Relating the top quark $\overline{\mathrm{MS}}$ and on-shell masses

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Outline

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Mass definitions

When we talk about the mass of a particle we have to define the renormalization scheme used.
Two schemes are particularly important

- $\overline{\mathrm{MS}}$ scheme (running mass)
- on-shell scheme

The value of the mass depends on the choice of the renormalization scheme!
We must be able to translate between the schemes.

Scheme definitions

One considers the renormalized quark propagator

$$S_F(q) = \frac{-i\, Z_2}{\not{q} - Z_m m + \Sigma(q, m)}$$

with $\Sigma(q, m)$ the quark two-point function.
For the $\overline{\mathrm{MS}}$ scheme we require

$$S_F(q) \quad \text{finite}$$

and for the on-shell scheme we require a pole at the position of the mass

$$S_F(q) \overset{q^2 \to M^2}{\longrightarrow} \frac{-i}{\not{q} - M}$$

Outline

Setup of the calculation

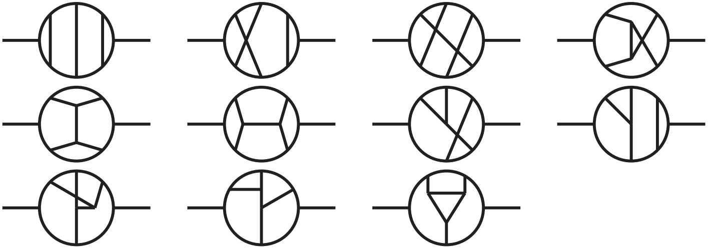
- Need to calculate mass renormalization constant Z_m^{OS} by calculating four-loop on-shell integrals
- Together with the renormalization constant in the $\overline{\text{MS}}$-scheme $Z_m^{\overline{\text{MS}}}$ [Chetyrkin '97; Larin, van Rittbergen, Vermaseren '97]
  we get

$$\left.\begin{aligned} m_{\text{bare}} &= Z_m^{\text{OS}} M \\ m_{\text{bare}} &= Z_m^{\overline{\text{MS}}} m \end{aligned}\right\} \Rightarrow m = M \frac{Z_m^{\text{OS}}}{Z_m^{\overline{\text{MS}}}}$$

  - 1-loop [Tarrach'81]
  - 2-loop [Gray,Broadhurst,Grafe,Schilcher'90]
  - 3-loop [Chetyrkin,Steinhauser'99; Melnikov, v. Ritbergen'00; Marquard,Mihaila,Piclum,Steinhauser'07]

Setup of the calculation

Need to calculate 4-loop on-shell diagrams of the form



where we have to consider all possible ways to route the external quark through the diagrams ⇒ 100 integral families.

Setup of the calculation

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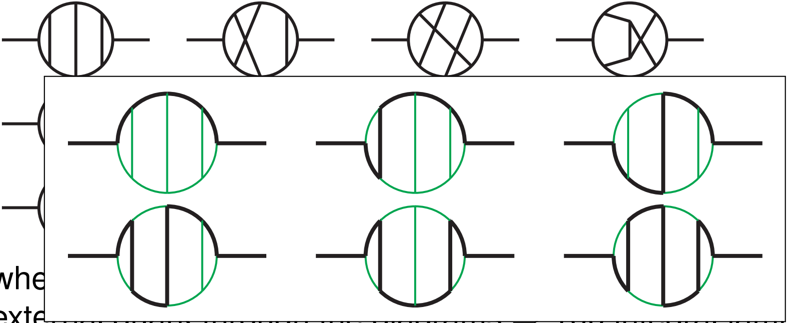

whe

exte … lies.

Setup of the calculation

Follow the *standard* procedure for multi-loop calculations

- exploit that the appearing integrals are not linear independent but related through Integration-By-Parts identities [Chetyrkin, Tkachov '81]
- reduce all appearing integrals to a small set of basis integrals using `FIRE` [Smirnov] or `Crusher` [PM,Seidel]
- evaluate the remaining basis integrals ($\mathcal{O}(350)$) using analytic or numerical techniques (Mellin-Barnes or Sector Decomposition)

$\overline{\text{MS}}$–on-shell relation at four-loop order

$\overline{\text{MS}} \to$ on-shell

$$
\begin{aligned}
m_t(m_t) &= M_t \Big(1 - 0.4244\,\alpha_s - 0.9246\,\alpha_s^2 - 2.593\,\alpha_s^3 \\
&\quad -(8.949 \pm 0.018)\,\alpha_s^4\Big) \\
&= 173.34 - 7.924 - 1.859 - 0.562 \\
&\quad - (0.209 \pm 0.0004)\ \text{GeV}
\end{aligned}
$$

[PM, Steinhauser, Smirnov, Smirnov, Wellmann ’16]

small remaining error due to numerical integration of the master integrals using FIESTA [A. Smirnov] for the sector decomposition.

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\begin{aligned}
M_b &= m_b(m_b) \Big(1 + 0.4244\,\alpha_s + 0.9401\,\alpha_s^2 + 3.045\,\alpha_s^3 \\
&\quad +(12.685 \pm 0.025)\,\alpha_s^4\Big) \\
&= 4.163 + 0.398 + 0.199 + 0.145 + (0.136 \pm 0.0003)\ \text{GeV}
\end{aligned}
$$

Threshold mass schemes

- Potential-subtracted mass

$$m^{\text{PS}} = M - \delta m(\mu_f)$$

$$\delta m(\mu_f) = -\frac{1}{2}\int_{|\vec{q}|<\mu_f} \frac{\mathrm{d}^3 q}{(2\pi)^3} V(\vec{q}) = \mu_f \frac{C_F \alpha_s}{\pi}(1 + \alpha_s \ldots)$$

need static potential @ three loops

[Smirnov,Smirnov,Steinhauser '09; Anzai,Kiyo,Sumino '09]

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- 1S mass

$$m^{1\text{S}} = M + \frac{1}{2}E_1^{\text{pt}}$$

$$E_1^{\text{pt}} = -\frac{C_F^2 M \alpha_s^2}{8}(1 + \alpha_s \ldots)$$

need binding energy @ three loops [Beneke et al]

PS mass $\leftrightarrow$ $\overline{\text{MS}}$ mass

$$
\begin{aligned}
m_t^{\text{PS}}(\mu_f = 80\text{ GeV}) &= 163.508 + (7.531 - 3.685) \\
&\quad +(1.607 - 0.989) + (0.495 - 0.403) \\
&\quad +(0.195 - 0.211 \pm 0.0004)\text{ GeV} \\
&= 163.508 + 3.847 + 0.618 + 0.092 \\
&\quad -(0.016 \pm 0.0004)\text{ GeV}
\end{aligned}
$$

- large cancellations between contributions from OS-MS and PS-OS
- good convergence
- 4-loop contribution 16 MeV

1S mass $\leftrightarrow$ $\overline{\text{MS}}$ mass

$$
\begin{aligned}
m_t^{1\text{S}} &= 163.508 + (7.531 - 0.428) + (1.588 - 0.368) \\
&\quad + (0.479 - 0.262) + (0.185 - 0.174 \pm 0.0004)\ \text{GeV} \\
&= 163.508 + 7.103 + 1.220 \\
&\quad + 0.217 + (0.011 \pm 0.0004)\ \text{GeV}\,.
\end{aligned}
$$

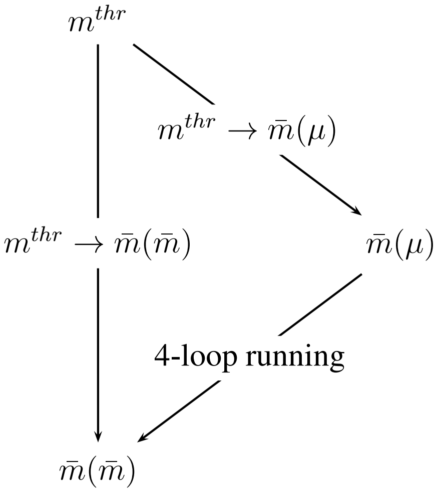
- large cancellations between contributions from OS-MS and 1S-OS
- good convergence
- 4-loop contribution 11 MeV

$m^{\text{thr}} \to \bar{m}(\bar{m})$

alternative error estimate by

- first calculating $\bar{m}(\mu)$
- and in a second step $\bar{m}(\bar{m})$

has to give the same result up to higher-order corrections



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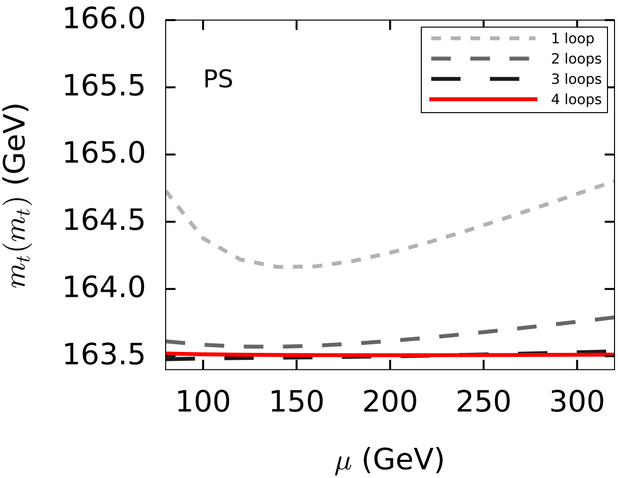

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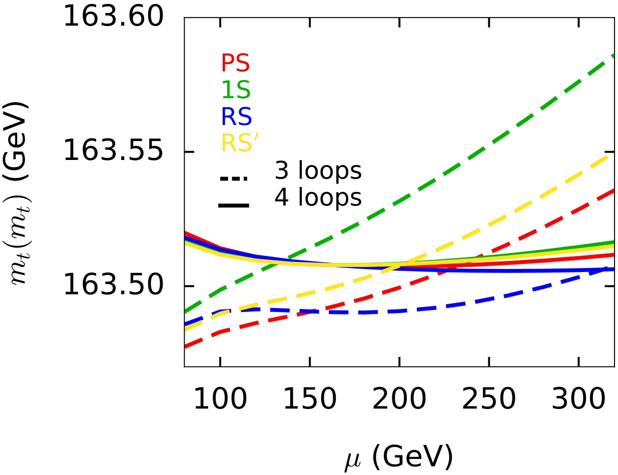


Beyond 4-loops

$$m_P = m(\mu)\left(1 + \sum_{n=1}^{\infty} c_n(\mu, \mu_m, m(\mu))\, \alpha_s^n(\mu)\right)$$

for large n

$$c_n(\mu, \mu_m, m(\mu_m)) \xrightarrow[n\to\infty]{} N c_n^{(\text{as})}(\mu, m(\mu_m)) \equiv N \frac{\mu}{m(\mu_m)}\, \tilde{c}_n^{(\text{as})}\,,$$

[Beneke '94 '99]

where

$$\tilde{c}_{n+1}^{(\text{as})} = (2b_0)^n \frac{\Gamma(n+1+b)}{\Gamma(1+b)} \left(1 + \frac{s_1}{n+b} + \frac{s_2}{(n+b)(n+b-1)} + \cdots\right)$$

b_0, b, s_1, s_2: Combinations of coefficients of the β-function.

Beyond 4-loops

Fit N to 4-loop term and take higher orders from asymptotic formula

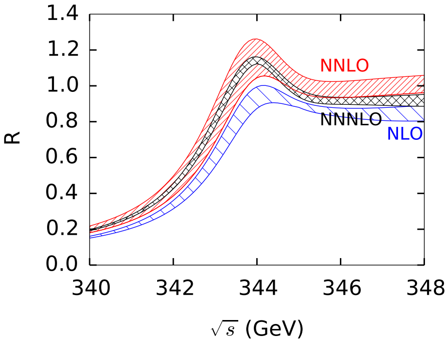
j	$\tilde{c}_j^{(\text{as})}$	$\tilde{c}_j^{(\text{as})}\alpha_s^j$
5	0.985499×10^2	0.001484
6	0.641788×10^3	0.001049
7	0.495994×10^4	0.000880
8	0.443735×10^5	0.000854
9	0.451072×10^6	0.000942
10	0.513535×10^7	0.001164

$$\begin{aligned}\delta^{(5+)} m_P &= 0.272^{+0.016}_{-0.041}\,(N) \pm 0.001\,(c_4) \pm 0.011\,(\alpha_s) \\ &\quad \pm 0.066\text{ (amb) GeV}\end{aligned}$$

[Beneke, PM, Nason, Steinhauser '16]

Outline

Total cross section

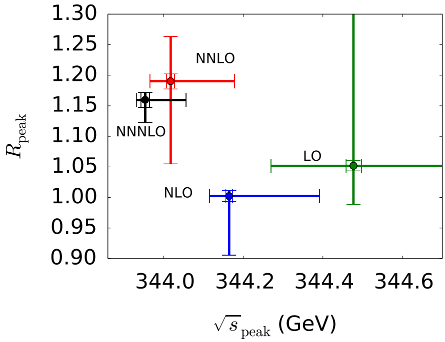


[Beneke,Kiyo,PM,Piclum,Steinhauser,Penin ’15]

$m_t = 171.5\,\mathrm{GeV}$, $\Gamma_t = 1.33\,\mathrm{GeV}$,
$50\,\mathrm{GeV} \leq \mu \leq 350\,\mathrm{GeV}$

- very good convergence of perturbative series below threshold
- above threshold -8% shift driven by large negative corrections to c_v

Peak height vs position



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$m_t = 171.5\,\mathrm{GeV}$, $\Gamma_t = 1.33\,\mathrm{GeV}$,
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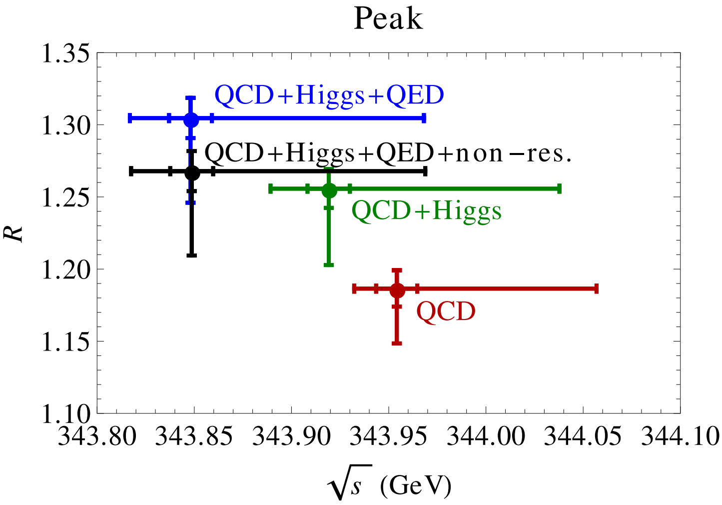
- inner error bars correspond to α_s error
- stabilization of peak position and height
- about 3% uncertainty on the peak height

Electro-weak effects



[Beneke,Maier,Piclum,Rauh '15]

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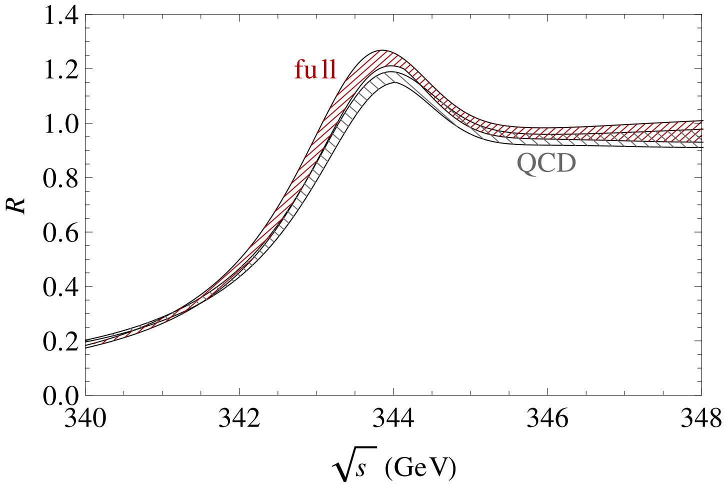
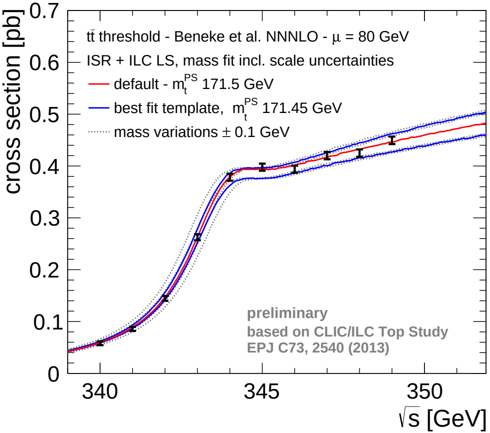


[Beneke,Maier,Piclum,Rauh '15]

available in code `QQbar_threshold`

[Beneke,Kiyo,Maier,Piclum '16]

Experimental study (prelim)



error budget:

- 32 MeV fit
- 45 MeV theory
- $\approx$ 10 MeV experimental systematics
- 32 MeV α_s

[Simon, LCWS '15]

Conclusions

- presented 4-loop corrections to the $\overline{\mathrm{MS}}$–on-shell relation
- four-loop contribution $\approx 200\,\mathrm{MeV}$
- five-loop and beyond add $\approx 250\,\mathrm{MeV}$ with an intrinsic ambiguity of $\approx 66\,\mathrm{MeV}$
- conversion between threshold masses and $\overline{\mathrm{MS}}$ mass well under control
- top-quark mass measurement at ILC with $\Delta M_t \approx 100\,\mathrm{MeV}$ feasible