

Theoretical results for top-quark cross sections and distributions

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- Top-antitop pair production
- Single-top production
- Cusp anomalous dimension

supported by NSF

Top-antitop pair production

QCD corrections are very significant for $t\bar{t}$

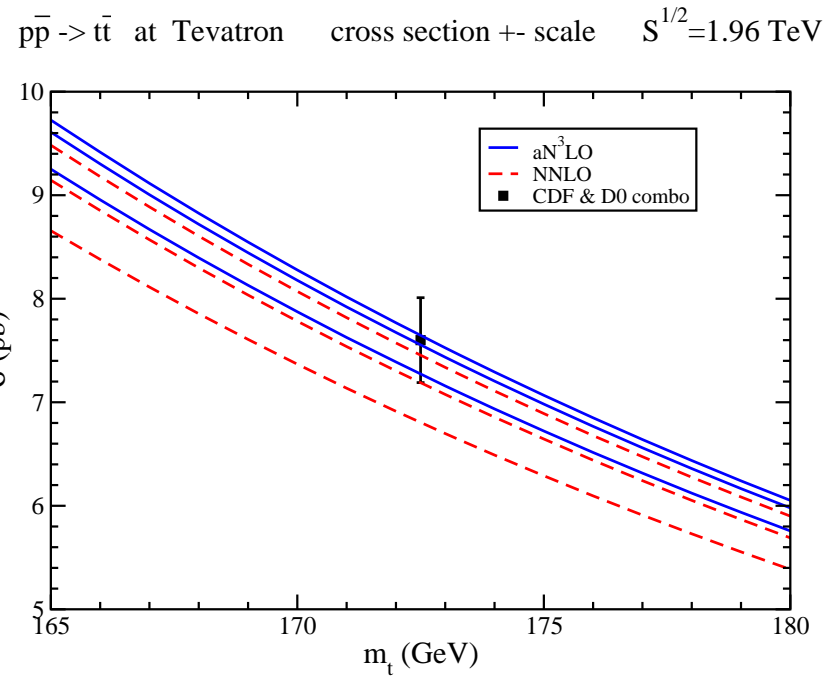
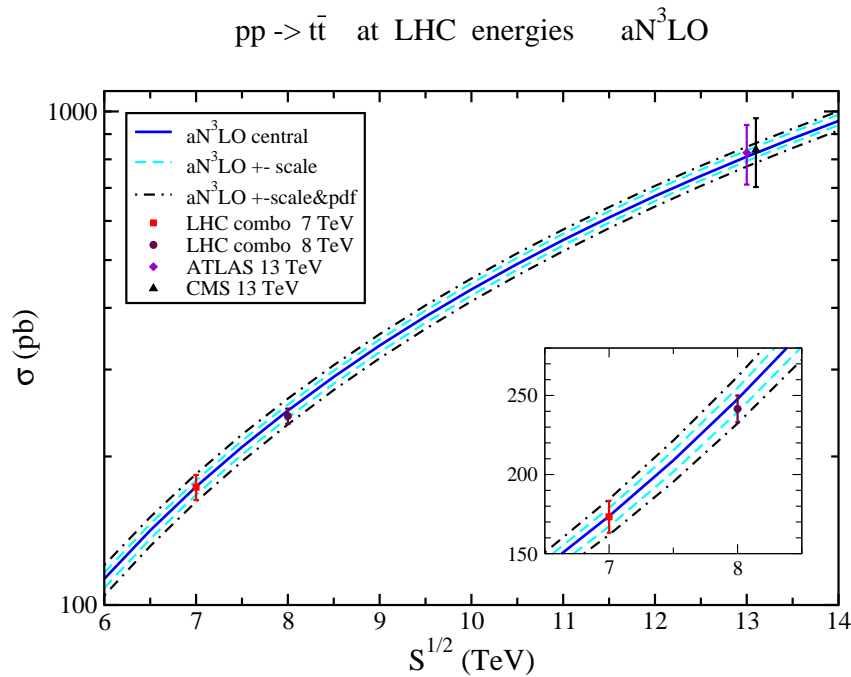
Soft-gluon corrections are important
and they approximate exact results very well

Calculate/resum these soft corrections at NNLL accuracy
for the double-differential cross section

Approximate $N^3\text{LO}$ ($aN^3\text{LO}$) predictions for cross sections are derived
by adding third-order soft corrections to fixed-order results

$aN^3\text{LO} = \text{LO} + \text{NLO} + \text{NNLO} + \text{approx } N^3\text{LO corrections}$

Top-pair cross sections at the LHC and the Tevatron



aN³LO total $t\bar{t}$ cross sections \pm scale \pm pdf uncertainties

LHC 13 TeV: 826^{+24+14}_{-16-18} pb

LHC 14 TeV: 975^{+28+16}_{-19-20} pb

$m_t = 173.3$ GeV and MMHT2014 pdf

Relative size of perturbative corrections

$$\sigma^{\text{aN}^3\text{LO}} = \sigma^{(0)} \left[1 + \frac{\sigma^{(1)}}{\sigma^{(0)}} + \frac{\sigma^{(2)}}{\sigma^{(0)}} + \frac{\sigma^{(\text{a}3)}}{\sigma^{(0)}} \right]$$

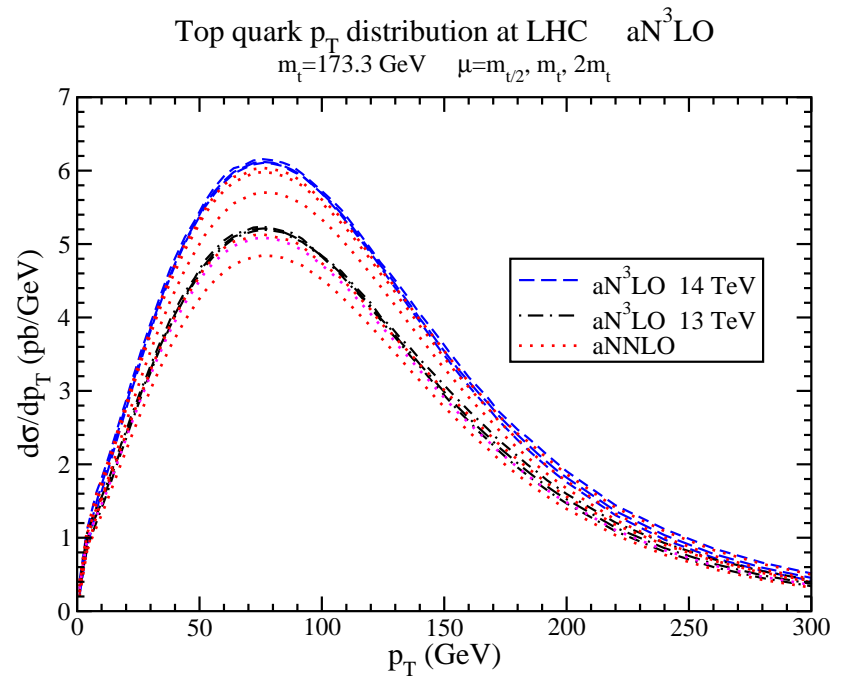
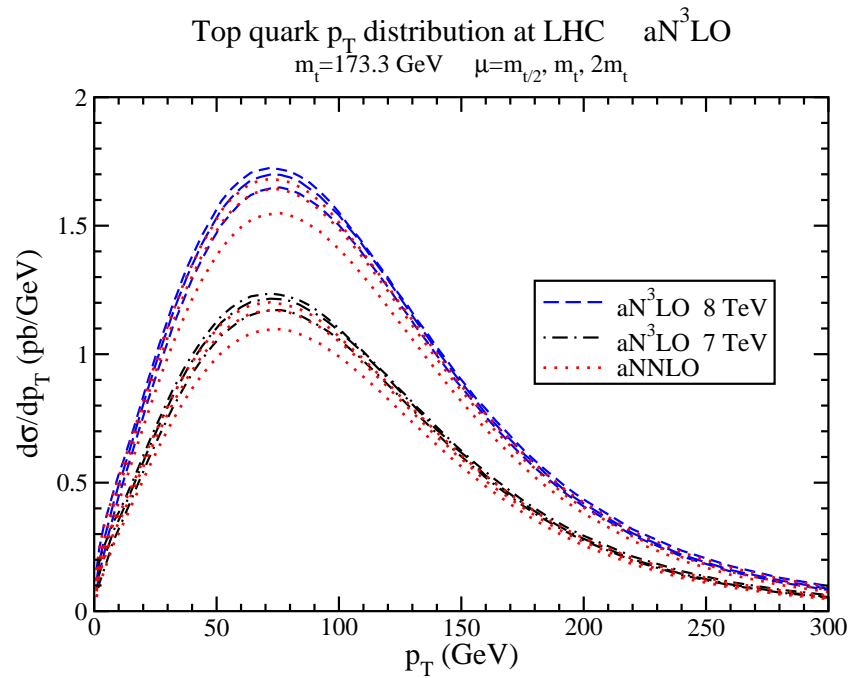
Fractional contributions to the perturbative series for the $t\bar{t}$ cross section					
corrections	Tevatron 1.96 TeV	LHC 7 TeV	LHC 8 TeV	LHC 13 TeV	LHC 14 TeV
$\sigma^{(1)}/\sigma^{(0)}$	0.236	0.470	0.476	0.493	0.496
$\sigma^{(2)}/\sigma^{(0)}$	0.106	0.178	0.177	0.172	0.170
$\sigma^{(\text{a}3)}/\sigma^{(0)}$	0.068	0.066	0.059	0.045	0.043
$\sigma^{\text{aN}^3\text{LO}}/\sigma^{(0)}$	1.410	1.714	1.712	1.710	1.709

Higher-order corrections are sizable for total cross sections and also for differential distributions

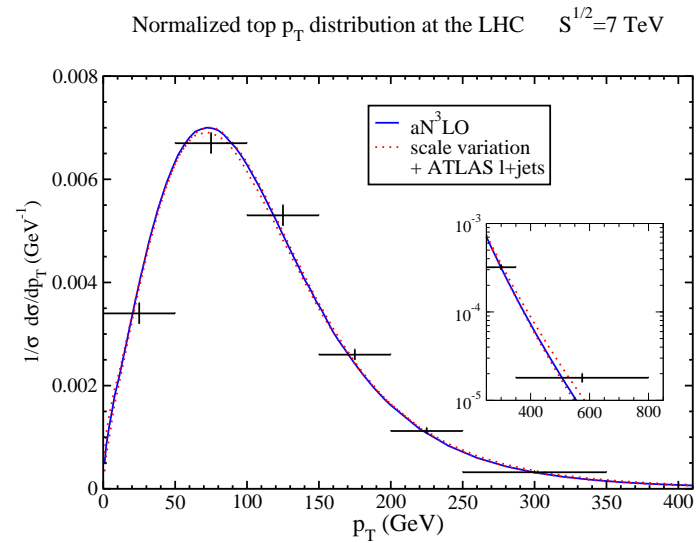
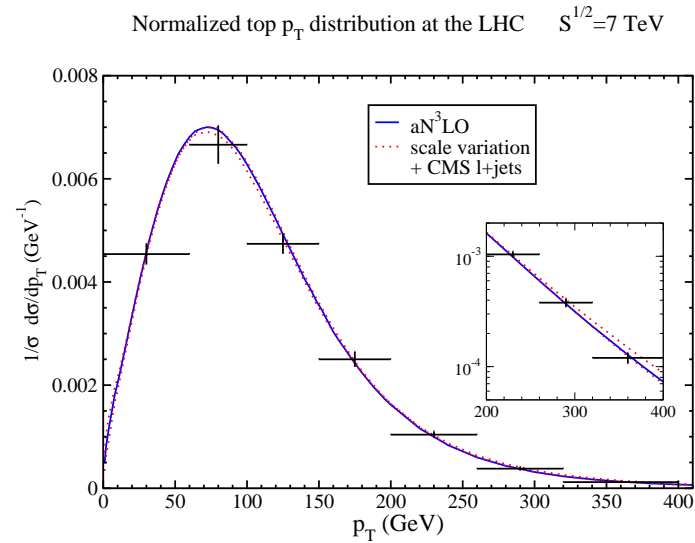
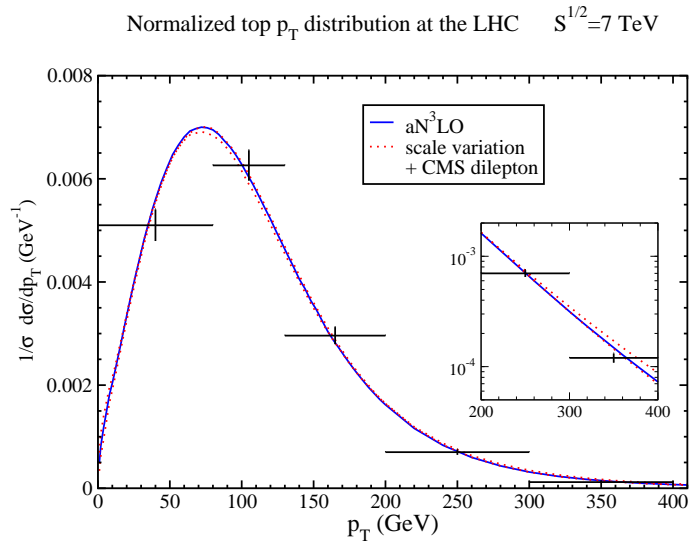
NNLO is not enough

aN³LO needed for precision physics

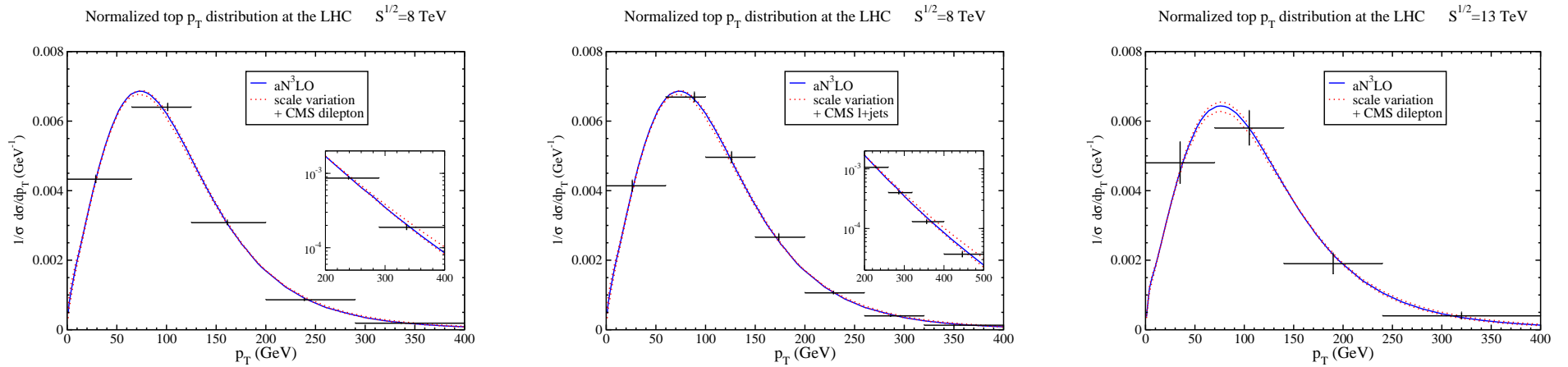
Top quark p_T distribution at the LHC



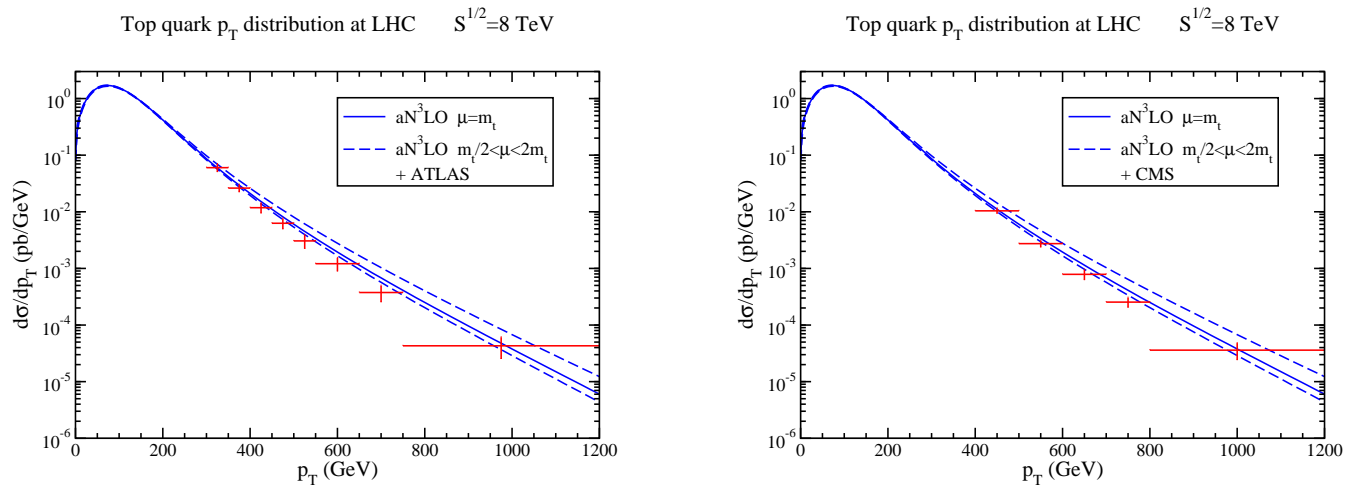
Normalized top quark p_T distribution at 7 TeV LHC



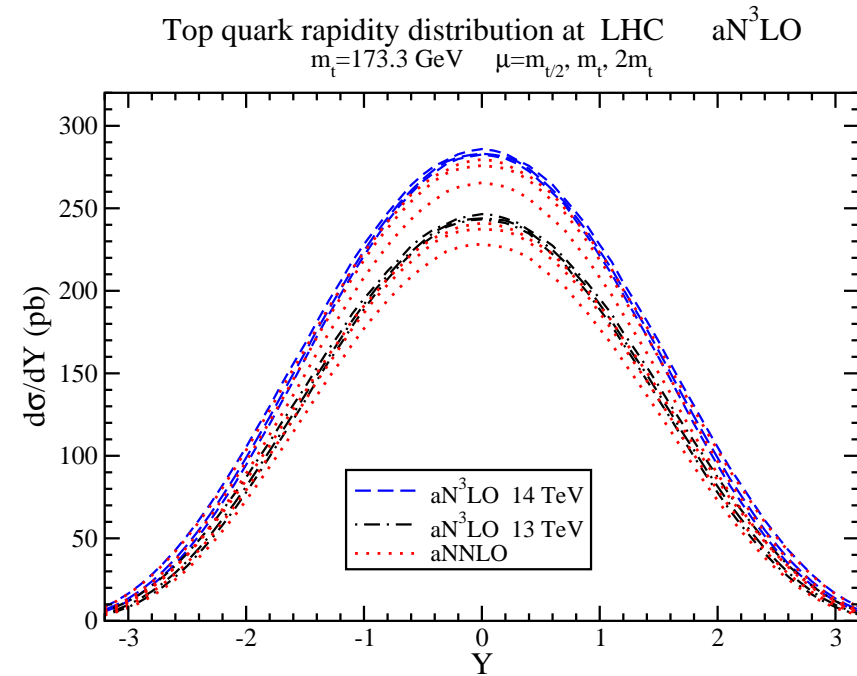
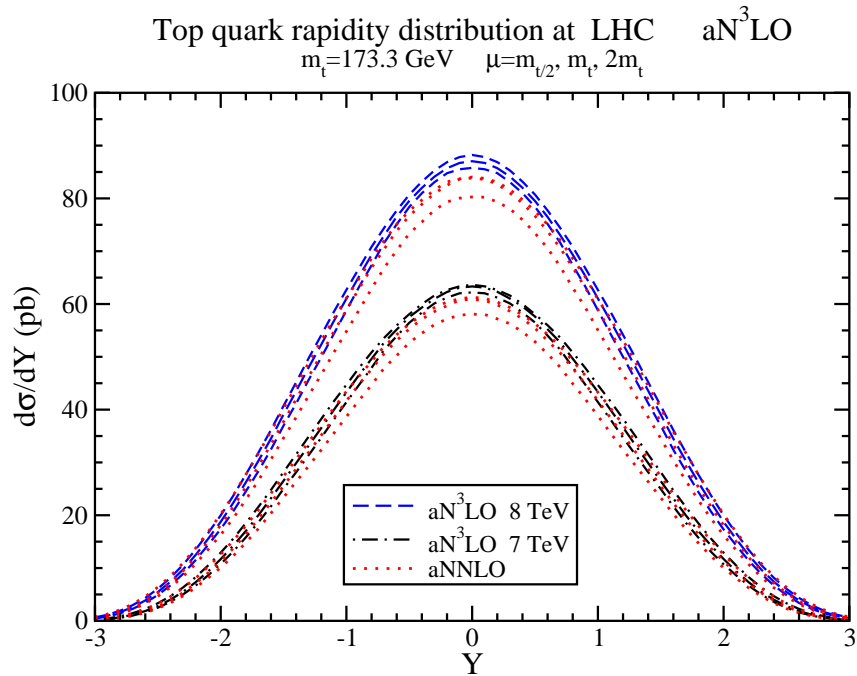
Normalized top quark p_T distribution at 8 and 13 TeV LHC



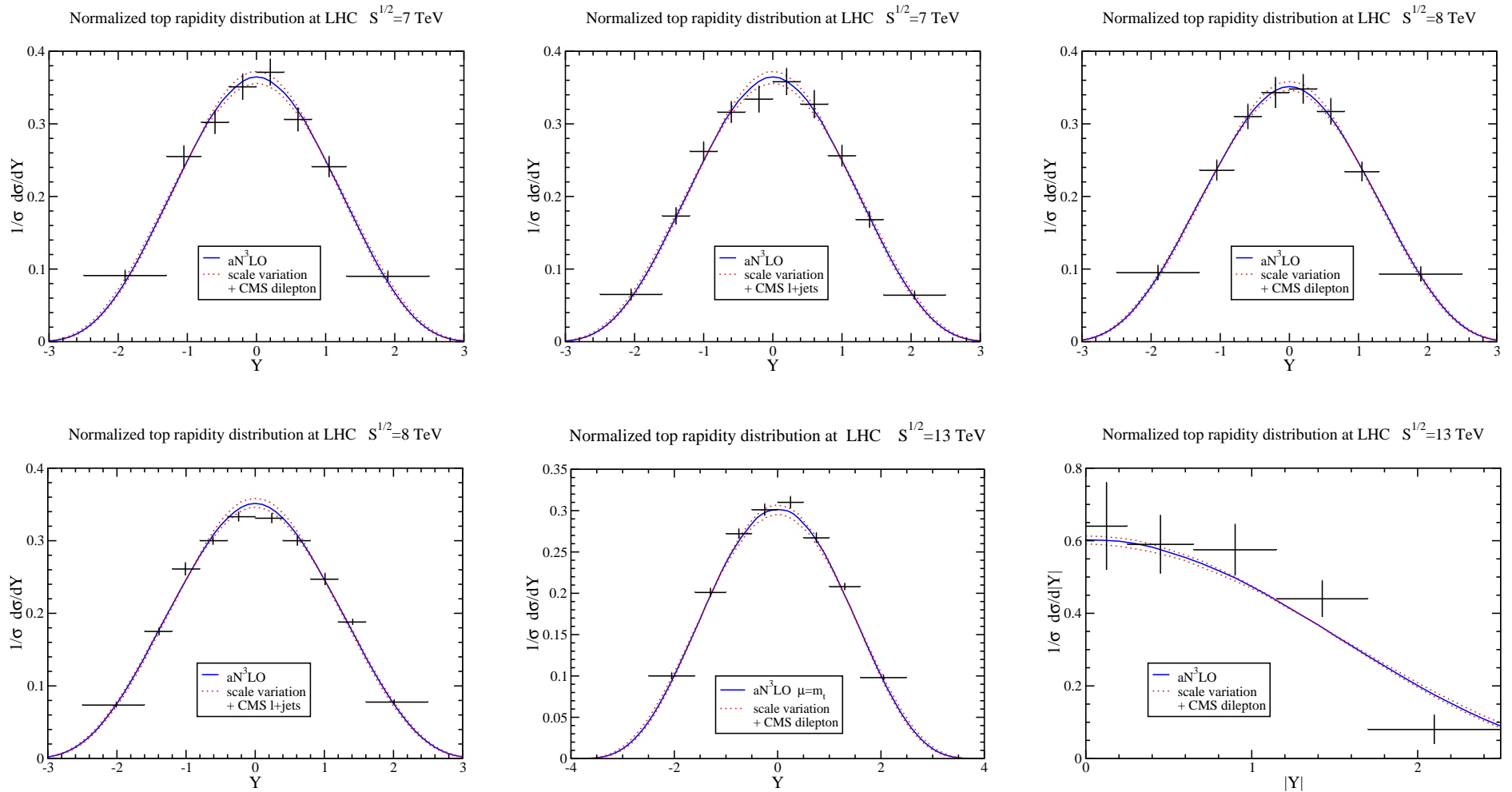
Boosted top quark p_T distribution at 8 TeV LHC



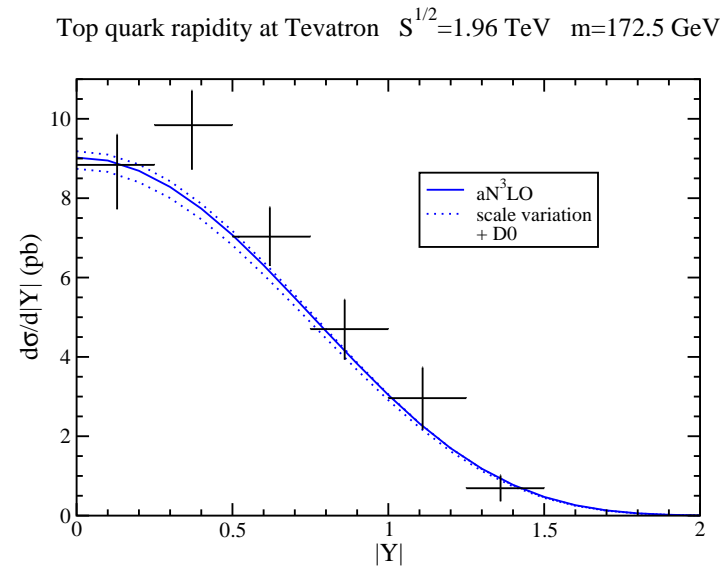
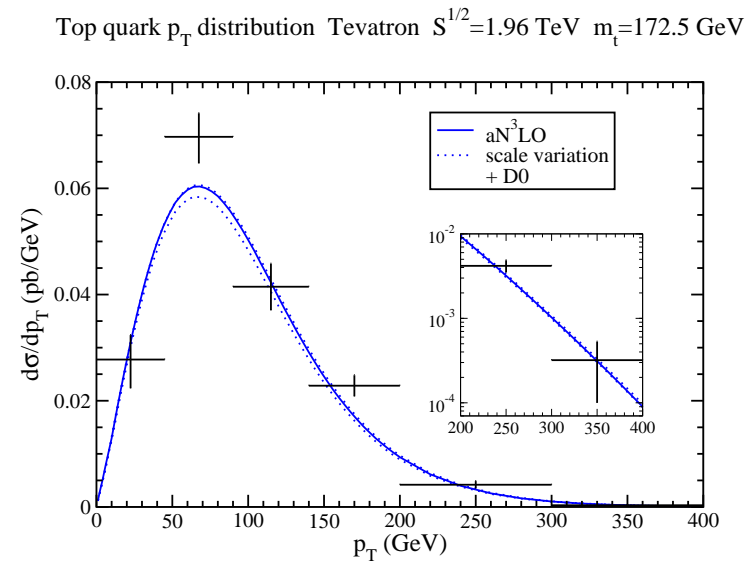
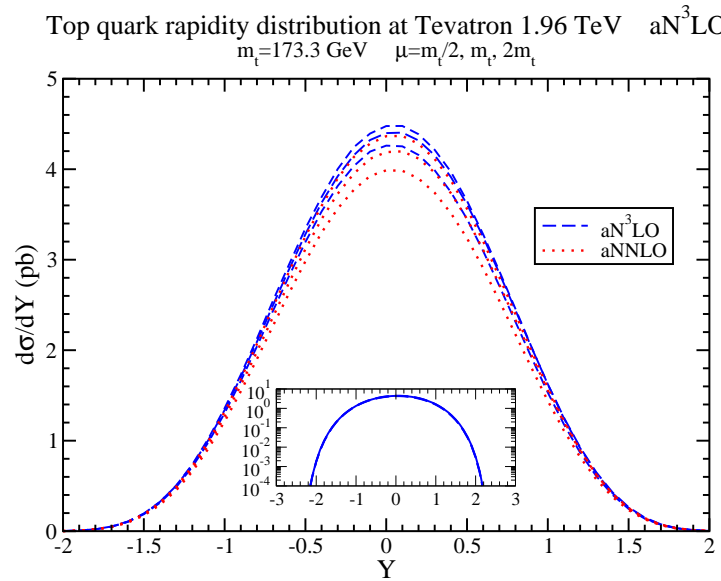
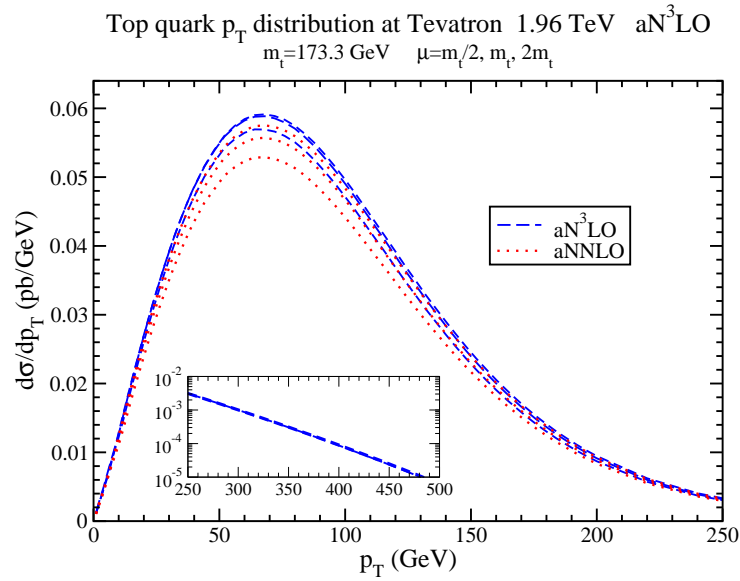
Top quark rapidity distribution at the LHC



Normalized top quark rapidity distribution at the LHC



Top quark p_T and rapidity distributions at the Tevatron



Top forward-backward asymmetry at the Tevatron

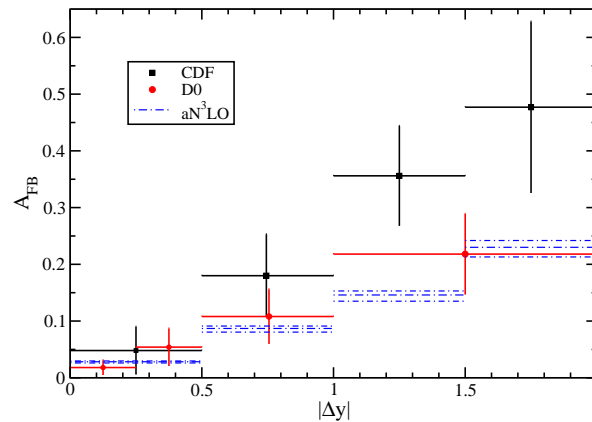
$$A_{\text{FB}} = \frac{\sigma(y_t > 0) - \sigma(y_t < 0)}{\sigma(y_t > 0) + \sigma(y_t < 0)} \equiv \frac{\Delta\sigma}{\sigma}$$

large corrections: **aN³LO/NNLO ratio is 1.05**

Top-quark asymmetry at the Tevatron		
aN ³ LO A_{FB} %	$p\bar{p}$ frame	$t\bar{t}$ frame
QCD only	6.0 ± 0.1	8.7 ± 0.2
QCD+EW	6.8 ± 0.3	10.0 ± 0.6

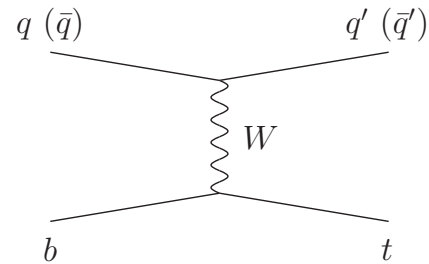
Top differential A_{FB} : $A_{\text{FB}}^{\text{bin}} = \frac{\sigma_{\text{bin}}^+(\Delta y) - \sigma_{\text{bin}}^-(\Delta y)}{\sigma_{\text{bin}}^+(\Delta y) + \sigma_{\text{bin}}^-(\Delta y)}$ with $\Delta y = y_t - y_{\bar{t}}$

A_{FB} at Tevatron $S^{1/2}=1.96$ TeV $m_t=173.3$ GeV

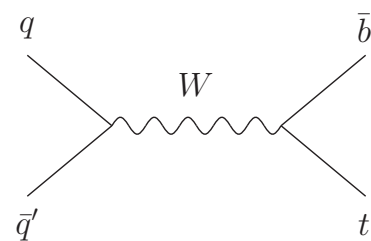


Single-top partonic processes at LO

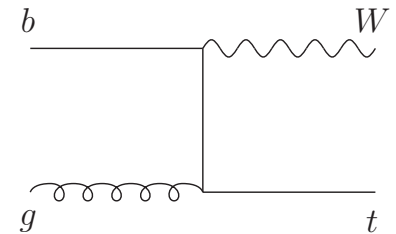
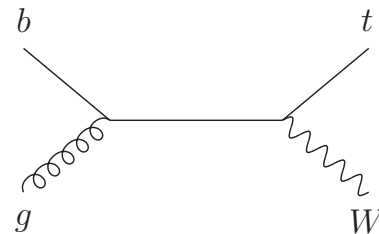
- **t channel:** $qb \rightarrow q't$ and $\bar{q}b \rightarrow \bar{q}'t$
dominant at Tevatron and LHC



- **s channel:** $q\bar{q}' \rightarrow \bar{b}t$
small at Tevatron and LHC

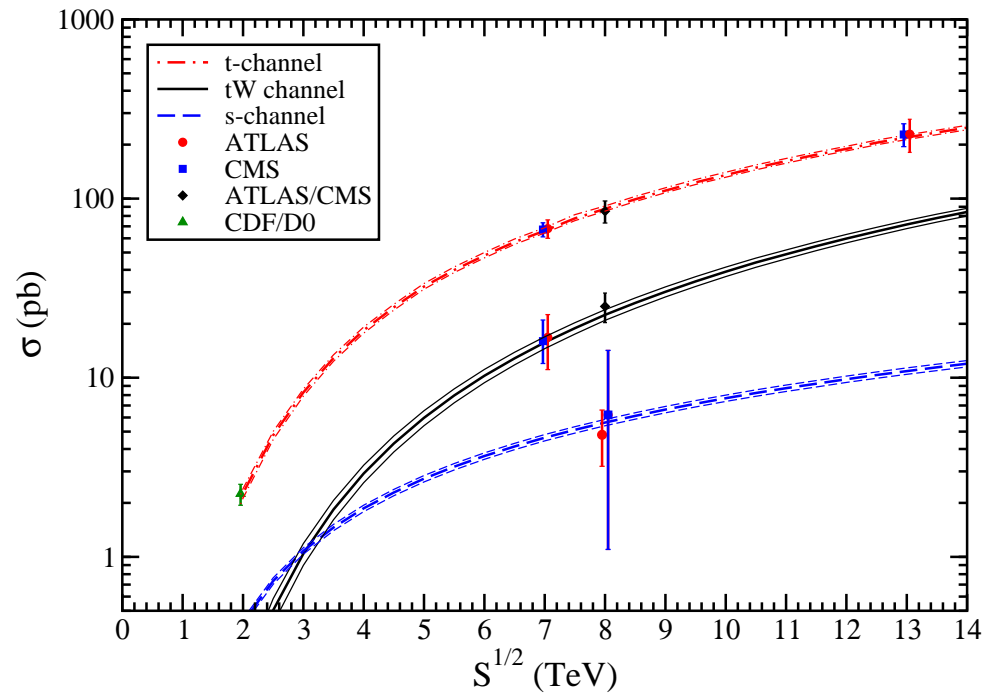


- **associated tW production:** $bg \rightarrow tW^-$
very small at Tevatron, significant at LHC



Single-top cross sections

aNNLO single-top cross sections +-scale&pdf $m_t=172.5$ GeV



Excellent agreement of theory with data for all three channels

Single top t -channel production at aNNLO

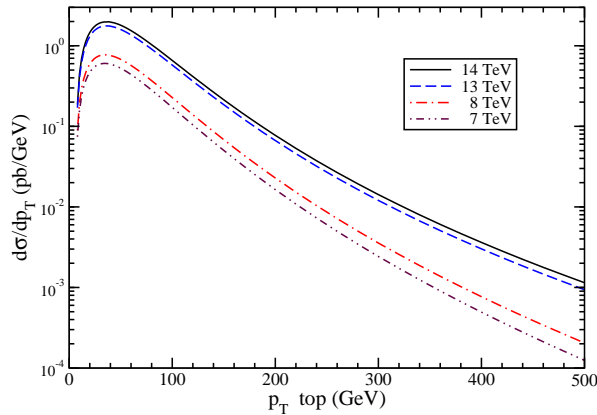
MMHT2014 NNLO pdf

$m_t = 173.3 \text{ GeV}$

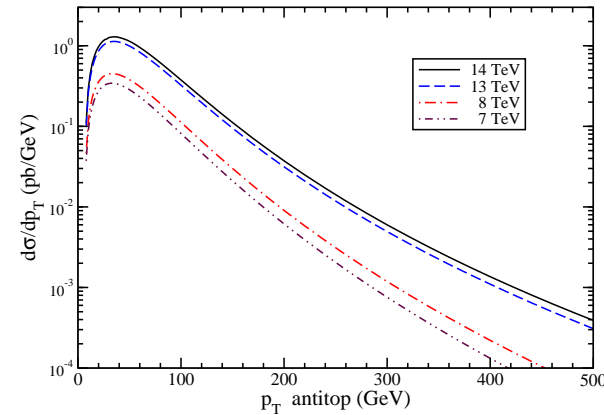
\pm scale \pm pdf errors

LHC	t	\bar{t}	Total (pb)
13 TeV	$138^{+3}_{-1} \pm 2$	$83^{+2}_{-1} \pm 1$	$221^{+5}_{-2} \pm 3$
14 TeV	$157^{+4}_{-1} \pm 2$	$95^{+2}_{-1} \pm 1$	$252^{+6}_{-2} \pm 3$

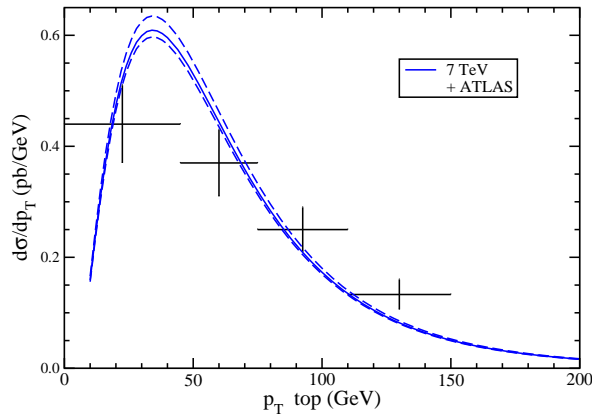
t-channel top p_T distribution at LHC aNNLO $m_t=173.3 \text{ GeV}$



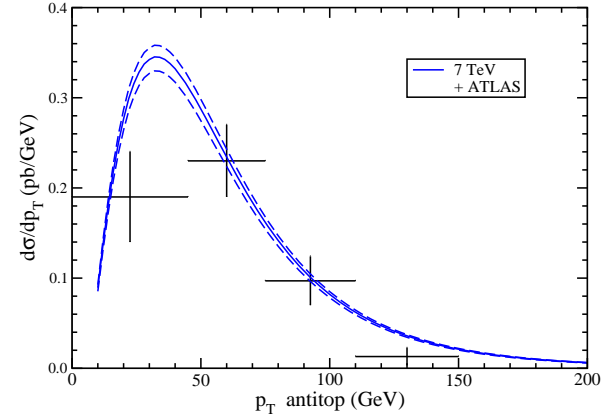
t-channel antitop p_T distribution at LHC aNNLO $m_t=173.3 \text{ GeV}$



t-channel top p_T distribution at LHC aNNLO $m_t=172.5 \text{ GeV}$

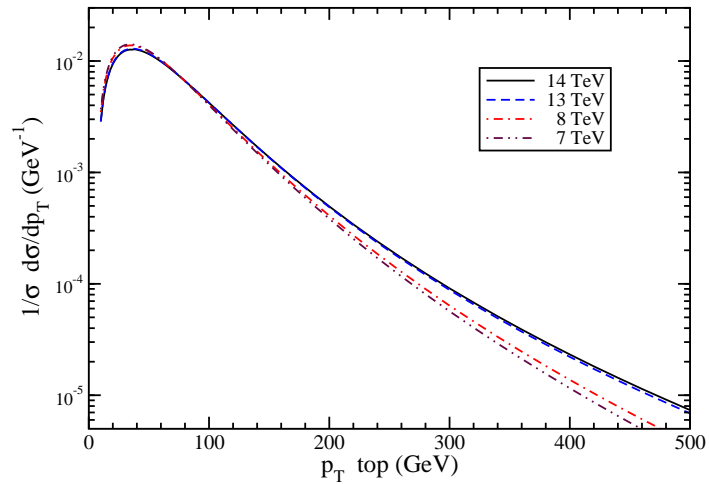


t-channel antitop p_T distribution at LHC aNNLO $m_t=172.5 \text{ GeV}$

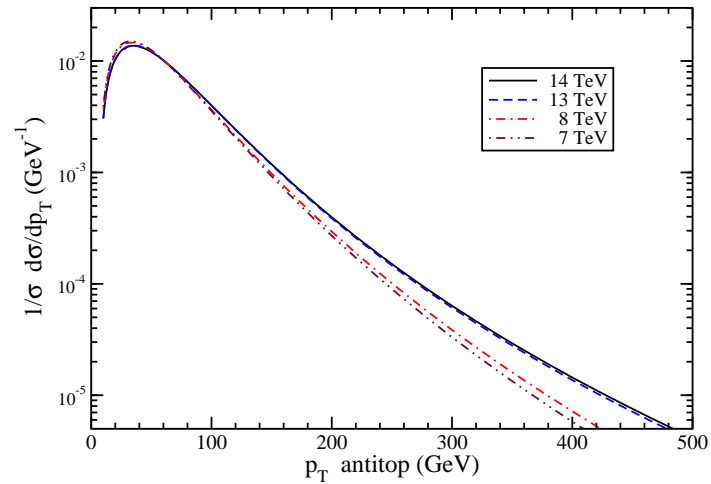


Single top t -channel aNNLO normalized p_T distributions at the LHC

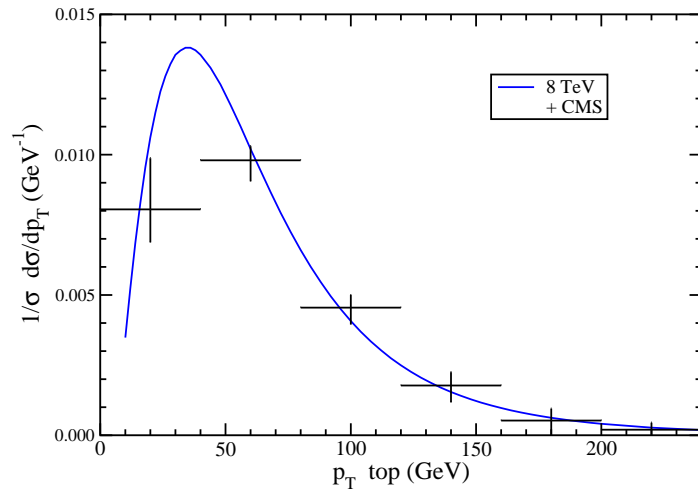
t-channel normalized top p_T distribution at LHC aNNLO $m_t=173.3$ GeV



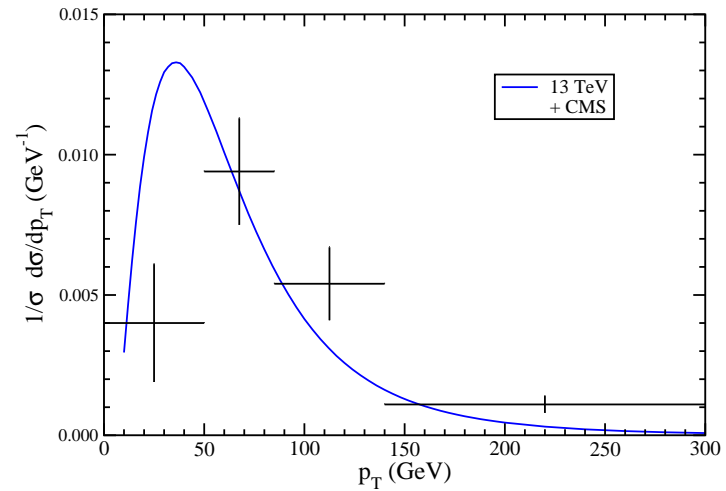
t-channel normalized antitop p_T distribution at LHC aNNLO $m_t=173.3$ GeV



t-channel normalized top p_T distribution at LHC aNNLO



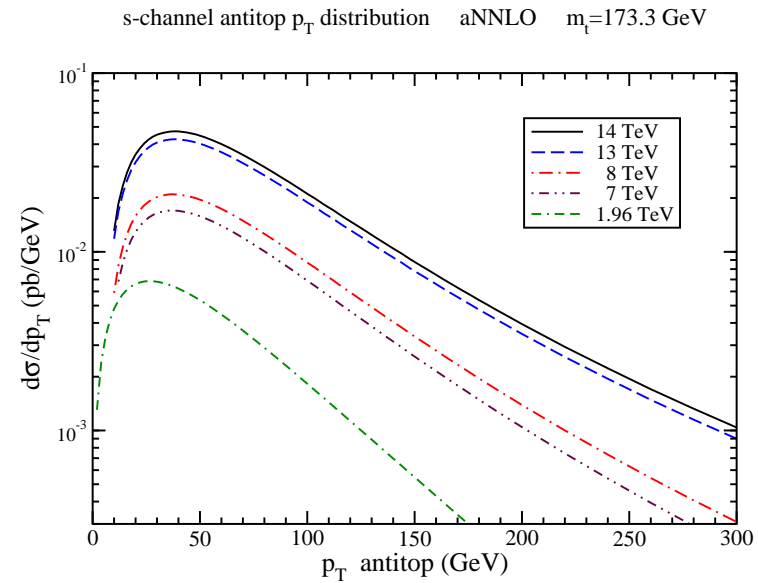
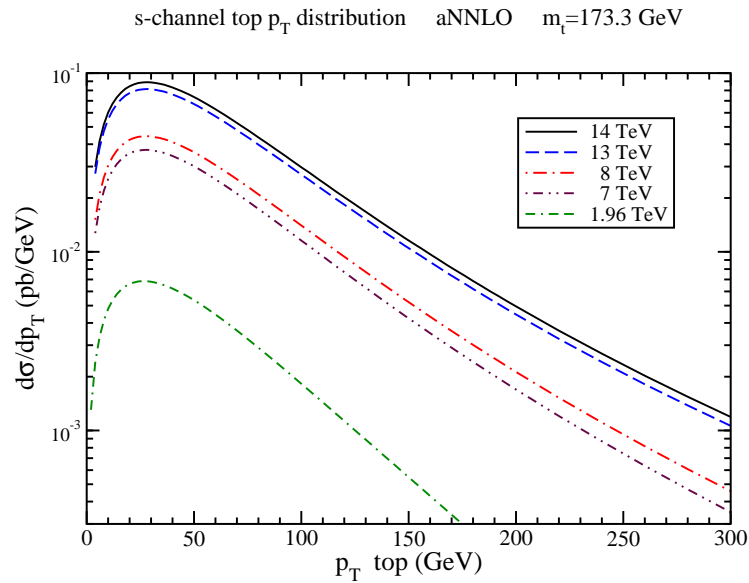
t-channel normalized top+antitop p_T distribution at LHC aNNLO



Single top s -channel production at aNNLO

LHC	t	\bar{t}	Total (pb)
13 TeV	$7.15 \pm 0.13^{+0.15}_{-0.17}$	$4.14 \pm 0.05 \pm 0.10$	$11.29 \pm 0.18 \pm 0.26$
14 TeV	$7.83 \pm 0.14 \pm 0.18$	$4.60 \pm 0.05 \pm 0.11$	$12.43 \pm 0.19 \pm 0.29$

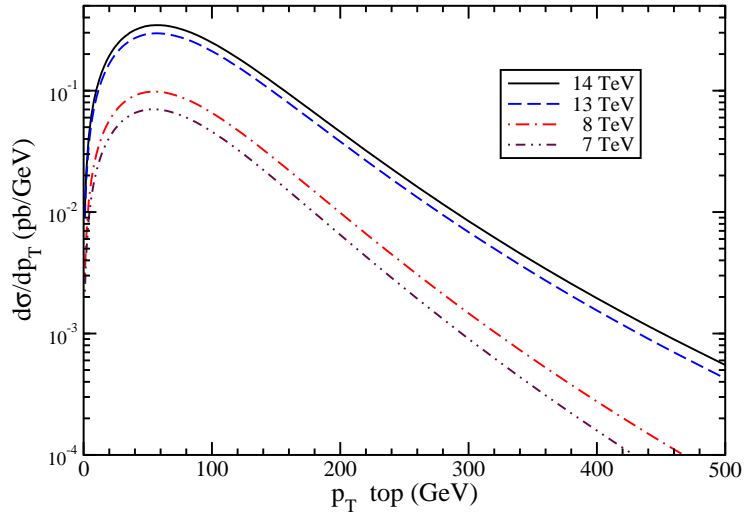
($m_t = 173.3$ GeV) \pm scale \pm pdf errors with MMHT2014 NNLO pdf



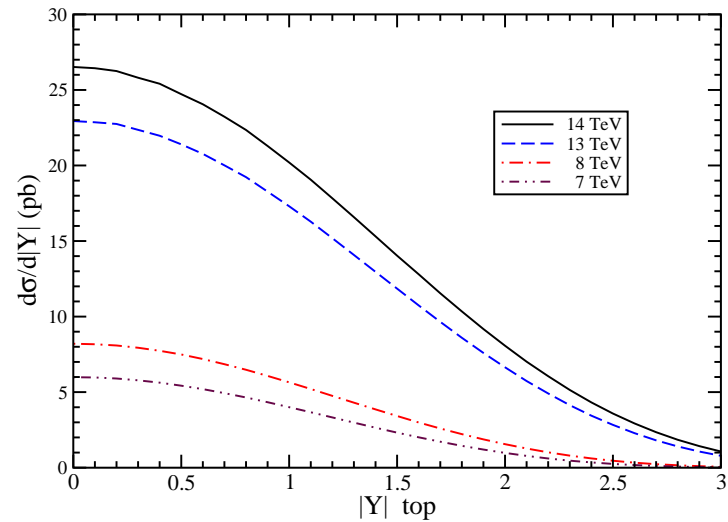
Associated tW^- production at aNNLO at the LHC

MMHT2014 NNLO pdf	LHC	tW^-	$tW^- + \bar{t}W^+$ (pb)
$m_t = 173.3$ GeV	13 TeV	$36.3 \pm 0.9 \pm 0.9$	$72.6 \pm 1.8 \pm 1.8$
\pm scale \pm pdf errors	14 TeV	$42.8 \pm 1.0 \pm 1.1$	$85.6 \pm 2.0 \pm 2.2$

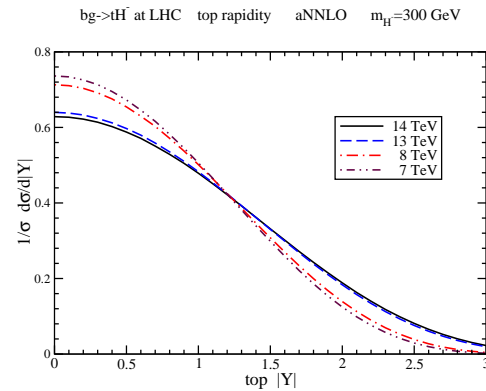
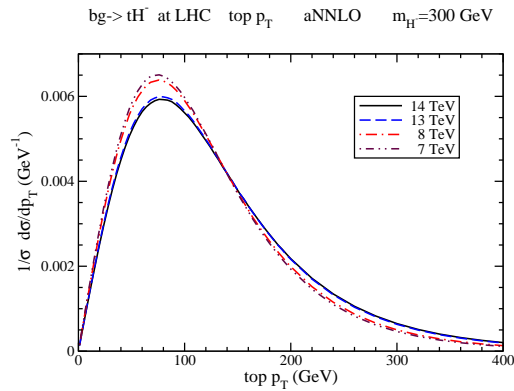
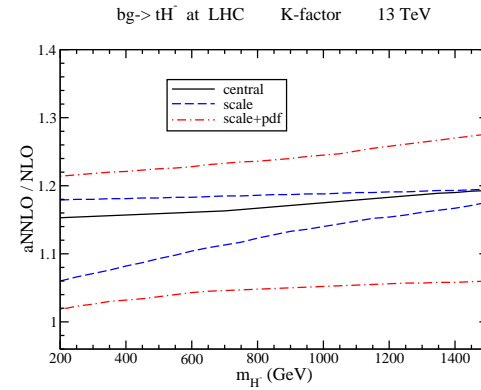
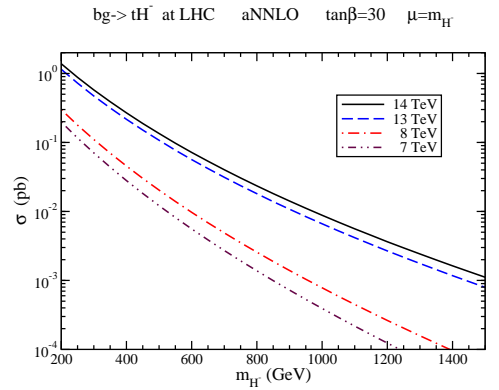
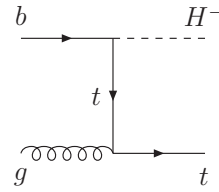
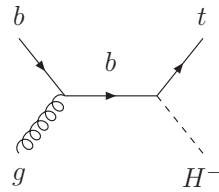
Top p_T distribution in tW^- production at LHC aNNLO $m_t=173.3$ GeV



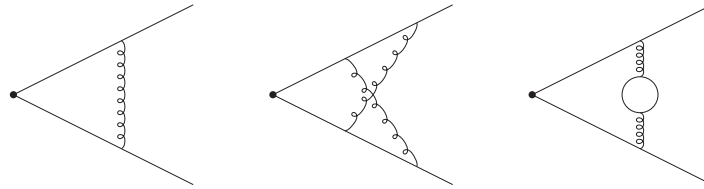
Top rapidity distribution in tW^- production at LHC aNNLO $m_t=173.3$ GeV



tH^- production



Cusp anomalous dimension



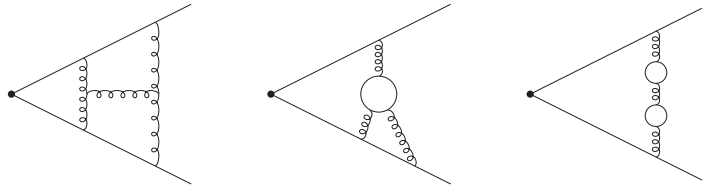
cusp angle $\theta = \cosh^{-1}(v_i \cdot v_j / \sqrt{v_i^2 v_j^2})$ **where** v_i **are velocity vectors**

$$\Gamma_{\text{cusp}} = \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{\pi} \right)^n \Gamma^{(n)}$$

$$\Gamma^{(1)} = C_F(\theta \coth \theta - 1)$$

$$\Gamma^{(2)} = \frac{K}{2} \Gamma^{(1)} + \frac{1}{2} C_F C_A \left\{ 1 + \zeta_2 + \theta^2 - \coth \theta \left[\zeta_2 \theta + \theta^2 + \frac{\theta^3}{3} + \text{Li}_2(1 - e^{-2\theta}) \right] + \coth^2 \theta \left[-\zeta_3 + \zeta_2 \theta + \frac{\theta^3}{3} + \theta \text{Li}_2(e^{-2\theta}) + \text{Li}_3(e^{-2\theta}) \right] \right\}$$

where $K = C_A(67/18 - \zeta_2) - 5n_f/9$



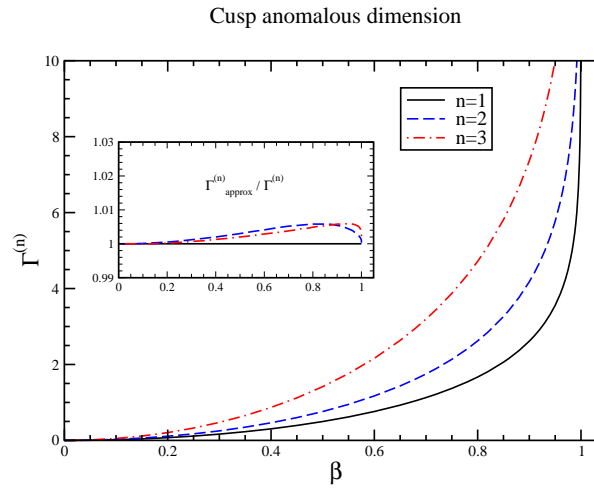
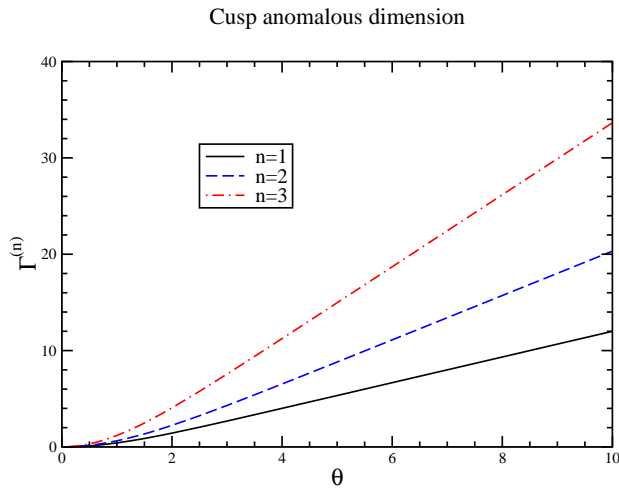
$$\Gamma^{(3)} = C^{(3)} + K'^{(3)}\Gamma^{(1)} + K \left[\Gamma^{(2)} - \frac{K}{2}\Gamma^{(1)} \right]$$

where $K'^{(n)} = K^{(n)}/C_F$

***n*-loop conjecture**

$$\Gamma^{(n)} = \sum_{k=1}^n \frac{(n-1)!}{(k-1)!(n-k)!} K'^{(k)} C^{(n-k+1)}.$$

$$\Gamma^{(n)} = C^{(n)} + \sum_{k=2}^n \frac{(n-1)!}{(k-1)!(n-k)!} F^{(k)} \Gamma^{(n-k+1)}$$



Numerical approximations with $n_f = 5$

$$\Gamma_{\text{approx}}^{(2)}(\beta) = -0.38649 \beta^2 + 1.72704 \Gamma^{(1)}(\beta)$$

$$\Gamma_{\text{approx}}^{(3)}(\beta) = 0.09221 \beta^2 + 2.80322 \Gamma^{(1)}(\beta)$$

with $\beta = \tanh(\theta/2)$

Summary

- cross sections and distributions for $t\bar{t}$ through aN³LO
 - top quark p_T and rapidity distributions
 - top quark forward-backward asymmetry
- cross sections and distributions for single-top production
- high-order corrections are very significant
- excellent agreement with LHC and Tevatron data
- cusp anomalous dimension at high orders