Open Effective Field Theories from Deeply Inelastic Reactions

[arXiv:1607.0239]

and

Lindblad Equation for Inelastic Loss of Ultracold Atoms

[arXiv:1607.08084]

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Some important processes in particle physics involve deeply inelastic reactions that produce particles with much larger momenta

- μ^- decay into $\nu_\mu\, e^-\, \bar{\nu}_e$
- positronium decay into two photons
- dark matter annihilation into $\gamma\gamma$, γZ^0, Z^0Z^0, W^+W^-
- axion scattering reactions that decrease axion number, such as $a\,a\,a\,a \to a\,a$

Many important loss processes for ultracold atoms also involve deeply inelastic reactions

Some important processes in particle physics involve deeply inelastic reactions that produce particles with much larger momenta

Q. What is the low-energy effective field theory from integrating out high-momentum particles from deeply inelastic reactions?

A. It is an open effective field theory in which time evolution of the density matrix is given by the Lindblad equation!

Open Effective Field Theories from Deeply Inelastic Reactions

- Open effective field theory and Lindblad equation
- Muon decay: a deeply inelastic reaction
- Density matrix puzzle
- Lindblad equation

Density Matrix

arbitrary state of many-body quantum system
can be described by density matrix ρ(t)

Time evolution: $$\frac{d}{dt}\rho = -i[H,\rho]$$

- linear in ρ
- preserves the trace of ρ (can choose Tr(ρ) = 1)
- Markovian: future determined by present
  does not depend also on past history

system average of operator O $$\langle \mathcal{O} \rangle = \mathrm{Tr}(\rho \mathcal{O})$$

Open Quantum System

subsystem and its environment

define density matrix $\rho(t)$ for subsystem
by tracing over the environment:

$$\rho = \mathrm{Tr}_{\mathrm{environment}}(\rho_{\mathrm{full}})$$

In general, time evolution of $\rho(t)$

- preserves the trace of ρ
- is non-Markovian: $d\rho/dt$ depends on $\rho(t)$
  and on past history $\rho(t'), t' < t$

See lecture notes by John Preskill on
Quantum Information and Computation

Lindblad equation

Lindblad 1976

Gorini, Kossakowski, Sudarshan 1976

density matrix ρ(t) for subsystem
of open quantum system

If time evolution of ρ(t)
- is linear in ρ
- preserves the trace of ρ
- is Markovian
- is completely positive

evolution equation must have the form

$$\frac{d}{dt}\rho = -i[H,\rho] - \tfrac{1}{2}\sum_n \left(L_n^\dagger L_n \rho + \rho L_n^\dagger L_n - 2 L_n \rho L_n^\dagger\right)$$

for some Hermitian operator *H*
and some operators L_n (Lindblad operators)

Lindbladian time evolution

$$\frac{d}{dt}\rho = -i\left[H,\rho\right] - \tfrac{1}{2}\sum_n \left(L_n^\dagger L_n \rho + \rho L_n^\dagger L_n - 2L_n \rho L_n^\dagger\right)$$

$$\frac{d}{dt}\rho = -i\left[H,\rho\right] - \left\{K,\rho\right\} + \sum_n L_n \rho L_n^\dagger$$

where $K = \tfrac{1}{2}\sum_n L_n^\dagger L_n$

evolution with nonhermitian Hamiltonian *H - i K*
plus additional Lindblad term

that ensures $\frac{d}{dt}\mathrm{Tr}(\rho) = 0$

Open Effective Field Theory

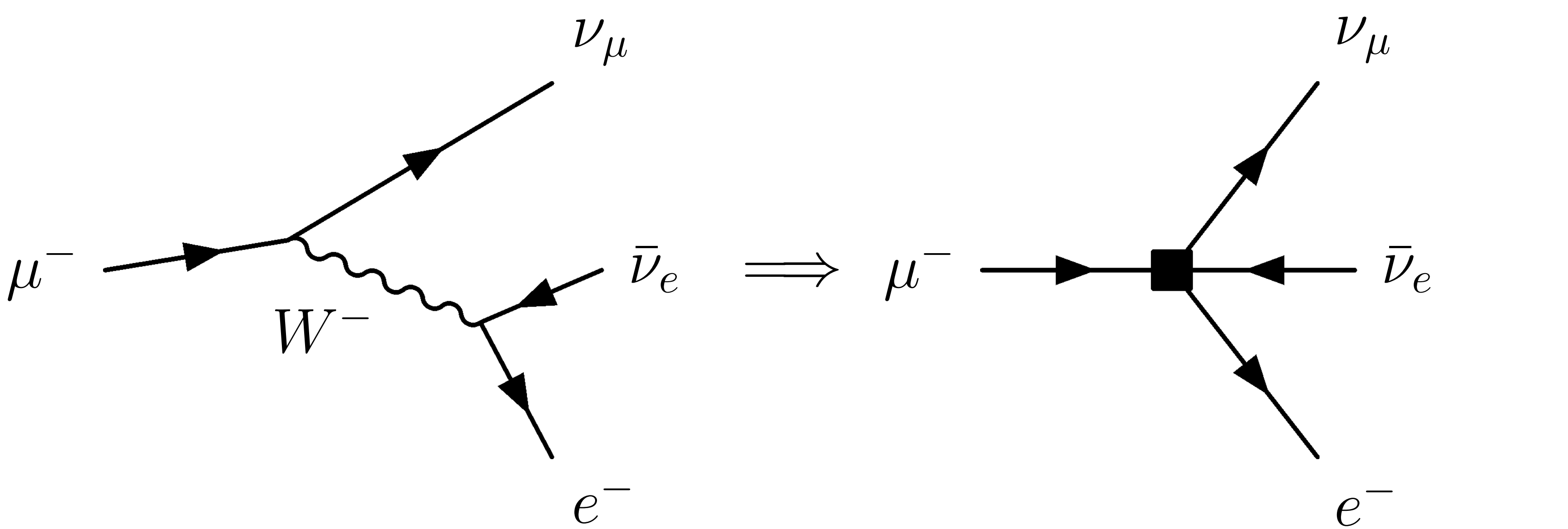
subsystem of an open quantum system
that is described by a field theory

Grozdanov and Polonyi arXiv:1305.3670
derive dissipative hydrodynamics using open EFT
for hydrodynamic modes of quantum field theory?

Burgess, Holman, Tasinato, and Williams
arXiv:1408.5002,1512.00169
open EFT for super-Hubble modes of
primordial quantum fluctuations in early universe
effective density matrix satisfies the Lindblad equation

Effective Field Theory for Muons

in the amplitude for μ decay
the virtual W can propagate only over short distances
of order $1/M_W$

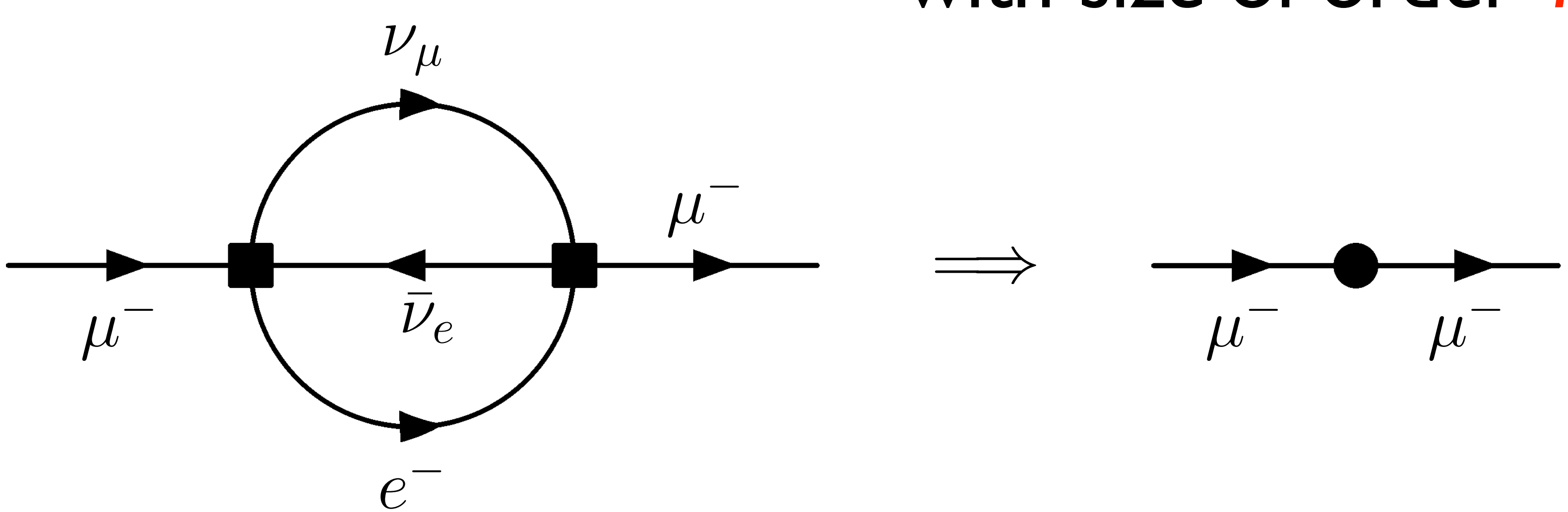


the amplitude can therefore be reproduced
by a 4-fermion contact interaction

$$\Delta\mathcal{L} = \frac{G_F}{\sqrt{2}}\,\bar{\nu}_\mu\gamma_\alpha(1-\gamma_5)\mu\;\bar{e}\gamma^\alpha(1-\gamma_5)\nu_e$$

Effective Field Theory for Muons

in the amplitude for $\mu^- \to \nu_\mu\, e^-\, \bar{\nu}_e \to \mu^-$
the high-momentum leptons
are created and annihilated in a localized region
with size of order $1/M_\mu$



their effects can therefore be reproduced
by local operators that are <u>anti-Hermitian</u>

$$\Delta\mathcal{H} = -\tfrac{i}{2}\Gamma_\mu\, \psi_\mu^\dagger \psi_\mu$$

Effective field theory

from integrating out high-momentum leptons
from decays of the muon
has a non-Hermitian Hamiltonian:

$$H_{eff} = H - i\,K$$

Its anti-Hermitian part is local

$$K = \tfrac{1}{2}\Gamma_\mu \int d^3r\, \psi_\mu^\dagger \psi_\mu + \ldots$$

Puzzle

K commutes with muon number N_μ

Is N_μ conserved in the effective field theory?

Does N_μ decrease exponentially at the rate Γ_μ?

Effective Field Theory for Muons

define effective density matrix
by tracing over states with high-momentum leptons
from decays of muons:

$$\rho = \mathrm{Tr}_{\mathrm{deep}}(\rho_{\mathrm{full}})$$

(and time average to remove high frequencies ~ M_μ)

Time evolution of ρ?

$$\frac{d}{dt}\rho \stackrel{?}{=} -i[H,\rho] - \{K,\rho\} \qquad K = \tfrac{1}{2}\Gamma_\mu N_\mu$$

does not conserve probability!

$$\frac{d}{dt}\mathrm{Tr}(\rho) = -\Gamma_\mu \mathrm{Tr}(N_\mu \rho)$$

Solution

time evolution of the effective density matrix

is given by the Lindblad equation!

$$\frac{d}{dt}\rho = -i[H,\rho] - \tfrac{1}{2}\Gamma_\mu\{N_\mu,\rho\} + \Gamma_\mu \int d^3r\, \psi_\mu \rho \psi_\mu^\dagger$$

probability is conserved! $$\frac{d}{dt}\mathrm{Tr}(\rho) = 0$$

Effective field theory

from integrating out high-momentum leptons

from decays of muons

is an open effective field theory!

probability for state with *n* muons: $P_n(t)$

Lindblad equation implies

$$\frac{d}{dt}P_n = -n\Gamma_\mu P_n + (n+1)\Gamma_\mu P_{n+1}$$

if $P_{n+1} = 0$, P_n decreases like $\exp(-n\Gamma_\mu t)$

number of muons: $\langle N_\mu \rangle \equiv \sum_n n P_n$

$$\frac{d}{dt}\langle N_\mu \rangle = -\Gamma_\mu \sum_n \left[n^2 P_n - n(n+1)P_{n+1}\right]$$
$$= -\Gamma_\mu \langle N_\mu \rangle$$

$\langle N_\mu \rangle$ decreases like $\exp(-\Gamma_\mu t)$ as expected

Generalization

A deeply inelastic reaction is a local process

The effective field theory
from integrating out high-momentum particles
from deeply inelastic reactions
has an effective Hamiltonian *H - i K*
whose anti-Hermitian part is local and positive:

$$K = \sum_n \gamma_n \int d^3r \, \Phi_n^\dagger \Phi_n$$

Generalization

An effective density matrix ρ
can be defined by tracing over states
that include high-momentum particles
from deeply inelastic reactions

Time evolution of the effective density matrix
is described by the Lindblad equation:

$$\frac{d}{dt}\rho = -i[H,\rho] - \{K,\rho\} + 2\sum_n \gamma_n \int d^3r\, \Phi_n \rho \Phi_n^\dagger$$

local Lindblad operators Φ_n
are determined by the anti-Hermitian terms
in the effective Hamiltonian

$$K = \sum_n \gamma_n \int d^3r\, \Phi_n^\dagger \Phi_n$$

Generalization

The effective field theory
from integrating out high-momentum particles
from deeply inelastic reactions
is an open effective field theory.

The time evolution of its density matrix
is described by the Lindblad equation.

Burgess, Holman, Tasinato, and Williams arXiv:1408.5002
"Now that we have a new hammer,
let's go find all those nails . . ."

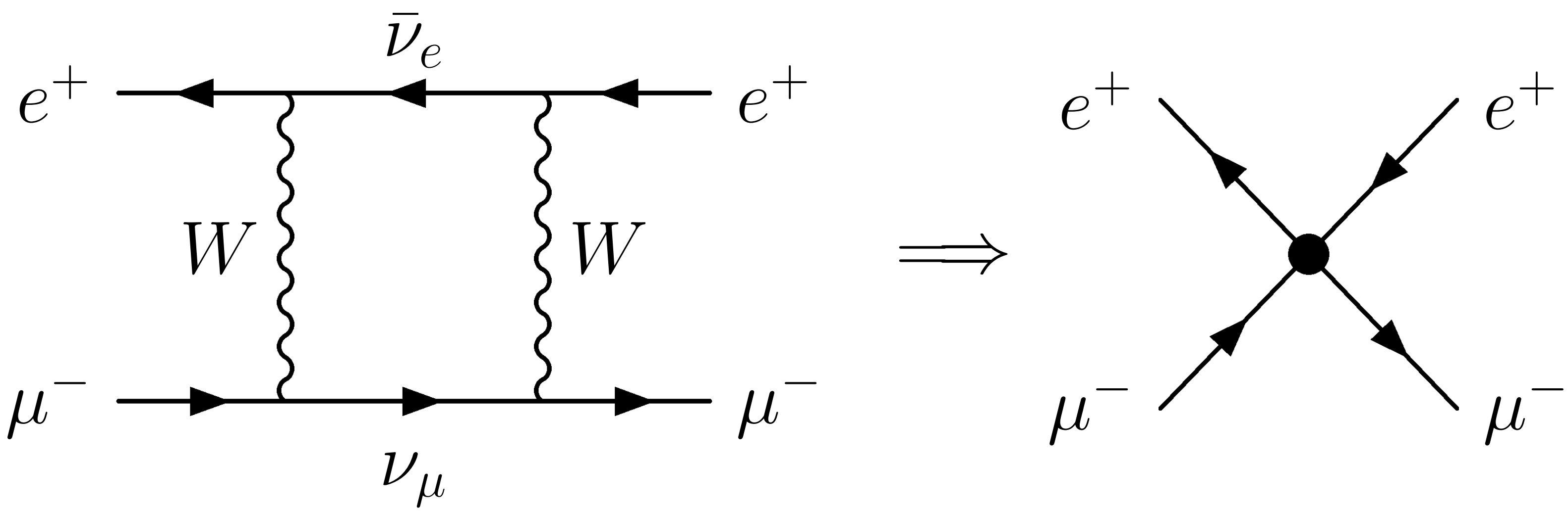
Effective Field Theory for Muons

in the amplitude for $\mu^- e^+ \rightarrow \nu_\mu \bar{\nu}_e \rightarrow \mu^- e^+$

the high-momentum neutrinos

are created and annihilated in a localized region

with size of order $1/M_\mu$



their effects can therefore be reproduced

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