# Exclusive Radiative Decays of Z Bosons in QCD Factorization

Stefan Alte, Johannes Gutenberg-Universität Mainz Yuval Grossman, Matthias König and Matthias Neubert



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"Exclusive Radiative Decays of W and Z Bosons in QCD Factorization", Grossman, König and Neubert (2015), JHEP **1504** (2015) 101, arXiv: **1501.06569** 

"Exclusive Radiative Z-Boson Decays to Mesons with Flavor-Singlet Components", Alte, König and Neubert (2016), JHEP **1602** (2016) 162, arXiv: **1512.09135** 

Bauer, Pirjol, Stewart (2001); Bauer et al. (2002); Beneke, Chapovsky, Diehl, Feldmann (2002)

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angle = -i\,f_M\,E\int_0^1\mathrm{dx}\,e^{i imes t\bar{n}\cdot k}\phi_M(x,\mu)$$

The LCDA  $\phi_M(x, \mu)$  can be interpreted as the **amplitude for finding a quark with longitudinal momentum fraction** x inside the meson  $\langle \mathcal{M}(k)|\bar{q}(t\bar{n})rac{\hbar}{2}(\gamma_5)[t\bar{n},0]q(0)|0
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$$\phi_M(x,\mu) = 6x(1-x) \left[ 1 + \sum_{n=1}^{\infty} a_n^M(\mu) C_n^{(3/2)}(2x-1) \right]$$

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- Gegenbauer coefficients are extracted from data, QCD sum rules or lattice QCD at  $\mu_0 = 1$  GeV

$$\phi_M(x,\mu\to\infty)=6x(1-x)$$

The LCDAs approach **asymptotic form** when evolving from the low hadronic scale  $\mu_0$  up to the factorization scale  $\mu = m_Z$ 

# **Renormalisation-Group Evolution**

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# Profit from high factorization scale: less sensitive to the poorly known hadronic parameters

$$i\mathcal{A}(Z \to M\gamma) = \pm \frac{eg f_{M}}{2\cos\theta_{W}} \left[ i\epsilon_{\mu\nu\alpha\beta} \frac{k^{\mu}q^{\nu} \varepsilon_{Z}^{\nu} \varepsilon_{\gamma}^{\nu}}{k \cdot q} F_{1}^{M} - \left( \varepsilon_{Z} \cdot \varepsilon_{\gamma}^{*} - \frac{q \cdot \varepsilon_{Z} k \cdot \varepsilon_{\gamma}^{*}}{k \cdot q} \right) F_{2}^{M} \right]$$

LO and NLO diagrams



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LO and NLO diagrams

Form factors in terms of Gegenbauer moments

$$F_{1}^{M} = \mathcal{Q}_{M} \underbrace{\sum_{n=0}^{\infty} C_{2n}^{(+)}(m_{Z}, \mu) a_{2n}^{M}(\mu)}_{P_{2}^{M}} \text{ even moments}$$

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#### Hard functions in moment space

Tree level  

$$C_{n}^{\pm}(m_{Z},\mu) = 1 + \frac{C_{F} \alpha_{S}(\mu)}{4\pi} c_{n}^{(\pm)} \left(\frac{m_{Z}}{\mu}\right) + \mathcal{O}(\alpha_{S}^{2})$$

$$c_{n}^{(\pm)} \left(\frac{m_{Z}}{\mu}\right) = \left[\frac{2}{(n+1)(n+2)} - 4H_{n+1} + 3\right] \left(\log \frac{m_{Z}^{2}}{\mu^{2}} - i\pi\right)$$

$$+ 4H_{n+1}^{2} - \frac{4(H_{n+1} - 1) \pm 1}{(n+1)(n+2)} + \frac{2}{(n+1)^{2}(n+2)^{2}} - 9$$

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One loop QCD  

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Resummation of large logarithms by choosing  $\mu = m_Z$ 

Decay mode	Branching ratio	asymptotic
$Z^0 \rightarrow \pi^0 \gamma$	$(9.80^{+0.09}_{-0.14}{}^{\mu}\pm0.03_{f}\pm0.61_{a_{2}}\pm0.82_{a_{4}})\cdot10^{-12}$	7.71
$Z^0 \rightarrow \rho^0 \gamma$	$(4.19 {}^{+0.04}_{-0.06}\mu\pm0.16_f\pm0.24_{a_2}\pm0.37_{a_4})\cdot 10^{-9}$	3.63
$Z^0 \rightarrow \omega \gamma$	$(2.82^{+0.03}_{-0.04} \ \mu \pm 0.15_{f} \pm 0.28_{a_{2}} \pm 0.25_{a_{4}}) \cdot 10^{-8}$	2.48
$Z^0 \rightarrow \phi \gamma$	$(1.04 + 0.01 - 0.02 \ \mu \pm 0.05_f \pm 0.07_{a_2} \pm 0.09_{a_4}) \cdot 10^{-8}$	0.86
$Z^0 \rightarrow J/\psi \gamma$	$(8.02^{+0.14}_{-0.15 \ \mu} \pm 0.20_{f} + 0.39_{\sigma}) \cdot 10^{-8}$	10.48
$Z^0 \rightarrow \Upsilon(1S) \gamma$	$(5.39^{+0.10}_{-0.10} \mu \pm 0.08_{f} {+0.11}_{-0.08} \sigma) \cdot 10^{-8}$	7.55
$Z^0 \rightarrow \Upsilon(4S) \gamma$	$(1.22 + 0.02 - 0.02 \mu \pm 0.13_f + 0.02 \sigma) \cdot 10^{-8}$	1.71
$Z^0 \to \Upsilon(nS) \gamma$	$(9.96^{+0.18}_{-0.19\ \mu}\pm 0.09_{f\ -0.15\ \sigma})\cdot 10^{-8}$	13.96

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What about  $Z^0 o \eta^{(\prime)} \gamma$ ?

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- Flavour structure of  $\eta$  and  $\eta'$ : FKS scheme Feldmann et al. (1998)

#### Two different sets for the mixing parameters

 $\begin{aligned} f_q &= (1.07 \pm 0.02) f_\pi \,, \quad f_s = (1.34 \pm 0.06) f_\pi \,, \quad \varphi = 39.3^\circ \pm 1.0^\circ \,\, \text{Feldmann et al. (1998)} \\ f_q &= (1.09 \pm 0.03) f_\pi \,, \quad f_s = (1.66 \pm 0.06) f_\pi \,, \quad \varphi = 40.7^\circ \pm 1.4^\circ \,\, \text{Escribano, Free (2005)} \end{aligned}$ 

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**Different sets** of **Gegenbauer moments** for the LCDAs  $\phi_q$ ,  $\phi_s$  and  $\phi_g$  are extracted from data for the  $\gamma^*\gamma \rightarrow \eta^{(\prime)}$  transition form factor taken by CLEO (1998) and BaBar (2011) Agaev et al. (2014); Kroll and Passek (2013)

Model	(i)	(ii)	(iii)
$10^9 { m Br}(Z  o \eta \gamma)$	$0.16\pm0.05$	$0.17\pm0.05$	$\textbf{0.16} \pm \textbf{0.05}$
$10^9 { m Br}(Z  o \eta' \gamma)$	$4.70\pm0.23$	$\textbf{4.77} \pm \textbf{0.24}$	$\textbf{4.73} \pm \textbf{0.24}$
Model	(iv)	(v)	(vi)
$\frac{Model}{10^9\mathrm{Br}(Z\to\eta\gamma)}$	(iv) $0.11 \pm 0.03$	(v) 0.10 ± 0.03	(vi) $0.010^{+0.014}_{-0.010}$

Models (i)-(iii): Agaev et al. (2014); Models (iv)-(vi): Kroll and Passek (2013)

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$\frac{Model}{10^9\mathrm{Br}(Z\to\eta\gamma)}$	(iv) $0.11 \pm 0.03$	(v) $0.10 \pm 0.03$	(vi) $0.010 \substack{+0.014 \\ -0.010}$

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Measurements of the decays  $Z^0 \rightarrow \eta^{(\prime)}\gamma$  at a **future** *Z*-factory could provide valuable information about the hadronic input parameters and the gluon LCDA in particular

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"*Exclusive Radiative Higgs Decays as Probes of Light-Quark Yukawa Couplings*", König and Neubert (2015), JHEP **1508** (2015) 012, arXiv: **1505.03870** 

*"Exclusive Weak Radiative Higgs Decays and Flavor-Changing Higgs-Top Couplings"*, Alte, König and Neubert, in preparation