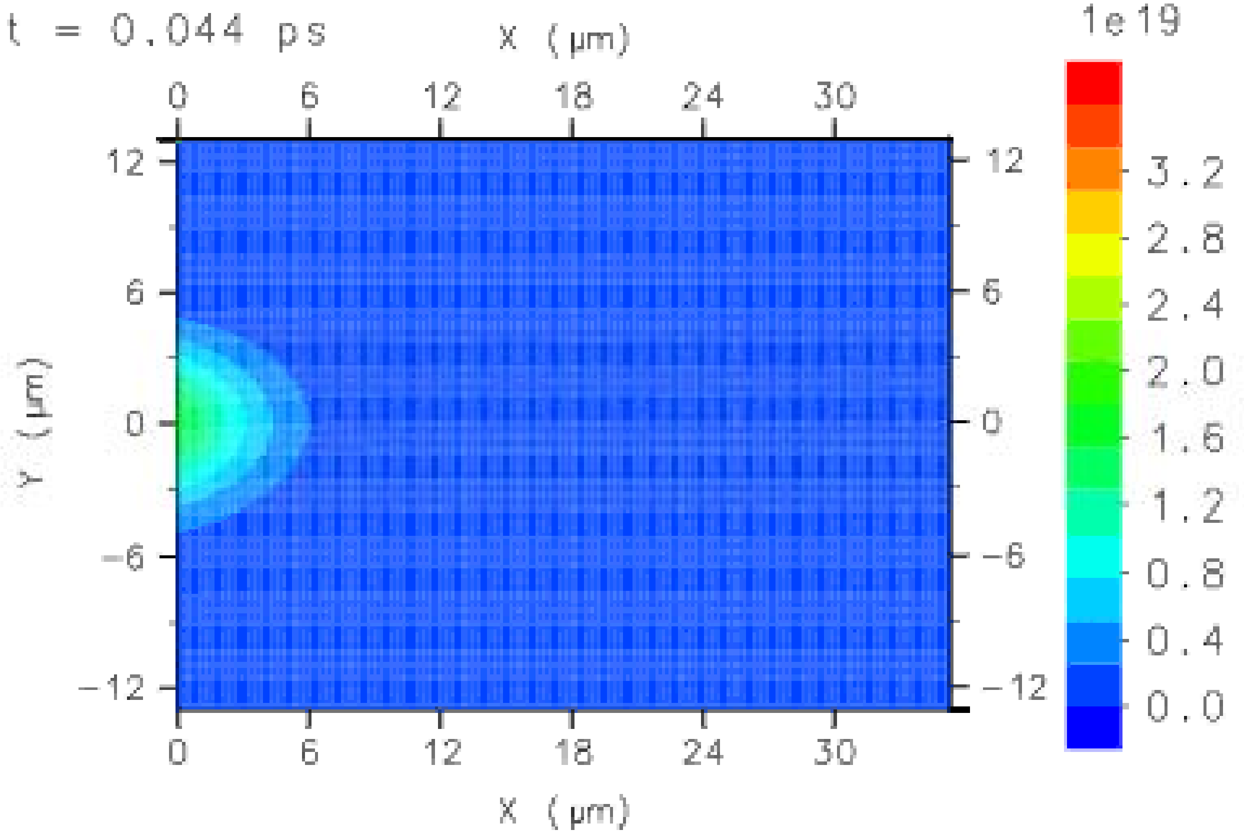
- Hosing appears with initial assymetries in laser pulse profile or starts on noise.



- Hosing is detrimental to propagation

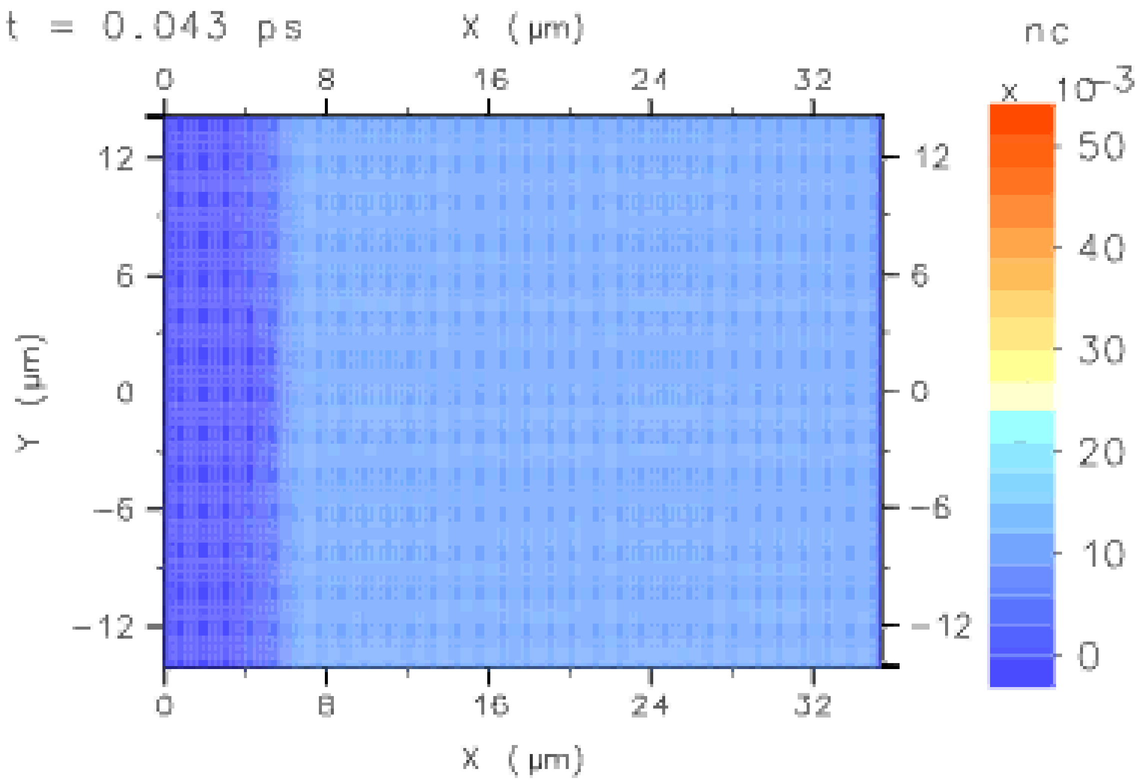
- Hosing is detrimental to acceleration (synchrotron radiation, phase mixing …)

Laser propagation: movie (2D PIC)



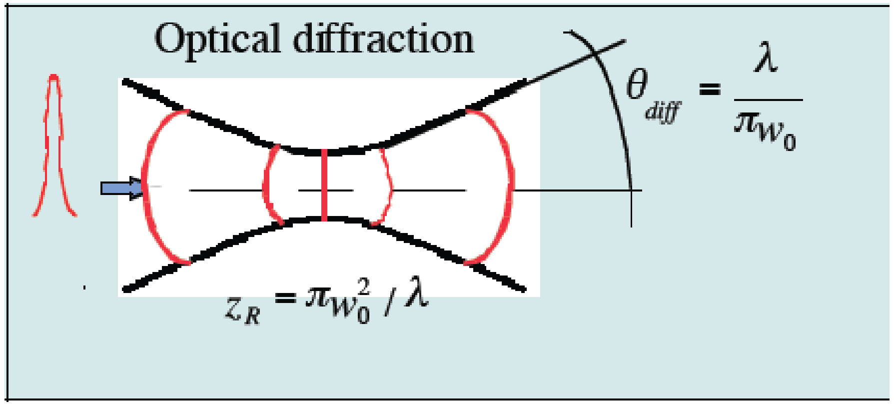
- Self-focusing
- Erosion and shortening
- Pump depletion
- A bit of hosing

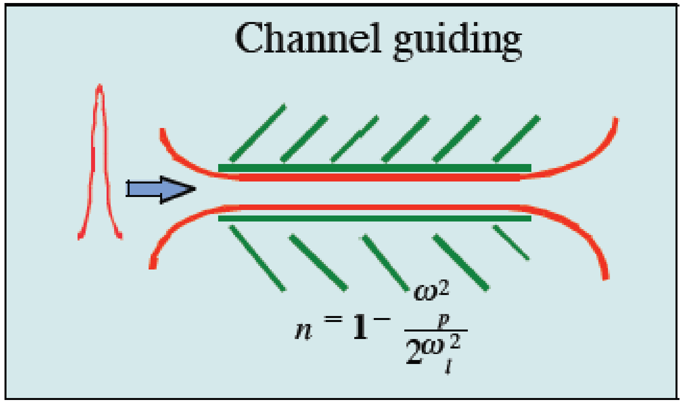
Laser propagation: plasma density



- Nonlinear wake formation
- Wavebreaking → trapping and acceleration
- Hosing: transverse oscillation of the electron beam

Using guiding devices



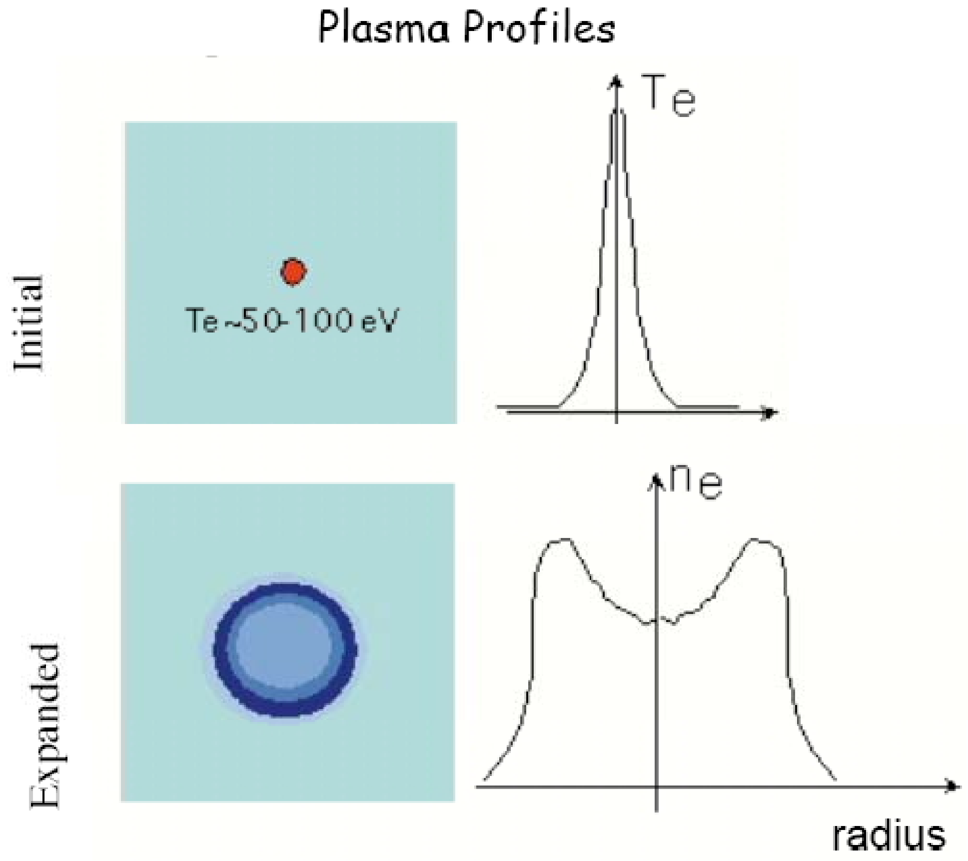


Guiding requires $d\eta/dr<0$

Index of refraction $$\eta = \sqrt{1-\left(\frac{\omega_p}{\omega}\right)^2}$$

$$\eta \approx 1-\frac{\omega_p^2}{2\gamma_\perp\omega^2} = 1-\frac{4\pi e^2 n_e(r)/m}{2(1+a^2)^{1/2}\omega^2} \approx 1-\frac{\omega_{po}^2}{2\omega^2}\left(1-\underbrace{\boxed{\frac{a^2}{2}}}_{\text{Relativistic}}+\overset{\text{Wake}}{\boxed{\frac{\delta n}{n}}}+\underbrace{\boxed{\frac{\Delta n}{n}}}_{\text{Channel}}\right)$$

Plasma channels

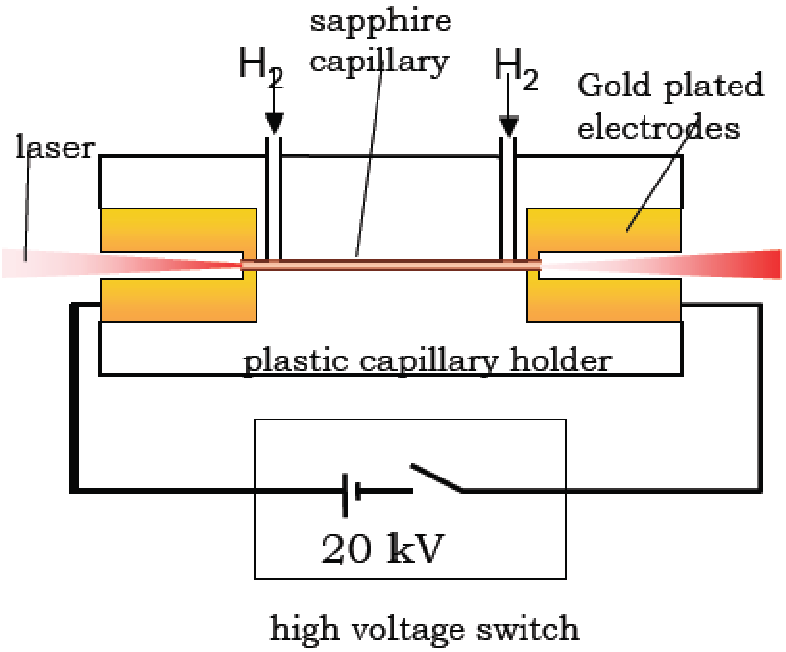


Step 1: Heat

Step 2: expand

- On-axis axicon (C.G. Durfee and H. Milchberg, PRL 71 (1993))
- Ignitor-Heater (P. Volfbeyn et al., Phys. Plasmas 6 (1999))

Capillary discharge



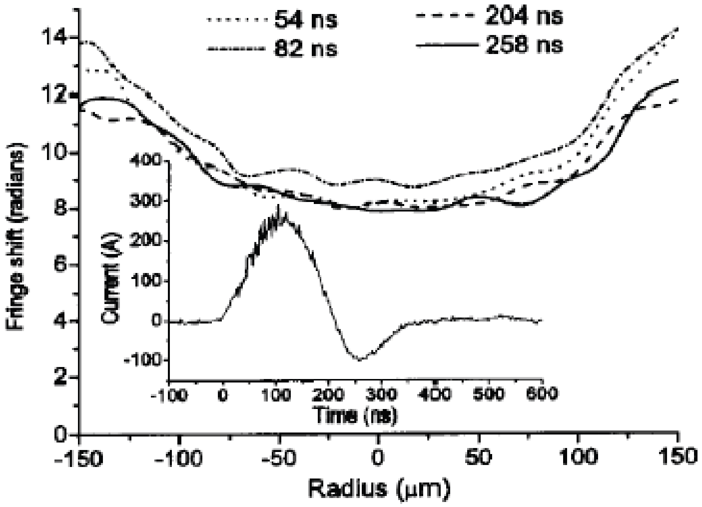
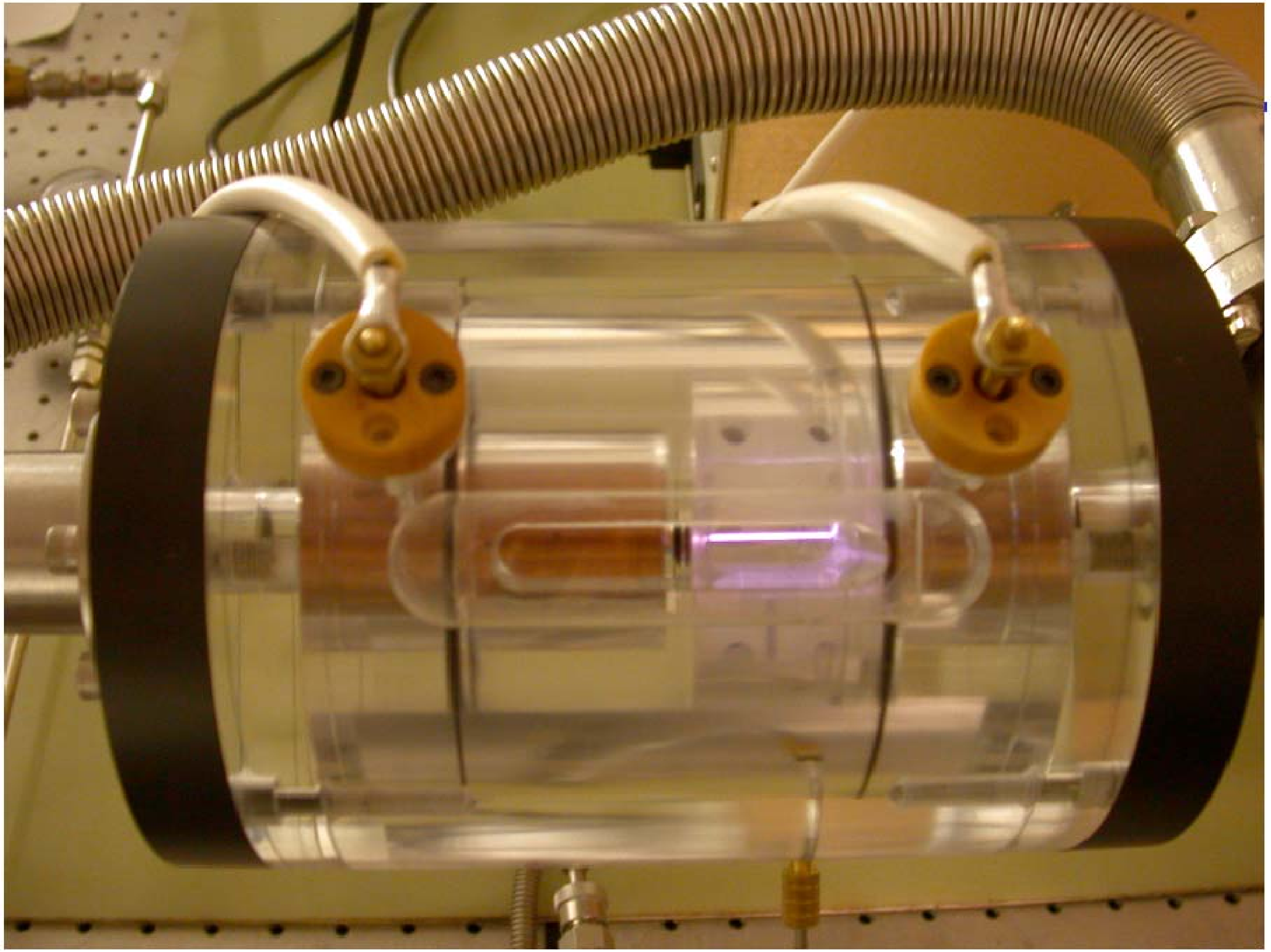


Figure taken from :
D. J. Spence, A. Butler, and S. M. Hooker,
J. Opt. Soc. Am. B, 20,p138, 2003

- Discharge allows lower density channels
- Lower density results in higher phase velocity
- Simulations indicate 3 cm, 10^{18} cm^{-3} optimal for 1 GeV

Picture of discharge

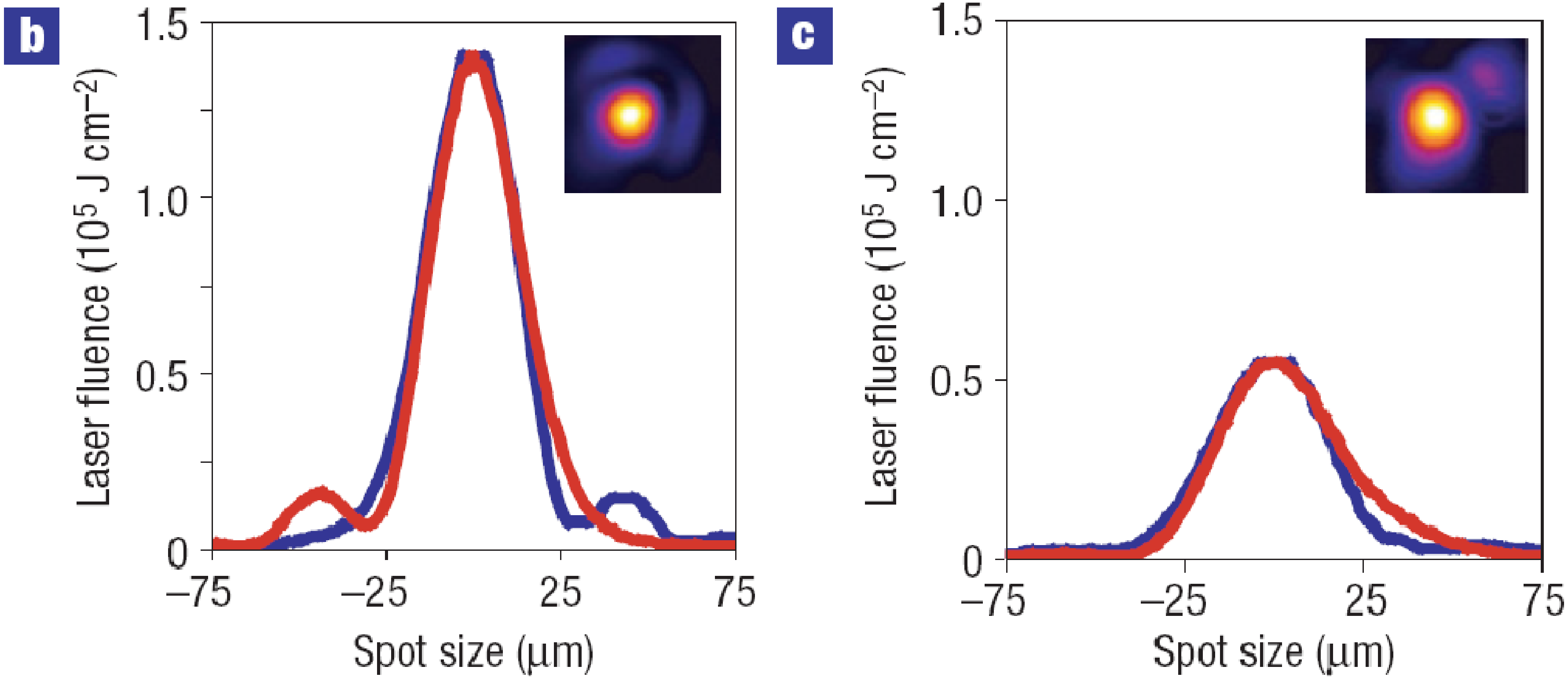


S. Hooker's group @ OXFORD, ENGLAND

Successful guiding in capillary discharge

ENTRANCE

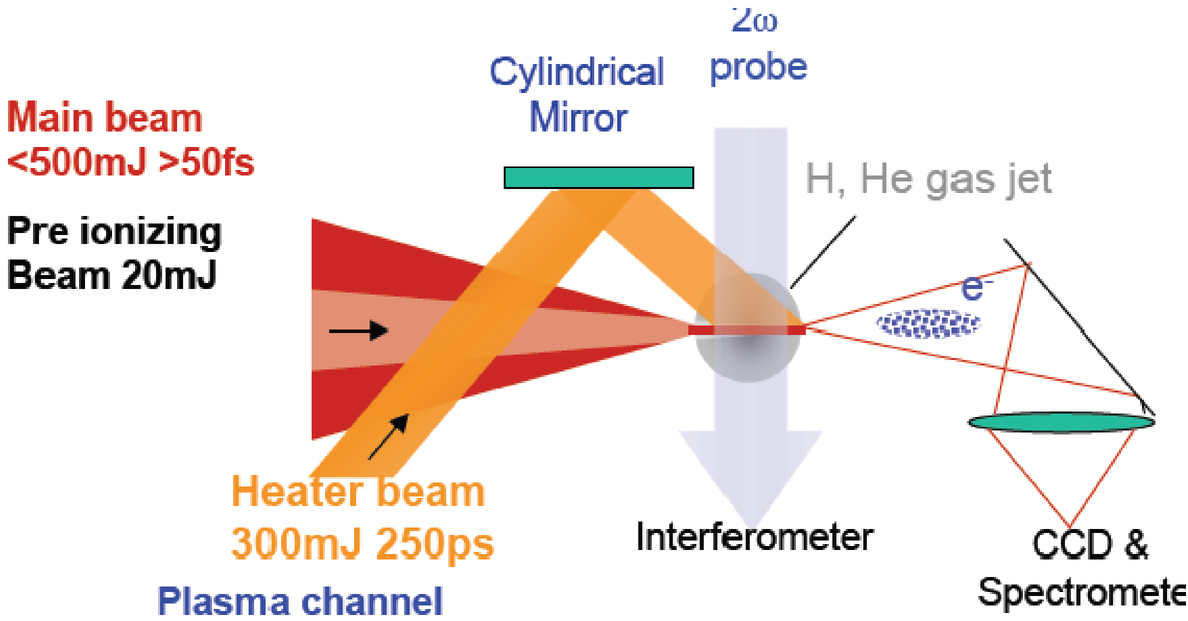
OUTPUT (3.3 cm)

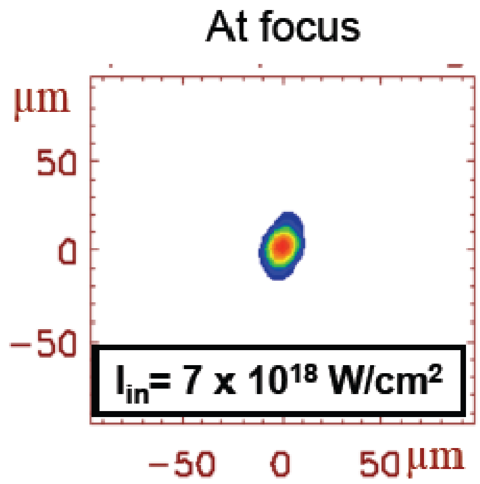


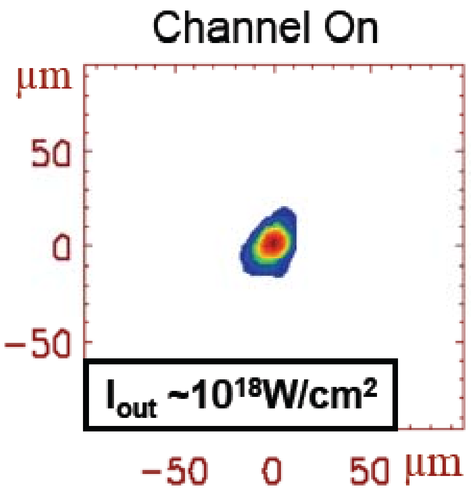
GUIDING OF A 25-30 µm SPOT SIZE

Leemans et al, Nat. Phys. 2006

Laser generated plasma channels (1)







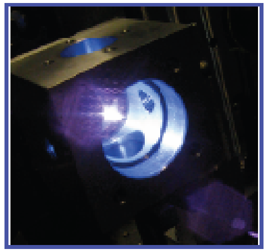
- Relativistic intensities (> 7×10^{18} W/cm^2)guided
- No dark current for P<4 TW

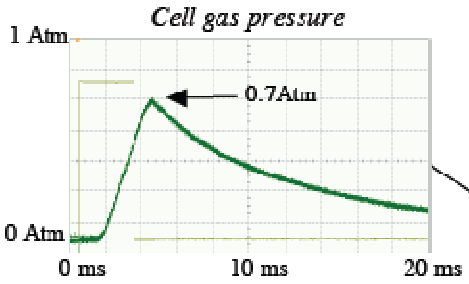
P. Volfbeyn, E. Esarey and W.P. Leemans, *Phys. Plasmas* 1999
C.G.R. Geddes et al., Nature 431, p. 543 (2004).

Laser generated plasma channels (2)

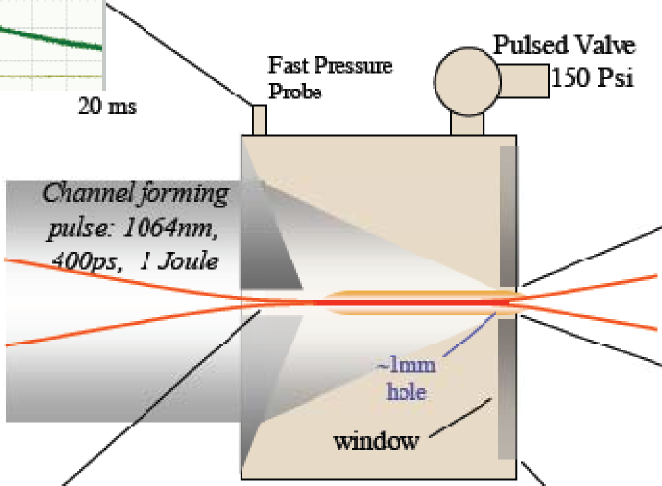
Differentially Pumped H_2 Channel Cell:

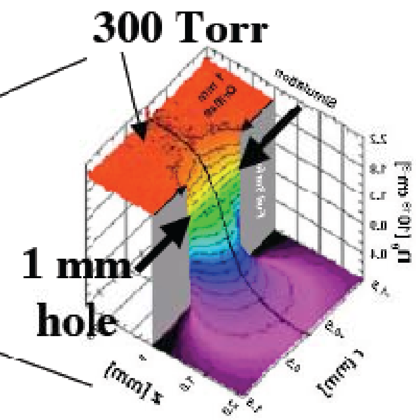
Towards controlled guiding at 10^{18} W/cm²

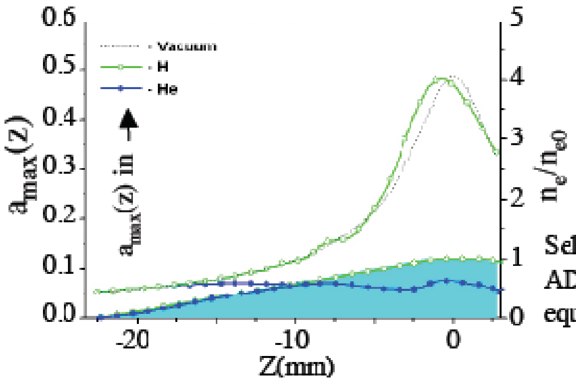




-Highly uniform, sharply bounded gas profile
-Easy Length Scalability
-Convenient, accurate density control

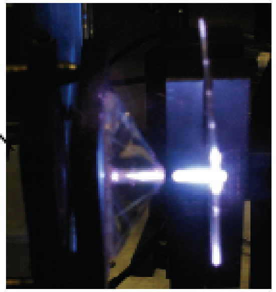






Self-consistent calculation, including ADK and relativistic cold plasma equations.

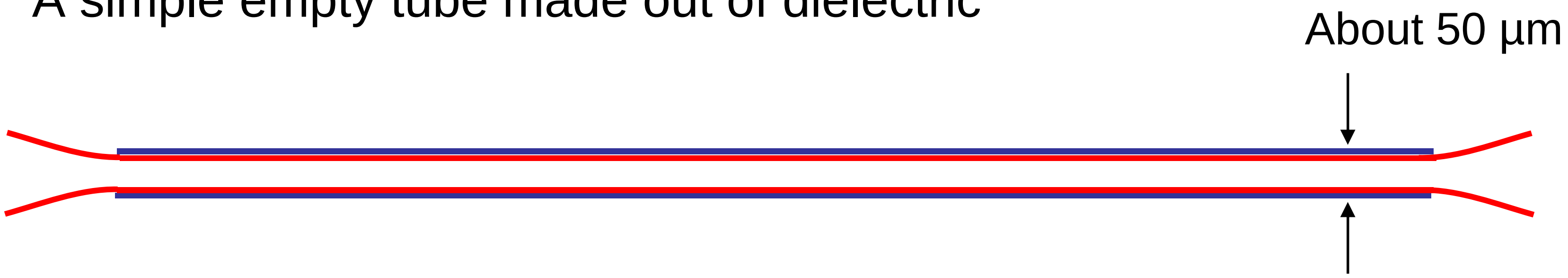
Disassembled Cell



Output window remains undamaged indefinitely

Guiding in capillary tubes

- A simple empty tube made out of dielectric



- Guiding over 12 cm, 60 % efficiency

- Problems:
  - capillary is fragile
  - Laser tends to destroy capillary
  - Nonlinearities destroy propagation

- Work in progress

Cros, et al Phys. Scripta T107 (2004) 125

CASE OF A BEAM DRIVER

Envelope equation of the e-beam

- σ is the R.M.S. beam radius $\sigma = \langle r^2 \rangle$
- transverse normalized emittance $\epsilon_\perp = \sqrt{\langle r^2 \rangle \langle p_\perp^2 \rangle - \langle r p_\perp \rangle^2}$

$$\frac{d^2\sigma}{dt^2} + \frac{K\sigma}{m_e\gamma_b} - \frac{\varepsilon_\perp^2}{m_e^2\gamma_b^2\sigma^3} - \frac{\langle rF_{i\perp}\rangle}{m_e\gamma_b\sigma} = 0$$

External linear forces

$$F_{e\perp} = -K(t)r$$

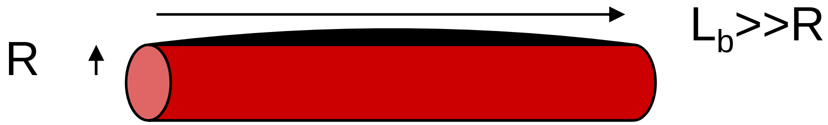
Emittance term

Internal forces
(space charge)

- Strictly valid for *long bunches:* $L_b \gg \sigma$

Space charge fields

- The electron beam: a uniform cylinder of electrons



- Integrate Poisson's equation: *defocusing* electric field

$$E_{i\perp}(r) = -\frac{e n_b r}{2\epsilon_0}$$

- Integrate Ampère's equation: *focusing* magnetic field

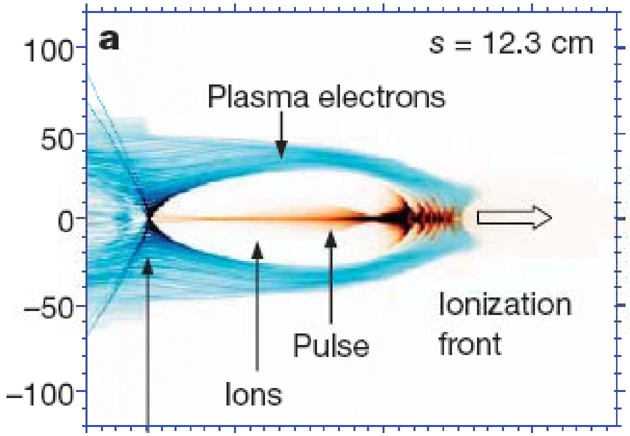
$$B_{i\theta}(r) = -\frac{e n_b v_b r}{2\epsilon_0 c^2}$$

- Space charge force is *defocusing*

$$F_{i\perp}(r) = -e(E_{i\perp}(r) - v_b B_{i\theta}) = \frac{m_e \omega_{pb}^2}{2\gamma_b^2} r$$

External forces: the blow-out (bubble) regime

- When $n_b >> n_0$: very nonlinear response of the plasma



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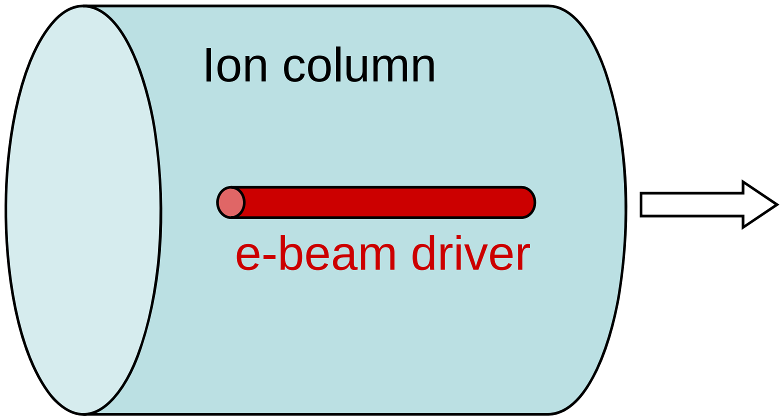


- Apply similar analysis to find, the focusing force of ion column:

$$F_{e\perp} = -\frac{m_e\omega_p^2}{2} r \qquad >> \qquad F_{i\perp}(r) = \frac{m_e\omega_{pb}^2}{2\gamma_b^2} r$$

Matched beam condition

- Neglecting the beam space charge forces, envelope equation in the blow-out regime

$$\frac{d^2\sigma}{dt^2} - \frac{\varepsilon_\perp^2}{m_e^2\gamma_b^2\sigma^3} + \frac{\omega_p^2}{2\gamma_b}\sigma = 0$$

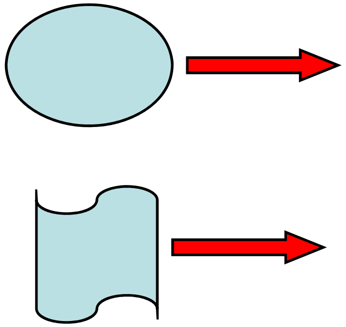
- The beam can self-focus. Analogy to the laser driver case:
  - Laser: self-focusing due to *changes in index of refraction*
  - Beam: self-focusing due to *induced plasma transverse fields*

- Matched beam condition on σ: last 2 terms cancel

$$\sigma_{matched} = \left(\frac{2\varepsilon_\perp^2}{m_e^2\gamma_b\omega_p^2}\right)^{1/4}$$

Other instabilities: hosing, erosion

- Hosing: radial motion of beam centroid



- Hosing appears with initial assymetries in e-beam profile or starts on noise.

- Hosing is detrimental to propagation

- Hosing is detrimental to acceleration (synchrotron radiation, phase mixing …)

Meter-scale simulation of beam driver in blow-out regime

SEE MOVIE

CONCLUSION:

- long scale propagation is possible
- no beam instabilities
- stable accelerating structure
- pump depletion: erosion

Summary

- Laser propagation:
  - Self-focusing can extend propagation (to a few mm, cm ?)
  - Subject to nonlinear pulse evolution (shortening, erosion)
  - Instabilities
  - Guiding devices can be used (for 1-10 cm propagation)

- E-beam propagation:
  - Self-focusing can extend propagation to meter scale
  - Subject to nonlinear pulse evolution (shortening erosion)