

# $\alpha_s$ *from the (revised) Aleph data for $\tau$ decay*

SANTIAGO PERIS (UAB)

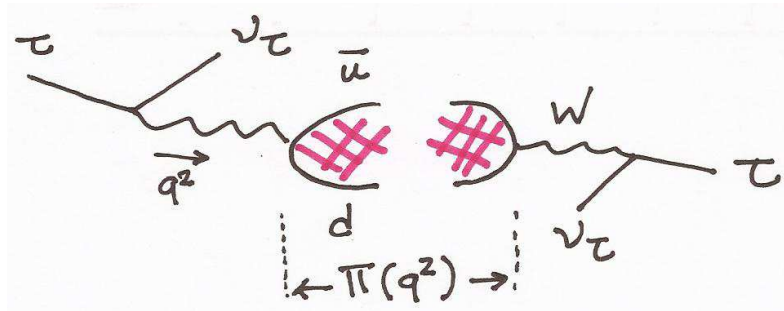
In collaboration with D. Boito, M. Golterman, K. Maltman and J. Osborne

ICHEP 2016, Chicago

Mainly based on Phys. Rev. **D91**, 034003 (2015) and refs. therein.

See also arXiv:1606.08898 [hep-ph] and 1606.08899 [hep-ph].

# QCD in $\tau$ decay

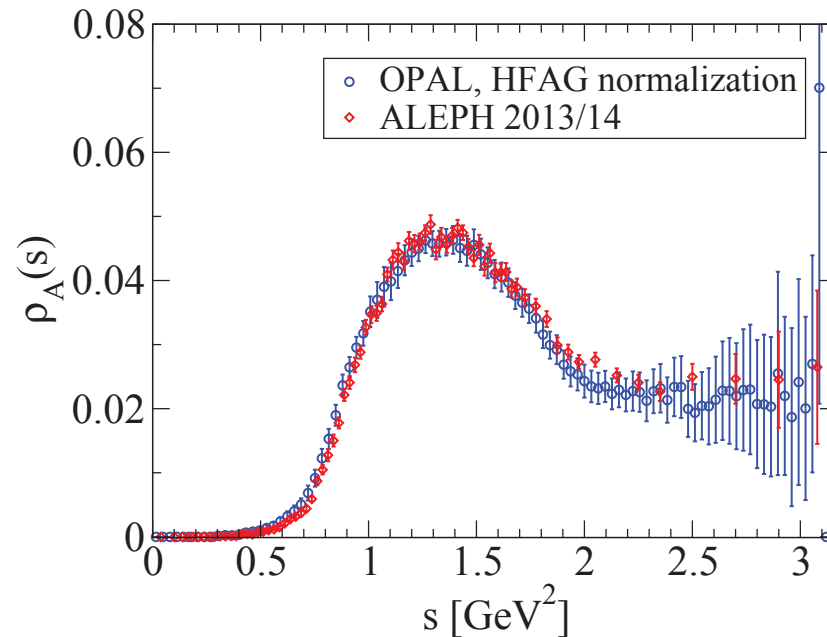
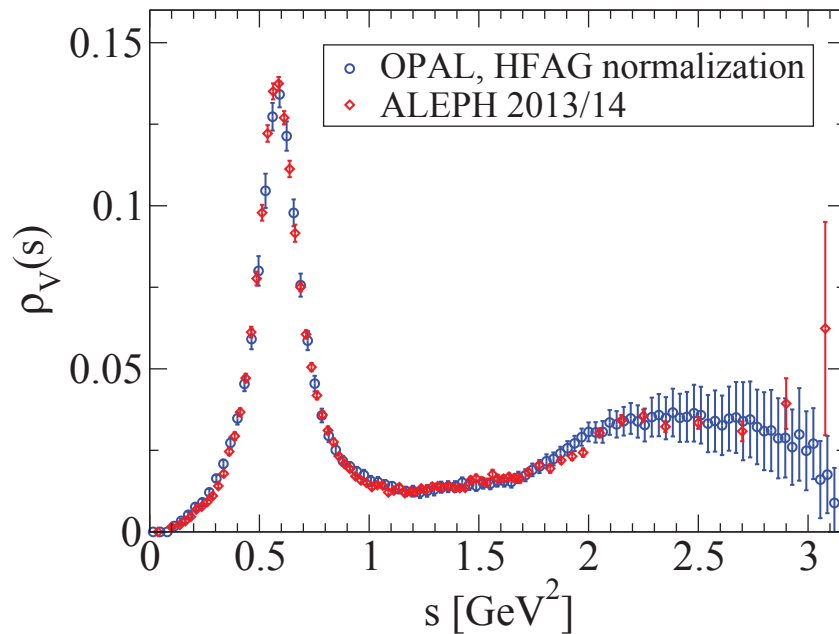


$$w_T(s; s_0) = (1 + 2\frac{s}{s_0}) \underbrace{(1 - \frac{s}{s_0})^2}_{\text{doubly pinched}}$$

$$w_L(s; s_0) = 2(\frac{s}{s_0}) \underbrace{(1 - \frac{s}{s_0})^2}_{\text{doubly pinched}}$$

$$s_0 = m_\tau^2 \quad \rho_{V,A} = \frac{1}{\pi} \text{Im} \Pi_{V,A}$$

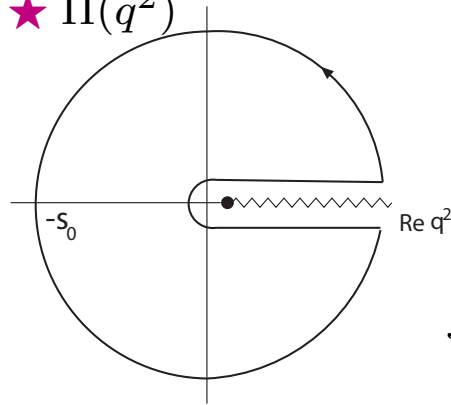
$$\frac{\Gamma(\tau \rightarrow \nu_\tau \mathbf{H}_{ud}(\gamma))}{\Gamma[\tau \rightarrow \nu_\tau e \bar{\nu}_e(\gamma)]} = 12\pi^2 |V_{ud}|^2 S_{EW} \int_0^{s_0} \frac{ds}{s_0} \left[ w_T(s; s_0) \rho_{V+A}^{(1+0)}(s) - w_L(s; s_0) \rho_A^{(0)}(s) \right]$$



# Theoretical Foundations

Shankar '77; Braaten-Narison-Pich '92

★  $\Pi(q^2)$



“Cauchy’s Theorem” ( $z = q^2$  ;  $\rho(t) = \frac{1}{\pi} \text{Im}\Pi$  ;  $w_n = \text{polynomial}$ ) :

$$\int_0^{s_0} dt w_n(t) \underbrace{\rho(t)}_{\text{exp.}} = \frac{1}{2i\pi} \oint_{|z|=s_0} dz w_n(z) \Pi(z)$$

$$= \frac{1}{2i\pi} \oint_{|z|=s_0} dz w_n(z) \left[ \underbrace{\Pi_{\text{OPE}}(z)}_{\mathcal{O}(\alpha_s^4)} + \underbrace{\Pi(z) - \Pi_{\text{OPE}}(z)}_{\Pi_{DV}(z)} \right]$$

★  $\Pi_{DV} \rightarrow 0 \iff \Pi_{\text{OPE}} \rightarrow \Pi$ .

(Cata-Golterman-S.P. '05)

However,  $\Pi_{\text{OPE}}$  expected asymptotic.

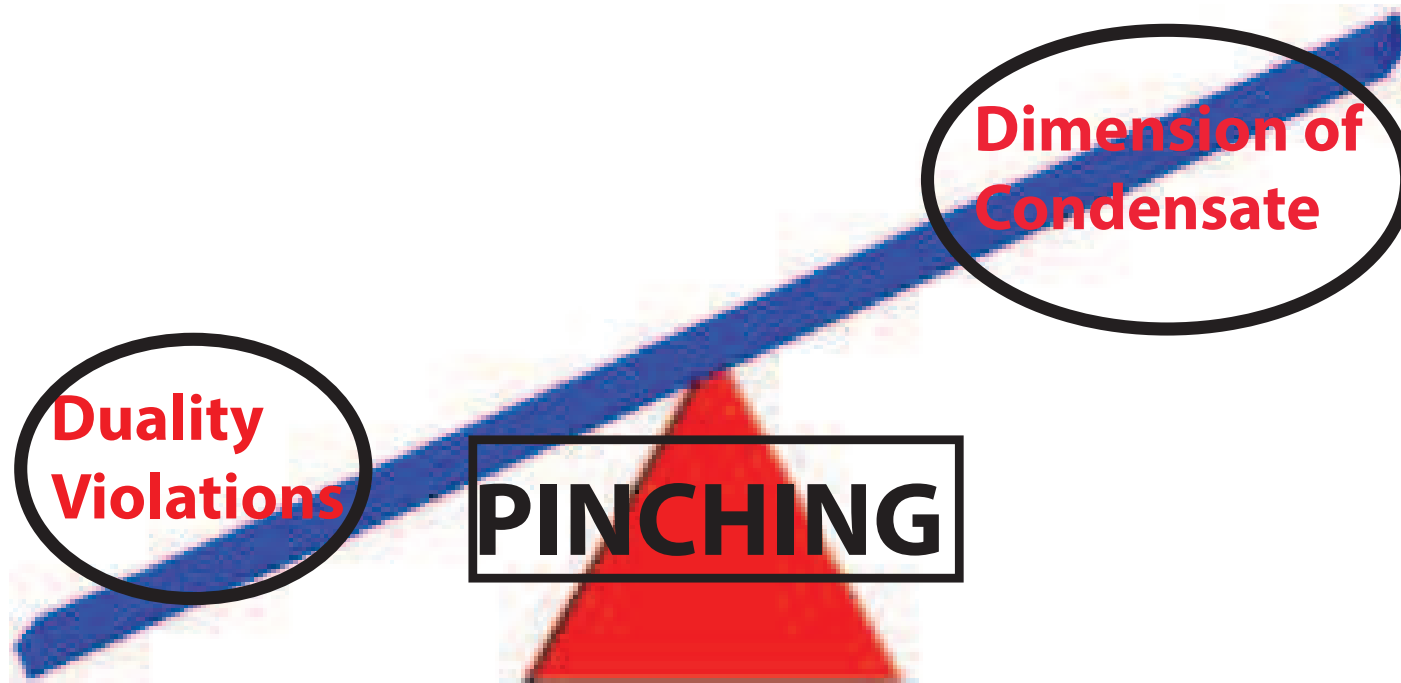
For  $s_0$  large:

$$-\frac{1}{2\pi i} \oint_{|z|=s_0} dz w(z) \Pi_{DV}(z) = - \underbrace{\int_{s_0}^{\infty} ds w(s)}_{\text{extrapolation!}} \underbrace{\frac{1}{\pi} \text{Im}\Pi_{DV}(s)}_{\sim \text{exp. damping}}$$

# Main Theoretical Message

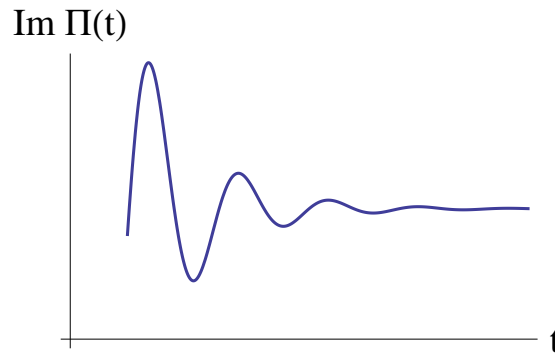
★ “Seesaw” mechanism at work:

Pinching: Not possible to suppress simultaneously DVs and NP condensate contributions.



# What is $\text{Im}\Pi_{DV}$ ?

Asymptotically at large  $t$ :



No answer from first principles, yet; just an educated guess.  
(to be checked against the data)

- **Oscillation Period**: assume Regge-type spectrum, i.e.  $M_n^2 \sim n$ ;

$$\Rightarrow \text{Im}\delta\Pi \sim \sin(\alpha + \beta q^2).$$

- **Amplitude**: Exp. damping from asymptotic expansions and renormalons,

$$\Pi(Q^2) \sim \sum_n n! (b\alpha_s)^{n+1} \Rightarrow \delta\Pi \sim e^{-1/b'\alpha_s} \sim \frac{\langle\alpha_s G^2\rangle}{q^4}$$

with the replacement  $\alpha_s \rightarrow 1/q^2$ , i.e.  $\delta\Pi \sim e^{-\gamma q^2}$   
 $\Rightarrow \text{Im}\delta\Pi \sim e^{-\gamma q^2} \sin(\alpha + \beta q^2).$

These properties can be explicitly verified in a (physically motivated) model.  
(Blok, Shifman and Zhang '97)

# A Change of Strategy (I)

Old Strategy: (LeDiberder-Pich '92)

- Use 5 pinched weights

$$w_{kl}(y) = (1 - y)^2(1 + 2y)(1 - y)^k y^l \quad , \quad y = s/s_0, \quad s_0 = m_\tau^2 \text{ (only)}$$

with  $(k, l) = \{(0, 0), (1, 0), (1, 1), (1, 2), (1, 3)\}$ .

- Fit to extract 4 param. :  $\alpha_s$  and  $C_{D=4,6,8}$ .
- Set (arbitrarily) OPE condensates  $C_{D=10,12,14,16} = 0$ .
- Set (unknown) Duality Violations =0.
- May use  $V$  and  $A$ , but assume  $V + A$  more reliable.

(Davier et al. '14)

$$\begin{aligned} \langle \frac{\alpha_s}{\pi} GG \rangle &= (-0.5 \pm 0.3) \times 10^{-2} \text{ GeV}^4, & \chi^2 = 0.43, p = 51\% & \quad V, \\ &(-3.4 \pm 0.4) \times 10^{-2} \text{ GeV}^4, & \chi^2 = 3.4, p = 7\% & \quad A, \\ &(-2.0 \pm 0.3) \times 10^{-2} \text{ GeV}^4, & \chi^2 = 1.1, p = 29\% & \quad V + A. \end{aligned}$$

- Check Weinberg sum rules.

# A Change of Strategy (II)

New Strategy (Boito et al. '11 and '12):

- Do not use  $w(y)$  with a term linear in  $y$ . (Beneke et al. '13)
- Do not assume any condensate is zero. (Let the data speak.)
- Do not assume that Duality Violations are zero. (Let the data speak.)

For  $s \geq s_{min}$  (Regge/asymptotic series model assumption):

$$\rho_{DV}^{V,A}(s) = e^{-\delta_{V,A} - \gamma_{V,A}s} \sin(\alpha_{V,A} + \beta_{V,A}s)$$

c.f. old strategy model assumption:  $e^{-\delta_{V,A}} = 0$ .

- Fit to  $\alpha_s$ ,  $C_{D=6,8}$  and DV parameters with 3 weights:

$$w_0 = 1, w_2 = 1 - y^2 \quad \text{and} \quad w_3 = (1 - y)^2(1 + 2y)$$

Use all data for  $s_0 \geq s_{min}$ , to be determined by the fit as well.

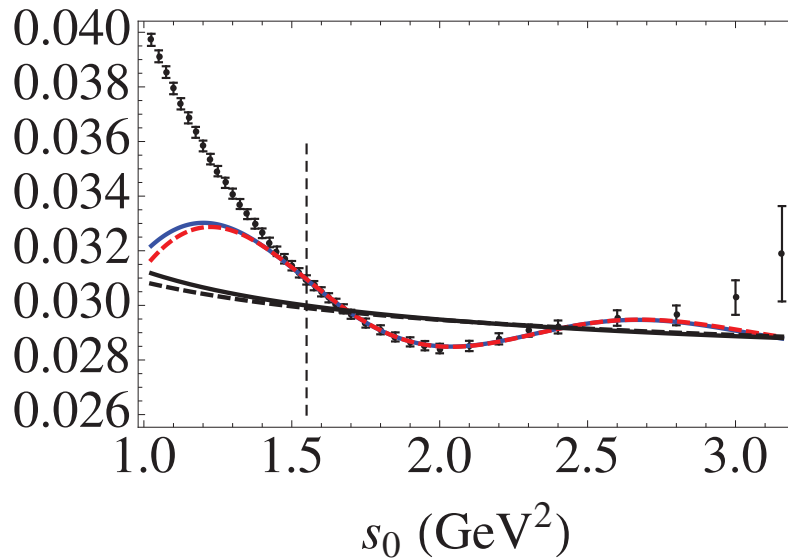
- Use  $V$  and  $A$ . Check spectral functions.
- Check Weinberg sum rules.

# Example: Fit to $w_0 = 1$ , $V$ channel (I).

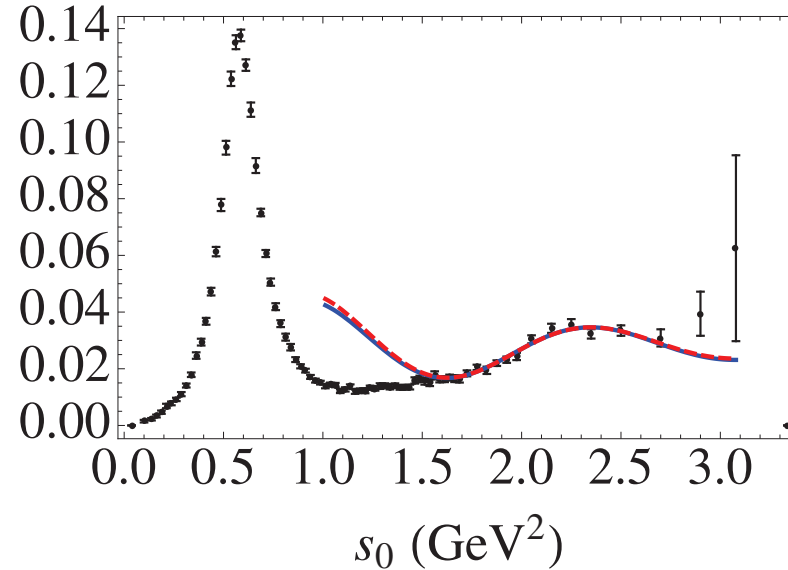
$$s_{min} = 1.55 \text{ GeV}^2, \quad \chi^2/dof = 24.5/16 \quad (p = 8\%) \quad (D = 0 \text{ FOPT, CIPT similar})$$

curves:      red=CIPT      blue=FOPT      black=no DV

$w_0 = 1$  spectral integral

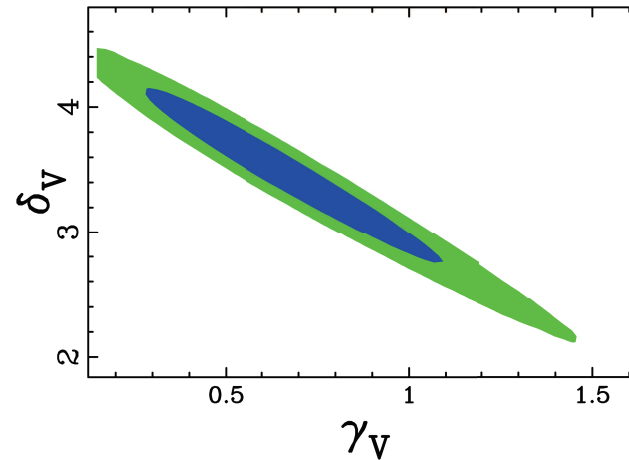
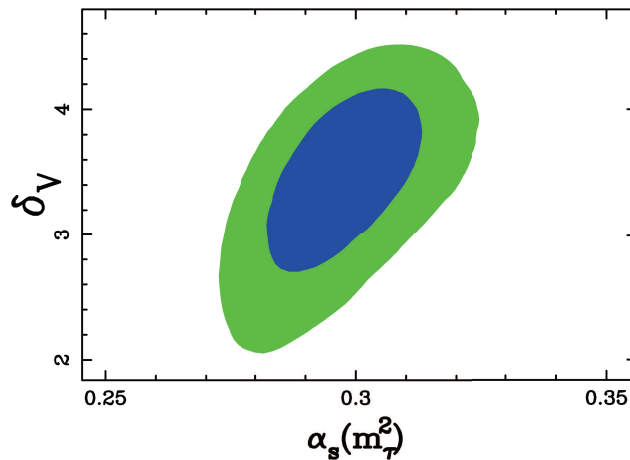
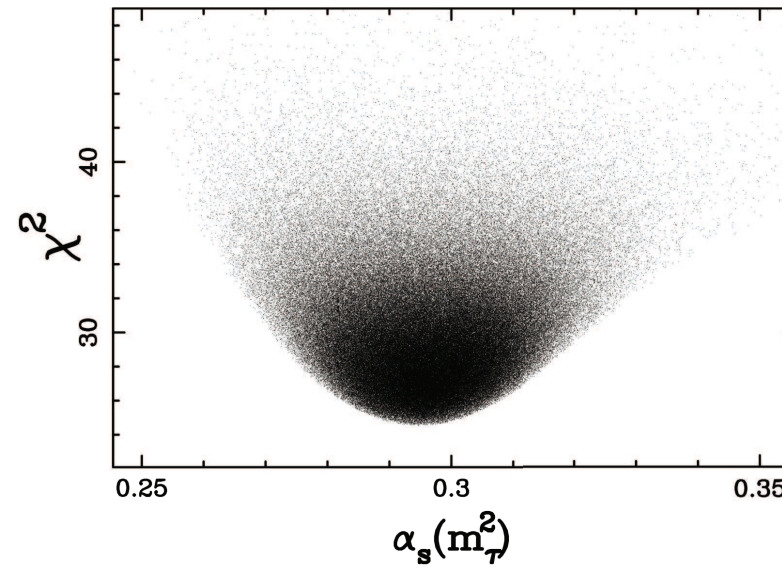


$V$  spectrum





# Example: Fit to $w_0 = 1$ , $V$ channel (II).



(68% and 95% contour plots), FOPT. **Clearly  $DV_s \neq 0$ .**

# *We did lots of other fits as well...*

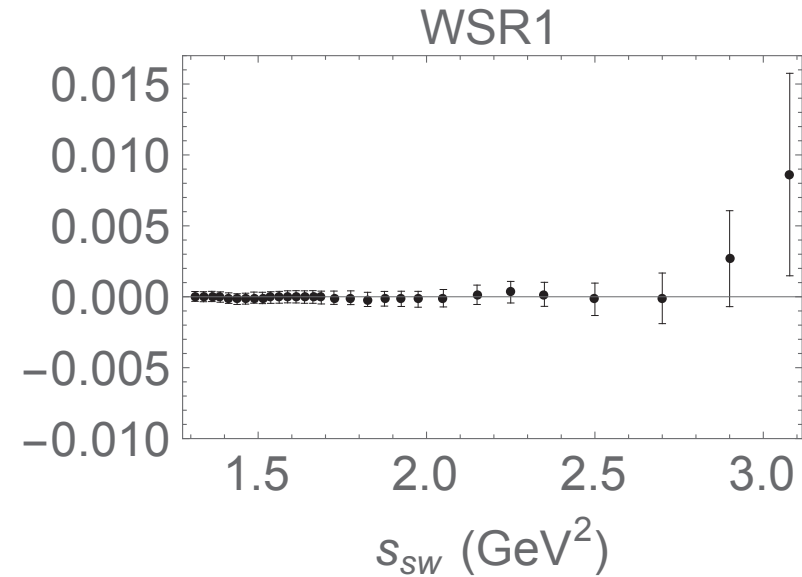
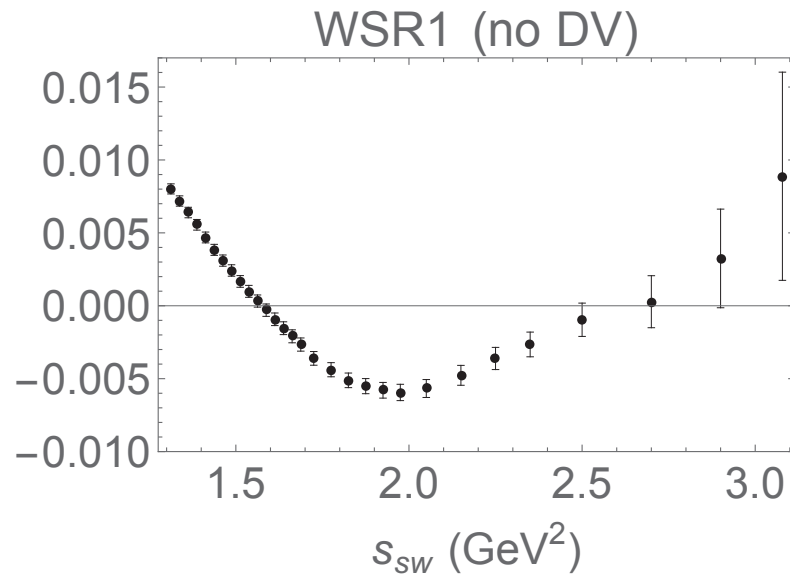
## Fits :

- V channel,  $w_0 = 1$ .
- V and A channels,  $w_0 = 1$ .
- V channel,  $w_0 = 1$  and  $w_2 = 1 - y^2$ .
- V and A channels,  $w_0 = 1$  and  $w_2 = 1 - y^2$ .
- V channel,  $w_0 = 1$ ,  $w_2 = 1 - y^2$  and  $w_3 = (1 - y)^2(1 + 2y)$ .
- V and A channels,  $w_0 = 1$ ,  $w_2 = 1 - y^2$  and  $w_3 = (1 - y)^2(1 + 2y)$ .

Consistent results in all cases.

# Classic Tests

1st Weinberg sum rule:  $\int_0^\infty ds \left( \rho_V^{(1)}(s) - \rho_A^{(1)}(s) \right) - 2f_\pi^2 = 0$



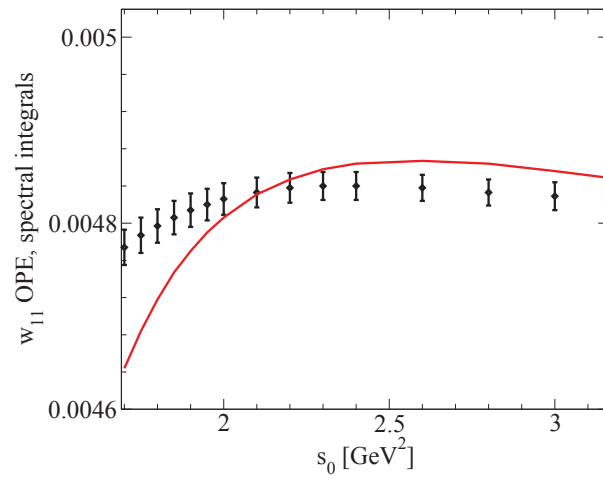
We also checked the 2nd Weinberg sum rule and the pion EM splitting sum rule.

# Further Tests

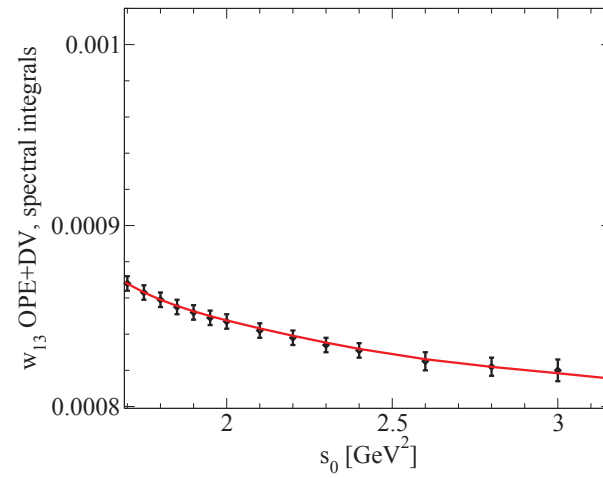
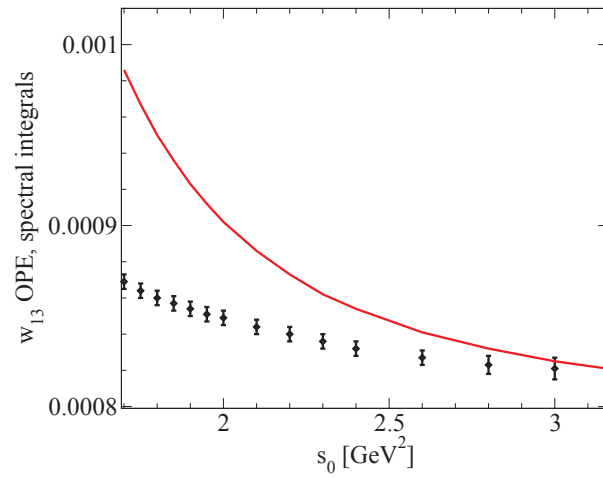
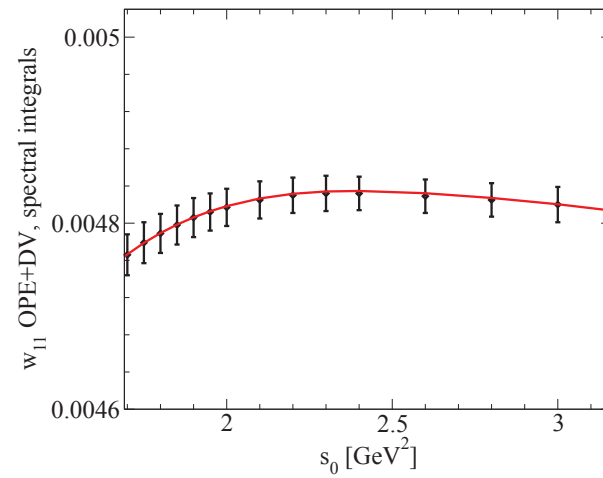
(Maltman-Yavin '08)

"Old vs. New" Strategy in other FESRs:  $w_{11}, w_{13}$ , etc...

Old



New



# Results

## Aleph:

$$\text{(FOPT)} \quad \alpha_s(m_\tau) = 0.296 \pm 0.010 \longrightarrow \alpha_s(m_Z) = 0.1155 \pm 0.0014$$

$$\text{(CIPT)} \quad \alpha_s(m_\tau) = 0.310 \pm 0.014 \longrightarrow \alpha_s(m_Z) = 0.1174 \pm 0.0019$$

- N.B. “Old Strategy” produces a shift, i.e.

$$\alpha_s(m_\tau) \sim +0.03 \text{ higher, (and } \sim \text{ half errors) (Davier et al. '14)}$$

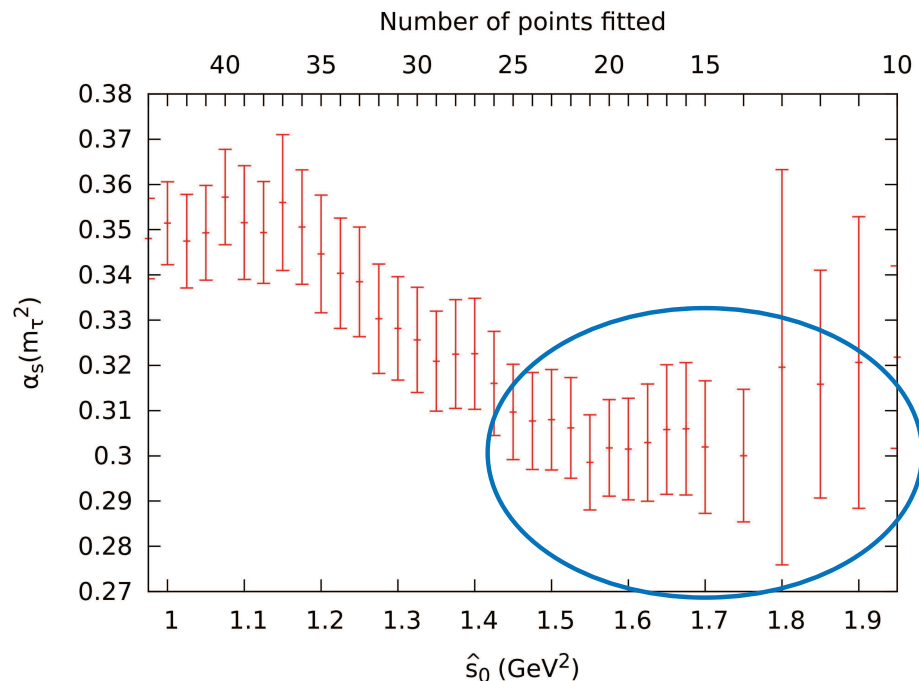
- Using **Aleph + Opal** data, we get:

$$\alpha_s(m_Z) = 0.1165 \pm 0.0012 \text{ (FOPT)} \qquad \alpha_s(m_Z) = 0.1188 \pm 0.0015 \text{ (CIPT)}$$

$$\text{(Current PDG world average: } \alpha_s(m_Z) = 0.1181 \pm 0.0013 \text{ )}$$

# Pich and Rodriguez-Sanchez '16

- New reanalysis with the Old Strategy  $\Rightarrow$  same conclusions (not a surprise).  
(and same  $s_0$  dependence problems)
- Criticism: when DVs are included they obtain



$\Leftarrow$  Stability as  $\hat{s}_0 = s_{min}$  increases,  
as expected for the DV ansatz.  
(So this is actually nice...)

## Question of Principle:

If modeling DVs is useful for determining condensates in  $V - A$

(as, e.g., in Glez-Alonso, Pich, Rguez-Sanchez '15,'16)

why wouldn't it be useful for  $\alpha_s$  in  $V + A$  as well ?

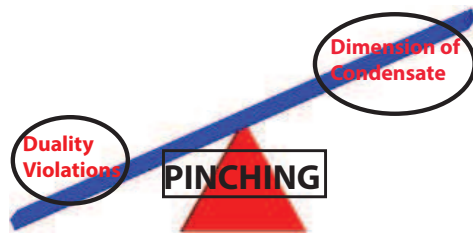
# Conclusions and Outlook

- DVs are clearly **visible** in the  $\tau$  data.

(DVs are not a question of principle, they exist in practice.)

- Pinching does not allow a simultaneous reduction of DVs and higher-dim condensates

(unlike what has been assumed so far in the “Old Strategy” Method).



This introduced an unquantified **systematic error**, and an  $s_0$ -dependence **mismatch**.

- I see no way to make progress without a better understanding of DVs and/or the OPE as a series expansion.

Resurgence ? (Shifman '14)

Functional Analysis Methods ? (Caprini, Golterman, S.P. '14)

# Conclusions and Outlook (II)

- We have introduced a **new strategy** based on an **educated guess** for DVs which **allows the data** to determine both the contribution from DVs and condensates.
- The new strategy **passes all known tests**, experimental and theoretical, performing **better** than the "Old Strategy".

N.B. The "Old Strategy" also uses **a model**:

$$\text{DV}_S = 0 \Leftrightarrow e^{-\delta} = 0 \quad \text{and} \quad \langle O_{10,12,14,16} \rangle = 0.$$

Not favored by data/present theoretical knowledge.

- **Better data (Babar and Belle)** will help significantly.  
(We are very excited about A. Lusiani's future spectral functions analysis...)