

# Role of low-energy observables in precision Higgs analyses



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- Introduction
- Global fit and input parameters
- Example: partial widths and quark masses
- Conclusions and outlook

Based on AAP, S. Pokorski, J.D. Wells, Z. Zhang, PRD91, 073001 (2015)

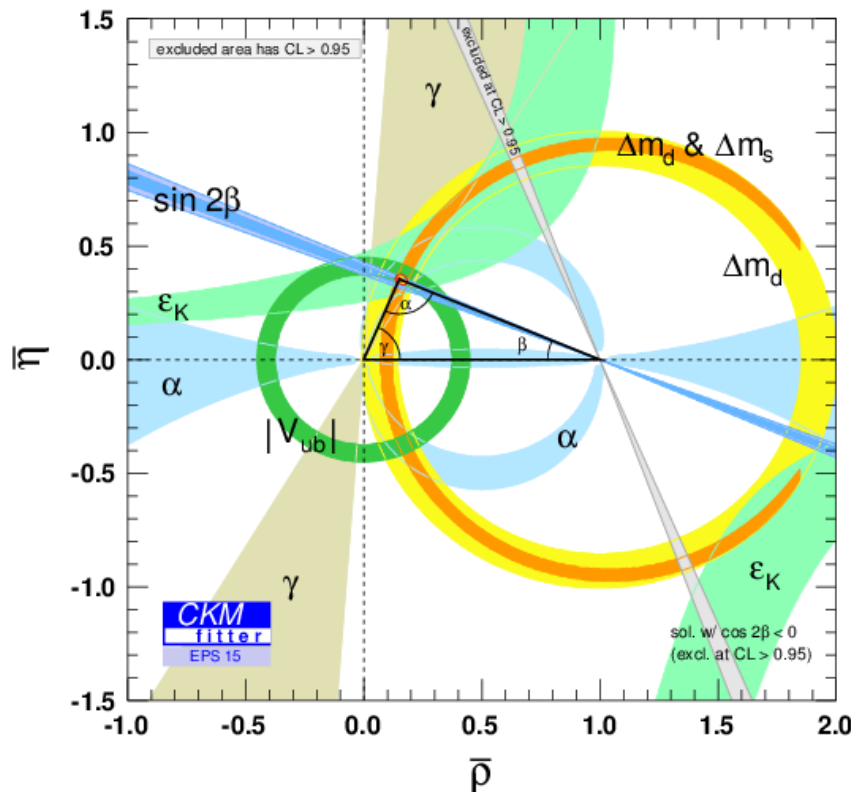
# Introduction

★ Long-long time ago (31 July 2000 at ICHEP in Osaka)...

- first measurement of CP-violation in B-system:  $\sin 2\beta = 0.34 \pm 0.20 \pm 0.05$

BaBar result

★ Indirect searches for New Physics via a combination of measurements



Observable	Central $\pm 1 \sigma$
$\sin 2\alpha$	-0.036 [+0.042 -0.082]
$\sin 2\alpha$ (meas. not in the fit)	-0.053 [+0.046 -0.146]
$\sin 2\beta$	0.692 [+0.018 -0.019]
$\sin 2\beta$ (meas. not in the fit)	0.771 [+0.017 -0.041]

Global fit by the  
CKMfitter collaboration  
(updated January 2015)



Geek

## We know CERN found the Higgs Boson Particle—now what?

By [Aja Romano](#)

Jun 24, 2014, 3:35pm CT



# Introduction: BSM Physics

## ★ Precision studies of the Higgs boson properties: BSM/NP discovery?

- flavor problem has NP solutions that affect Higgs partial rates

SM:

$$y_u Q_L \tilde{H} u_R + h.c.$$

$$m_u = y_u \frac{v}{\sqrt{2}}$$

quark mass

$$\frac{y_u}{\sqrt{2}}$$

Higgs-fermion coupling

BSM:

$$\left( \frac{H^\dagger H}{\Lambda^2} \right)^n \bar{Q}_L \tilde{H} u_R + h.c.$$

$$m_u \sim \frac{1}{\Lambda^2} \left( \frac{v}{\sqrt{2}} \right)^3$$

quark mass (top)

$$\sim \frac{3}{\Lambda^2} \left( \frac{v}{\sqrt{2}} \right)^2$$

Higgs-fermion coupling (top)

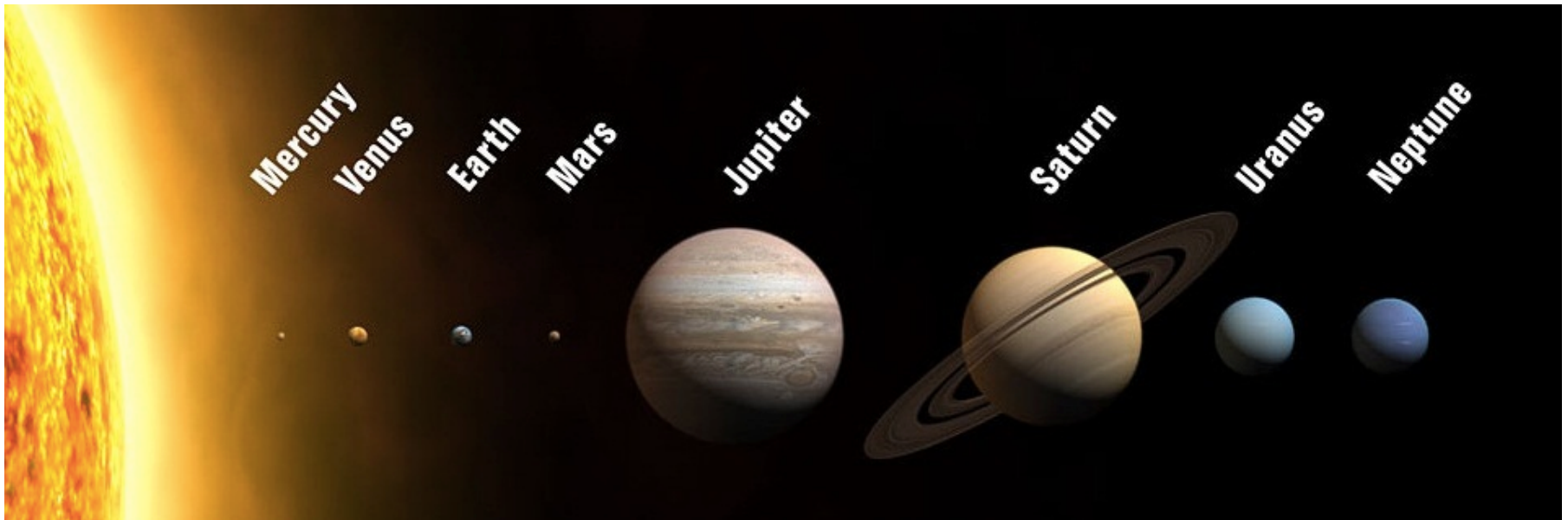
Guidice, Lebedev;  
Bauer et al

BSM (2HDM):

$$\mathcal{L}_2 = -y_\psi \bar{\psi}_L \psi_R \phi_1 - y_\chi \bar{\chi}_L \chi_R \phi_2 + h.c. \rightarrow \frac{m_\chi}{m_\psi} = \frac{y_\chi}{y_\psi} \frac{v_2}{v_1} = \frac{y_\chi}{y_\psi} \tan \beta \gg 1$$

Blechman, AAP, Yeghiyan, JHEP 1011 (2010) 075

Are there new phenomena behind flavor problem? Or not?



Why is  $M_{\text{Jupiter}} \gg M_{\text{Mercury}}$ ?

# Introduction: experiment

## ★ Precision studies of the Higgs boson properties: NP discovery?

- experiment: expect (sub)percent-level measurements

Table. 1.

List of the main observables and expected accuracy at FCC-ee and CEPC with 2 Million/1 Million Higgs boson respectively

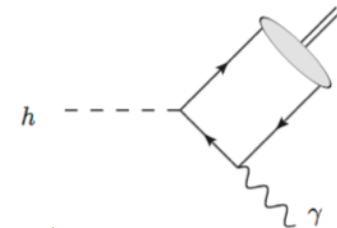
	FCC-ee 240GeV	CEPC 250GeV
Higgs mass	-	5.4 MeV
$\sigma(ZH)$	0.4%	0.7%
$\sigma(ZH) \times Br(H \rightarrow bb)$	0.2%	0.4%
$\sigma(ZH) \times Br(H \rightarrow cc)$	1.2%	2.1%
$\sigma(ZH) \times Br(H \rightarrow gg)$	1.4%	1.8%
$\sigma(ZH) \times Br(H \rightarrow WW)$	0.9%	1.3%
$\sigma(ZH) \times Br(H \rightarrow ZZ)$	3.1%	5.1%
$\sigma(ZH) \times Br(H \rightarrow \tau\tau)$	0.7%	1.2%
$\sigma(ZH) \times Br(H \rightarrow \gamma\gamma)$	3.0%	8.0%
$\sigma(ZH) \times Br(H \rightarrow \mu\mu)$	13%	18%
$\sigma(vvH) \times Br(H \rightarrow bb)$	2.2%	3.8%

Ruan, 1411.5606

- far future: exclusive decays?

$$\mathcal{B}(h \rightarrow J/\psi\gamma) = (2.95 \pm 0.17) \times 10^{-6}$$

$$\mathcal{B}(h \rightarrow \Upsilon(1S)\gamma) = (4.61^{+1.76}_{-1.23}) \times 10^{-9}$$



- need to know the SM values very well

Kagan et al 1406.1722

Bodwin et al 1407.6695

# Introduction

## ★ Precision studies of the Higgs boson properties: NP discovery?

- need to know the SM values very well ( $\alpha_s = \alpha_s(m_H)/\pi \sim 0.0336$ )

$$\Gamma(H \rightarrow \bar{f}f) = \frac{G_F M_H}{4\sqrt{2}\pi} m_f^2 \tilde{R}(s = M_H^2),$$

$$\tilde{R} = 1 + 5.6668a_s + 29.147a_s^2 + 41.758a_s^3 - 825.7a_s^4$$

Baikov, Chetyrkin, Kuhn  
PRL 96 012003 (2006)

## ★ Appreciable dependence upon input parameters, e.g. for $\Gamma_{H \rightarrow b\bar{b}}$

	LO	NLO	N <sup>2</sup> LO	N <sup>3</sup> LO	N <sup>4</sup> LO	Total
$\Gamma_i$ (KeV)	1924.28	391.74	72.38	3.73	-2.65	2389.48
$\Gamma_i/\Gamma_{\text{tot}}$	80.53%	16.39%	3.03%	0.16%	-0.11%	

S.Q. Wang et al  
arXiv:1308.6364 [hep-ph]

# Introduction

## ★ Precision studies of the Higgs boson properties: NP discovery?

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$$\tilde{R} = 1 + 5.6668a_s + 29.147a_s^2 + 41.758a_s^3 - 825.7a_s^4$$

Baikov, Chetyrkin, Kuhn

## ★ SM results depend on several external input parameters

- quark masses are extracted from low energy data

$$m_c(m_c) = 1.275(25) \text{ GeV}, \quad m_b(m_b) = 4.18(3) \text{ GeV}$$

- lattice?
- QCD Sum Rules? PDG?
- correlations among input parameters?
- inflation of systematic errors?

Lepage, Mackenzie, Peskin

## ★ Maybe there is a better way to arrange the calculation?



the central value of  $m_c$  can differ by a much larger amount depending on which algorithm (all of which are formally equally good) is used to determine  $m_c$  from the data. This leads to a systematic error from perturbation theory of around 20 MeV for the  $c$  quark and 25 MeV for the  $b$  quark. Electromagnetic effects, which also are important at this precision, are often not included. For this reason, we inflate the errors on the continuum extractions of  $m_c$  and  $m_b$ . The average values of  $m_c$  and  $m_b$  from continuum determinations are (see Sec. G for the 1S scheme)

$$\overline{m}_c(\overline{m}_c) = (1.275 \pm 0.025) \text{ GeV}$$

$$\overline{m}_b(\overline{m}_b) = (4.18 \pm 0.03) \text{ GeV}, \quad m_b^{1\text{S}} = (4.65 \pm 0.03) \text{ GeV}.$$

PDG (Quark Masses review)

# Global fit

★ A powerful way to extract parameters is a global fit

- need to minimize the chi-square function

$$\chi^2 = \sum_{ij} \left[ \hat{O}_i^{\text{th}}(\{\mathcal{I}_k\}) - \hat{O}_i^{\text{expt}} \right] V_{ij}^{-1} \left[ \hat{O}_j^{\text{th}}(\{\mathcal{I}_k\}) - \hat{O}_j^{\text{expt}} \right]$$

- ... which includes calculation inputs...

$$\{\mathcal{I}_k\} \equiv \{\hat{O}_k^{\text{in}}\} \cup \{m_Q(m_Q)\} \cup \{p_k^{\text{other}}\}$$

- ... and fit observables

$$\{\hat{O}_i\} \equiv \{\hat{O}_i^{\text{in}}\} \cup \{\hat{O}_i^{\text{high}}\} \cup \{\hat{O}_i^{\text{low}}\}$$

input observables:  $\{\hat{O}_k^{\text{in}}\} \equiv \{m_Z, G_F, \alpha(m_Z), m_t, \alpha_s(m_Z), m_H\}$

★ In what follows, let us concentrate on  $H \rightarrow \text{bb/cc}$  partial widths

# Eliminating quark masses

★ Let us understand theoretical uncertainties of input observables

- concentrate on quark masses from low-energy observables

$$\hat{O}_1^{\text{low}} = \hat{O}_1^{\text{low}}[\{\hat{O}_k^{\text{in}}\}, \{m_Q(m_Q)\}],$$

$$\hat{O}_2^{\text{low}} = \hat{O}_2^{\text{low}}[\{\hat{O}_k^{\text{in}}\}, \{m_Q(m_Q)\}],$$

- ... which can be solved for quark masses

$$m_c(m_c) = m_c(m_c)[\{\hat{O}_k^{\text{in}}\}, \hat{O}_1^{\text{low}}, \hat{O}_2^{\text{low}}],$$

$$m_b(m_b) = m_b(m_b)[\{\hat{O}_k^{\text{in}}\}, \hat{O}_1^{\text{low}}, \hat{O}_2^{\text{low}}].$$

- ... which can be then eliminated from the Higgs observables

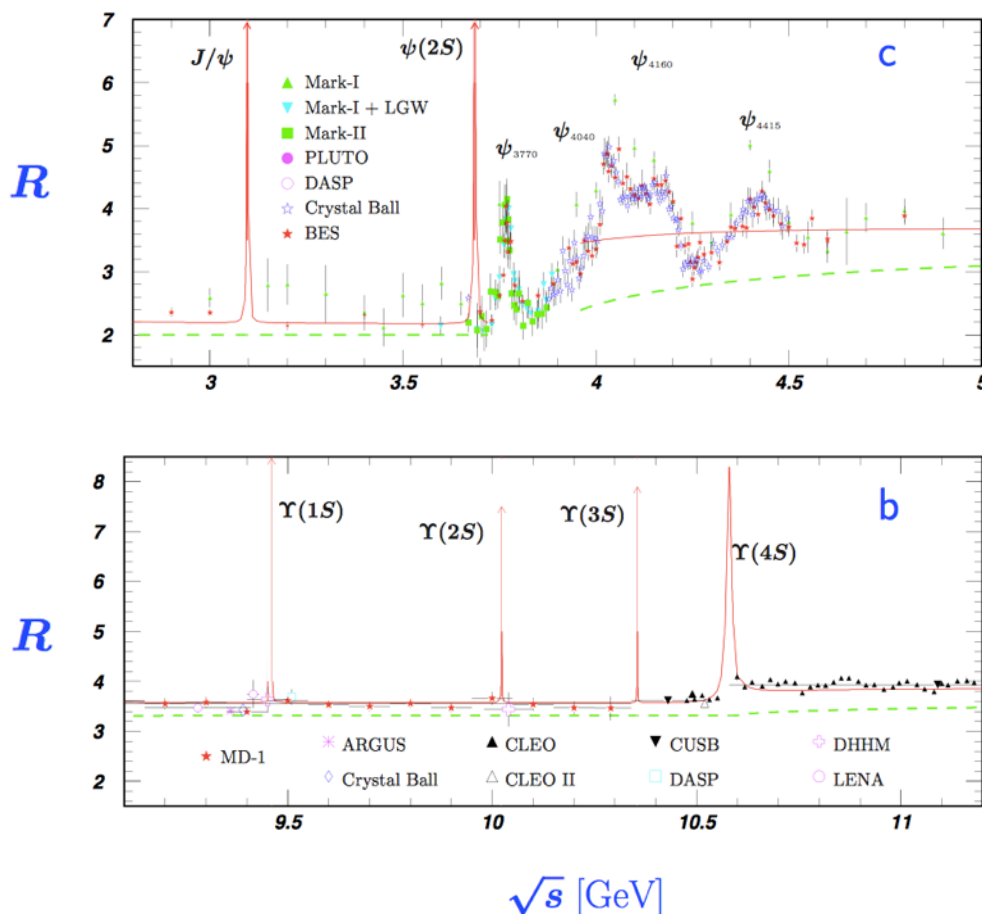
$$\hat{O}_i^{\text{high}} = \hat{O}_i^{\text{high}}[\{\hat{O}_k^{\text{in}}\}, \{m_Q(m_Q)\}] = \hat{O}_i^{\text{high}}[\{\hat{O}_k^{\text{in}}\}, \hat{O}_1^{\text{low}}, \hat{O}_2^{\text{low}}].$$

★ We choose to deal with  $\hat{O}_1^{\text{low}}, \hat{O}_2^{\text{low}} = \mathcal{M}_1^c, \mathcal{M}_2^b$ .

# Eliminating quark masses

★ Find low-energy observables that are sensitive to quark masses

- (moments of) semileptonic b/c decay rates, QQ production rates, etc.



$$\mathcal{M}_n^Q \equiv \int \frac{ds}{s^{n+1}} R_Q(s),$$

where

$$R_Q \equiv \frac{\sigma(e^+e^- \rightarrow Q\bar{Q}X)}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$

We shall use:  $\mathcal{M}_1^c$  and  $\mathcal{M}_2^b$

# Quark masses and moments of $R(s)$

★ Moments of  $R(s)$  can be obtained from experimental data

$$\mathcal{M}_n^Q \equiv \int \frac{ds}{s^{n+1}} R_Q(s),$$

- where, e.g. in the narrow width approximation (NWA),

$$\begin{aligned} R(t) &\equiv 4\pi \operatorname{Im} \Pi(t + i\epsilon) \\ &= \pi \frac{N}{Q_c^2 \alpha^2} \sum_{J/\psi} M_\psi \Gamma_{\psi \rightarrow e^+ e^-} \delta(t - M_\psi^2) \end{aligned}$$

$$\text{with } (q^2 g_{\mu\nu} - q_\mu q_\nu) \Pi_Q(q^2) = -i \int d^4x e^{iq \cdot x} \langle 0 | T j_\mu(x) j_\nu^\dagger(0) | 0 \rangle$$

★ At the moment need to use a combination of NWA, data, and pQCD results

★ What does it have to do with the quark masses?



# Quark masses and moments of $R(s)$

★ Assuming (global) quark-hadron duality, can calculate moments in QCD

- moments are related to derivatives of the correlation function

$$\mathcal{M}_n^Q = \frac{12\pi^2}{n!} \left( \frac{d}{dq^2} \right)^n \Pi_Q(q^2)|_{q^2=0}$$

- use operator-product expansion of the correlation function

$$(q^2 g_{\mu\nu} - q_\mu q_\nu) \Pi_Q(q^2) = -i \int d^4x e^{iq \cdot x} \langle 0 | T j_\mu(x) j_\nu^\dagger(0) | 0 \rangle$$

- ... to get the "QCD expression" for the moments

$$\begin{aligned} \mathcal{M}_n^Q = & \frac{(Q_Q/(2/3))^2}{(2m_Q(\mu))^{2n}} \sum_{i,j} \bar{C}_{n,i}^{(j)}(n_f) \left( \frac{\alpha_s(\mu)}{\pi} \right)^i \ln^j \frac{m_Q(\mu)^2}{\mu^2} \\ & + \mathcal{M}_n^{Q,np}, \end{aligned}$$

★ Scales at which  $m_Q$  and  $\alpha_s$  are renormalized should be considered independently to avoid bias in the uncertainty estimate

Dehnadi, Hoang, Mateu,  
Zebarjad, 1102.2264

# Quark masses and moments of $R(s)$

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- ... to get the "QCD expression" for the moments

$$\begin{aligned} \mathcal{M}_n^Q &= \frac{(Q_Q/(2/3))^2}{(2m_Q(\mu_m))^{2n}} \sum_{i,a,b} C_{n,i}^{(a,b)}(n_f) \left( \frac{\alpha_s(\mu_\alpha)}{\pi} \right)^i \\ &\times \ln^a \frac{m_Q(\mu_m)^2}{\mu_m^2} \ln^b \frac{m_Q(\mu_m)^2}{\mu_\alpha^2} + \mathcal{M}_n^{Q,\text{np}}. \end{aligned}$$

★ Calculated moments exhibit dependence on scales  $\mu_m$  and  $\mu_\alpha$

- thus, Higgs observables will be sensitive to them as well

# Non-perturbative corrections

## ★ Sensitivity of moments to non-perturbative parameters?

- depends on what moment we are dealing with

$$\mathcal{M}_n^Q = \frac{12\pi^2}{n!} \left( \frac{d}{dq^2} \right)^n \Pi_Q(q^2)|_{q^2=0}$$

- ... but for low-order moments dependence is not large

$$\mathcal{M}_1^{c,\text{np}} = \frac{\langle \frac{\alpha_s}{\pi} G^2 \rangle}{(2m_c^{\text{pole}})^6} \left[ -16.042 - 168.07 \frac{\alpha_s(\mu)}{\pi} + \mathcal{O}(\alpha_s^2) \right]$$

$$\text{For } \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle = 0.006 \pm 0.012 \text{ GeV}^4$$

$$\text{get } \mathcal{M}_1^{c,\text{np}} = -0.0001_{-0.0014}^{+0.0006} \text{ GeV}^{-2}$$

★ For b-quark the dependence is negligible, for c-quark it is sub-percent

# Eliminating quark masses

★ Let us understand theoretical uncertainties of input observables

- concentrate on quark masses from low-energy observables

$$\mathcal{M}_1^c = \mathcal{M}_1^c[\alpha_s(m_Z), m_c(m_c), \mu_m^c, \mu_\alpha^c, \mathcal{M}_1^{c,\text{np}}]$$

$$\mathcal{M}_2^b = \mathcal{M}_2^b[\alpha_s(m_Z), m_b(m_b), \mu_m^b, \mu_\alpha^b],$$

- ... which can be solved for quark masses

$$m_c(m_c) = m_c(m_c)[\alpha_s(m_Z), \mathcal{M}_1^c, \mu_m^c, \mu_\alpha^c, \mathcal{M}_1^{c,\text{np}}],$$

$$m_b(m_b) = m_b(m_b)[\alpha_s(m_Z), \mathcal{M}_2^b, \mu_m^b, \mu_\alpha^b],$$

- ... which can be then eliminated from the Higgs partial widths in favor of direct observables (moments in our case)

$$\begin{aligned}\Gamma_{H \rightarrow b\bar{b}} &= \Gamma_{H \rightarrow b\bar{b}}[\{\hat{\mathcal{O}}_k^{\text{in}}\}, m_b(m_b), \mu_H^b] \\ &= \Gamma_{H \rightarrow b\bar{b}}[\{\hat{\mathcal{O}}_k^{\text{in}}\}, \mathcal{M}_2^b, \mu_m^b, \mu_\alpha^b, \mu_H^b],\end{aligned}$$

$$\begin{aligned}\Gamma_{H \rightarrow c\bar{c}} &= \Gamma_{H \rightarrow c\bar{c}}[\{\hat{\mathcal{O}}_k^{\text{in}}\}, m_c(m_c), \mu_H^c] \\ &= \Gamma_{H \rightarrow c\bar{c}}[\{\hat{\mathcal{O}}_k^{\text{in}}\}, \mathcal{M}_1^c, \mu_m^c, \mu_\alpha^c, \mu_H^c, \mathcal{M}_1^{c,\text{np}}],\end{aligned}$$

# Numerics: input parameters

★ Now let us see what we can get out of this numerically

$$\mathcal{M}_1^c = 0.2121(20)(30) \text{ GeV}^{-2} [17],$$

$$\mathcal{M}_2^b = 2.819(27) \times 10^{-5} \text{ GeV}^{-4} [45],$$

$$\alpha_s(m_Z) = 0.1185(6) [14],$$

$$m_H = 125.7(4) \text{ GeV} [14],$$

$$m_t = 173.21(51)(71) \text{ GeV} [14],$$

$$m_Z = 91.1876(21) \text{ GeV} [14],$$

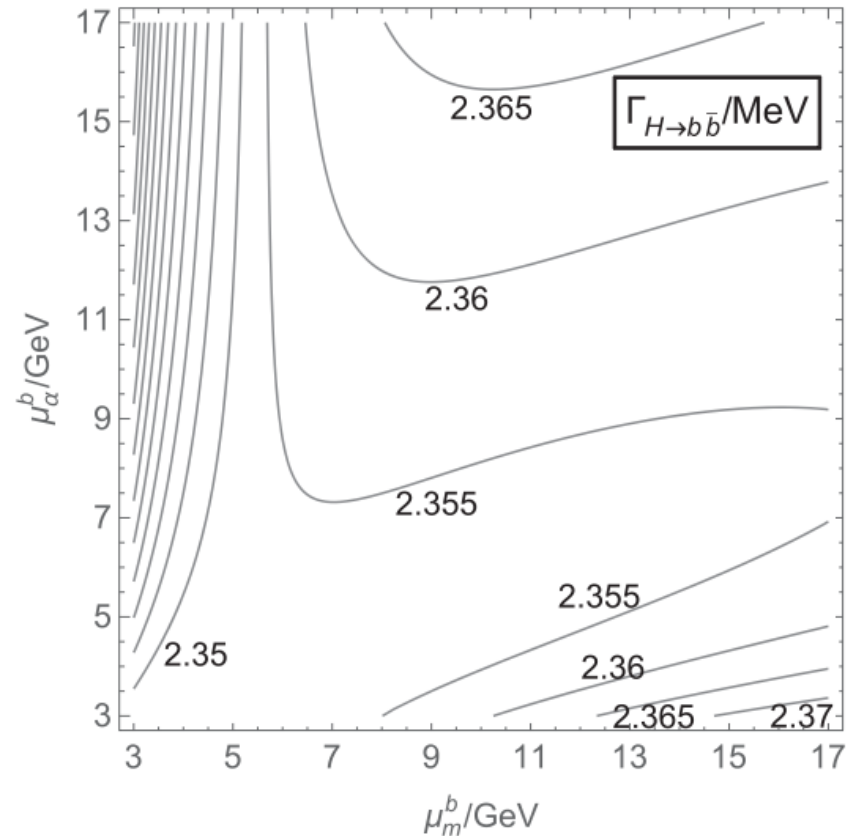
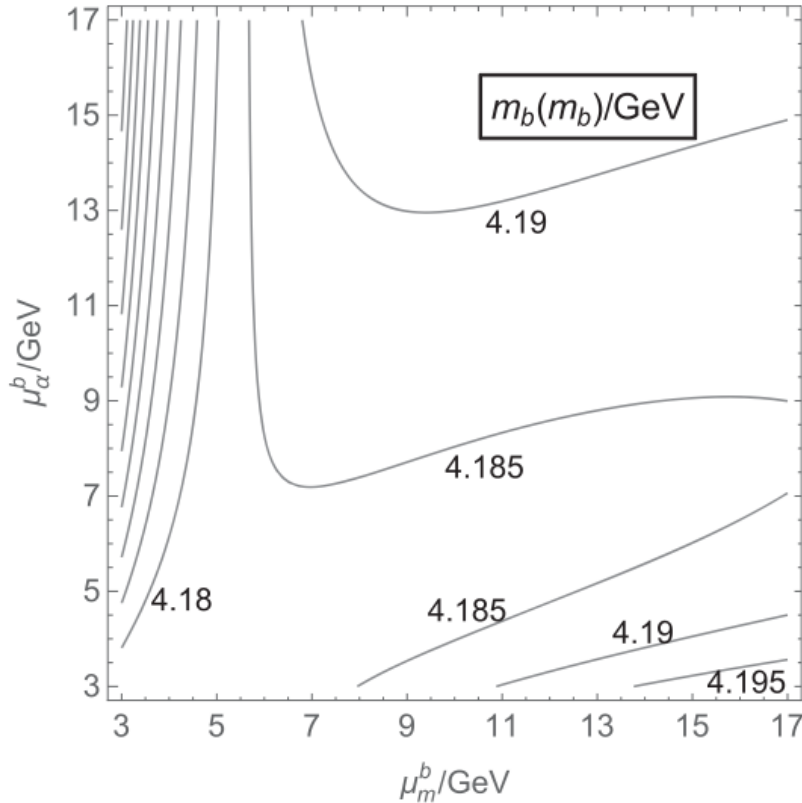
$$\alpha(m_Z) = 1/127.940(14) [14],$$

$$G_F = 1.1663787(6) \times 10^{-5} \text{ GeV}^{-2} [14].$$



# Parametric dependence

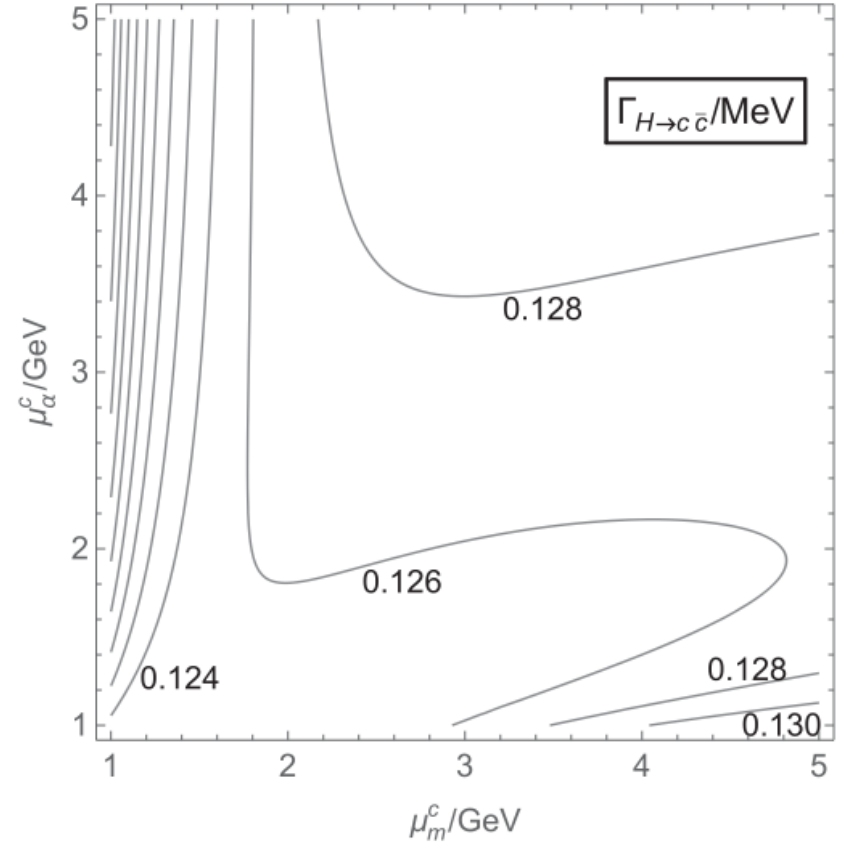
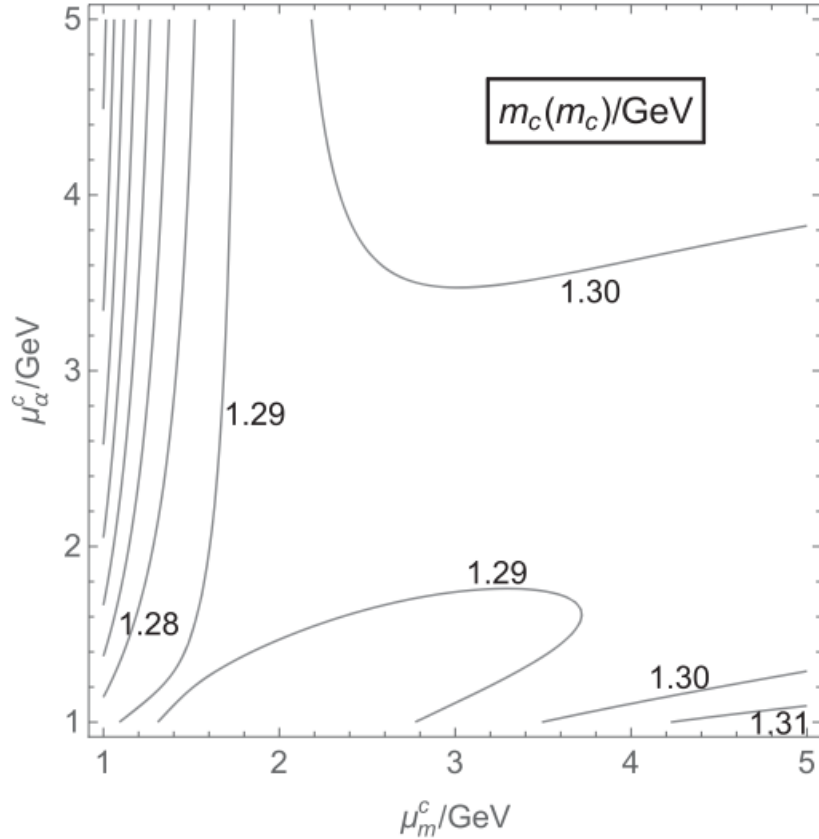
PRD91, 073001 (2015)



$$\begin{aligned}\Gamma_{H \rightarrow b\bar{b}} &= \Gamma_{H \rightarrow b\bar{b}}[\{\hat{\mathcal{O}}_k^{\text{in}}\}, m_b(m_b), \mu_H^b] \\ &= \Gamma_{H \rightarrow b\bar{b}}[\{\hat{\mathcal{O}}_k^{\text{in}}\}, \mathcal{M}_2^b, \mu_m^b, \mu_\alpha^b, \mu_H^b],\end{aligned}$$

# Parametric dependence

PRD91, 073001 (2015)



$$\begin{aligned}\Gamma_{H \rightarrow c\bar{c}} &= \Gamma_{H \rightarrow c\bar{c}}[\{\hat{\mathcal{O}}_k^{\text{in}}\}, m_c(m_c), \mu_H^c] \\ &= \Gamma_{H \rightarrow c\bar{c}}[\{\hat{\mathcal{O}}_k^{\text{in}}\}, \mathcal{M}_1^c, \mu_m^c, \mu_\alpha^c, \mu_H^c, \mathcal{M}_1^{c,\text{np}}],\end{aligned}$$

# Parametric dependence

## ★ How to deal with low-energy scale uncertainties?

- vary scales, BLM/principle of maximum conformality, convergence test, etc?

PRD91, 073001 (2015)

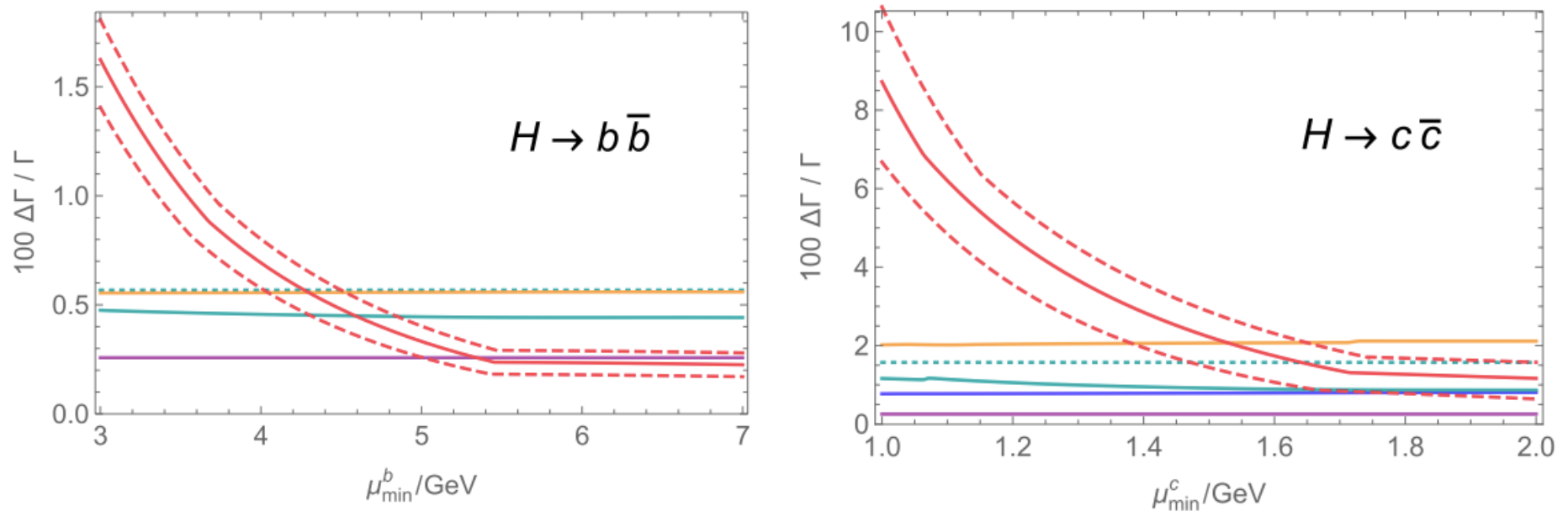


FIG. 2 (color online). Percent relative uncertainties in  $\Gamma_{H \rightarrow c \bar{c}}$  (left) and  $\Gamma_{H \rightarrow b \bar{b}}$  (right) as functions of  $\mu_{\min}$  from various sources: perturbative uncertainty with  $\mu_{\max}^c = 4$  GeV,  $\mu_{\max}^b = 15$  GeV (red solid) or alternatively  $\mu_{\max}^c = 3, 5$  GeV,  $\mu_{\max}^b = 13, 17$  GeV (red dashed), parametric uncertainties from  $\mathcal{M}_1^c$  or  $\mathcal{M}_2^b$  (orange),  $\alpha_s(m_Z)$  (cyan solid),  $\mathcal{M}_1^{c, \text{np}}$  (blue, for  $\Gamma_{H \rightarrow c \bar{c}}$  only), and  $m_H$  (purple). The parametric uncertainty from  $\alpha_s(m_Z)$  incorrectly calculated assuming no correlation with  $m_Q$  (cyan dotted) is also shown for comparison.

# Conclusions

- We are entering the era of precision Higgs studies
    - Higgs as part of the Intensity Frontier?
      - observation of discrepancies in Higgs observables from New Physics
      - understanding of uncertainties of low-energy inputs
  - Global fits to Higgs/LE observables to avoid using "processed numbers"
    - study Higgs partial widths with direct inputs from low energy
      - issues in proper selection of renormalization scale
      - ...which result in uncertainties of partial widths
- $$\frac{\Delta\Gamma_{H\rightarrow c\bar{c}}}{\Gamma_{H\rightarrow c\bar{c}}} \simeq \frac{\Delta m_c(m_c)}{10 \text{ MeV}} \times 2.1\%,$$
- $$\frac{\Delta\Gamma_{H\rightarrow b\bar{b}}}{\Gamma_{H\rightarrow b\bar{b}}} \simeq \frac{\Delta m_b(m_b)}{10 \text{ MeV}} \times 0.56\%.$$
- New data from Belle-II on  $R(s)$  is welcome - reduce uncertainty
  - More data from ATLAS/CMS then Fcc-ee/CEPC on Higgs partial widths
  - Maybe Higgs will show us the first glimpses of New Physics...
  - ...but then again, maybe not.

# Thank you for your attention!

JOURNAL OF APPLIED BEHAVIOR ANALYSIS

1974, 7, 497

NUMBER 3 (FALL 1974)

*THE UNSUCCESSFUL SELF-TREATMENT OF  
A CASE OF "WRITER'S BLOCK"*<sup>1</sup>

DENNIS UPPER

VETERANS ADMINISTRATION HOSPITAL, BROCKTON, MASSACHUSETTS

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REFERENCES

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<sup>1</sup>Portions of this paper were not presented at the 81st Annual American Psychological Association Convention, Montreal, Canada, August 30, 1973. Reprints may be obtained from Dennis Upper, Behavior Therapy Unit, Veterans Administration Hospital, Brockton, Massachusetts 02401.

*Received 25 October 1973.  
(Published without revision.)*

Hopefully, I did better than him...



“Uncertainties from  $m_Q$ ” are decomposed into concrete sources.

Uncertainty source	$\Delta\Gamma_{H\rightarrow c\bar{c}}/\Gamma_{H\rightarrow c\bar{c}}$	$\Delta\Gamma_{H\rightarrow b\bar{b}}/\Gamma_{H\rightarrow b\bar{b}}$
$\mathcal{M}_n^Q$ measurement <sup>†</sup>	2%	0.6%
$\mathcal{M}_n^Q$ calculation	see next 3 slides	
$\alpha_s$ (vs. no correlation)	1% (1.6%)	0.5% (0.6%)
$\mathcal{M}_n^{Q,np}$	<0.8%	→0
$m_H$	<0.3%	<0.3%