2+1+1 Polyakov Linear-Sigma Model at finite Temperature and Density

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Sigma Model

Sigma model was introduced by Gell-Mann and Levy in 1960

The name σ-model comes from a field corresponding to the spinless meson scalar σ introduced earlier by Schwinger.

Il Nuovo Cimento 16, 705 (1960)

Sigma-Model is a physical system with the Lagrangian

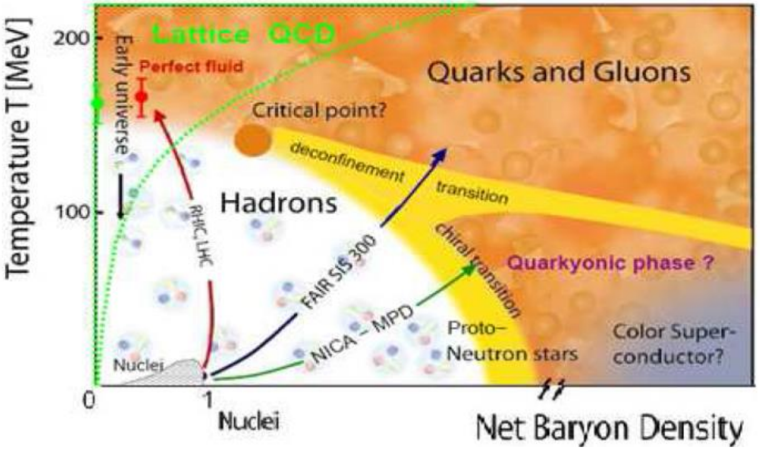
$$\mathcal{L}(\phi_1, \phi_2, \ldots, \phi_n) = \sum_{i=1}^{n} \sum_{j=1}^{n} g_{ij}\, \mathrm{d}\phi_i \wedge *\mathrm{d}\phi_j$$ ***Wedge Product***

fields φ_i represent map from a base manifold space-time (worldsheet) to a target (Riemannian) manifold of the scalars linked together by internal symmetries,
the scalars gij determines linear and non-linear properties.

Importance of Linear-Sigma Model

- LSM is one of lattice QCD alternatives
- It doesn’t need supercomputers like lattice QCD, you can use suitable PC and easily computational techniques.
- Various symmetry-breaking scenarios can be investigated in a more easy way, for instance,

- Various properties of strongly interacting matter can be studied
  - QCD Equation of State
  - Chiral phase structure of masses
  - QGP transport properties
  - ….etc.



LSM (axial)vector Symmetries

Chiral symmetry of vector field under unitary transformatio $\vec{\Phi} \Longrightarrow e^{-i\,\theta^a\,T^a_{ij}}\,\vec{\Phi}$

θ^a corresponding to rotational angle and T^a_{ij} being a matrix generating transformation.

Vector transformation

$$\Psi \Longrightarrow e^{-i\,\frac{\vec{\tau}}{2}\vec{\theta}}\,\Psi \approx \left(1 - i\frac{\vec{\tau}}{2}\vec{\theta}\right)\Psi$$

$$\bar{\Psi} \Longrightarrow e^{+i\,\frac{\vec{\tau}}{2}\vec{\theta}}\,\bar{\Psi} \approx \left(1 + i\frac{\vec{\tau}}{2}\vec{\theta}\right)\bar{\Psi}$$

Axial-vector transformation

$$\Psi \Longrightarrow e^{-i\gamma_5\,\frac{\vec{\tau}}{2}\vec{\theta}}\,\Psi \approx \left(1 - i\gamma_5\frac{\vec{\tau}}{2}\vec{\theta}\right)\Psi$$

$$\bar{\Psi} \Longrightarrow e^{-i\,\gamma_5\frac{\vec{\tau}}{2}\vec{\theta}}\,\bar{\Psi} \approx \left(1 - i\gamma_5\frac{\vec{\tau}}{2}\vec{\theta}\right)\bar{\Psi}$$

For fermions: Dirac Lagrangian describing free Fermion particle of mass m is given by

$$\mathcal{L}_D = \bar{\psi}(i\gamma_\mu\partial^\mu - m^2)\psi$$

Under vector transformation, the Lagrangian is invariant
But for axialvector transformation,

Int.J.Mod.Phys. E6 (1997) 203

$$\Lambda_A: \quad m\,\bar{\psi}\,\psi \Longrightarrow e^{-i\,\gamma^5\frac{\vec{\tau}}{2}\vec{\theta}}\,m\,\bar{\psi}\,\psi \approx \left(1 - i\gamma^5\frac{\vec{\tau}}{2}\vec{\theta}\right)m\,\bar{\psi}\,\psi,$$

$$= m\,\bar{\psi}\,\psi - 2im\vec{\theta}\left(\bar{\psi}\gamma_5\frac{\vec{\tau}}{2}\psi\right)$$

LSM chiral Symmetries

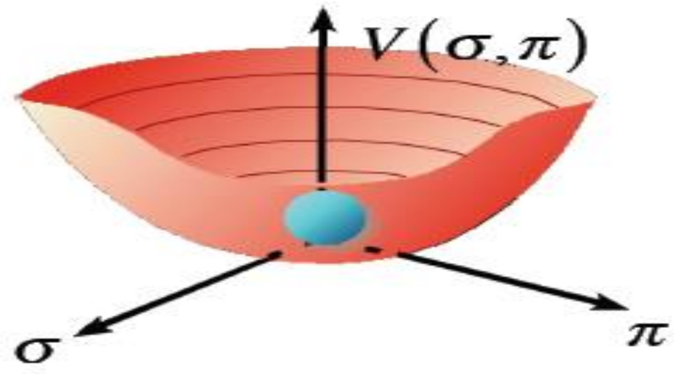
Lagrangian of massless fermions is invariant under chiral transformations BUT massive one make an spontaneous symmetry breaking.

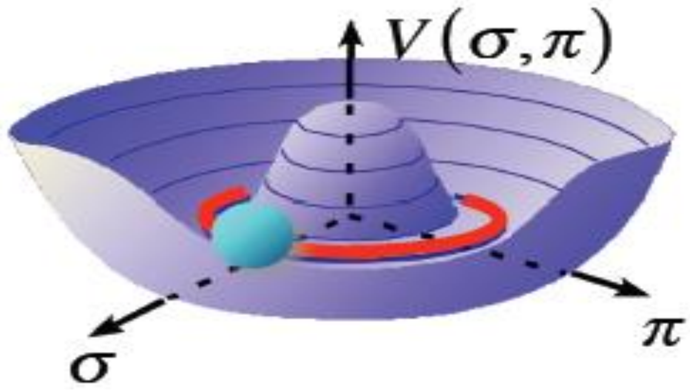
Spontaneous symmetry breaking

if Lagrangian symmetry is not realized in the ground state

Minimum energy configuration is given as shown in the potential energy

Minimum energy (density) is given by an any point on the circle (1,1)





Ground state is right in the middle (0,0) and the potential plus ground state are still invariant under rotations

Ground state is away from the center. The point at the center is a local maximum of the potential and thus unstable

LSM (pseudo)scalar Symmetries

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(scalar Meson) Sigma like state $J^p = 0^+$	(pseudoscalar Meson) Pion like state $J^p = 0^-$
$\sigma = \bar{\psi}\psi$	$\pi = i\bar{\psi}\bar{\tau}\gamma_5\psi$

Gell-Mann & Levy obtained the invariant form if the squares of two states are summed

$$\Lambda_V:\quad \pi^2 \to \pi^2, \qquad \sigma^2 \to \sigma^2$$

$$\Lambda_A:\quad \pi^2 \to \pi^2 + 2\sigma\theta\pi, \qquad \sigma^2 \to \sigma^2 - 2\sigma\theta\pi$$

$$(\pi^2 + \sigma^2) \xleftrightarrow{\Lambda_V, \Lambda_A} (\pi^2 + \sigma^2)$$

$$\mathcal{L} = i\bar{\psi}\gamma_\mu\partial_\mu\psi + \frac{1}{2}(\partial_\mu\pi\partial^\mu\pi + \partial_\mu\sigma\partial^\mu\sigma) - g_\pi\left[(i\bar{\psi}\gamma_5\bar{\tau}\psi)\bar{\pi} + (i\bar{\psi}\psi)\sigma\right] - \frac{\lambda}{4}\left((\bar{\pi}^2 + \sigma^2) - f_\pi^2\right)$$

K. E Of nucleons

K. E Of Mesons

interaction term between nucleons and the mesons

Pion nucleon Potential

Nucleon mass term

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U(Nf)r × U(Nf)ℓ linear-sigma model

Chiral Lagrangian of U(Nf)r × U(Nf)ℓ linear-sigma model for any Nf flavors

$$\mathcal{L}_{chiral} = \mathcal{L}_q + \mathcal{L}_m$$

Fermionic Part ⇒

$$\mathcal{L}_q = \sum_f \overline{\psi}_f (i\gamma^\mu D_\mu - g T_a(\sigma_a + i\gamma_5 \pi_a))\psi_f$$

Mesonic Part ⇒

$$\begin{aligned}\mathcal{L}_m \; = \; & \mathrm{Tr}(\partial_\mu \Phi^\dagger \partial^\mu \Phi - m^2 \Phi^\dagger \Phi) - \lambda_1 [\mathrm{Tr}(\Phi^\dagger \Phi)]^2 \\ & - \lambda_2 \mathrm{Tr}(\Phi^\dagger \Phi)^2 + c[\mathrm{Det}(\Phi) + \mathrm{Det}(\Phi^\dagger)] \\ & + \mathrm{Tr}[H(\Phi + \Phi^\dagger)],\end{aligned}$$

ϕ is a complex Nf × Nf matrix parameterizing the scalar and pseudoscalar mesons,

$$\sigma = \bar{\psi}\psi$$

where σ_a are the scalar ($J^P = 0+$) fields and π_a are the pseudoscalar ($J^P = 0^-$) fields.

Phys. Rev. D 62, 085008 (2000).

U(Nf)r × U(Nf)ℓ linear-sigma model

The Nf×Nf matrix H breaks the symmetry, explicitly

$$H = T_a h_a$$

where h_a are nine external fields and T_a= $\hat{\lambda}_a$ /2 are generators of U(3) with $\hat{\lambda}_a$ are Gell-Mann matrices

The T_a are normalized such that $\mathrm{Tr}(T_a T_b) = \delta_{ab}/2$ and obey the $U(3)$

$$[T_a, T_b] = i\, f_{abc} T_c \ ,$$
$$\{T_a, T_b\} = d_{abc} T_c \ ,$$

where f_{abc} and d_{abc} are the standard antisymmetric and symmetric structure constants of SU(Nf), respectively, and

$$a, b, c = 1, \ldots, N_f^2 - 1$$

Physical scalar and pseudoscalar meson fields

The complex $N_f \times N_f$ matrix reads $$\Phi = T_a\, \phi_a = T_a\, (\sigma_a + i\pi_a)$$

By using Pauli and Gell-mann Matrices, we find

For $N_f = 2$,

$$T_a\sigma_a = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}}\sigma_0 + \frac{1}{\sqrt{2}}a_0^0 & a_0^+ \\ a_0^- & \frac{1}{\sqrt{2}}\sigma_0 - \frac{1}{\sqrt{2}}a_0^0 \end{pmatrix},$$

$$T_a\pi_a = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}}\pi_0 + \frac{1}{\sqrt{2}}\pi^0 & \pi^+ \\ \pi^- & \frac{1}{\sqrt{2}}\pi_0 - \frac{1}{\sqrt{2}}\pi^0 \end{pmatrix}.$$

For $N_f = 3$,

$$T_a\sigma_a = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}}a_0^0 + \frac{1}{\sqrt{6}}\sigma_8 + \frac{1}{\sqrt{3}}\sigma_0 & a_0^+ & \kappa^+ \\ a_0^- & -\frac{1}{\sqrt{2}}a_0^0 + \frac{1}{\sqrt{6}}\sigma_8 + \frac{1}{\sqrt{3}}\sigma_0 & \kappa^0 \\ \kappa^- & \bar{\kappa}^0 & -\frac{2}{\sqrt{3}}\sigma_8 + \frac{1}{\sqrt{3}}\sigma_0 \end{pmatrix},$$

$$T_a\pi_a = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\pi_8 + \frac{1}{\sqrt{3}}\pi_0 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\pi_8 + \frac{1}{\sqrt{3}}\pi_0 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{3}}\pi_8 + \frac{1}{\sqrt{3}}\pi_0 \end{pmatrix}.$$

For $N_f = 4$,

$$T_a\,\sigma_a = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{a0}{\sqrt{2}} + \frac{\sigma 0}{2} + \frac{\sigma 15}{2\sqrt{3}} + \frac{\sigma 8}{\sqrt{6}} & a_0^+ & k^+ & D_0^{*0} \\ a_0^- & -\frac{a0}{\sqrt{2}} + \frac{\sigma 0}{2} + \frac{\sigma 15}{2\sqrt{3}} + \frac{\sigma 8}{\sqrt{6}} & k^0 & D_0^{*-} \\ k^- & k00 & \frac{\sigma 0}{2} + \frac{\sigma 15}{2\sqrt{3}} - \frac{2\,\sigma 8}{\sqrt{6}} & D_{S0}^{*-} \\ \bar{D}_0^{*0} & D_0^{*+} & D_{S0}^{*+} & \frac{\sigma 0}{2} - \frac{3\,\sigma 15}{2\sqrt{3}} \end{pmatrix}$$

$$T_a\,\pi_a = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\pi 00}{\sqrt{2}} + \frac{\pi 0}{\sqrt{3}} + \frac{\pi 15}{2\sqrt{3}} + \frac{\pi 8}{\sqrt{6}} & \pi^+ & K^+ & D^0 \\ \pi^- & -\frac{\pi 00}{\sqrt{2}} + \frac{\pi 0}{\sqrt{3}} + \frac{\pi 15}{2\sqrt{3}} + \frac{\pi 8}{\sqrt{6}} & K^0 & D^- \\ K^- & \bar{K}^0 & \frac{\pi 0}{\sqrt{3}} + \frac{\pi 15}{2\sqrt{3}} - \frac{2\,\pi 8}{\sqrt{6}} & D_S^- \\ \bar{D}^0 & D^+ & D_S^+ & \frac{\pi 0}{\sqrt{3}} - \frac{3\,\pi 15}{2\sqrt{3}} \end{pmatrix}$$

Mean-field approximation

To calculate the grand potential in mean-field approximation, we start from the partition function.

In thermal equilibrium, the grand-canonical partition-function can be defined by using a path integral over the quark, antiquark and meson field.

$$
\begin{aligned}
\mathcal{Z} &= \mathrm{Tr}\exp[-(\hat{\mathcal{H}} - \sum_f \mu_f \hat{\mathcal{N}}_f)/T] \\
&= \int \prod_a \mathcal{D}\sigma_a \mathcal{D}\pi_a \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp\left[\int_x (\mathcal{L} + \sum_f \mu_f \bar{\psi}_f \gamma^0 \psi_f)\right],
\end{aligned}
$$

where $\int_x \equiv i \int_0^{1/T} dt \int_V d^3x$ and μ_f It the chemical potential

For N_f = 2, f = u, d, if N_f = 3, f = u, d and s and if N_f = 4 f = u, d, s and c

Mean-field approximation

Thermodynamic potential density

For $N_f = 2$,

$$\Omega = \mathcal{U}(\phi, \phi^*) + U(\sigma_l) + \Omega_{\bar{q}q}(\phi, \phi^*, \sigma_l).$$

For $N_f = 3$,

$$\Omega = \mathcal{U}(\phi, \phi^*) + U(\sigma_l, \sigma_s) + \Omega_{\bar{q}q}(\phi, \phi^*, \sigma_l, \sigma_s).$$

For $N_f = 4$,

$$\Omega = \mathcal{U}(\phi, \phi^*) + U(\sigma_l, \sigma_s, \sigma_c) + \Omega_{\bar{q}q}(\phi, \phi^*, \sigma_l, \sigma_s, \sigma_c).$$

Polyakov-loop Potential

Pure mesonic potential

quarks and antiquarks potential

Pure mesonic potential

For $N_f = 2$,

$$U(\sigma) \;=\; \frac{\lambda}{4}(\sigma^2 - v^2)^2 - c\sigma \;.$$

where g, λ, v, and c, are model parameters

Phys.Rev.D76:074023,2007

For $N_f = 3$,

$$U(\sigma_l, \sigma_s) \;=\; -h_l\sigma_l - h_s\sigma_s + \frac{m^2\,(\sigma_l^2 + \sigma_s^2)}{2} - \frac{c\,\sigma_l^2\sigma_s}{2\sqrt{2}} + \frac{\lambda_1\,\sigma_l^2\sigma_s^2}{2} + \frac{(2\lambda_1 + \lambda_2)\sigma_l^4}{8} + \frac{(\lambda_1 + \lambda_2)\sigma_s^4}{4}$$

where ***m_2, h_l, h_s, λ_1, λ_2, c, and g*** are model parameters

Phys.Rev. D81 (2010) 074013

For $N_f = 4$,

$$U(\sigma_l, \sigma_s, \sigma_c) = \frac{1}{8}\Big(-8 h_c \sigma_c + 2\lambda_1 \sigma_c^4 + 2\lambda_2 \sigma_c^4 - 8 h_l \sigma_l + 4\lambda_1 \sigma c^2 \sigma l^2 + 2\lambda_1 \sigma l^4 + \lambda_2 \sigma_l^4 - 2\left(4 h_s + c\,\sigma_c \sigma_l^2\right)\sigma s + 4\lambda_1\left(\sigma_c^2 + \sigma_l^2\right)\sigma_s^2 + 2(\lambda_1 + \lambda_2)\sigma_s^4 + 4 m^2\left(\sigma_c^2 + \sigma_l^2 + \sigma_s^2\right)\Big)$$

Quarks-antiquarks potential

$$\Omega_{\bar{q}q}(\phi,\phi^*,\sigma_f) = -2T\sum_f \int_0^\infty \frac{d^3\vec{P}}{(2\pi)^3}\Bigg\{\ln\left[1+3\left(\phi+\phi^* e^{-\frac{E_f-\mu_f}{T}}\right)\times e^{-\frac{E_f-\mu_f}{T}} + e^{-3\frac{E_f-\mu_f}{T}}\right]$$
$$+\ln\left[1+3\left(\phi^*+\phi e^{-\frac{E_f+\mu_f}{T}}\right)\times e^{-\frac{E_f+\mu_f}{T}} + e^{-3\frac{E_f+\mu_f}{T}}\right]\Bigg\},$$

where the dispersion relation $E_f = \sqrt{\vec{P}^2 + m_f^2}$

Phys. Rev. D 77, 114028 (2008).

Mass of the quark flavors couples to the sigma field via Yukawa coupling g

For N_f = 2, f = l = u, d $m_l = g\sigma_l$

For N_f = 3, f = u, d and s $m_l = g\sigma_l/2 \text{ and } m_s = g\sigma_s/\sqrt{2}$

For N_f = 4 f = u, d, s, and c $m_l = g\sigma_l/2,\ m_s = g\sigma_s/\sqrt{2} \text{ and } m_c = g\sigma_c/\sqrt{2}$

Polyakov-loop Potential

Polyakov-loop potential introduces the gluons degrees-of-freedom and the dynamics of the quark-gluon interactions to the QCD matter.

Polynomial-logarithmic parameterization of the Polyakov-loop potential, they even included higher-order terms,

$$\frac{\mathcal{U}_{\rm PolyLog}(\phi,\phi^*,T)}{T^4} = \frac{-a(T)}{2}\,\phi^*\phi + b(T)\,\ln\left[1-6\,\phi^*\phi+4\left(\phi^{*3}+\phi^3\right)-3\left(\phi^*\phi\right)^2\right] + \frac{c(T)}{2}\left(\phi^{*3}+\phi^3\right)+d(T)\left(\phi^*\phi\right)^2.$$

$$x(T) = \frac{x_0+x_1\,(T0/T)+x_2\,(T0/T)^2}{1+x_3\,(T0/T)+x_4\,(T0/T)^2}, \qquad b(T) = b_0\,(T0/T)^{b_1}\left(1-e^{b_2(T0/T)^{b_3}}\right),$$

where the coefficients x=a, c and d

Phys. Rev. D 88, 074502 (2013).

Chiral and deconfinement order-parameters

The thermodynamic potential

$$\Omega = \mathcal{U}(\phi, \phi^*) + U(\sigma_l, \sigma_s, \sigma_c) + \Omega_{\bar{q}q}(\phi, \phi^*, \sigma_l, \sigma_s, \sigma_c).$$

has the parameters

- **chiral condensates** $\sigma_l, \sigma_s, \sigma_c$
- **deconfinement order-parameters** ϕ and ϕ^*

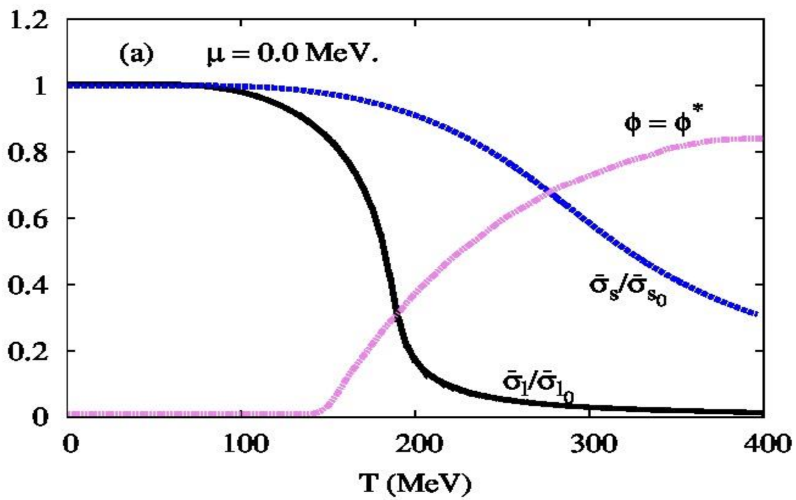
The analysis of the order parameters is given by minimizing the real part of thermodynamic potential $\mathrm{Re}\,\Omega$

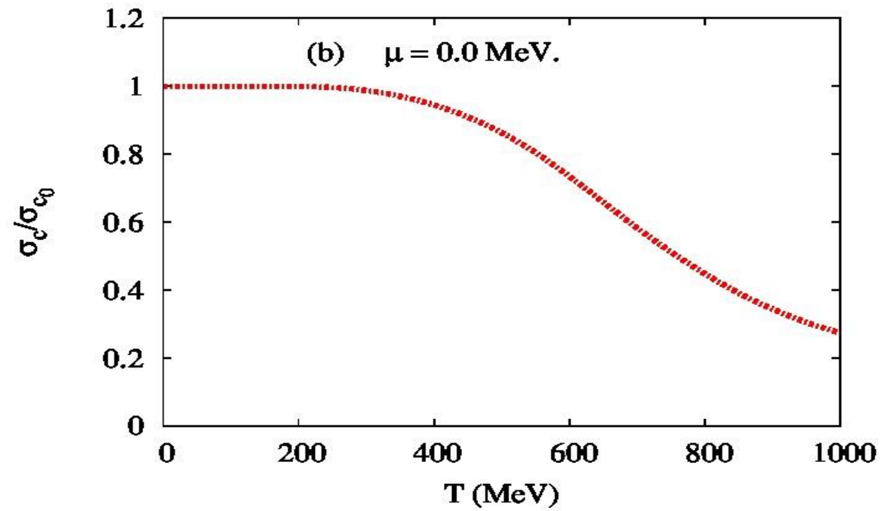
The solutions of these equations can be determined by minimizing the real potential at a saddle point,

$$\frac{\partial \Omega}{\partial \bar{\sigma_l}} = \frac{\partial \Omega}{\partial \bar{\sigma_s}} = \frac{\partial \Omega}{\partial \bar{\sigma_c}} = \frac{\partial \Omega}{\partial \bar{\phi}} = \frac{\partial \Omega}{\partial \bar{\phi}^*}\bigg|_{min} = 0.$$

The behavior of the chiral condensate $\sigma_l, \sigma_s, \sigma_c$ **and the Polyakov-loop expectation values** ϕ and ϕ^* **as functions of T and μ shall be presented**

Results





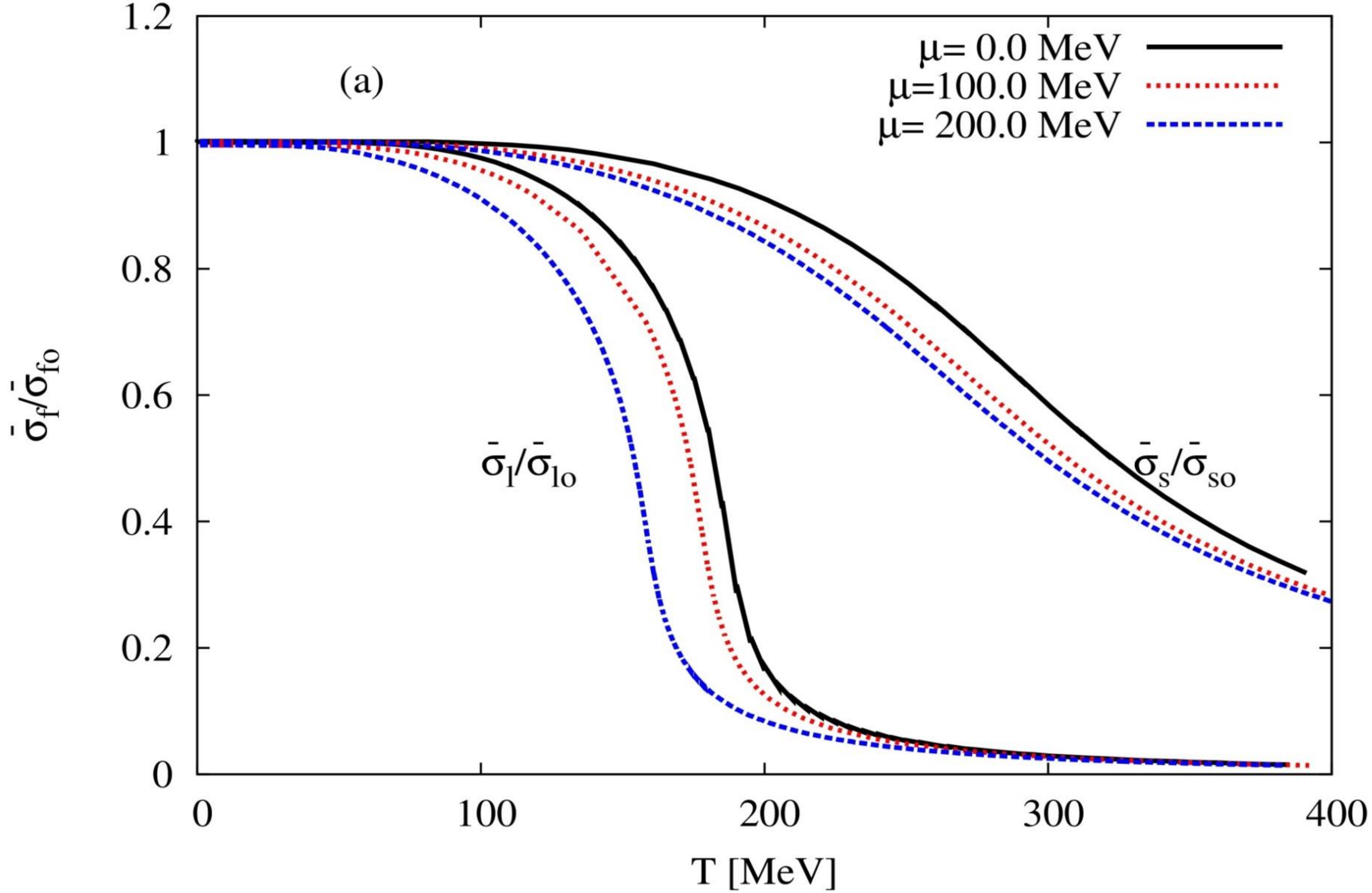
Normalized chiral condensates, σl/σl0 and σ_s/σ_{s0} which are correspondent to light and strange quarks, respectively are given as functions of temperature.

normalized chiral condensate, $\sigma_c/_{c0}$ correspondent to charm quark as functions of temperature.

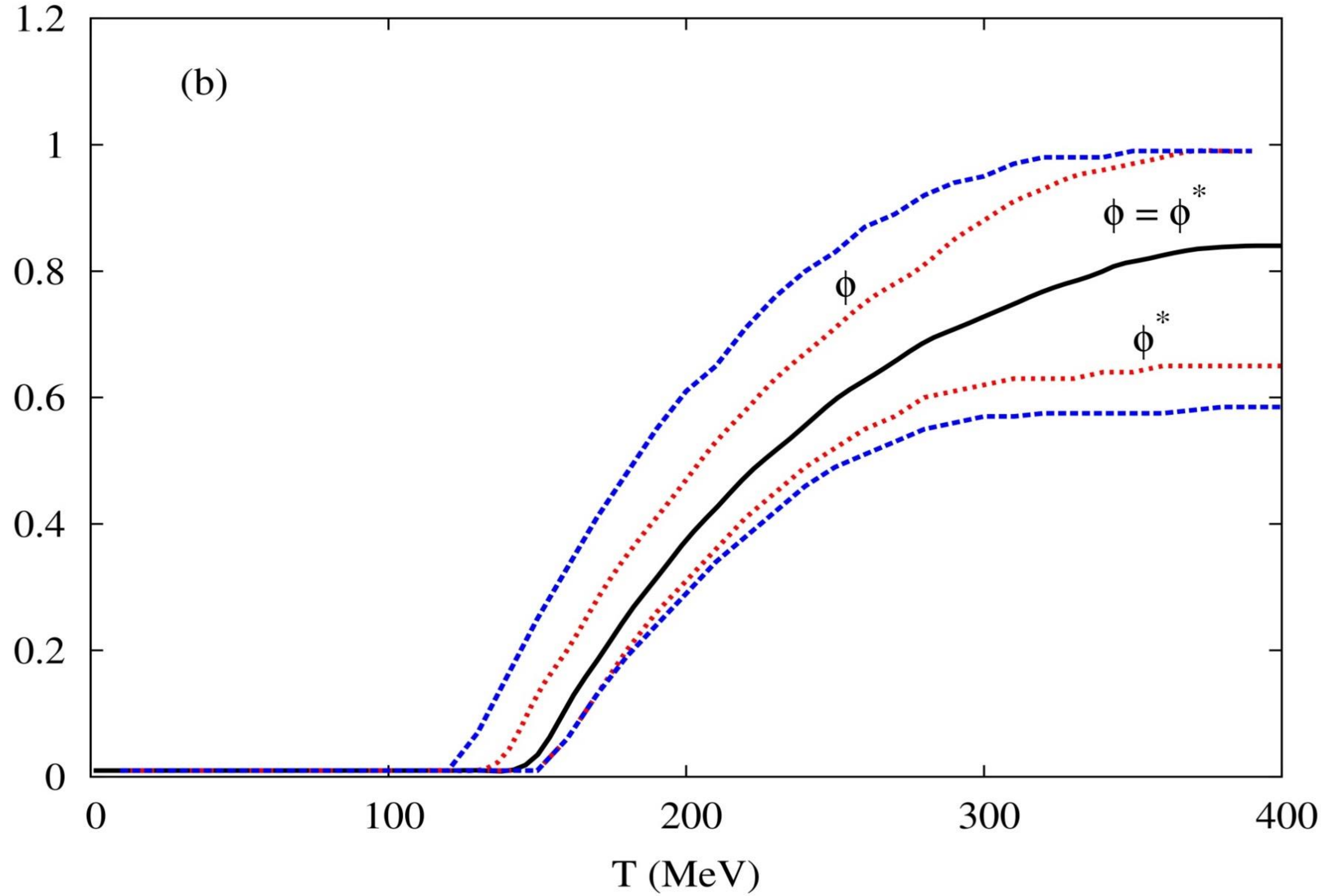
At μ = 0, the (thermal) expectation values of Polyakov-loops are identical, i.e. $\phi = \phi^*$

Constituent quark mass
m_l ~ 300 MeV,
m_s ~ 433 MeV, and
mc ~ 1750 MeV.

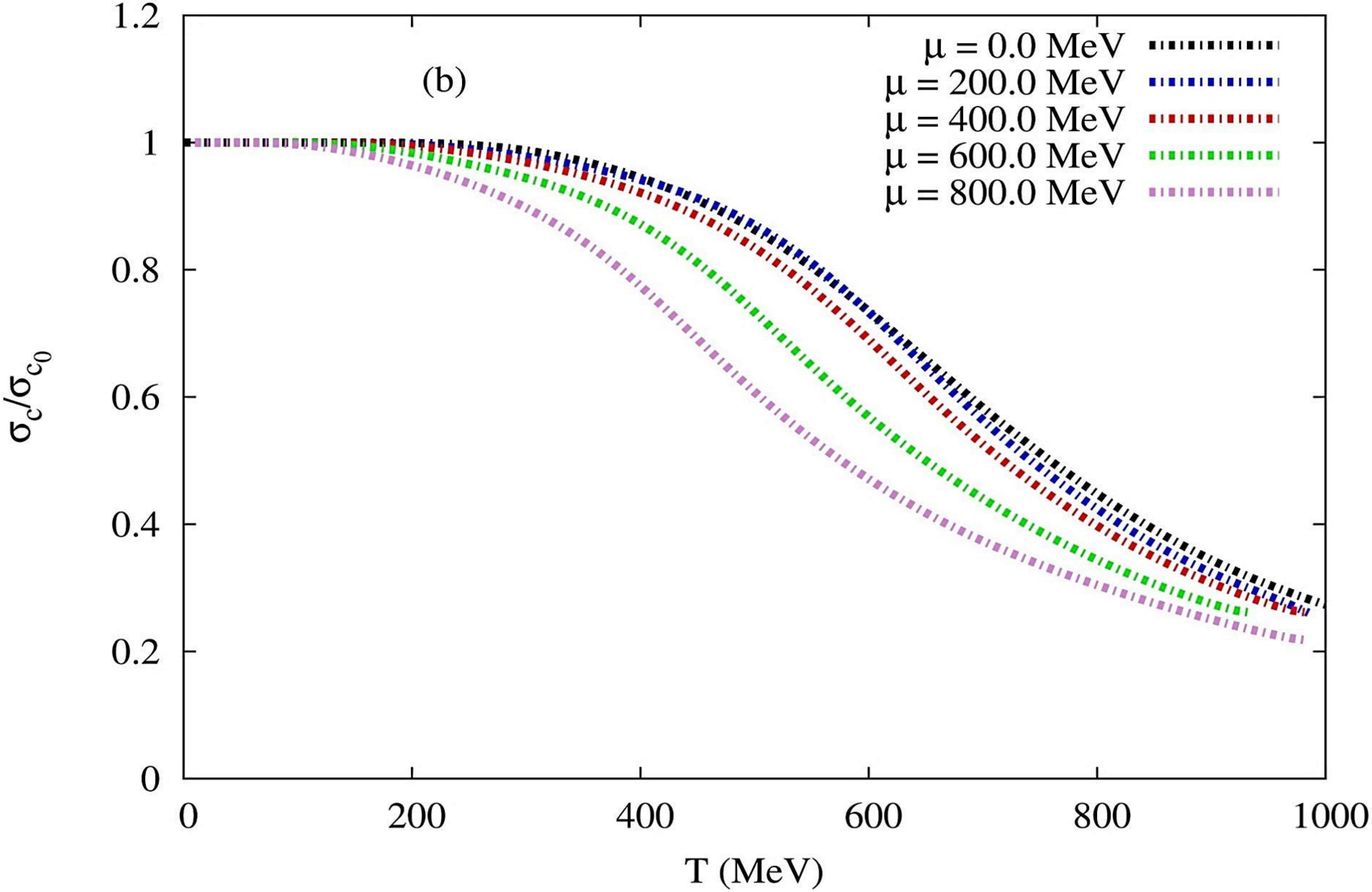
Results of l and s condensates at finite mu



Results of Polyakov-loop variables at finite mu



Results of c condensates and finite mu



Summary and Outlook

- Constructing PLSM Lagrangian in SU(2), SU(3) and SU(4)
- Introducing Ployakov-Loop Corrections to LSM
- Constructing SU(4) PLSM formalism at finite T and mu
- Estimating T- and mu-dependence of chiral and deconfinement order-parameters

- Estimate T- and mu-inmedium effects on hadrons with c-quarks
- Comparing with lattice QCD simulations
- Electromagnetic effects at finite T and mu

Thanks for your attention!