Neutrino-Nucleon Interactions and Lattice QCD

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People

**Thesis advising:**
Richard Hill, Andreas Kronfeld

**Deuterium Fitting:**
M. Betancourt, R. Gran

**Lattice Calculation:**

*(Fermilab Lattice & MILC Collaborations)*
Outline

- Motivation from neutrino physics
  - Importance of free-nucleon cross section
  - Where we can contribute

- z expansion
  - Introduction
  - Deuterium bubble chamber analysis

- (Preliminary) Lattice QCD axial form factor computation
Motivation

Oscillation experiments monitor flux by counting interactions assuming a cross section

$$\Phi(E_\nu) = \frac{N(E_\nu)}{\sigma_A(E_\nu)}$$

Ratio of flux at near/far detector cancels some systematics, but not sufficient for next generation precision measurements

$$\sigma_A \sim \sigma_{CCQE} \otimes \text{(nucl. models)}$$

- Large nuclear targets $\Rightarrow$ measurements of oscillation parameters depends on nuclear models
- **Nuclear effects entangled** with nucleon amplitudes $\Rightarrow$ factorization is oversimplification
- Model-dependent shape parameterization introduces systematic uncertainties and underestimates errors
Cross Sections

(Figure from LBNE, 1307.7335 [hep-ex])

- Measurements of neutrino parameters require precise knowledge of cross sections
- \( \sigma_{CCQE}(E_\nu, Q^2) \) is quadratic function of form factors: \( F_{1V}, F_{2V}, F_A, F_P \)
- Uncertainty on \( F_A(Q^2) \) is primary contribution to systematic errors
  - \( F_{1V}, F_{2V} \) known from \( e - p \) scattering
  - \( F_P \) suppressed by lepton mass, approximated by PCAC
Discrepancies in the Axial-Vector Form Factor

Most analyses assume the Dipole axial form factor (Llewellyn-Smith, 1971):

$$F_A^{\text{dipole}}(Q^2) = g_A \frac{1}{\left(1 + \frac{Q^2}{m_A^2}\right)^2}$$

Dipole is an ansatz:
unmotivated in interesting $Q^2$ (4-momentum) region
$\implies$ uncontrolled systematics and underestimated uncertainties

Large variation in $m_A$ over many experiments:

- $m_A = 1.026 \pm 0.021$ (Bernard et al., 0107088[hep-ph])
- $m_A^{\text{eff}} = 1.35 \pm 0.17$ (MiniBooNE, 1002.2680[hep-ph])

Essential to replace with model-independent parameterization
The $z$ expansion (Bhattacharya, Hill, Paz arXiv:1108.0423 [hep-ph]) is a conformal mapping which takes the kinematically allowed region ($t \leq 0$) to within $|z| < 1$

$$z(t; t_0, t_c) = \frac{\sqrt{t_c - t} - \sqrt{t_c - t_0}}{\sqrt{t_c - t} + \sqrt{t_c - t_0}}$$

$$F_A(z) = \sum_{n=0}^{\infty} a_n z^n$$

($t = q^2 = -Q^2$, $t_c = 9m^2_\pi$)
Advantages of $z$ Expansion

$z$ expansion is a model-independent description of the axial form factor

- Motivated by analyticity arguments
- Only a few coefficients needed to accurately represent form factor
- Provides a prescription for introducing more parameters as data improves
- Allows quantification of systematic errors
- Coefficient falloff required by perturbative QCD

$z$ expansion has been implemented in GENIE, publicly available
Deuterium Fitting

Analysis in *Phys. Rev. D* 93, 113015 (1603.03048 [hep-ph])  
ASM, M. Betancourt, R. Gran, R. Hill

Reanalyzed deuterium bubble chamber data by replacing dipole with $z$ expansion framework

Three datasets:

- **ANL 1982**: 1737 events, 0.5 GeV [peak]
- **BNL 1981**: 1138 events, 1.6 GeV [average]
- **FNAL 1983**: 362 events, 20 GeV [peak], 27 GeV [average]

Shape-only fits to QE differential cross section data

Gaussian priors used on $z$ expansion coefficients

Sum rules applied to enforce large $Q^2$ falloff
Deuterium Fits - Differential Cross Section

Dipole:

<table>
<thead>
<tr>
<th>$\chi^2/N_{\text{bins}}$</th>
<th>$m_A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>58.6/49</td>
<td>1.02(5)</td>
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</table>

$z$ Expansion:

<table>
<thead>
<tr>
<th>$\chi^2/N_{\text{bins}}$</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$a_3$</th>
<th>$a_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>60.9/49</td>
<td>2.25(10)</td>
<td>0.2(0.9)</td>
<td>-4.9(2.4)</td>
<td>2.7(2.7)</td>
</tr>
</tbody>
</table>

Dipole:

<table>
<thead>
<tr>
<th>$\chi^2/N_{\text{bins}}$</th>
<th>$m_A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>70.9/49</td>
<td>1.05(4)</td>
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</table>

$z$ Expansion:

<table>
<thead>
<tr>
<th>$\chi^2/N_{\text{bins}}$</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$a_3$</th>
<th>$a_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>73.4/49</td>
<td>2.24(10)</td>
<td>0.6(1.0)</td>
<td>-5.4(2.4)</td>
<td>2.2(2.7)</td>
</tr>
</tbody>
</table>
Final Fits

Calculated observables:

\[ r_A^2 = 0.46(22) \text{ fm}^2, \quad \sigma_{\nu n \rightarrow \mu p}(E_\nu = 1 \text{ GeV}) = 10.1(0.9) \times 10^{-39} \text{ cm}^2 \]

compared with Bodek et al. [Eur. Phys. J. C 53, 349]:

\[ r_A^2 = 0.453(13) \text{ fm}^2, \quad \sigma_{\nu n \rightarrow \mu p}(E_\nu = 1 \text{ GeV}) = 10.63(0.14) \times 10^{-39} \text{ cm}^2 \]

Dipole model significantly underestimates error from nucleon form factor
Necessary to use model-independent approach to properly understand systematic uncertainties from nucleon form factors

- $z$ expansion is the appropriate model-independent approach

Need another first-principles approach to determine form factor

- Will aid future construction of nuclear models, understanding of nuclear physics
- Lattice QCD provides needed handle

Combination of Lattice QCD and $z$ expansion already very successful in CKM matrix element computations
Lattice QCD in Neutrino Physics

- LQCD measurements becoming more accurate, precise
  ⇒ now able to inform neutrino experiment

- LQCD enables clean measurement of form factors
  (no nuclear corrections, no experiment systematics)

- Offers way of breaking measurement degeneracy between
  nuclear models, nucleon form factors

- Less explosive than hydrogen!
Current Lattice Effort

LQCD calculation of form factors underway by MILC/Fermilab Lattice Collaborations

Lattice computation involves several stages, building up to result:
2-point functions = masses, overlap factors

\[
\lim_{t \to \infty} \langle N(0)|N(t)\rangle \sim |a|^2 e^{-m_N t}
\]
Lattice QCD Axial Form Factor

Use 2-point functions to calculate 3-point functions $= \text{form factors}$

$$\lim_{T, t \to \infty} \langle N'(0)| A_{\mu}(x, t) | N(T) \rangle \sim F_A(Q^2) |a|^2 e^{-m_N t} e^{-m_N (T-t)} e^{-iq \cdot x}$$

Results blinded by constant factor
Conclusions

Neutrino physics is subject to
  underestimated and model-dependent systematics
→ To reduce systematics from modeling,
  need to understand nuclear physics
→ To understand nuclear physics, need to understand
  nucleon-level cross sections from an ab initio calculation

- $z$ expansion removes model assumptions and permits better
  understanding of systematic errors
  - $z$ expansion available in GENIE “trunk” version,
    will be fully supported in v2.12
- hydrogen (deuterium) targets have relatively small nuclear effects
- LQCD offers a way to access nucleon form factors directly

Thanks for listening!
z Expansion in GENIE

To be officially released in production version 2.12

Currently available in GENIE “trunk” version
3-Point Functions: Normalization of $A_\mu$/Blinding

Calculate form factor:

$$\frac{\langle N| Z_A A_\mu | N \rangle}{\langle 0| Z_A A_\mu | \pi^a \rangle} \bigg|_{q=0} \propto \frac{g_A}{f_\pi}$$

Benefits from statistical cancellation, exact renormalization

Normalize with $f_\pi$ computed from MILC computation of $f_\pi$,
Phys. Rev. D 90, 074509 (1407.3772 [hep-lat])

$F_A$ at nonzero momentum computed as ratio of nuclear matrix elements:

$$\frac{\langle N(0)| Z_A A_{\perp\mu}(q) | N(q) \rangle}{\langle N(0)| Z_A A_\mu(0) | N(0) \rangle} \propto \frac{F_A(Q^2)}{g_A}$$
3-Point Functions: Blinding

Value of $g_A$ well-known from neutron beta decay experiments

$\Rightarrow$ Blinding implemented as a factor multiplying 3-point function

$$\beta \langle N(0) | Z_A A_{\mu}(q) | N(q) \rangle \sim \beta F_A(Q^2)$$

Blinding known only to few members of collaboration, not to me
Acceptance Corrections

Acceptance correction for fixing errors from hand scanning $Q^2$ dependent correction, correlated between bins:

$$\frac{dN}{e(Q^2)} \rightarrow \frac{dN}{e(Q^2) + \eta de(Q^2)}, \quad \eta = 0 \pm 1$$

For ANL, BNL, FNAL respectively, $\eta = -1.9, -1.0, +0.01$; minimal improvement of goodness of fit.
Deuterium Corrections

Corrections assumed to be $E_\nu$ independent
Two corrections tested:
Singh Nucl. Phys. B 36, 419,
Shen 1205.4337 [nucl-th]

Central values of Shen, Singh are consistent with each other
Final fit done with Singh, inflated error bars
Nuclear Effects

Nuclear effects not well understood
→ Models which are best for one measurement are worst for another
Need to break $F_A$/nuclear model entanglement

$\nu_\mu \rightarrow \mu^-$

$A \rightarrow A'$

(assumed $m_A = 0.99$ GeV)

<table>
<thead>
<tr>
<th>NuWro Model</th>
<th>RFG [GENIE]</th>
<th>RFG+ TEM</th>
<th>assorted others</th>
</tr>
</thead>
<tbody>
<tr>
<td>leptonic(rate)</td>
<td>3.5</td>
<td>2.4</td>
<td>2.8-3.7</td>
</tr>
<tr>
<td>leptonic(shape)</td>
<td>4.1</td>
<td>1.7</td>
<td>2.1-3.8</td>
</tr>
<tr>
<td>hadronic(rate)</td>
<td>1.7[1.2]</td>
<td>3.9</td>
<td>1.9-3.7</td>
</tr>
<tr>
<td>hadronic(shape)</td>
<td>3.3[1.8]</td>
<td>5.8</td>
<td>3.6-4.8</td>
</tr>
</tbody>
</table>

(Minerva collaboration, 1305.2243,1409.4497[hep-ph])
Neither $z$ expansion, nor dipole can properly explain shape of data.

Difficult to extract form factor from scattering data, uncontrolled systematics introduced in process.
Normalization Degeneracy

Despite apparent similarity of dipole/z expansion cross sections, form factors quite different

Consequence of self-consistency: cross section prediction

$$\frac{dN}{dE} \propto \frac{1}{\sigma} \frac{d\sigma}{dQ^2}$$

Cut of low-$Q^2$ data & floating normalization hide cross section differences