Enhancement of Wino Dark Matter Annihilation through the Radiative Formation of Bound States

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Outline of this talk

- Wino WIMP dark matter and annihilation
  - Direct wino-pair annihilation
  - Wino bound state production and subsequent annihilation
    - **New mechanism for monochromatic gamma ray signal**
- Nonrelativistic Effective Field Theory
- Zero-Range Effective Field Theory
- Wino bound state formation by two photon transition

Wino WIMP dark matter

- Extend the Standard Model to include one electroweak SU(2) x U(1) multiplet:
  - Triplet under SU(2) with zero hypercharge

$$\tilde{w} = \begin{pmatrix} \tilde{w}^{+} & \tilde{w}^{0} & \tilde{w}^{-} \end{pmatrix}$$

- Or, the MSSM in the region of parameters where the Lightest Supersymmetric Particle (LSP) is a wino-like neutralino
  - Wino: SUSY partner of the W boson

- Wino masses: neutral wino $M \sim$ few TeV, charged winos $M + \delta$
  - Electroweak radiative corrections give δ = 170 MeV and varies very little with M

- The neutral wino is the WIMP dark matter candidate
  - $M > 1$ TeV for the relic density to be compatible with observed dark matter density

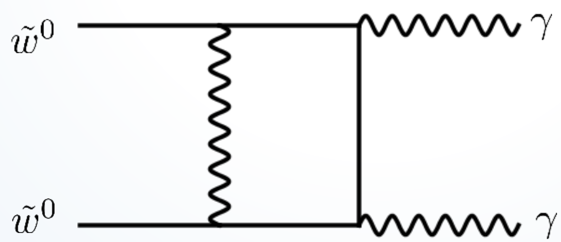
Wino interactions and nonperturbative effects

- A pair of neutral winos can annihilate into a pair of electroweak gauge bosons

$$\left.\begin{aligned}\tilde{w}^0\tilde{w}^0 &\to \gamma\gamma \\ &\to \gamma Z^0\end{aligned}\right\} \text{Monochromatic } \gamma\text{-ray signal}$$

$$\left.\begin{aligned}\tilde{w}^0\tilde{w}^0 &\to Z^0 Z^0 \\ &\to W^+ W^-\end{aligned}\right\} \text{Continuous } \gamma\text{-ray and positron signal}$$

- Leading-order cross-section for $\tilde{w}^0\tilde{w}^0 \to \gamma\gamma$:



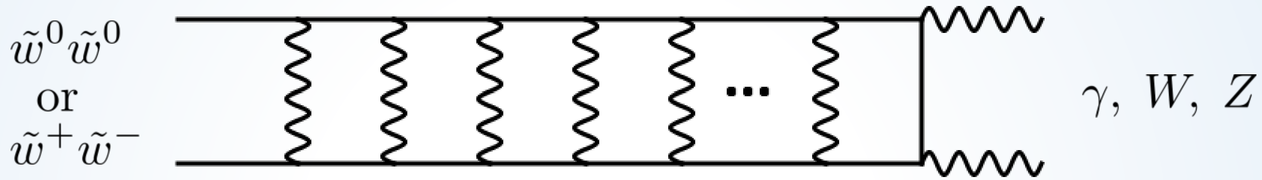
$$(\sigma_{\text{ann}} v)_{\text{LO}} \sim \frac{\alpha^2 \alpha_2^2}{m_W^2}$$

- Exceeds unitarity bound $\sigma_{\text{ann}} v < 4\pi / v M^2$ for sufficiently large M
  - Higher-order diagrams must be included in the cross-section

J. Hisano, S. Matsumoto, M.M. Nojiri and O. Saito *Phys. Rev.* **D 71** (2005)

Wino interactions and nonperturbative effects

- Transitions between wino pairs, $\tilde{w}^0\tilde{w}^0$ and $\tilde{w}^+\tilde{w}^-$, involve exchange of EW gauge bosons:



- The ladder diagrams must be summed to all orders
  - Each 'rung' of the ladder gives a factor of $\alpha_2 M/m_W$
  - For large M , $\alpha_2 M \sim m_W$
  - The cross sections for $\tilde{w}^0\tilde{w}^0 \rightarrow \gamma\gamma$, γZ , ZZ , WW receive large corrections: "Sommerfeld Enhancement"
- Difficult to calculate in the fundamental field theory
- Calculate with Nonrelativistic Effective Field Theory

J. Hisano, S. Matsumoto, M.M. Nojiri and O. Saito *Phys. Rev.* **D 71** (2005)

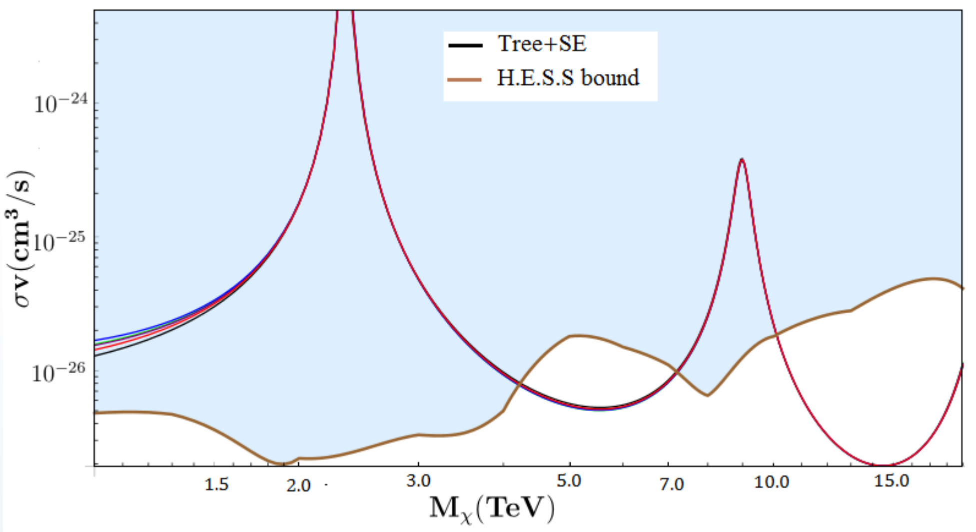
Nonrelativistic Effective Field Theory (NREFT)

- Ladder diagrams from exchange of electroweak gauge bosons between a pair of winos can be summed to all orders by solving the Schrödinger equation
  - Neutral wino pairs and charged wino pairs are coupled channels interacting through the potential

$$V(r) = \begin{pmatrix} 0 & -\sqrt{2}\alpha_2 \frac{e^{-m_W r}}{r} \\ -\sqrt{2}\alpha_2 \frac{e^{-m_W r}}{r} & 2\delta - \frac{\alpha}{r} - \alpha_2 \cos^2\theta_W \frac{e^{-m_Z r}}{r} \end{pmatrix}$$

Channel	Threshold Energy
$\tilde{w}^0\tilde{w}^0$	0
$\tilde{w}^+\tilde{w}^-$	2δ

- Sequence of critical masses where the neutral winos form a zero-energy resonance:
  - Resonance near the neutral wino scattering threshold: "Resonance Enhancement" or "Sommerfeld Enhancement"

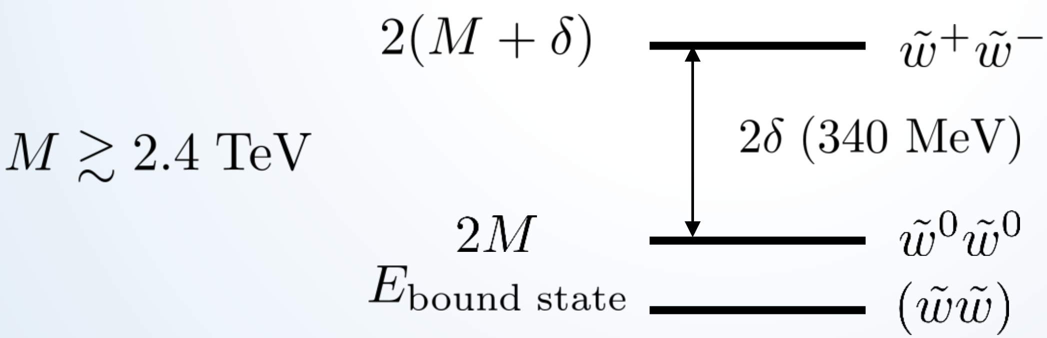


M. Baumgart, I. Z. Rothstein, and V. Vaidya, *JHEP* **1504** (2015)

Wino bound states

- There are critical values of the wino mass where a pair of neutral winos form a zero-energy resonance
  - Critical masses are determined by the mass splitting δ and electroweak parameters
  - For δ = 170 MeV, the first critical mass is M_* = 2.4 TeV

- When M is above M_* , the resonance is a bound state, denoted $(\tilde{w}\tilde{w})$



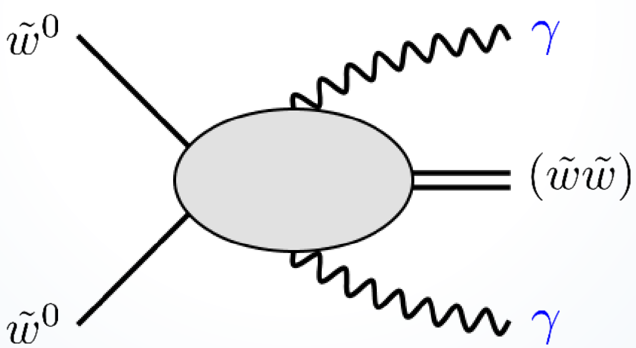
New mechanism for monochromatic gamma ray signal

- Formation of bound state in neutral wino scattering through radiative transition

$$\tilde{w}^0\tilde{w}^0 \;\to\; (\tilde{w}\tilde{w}) + \text{soft photons}$$

- followed by annihilation of bound state into two hard photons

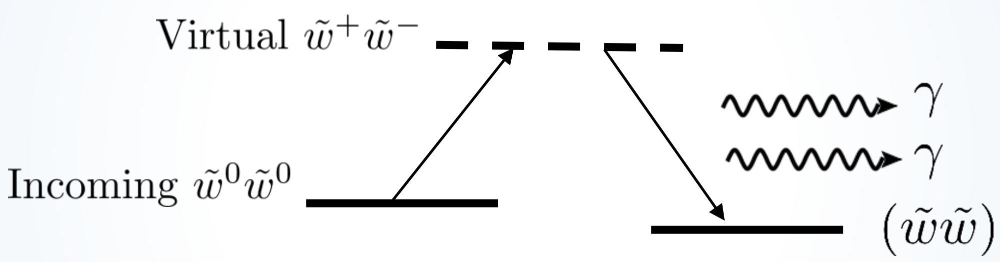
$$(\tilde{w}\tilde{w}) \;\to\; \gamma\gamma$$



- <u>Annihilation rate = bound state production rate</u>
  - Wino bound state is unstable and decays with probability one

Production of wino-pair bound state

- To conserve energy, the wino pair must radiate photons
  - Single-photon emission: forbidden by parity
  - Double-photon emission: allowed



- Extremely difficult to calculate using the Nonrelativistic Effective Field Theory
- Calculate with Zero-Range Effective Field Theory

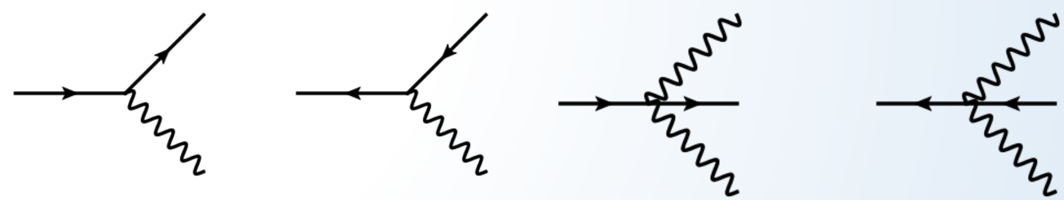
Zero-Range Effective Field Theory

- Kinetic Lagrangian for wino fields:

$$\mathcal{L}_{\text{kinetic}} = w_0^\dagger \left(i\partial_0 + \frac{\boldsymbol{\nabla}^2}{2M} \right) w_0 + \sum_\pm w_\pm^\dagger \left(iD_0 + \frac{\boldsymbol{D}^2}{2M} - \delta \right) w_\pm$$

  - EM covariant derivative for charged winos: $D_0 w_\pm = (\partial_0 \pm ieA_0) w_\pm \quad , \quad \boldsymbol{D} w_\pm = (\boldsymbol{\nabla} \mp ie\boldsymbol{A}) w_\pm$
  - Propagators for wino fields:

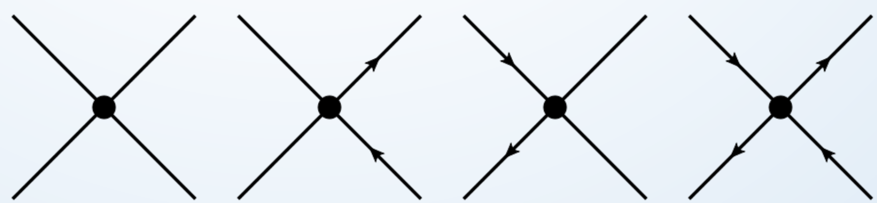
  - Photon emission vertices:



- Interaction Lagrangian for zero-range interactions between winos

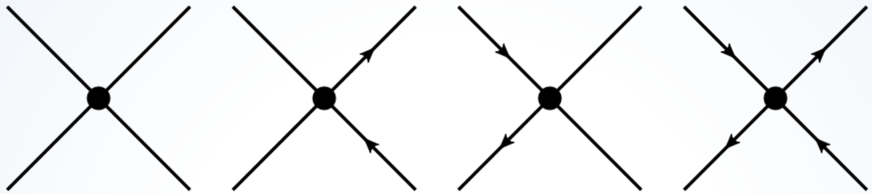
$$\begin{aligned}\mathcal{L}_{\text{zero-range}} = & -\tfrac{1}{4}\lambda_{00}({w_0}^\dagger w_0)^2 - \lambda_{11}({w_+}^\dagger w_+)({w_-}^\dagger w_-) \\ & -\tfrac{1}{2}\lambda_{01}\left[({w_+}^\dagger w_0)({w_-}^\dagger w_0) + ({w_0}^\dagger w_+)({w_0}^\dagger w_-)\right]\end{aligned}$$

  - Zero-range vertices:



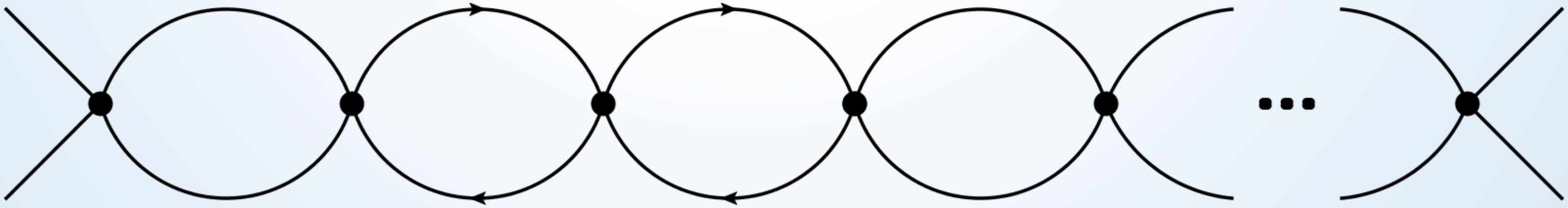
Zero-Range Effective Field Theory

- Wino pairs interact through local, zero-range contact interactions



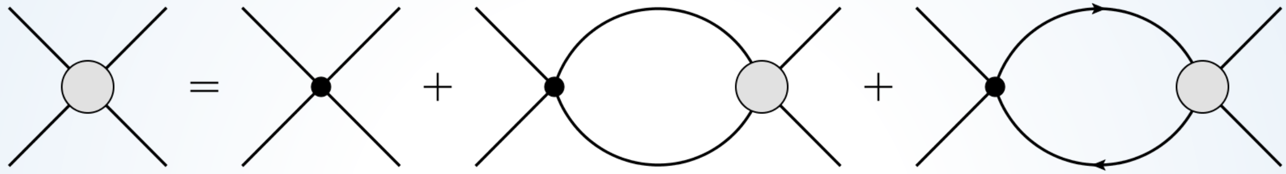
  - Three scattering parameters with units of momentum: γ_{00}, γ_{01}, γ_{11}
  - Other parameters: wino mass M, energy gap δ , EM coupling α

- Contact interactions must be treated nonperturbatively by summing bubble diagrams to all orders



Zero-Range Effective Field Theory

- Calculate scattering amplitudes analytically by solving the coupled-channel Lippmann-Schwinger integral equations



- Scattering amplitude for neutral-wino elastic scattering with center-of-mass energy E

$$f_{00}(k) = \left(-\gamma_{00} - ik - \gamma_{01}^2 \left[-\gamma_{11} + \sqrt{2M\delta - k^2} \right]^{-1} \right)^{-1}$$

Relative momentum $k = \sqrt{ME}$

- Zero energy limit: $f_{00}(k \to 0) \to -a_0$
- Neutral wino scattering length: $a_0 = \left(\gamma_{00} - \gamma_{01}^2 / (\gamma_{11} - \sqrt{2M\delta})\right)^{-1}$
- The scattering length diverges at the critical mass $a_0(M_*) = \pm\infty$
- For $M > M_*$, bound state with energy $E = -\gamma^2/M$ where binding γ momentum satisfies

$$\gamma = \gamma_{00} - \gamma_{01}^2 \left(\gamma_{11} - \sqrt{2M\delta + \gamma^2}\right)^{-1}$$

NREFT to Zero-Range EFT

- Determine three scattering parameters $\gamma_{00}, \gamma_{01}, \gamma_{11}$ by fitting neutral-wino elastic scattering amplitude

$$f_{00}(k) \;=\; \frac{1}{k\cot\delta_0(k) - ik}$$

  - Neutral-wino S-wave phase shift: $\delta_0(k)$

- Calculate $\delta_0(k)$ by solving Schrödinger's equation in the NREFT

- Fit energy dependence to analytic result from Zero-Range EFT:

$$k\cot\delta_0(k) \;=\; -\gamma_{00} - \gamma_{01}^2\left[-\gamma_{11} + \sqrt{2M\delta - k^2}\right]^{-1}$$

  - Fit results:

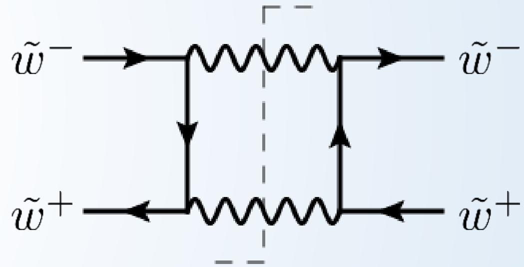
$$\begin{aligned} \gamma_{00} &= -177\ \text{GeV} \\ \gamma_{01} &= +91\ \text{GeV} \\ \gamma_{11} &= -43\ \text{GeV} \end{aligned}$$

NREFT to Zero-Range EFT

- Inelastic annihilation processes: $\tilde{w}^0\tilde{w}^0 \rightarrow \gamma\gamma\ ,\ \gamma Z\ ,\ ZZ\ ,\ WW$
  - Accounted for by analytically continuing the scattering parameters to complex values

$$\begin{pmatrix} \tilde{\gamma}_{00} & \tilde{\gamma}_{01} \\ \tilde{\gamma}_{01} & \tilde{\gamma}_{11} \end{pmatrix} = \begin{pmatrix} \gamma_{00} & \gamma_{01} \\ \gamma_{01} & \gamma_{11} \end{pmatrix} + i\beta \begin{pmatrix} 6 & \sqrt{2} \\ \sqrt{2} & 4 \end{pmatrix}$$

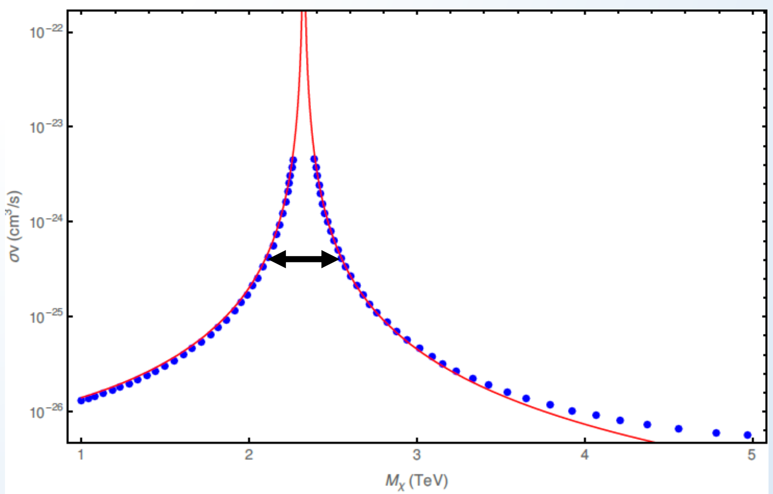
  - Matrix from annihilation contribution to



$$\mathrm{Im}\begin{bmatrix} \mathcal{M}[\tilde{w}^0\tilde{w}^0 \rightarrow \tilde{w}^0\tilde{w}^0] & \mathcal{M}[\tilde{w}^0\tilde{w}^0 \rightarrow \tilde{w}^+\tilde{w}^-] \\ \mathcal{M}[\tilde{w}^+\tilde{w}^- \rightarrow \tilde{w}^0\tilde{w}^0] & \mathcal{M}[\tilde{w}^+\tilde{w}^- \rightarrow \tilde{w}^+\tilde{w}^-] \end{bmatrix}$$

in the fundamental theory

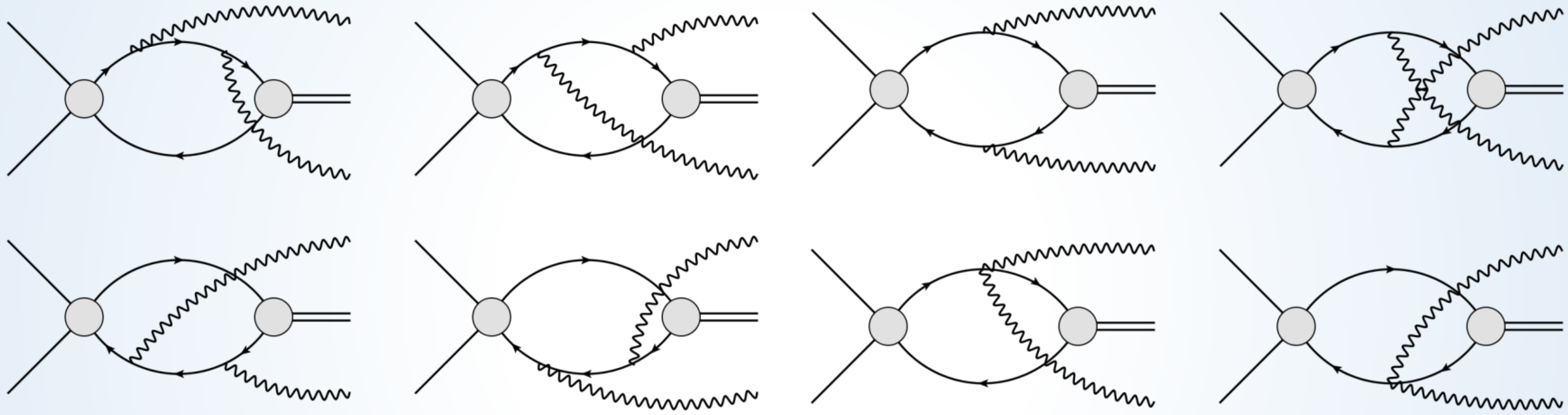
  - Coefficient β determined by fitting width of $\sigma_{\mathrm{ann}}v$ vs M in NREFT
    - $\beta = 0.5 \times 10^{-4}$ GeV



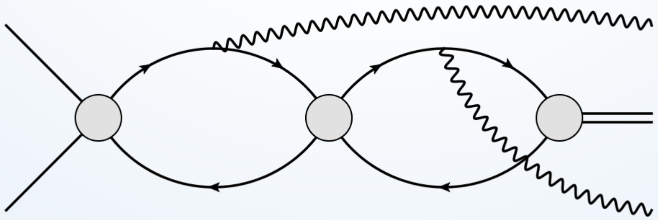
J. Hisano, S. Matsumoto, M.M. Nojiri and O. Saito *Phys. Rev.* **D 71** (2005) M. Baumgart, I. Z. Rothstein, and V. Vaidya, *JHEP* **1504** (2015)

Bound state production

- At leading order in α, diagrams that contribute to $\tilde{w}^0\tilde{w}^0 \rightarrow (\tilde{w}\tilde{w}) + \gamma\gamma$ are

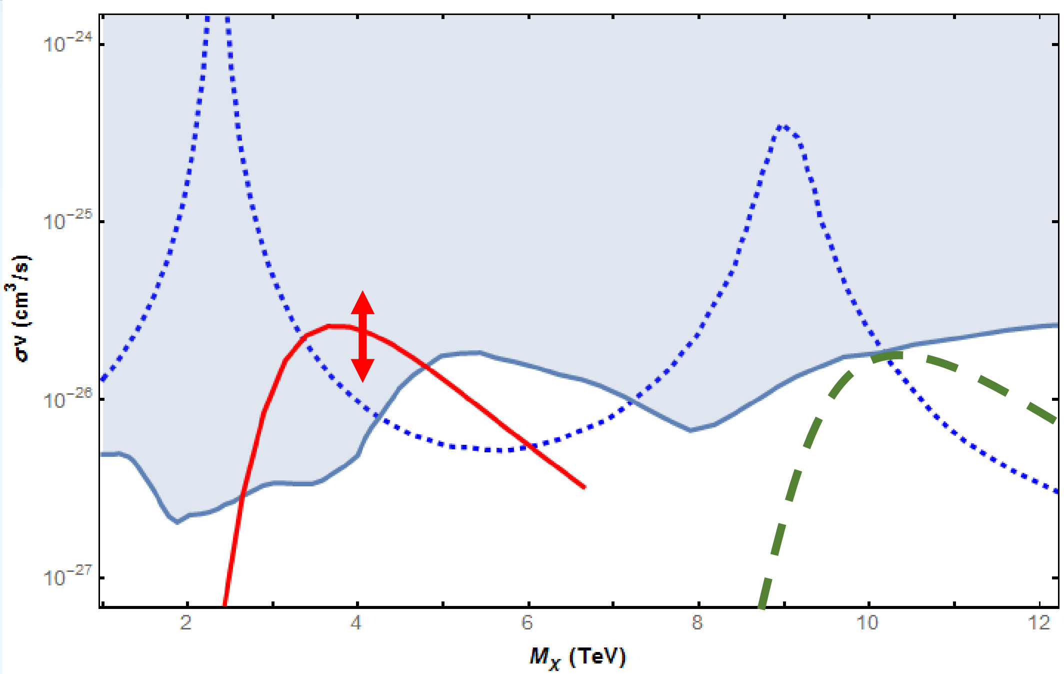


- Two-bubble diagrams with one photon attached to each bubble vanish by parity:



$$= 0$$

Bound state production rate



- Blue curve: neutral-wino annihilation cross section to $\gamma\gamma$, γZ
- Red curve: preliminary result for bound state production rate at first critical mass (normalization not yet finalized)
- Green curve: Future calculation of bound state production rate at second critical mass
- Grey region: Excluded from HESS

M. Baumgart, I. Z. Rothstein, and V. Vaidya, *JHEP* **1504** (2015) HESS, A. Abramowski *et al.*, *Phys. Rev. Lett.* **110** (2013)

Zero-Range Effective Field Theory

- The coupled Lippmann-Schwinger integral equations:

$$i\hat{\mathcal{A}}(E) \;=\; -i\hat{\lambda} + (-i\hat{\lambda})\;\hat{I}(E)\;(i\hat{\mathcal{A}}(E))$$

  - Represented diagrammatically: