Field-strength correlators for QCD in a magnetic background

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- M. D'Elia, E. Meggiolaro, M. Mesiti, F. Negro, Phys. Rev. D **93**, 054017 (2016)

Outline

We consider the properties of the gauge-invariant two-point correlation functions of the gauge-field strengths for QCD in the presence of a magnetic background field at zero temperature.

In particular:

- We discuss the general structure of the correlators in this case.
- We provide the results of an exploratory lattice study for $N_f = 2$ QCD discretized with unimproved staggered fermions.
- We provide evidence for the emergence of anisotropies in the nonperturbative part of the correlators and for an increase of the so-called *gluon condensate* as a function of the external magnetic field.

QCD in the presence of strong magnetic fields

The study of strong interactions in the presence of strong magnetic fields has attracted an increasing interest in the last few years (see, e.g., [Kharzeev *et al.*, Lect. Notes Phys. **871**, 2013]).

From a phenomenological point of view, the physics of some compact astrophysical objects, like magnetars, of noncentral heavy ion collisions and of the early Universe involve the properties of quarks and gluons in the presence of magnetic backgrounds going from 10^{10} Tesla up to 10^{16} Tesla ($|e|B \sim 1$ GeV^2).

From a purely theoretical point of view, one emerging feature is that gluon fields, even if not directly coupled to electromagnetic fields, can be significantly affected by them: effective QED-QCD interactions, induced by quark loop contributions, can be important, because of the nonperturbative nature of the theory . . .

Gauge-invariant two-point field-strength correlators

In the present study, we consider the gauge-invariant two-point field-strength correlators, defined as (see, e.g., [Di Giacomo, Dosch, Shevchenko & Simonov, Phys. Rep. **372**, 2002])

$$\mathcal{D}_{\mu\rho,\nu\sigma}(x) = g^2 \langle \mathrm{Tr}[G_{\mu\rho}(0) S(0,x) G_{\nu\sigma}(x) S^\dagger(0,x)] \rangle,$$

where $G_{\mu\rho} = T^a G^a_{\mu\rho}$ is the field-strength tensor and $S(0,x)$ is the parallel transport from 0 to x along a straight line, which is needed to make the correlators gauge invariant.

Such correlators were first considered to take into account the nonuniform distributions of the vacuum condensates.

Then, they have been widely used to parametrize the nonperturbative properties of the QCD vacuum, especially within the framework of the so-called *Stochastic Vacuum Model*.

The question that we approach here is:

How are these correlators modified by the background field?

Field correlators in the presence/absence of external fields

The most general parametrization for the correlators reads

$$\mathcal{D}_{\mu\rho,\nu\sigma} = \sum_n f_n T^{(n)}_{\mu\rho,\nu\sigma},$$

where: i) $T^{(n)}_{\nu\sigma,\mu\rho} = T^{(n)}_{\mu\rho,\nu\sigma}$, and ii) $T^{(n)}_{\rho\mu,\nu\sigma} = T^{(n)}_{\mu\rho,\sigma\nu} = -T^{(n)}_{\mu\rho,\nu\sigma}$.
A class of tensors satisfying such properties is written as

$$T^{(A,B)}_{\mu\rho,\nu\sigma} \equiv A_{\mu\nu} B_{\rho\sigma} - A_{\rho\nu} B_{\mu\sigma} - A_{\mu\sigma} B_{\rho\nu} + A_{\rho\sigma} B_{\mu\nu},$$

with: $A_{\nu\mu} = A_{\mu\nu}$, $B_{\nu\mu} = B_{\mu\nu}$; or: $A_{\nu\mu} = -A_{\mu\nu}$, $B_{\nu\mu} = -B_{\mu\nu}$.

In the absence of external background fields:

$$\mathcal{D}_{\mu\rho,\nu\sigma} = f_1 T^{(1)}_{\mu\rho,\nu\sigma} + f_2 T^{(2)}_{\mu\rho,\nu\sigma},$$

where

$$\begin{aligned} T^{(1)}_{\mu\rho,\nu\sigma} \equiv \quad & \tfrac{1}{2} T^{(\delta,\delta)}_{\mu\rho,\nu\sigma} &= \ & \delta_{\mu\nu}\delta_{\rho\sigma} - \delta_{\mu\sigma}\delta_{\rho\nu}, \\ T^{(2)}_{\mu\rho,\nu\sigma} \equiv \quad & T^{(xx,\delta)}_{\mu\rho,\nu\sigma} &= \ & x_\mu x_\nu \delta_{\rho\sigma} - x_\mu x_\sigma \delta_{\rho\nu} + x_\rho x_\sigma \delta_{\mu\nu} - x_\rho x_\nu \delta_{\mu\sigma}, \end{aligned}$$

and $f_1 \equiv \mathcal{D} + \mathcal{D}_1$ and $f_2 \equiv \frac{\partial \mathcal{D}_1}{\partial x^2}$ are two scalar functions of x^2.

Field correlators in a constant magnetic background

In the presence of an external background field $F_{\mu\nu}$, instead, many additional rank-2 tensors appear, like:
$F_{\mu\nu}$ itself, $H_{\mu\nu} \equiv h_\mu x_\nu - h_\nu x_\mu$ $(h_\mu \equiv F_{\mu\nu} x_\nu)$,
$F^{(2)}_{\mu\nu} \equiv F_{\mu\alpha} F_{\alpha\nu}$, $M_{\mu\nu} \equiv p_\mu x_\nu + p_\nu x_\mu$ $(p_\mu \equiv F^{(2)}_{\mu\nu} x_\nu = F_{\mu\alpha} h_\alpha)$ ...

Correspondingly, many more terms appear in the parametrization with new rank-4 tensors like:

$$\frac{1}{2} T^{(F,F)}_{\mu\rho,\nu\sigma}, \quad T^{(F,H)}_{\mu\rho,\nu\sigma}, \quad T^{(\delta,F^{(2)})}_{\mu\rho,\nu\sigma}, \quad T^{(xx,F^{(2)})}_{\mu\rho,\nu\sigma}, \quad T^{(\delta,hh)}_{\mu\rho,\nu\sigma}, \quad T^{(\delta,M)}_{\mu\rho,\nu\sigma} \quad \dots$$

Moreover, for a magnetic field directed along the z axis:
breaking of the $SO(4)$ symmetry $\Longrightarrow f_n = f_n(x^2 + y^2, z^2 + t^2)$.

All that makes a numerical analysis based on the most general parametrization of the correlator quite involved and not easily affordable ...

On the other hand, in our present investigation on the lattice, we shall consider only the 24 correlators of the kind

$$\mathcal{D}_{\mu\nu,\xi}(d) \equiv \mathcal{D}_{\mu\nu,\mu\nu}(x = d\hat{\xi}),$$

with x along one of the 4 lattice basis vectors ($\hat{\xi} = \hat{x},\ \hat{y},\ \hat{z},\ \hat{t}$).
In the absence of external background fields $\Longrightarrow$ $SO(4)$ symmetry $\Longrightarrow$ the 24 correlators are grouped into 2 equivalence classes, $\mathcal{D}_{\parallel}$ (when $\xi = \mu$ or $\xi = \nu$) and $\mathcal{D}_{\perp}$ (when $\xi \neq \mu$ and $\xi \neq \nu$):

$$\mathcal{D}_{\parallel} = \mathcal{D} + \mathcal{D}_1 + x^2 \frac{\partial \mathcal{D}_1}{\partial x^2}, \qquad \mathcal{D}_{\perp} = \mathcal{D} + \mathcal{D}_1.$$

In the presence of a constant and uniform magnetic field $\vec{B} = B\hat{z}$ (i.e., $F_{xy} \neq 0$):

$$SO(4) \to SO(2)_{xy} \otimes SO(2)_{zt}.$$

This residual symmetry implies two equivalence relations, $\hat{x} \sim \hat{y}$ (*transverse* directions) and $\hat{z} \sim \hat{t}$ ("*parallel*" directions) . . .

Class Name	Elements $(\mu\nu, \xi)$
$\mathcal{D}_{\parallel}^{tt,t}$	(12,1) , (12,2)
$\mathcal{D}_{\perp}^{tt,p}$	(12,3) , (12,4)
$\mathcal{D}_{\parallel}^{tp,t}$	(13,1) , (14,1) , (23,2) , (24,2)
$\mathcal{D}_{\parallel}^{tp,p}$	(13,3) , (14,4) , (23,3) , (24,4)
$\mathcal{D}_{\perp}^{tp,t}$	(13,2) , (14,2) , (23,1) , (24,1)
$\mathcal{D}_{\perp}^{tp,p}$	(13,4) , (14,3) , (23,4) , (24,3)
$\mathcal{D}_{\perp}^{pp,t}$	(34,1) , (34,2)
$\mathcal{D}_{\parallel}^{pp,p}$	(34,3) , (34,4)

Table : The 8 equivalence classes of linearly independent correlation functions in which the 24 components of the correlator $\mathcal{D}_{\mu\nu,\xi}(d) \equiv \mathcal{D}_{\mu\nu,\mu\nu}(x = d\hat{\xi})$ can be grouped.

The superscript t stands for the $\hat{x}, \hat{y}$ (*transverse* to $\vec{B}$) directions.

The superscript p stands for the $\hat{z}, \hat{t}$ (“*parallel*” to $\vec{B}$) directions.

Parametrization of the correlators vs. the distance d

In the absence of external field ($B = 0$), the correlators were directly determined by numerical simulations on the lattice [Di Giacomo, Panagopoulos, 1992; Di Giacomo, EM, Panagopoulos, 1997; D'Elia, Di Giacomo, EM, 1997 & 2003], using the following parametrization vs. the distance d:

$$\mathcal{D} = \frac{a_0}{d^4} + A_0 e^{-\mu d}, \qquad \mathcal{D}_1 = \frac{a_1}{d^4} + A_1 e^{-\mu d},$$

that is, in terms of $\mathcal{D}_{\parallel}$ and $\mathcal{D}_{\perp}$:

$$\mathcal{D}_{\parallel} = \left[A_0 + A_1\left(1 - \frac{1}{2}\mu d\right)\right] e^{-\mu d} + \frac{a_0 - a_1}{d^4}, \qquad \mathcal{D}_{\perp} = (A_0 + A_1)\, e^{-\mu d} + \frac{a_0 + a_1}{d^4}.$$

The terms $\sim 1/d^4$ are of perturbative origin and (according to the *Operator Product Expansion*) are necessary to describe the short distance behavior of the correlators.

The exponential terms represent the nonperturbative contributions: in particular, the coefficients A_0 and A_1 can be directly linked to the *gluon condensate* of the QCD vacuum (see below ...).

Inspired by this, we have used for the 8 functions $\mathcal{D}$ in the case $B \neq 0$ the following parametrization:

$$\mathcal{D}_{\parallel}^{tt,t} = \left[A_0^{tt} + A_1^{tt}\left(1 - \frac{1}{2}\mu^{tt,t}d\right)\right] e^{-\mu^{tt,t}d} + \frac{a_{\parallel}^{tt,t}}{d^4},$$

$$\mathcal{D}_{\perp}^{tt,p} = (A_0^{tt} + A_1^{tt})e^{-\mu^{tt,p}d} + \frac{a_{\perp}^{tt,p}}{d^4},$$

$$\mathcal{D}_{\parallel}^{tp,t} = \left[A_0^{tp} + A_1^{tp}\left(1 - \frac{1}{2}\mu^{tp,t}d\right)\right] e^{-\mu^{tp,t}d} + \frac{a_{\parallel}^{tp,t}}{d^4},$$

$$\mathcal{D}_{\parallel}^{tp,p} = \left[\tilde{A}_0^{tp} + \tilde{A}_1^{tp}\left(1 - \frac{1}{2}\mu^{tp,p}d\right)\right] e^{-\mu^{tp,p}d} + \frac{a_{\parallel}^{tp,p}}{d^4},$$

$$\mathcal{D}_{\perp}^{tp,t} = (A_0^{tp} + A_1^{tp})e^{-\mu^{tp,t}d} + \frac{a_{\perp}^{tp,t}}{d^4},$$

$$\mathcal{D}_{\perp}^{tp,p} = (\tilde{A}_0^{tp} + \tilde{A}_1^{tp})e^{-\mu^{tp,p}d} + \frac{a_{\perp}^{tp,p}}{d^4},$$

$$\mathcal{D}_{\perp}^{pp,t} = (A_0^{pp} + A_1^{pp})e^{-\mu^{pp,t}d} + \frac{a_{\perp}^{pp,t}}{d^4},$$

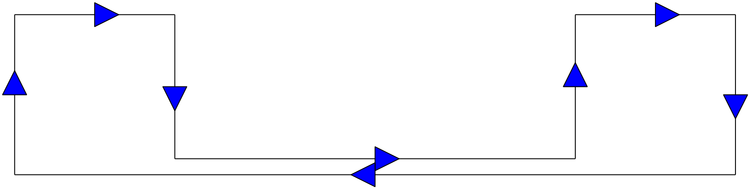
$$\mathcal{D}_{\parallel}^{pp,p} = \left[A_0^{pp} + A_1^{pp}\left(1 - \frac{1}{2}\mu^{pp,p}d\right)\right] e^{-\mu^{pp,p}d} + \frac{a_{\parallel}^{pp,p}}{d^4}.$$

Numerical investigation: technical details

The correlator has been discretized through the following lattice observable [Di Giacomo, EM, Panagopoulos, 1997]:

$$\mathcal{D}^L_{\mu\nu,\xi}(d) = \left\langle \mathrm{Tr}\left[\Omega^\dagger_{\mu\nu}(x) S(x, x+d\hat{\xi}) \Omega_{\mu\nu}(x+d\hat{\xi}) S^\dagger(x, x+d\hat{\xi})\right]\right\rangle ,$$

where $\Omega_{\mu\nu}(x)$ stands for the traceless anti-Hermitian part of the corresponding plaquette: $\Omega_{\mu\nu} \equiv \frac{1}{2}(\Pi_{\mu\nu} - \Pi^\dagger_{\mu\nu}) - \frac{1}{6}\mathrm{Tr}[\Pi_{\mu\nu} - \Pi^\dagger_{\mu\nu}]\mathbf{I}$.



Of course $\mathcal{D}^L_{\mu\nu,\xi}(d) \to a^4 \mathcal{D}_{\mu\nu,\xi}(d)$ when the lattice spacing $a \to 0$.

We have considered $N_f = 2$ QCD discretized via unimproved rooted staggered fermions and the standard plaquette action for the pure-gauge sector.
The background magnetic field $\vec{B} = B\hat{z}$ couples to the quark electric charges ($q_u = 2|e|/3$ and $q_d = -|e|/3$) and its introduction corresponds to additional $U(1)$ phases entering the elementary parallel transports in the discretized lattice version.
Periodicity constraints impose to quantize B as follows:

$$|e|B = 6\pi b/(a^2 L_x L_y)\,, \qquad b \in \mathbb{Z}\,.$$

In order to remove ultraviolet fluctuations, following previous studies of the gauge-field correlators, a *cooling* technique has been used which, acting as a diffusion process, smooths out short-distance fluctuations without touching physics at larger distances: for a correlator at a given distance d, this shows up as an approximate *plateau* in the dependence of the correlator on the number of cooling steps, whose location defines its value . . .

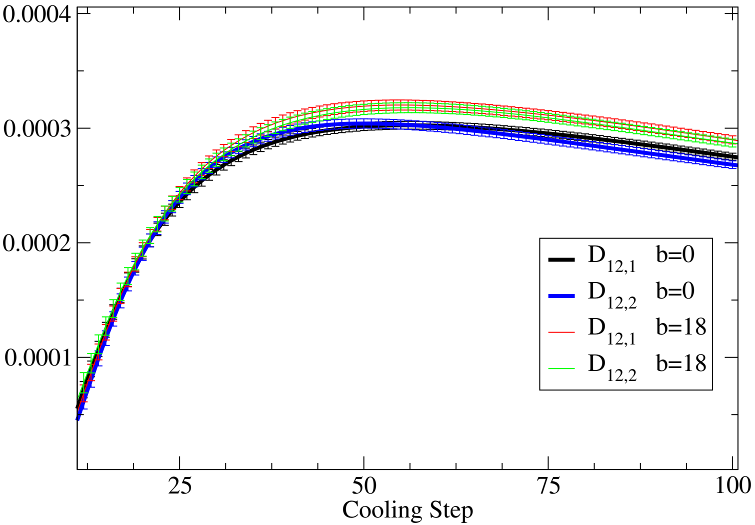


Figure : Effect of cooling on $\mathcal{D}_{12,1}$ and $\mathcal{D}_{12,2}$ (evaluated for $d/a = 8$).

Numerical investigation: results and analysis

- Numerical simulations have been performed on a 24^4 lattice by means of the *Rational Hybrid Monte Carlo* (RHMC) algorithm [Gottlieb *et al.*, 1987; Kennedy *et al.*, 1999] implemented on GPU cards, with statistics of $O(10^3)$ *molecular-dynamics time units* for each b ($0 < b < 27$).
- The bare parameters have been set to $\beta = 5.55$ and $am = 0.0125$, corresponding to a lattice spacing $a \simeq 0.125$ fm and to a pseudo-Goldstone pion mass $m_\pi \simeq 480$ MeV.
- The correlators have been measured on about 100 configurations for each explored value of $|e|B$, chosen once every 20 molecular-dynamics trajectories.

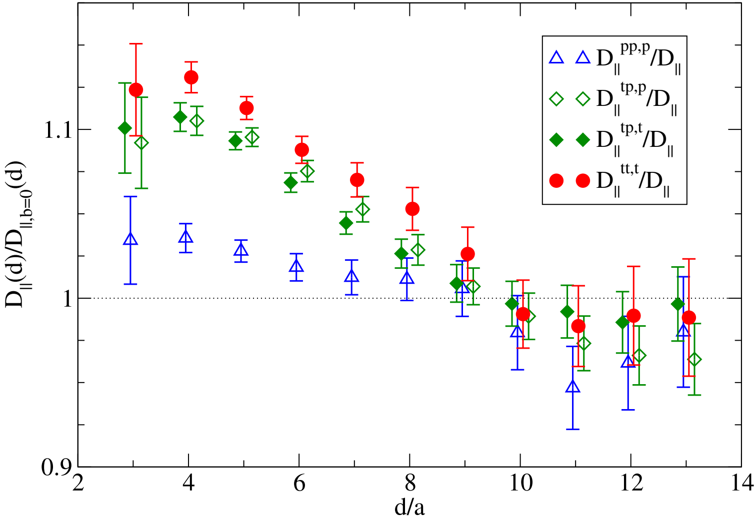


Figure : Effect of the magnetic field ($|e|B = 1.46\ \mathrm{GeV}^2$) on $\mathcal{D}_{\parallel}^{\mathrm{class}}(d)$.

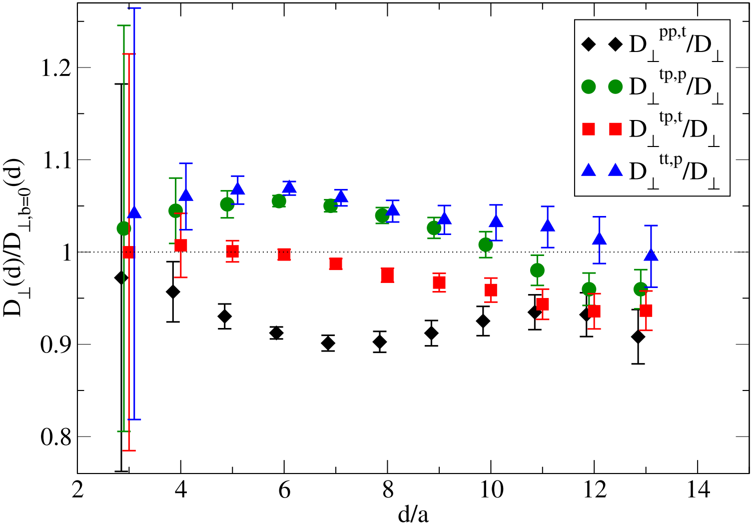


Figure : Effect of the magnetic field ($|e|B = 1.46\ \mathrm{GeV}^2$) on $\mathcal{D}_\perp^{\rm class}(d)$.

For each value of $|e|B$, we have fitted the correlators with the above-written parametrization, including distances in the range $3 \leqslant d/a \leqslant 8$, thus obtaining an estimate for all parameters.
From this first step, it has emerged that the 8 parameters pertaining to the perturbative part of the correlation functions satisfy, within errors, the following equalities:

$$a_{\parallel}^{tt,t} \simeq a_{\parallel}^{tp,t} \simeq a_{\parallel}^{tp,p} \simeq a_{\parallel}^{pp,p} \equiv a_{\parallel} \ ,$$
$$a_{\perp}^{tt,p} \simeq a_{\perp}^{tp,t} \simeq a_{\perp}^{tp,p} \simeq a_{\perp}^{pp,t} \equiv a_{\perp} \ ,$$

and, moreover, their dependence on $|e|B$ is negligible.
For example, for $|e|B = 1.46\ \mathrm{GeV}^2$ (and $B = 0$) one finds:

$a_{\parallel}(B=0)$	$a_{\parallel}^{tt,t}$	$a_{\parallel}^{tp,t}$	$a_{\parallel}^{tp,p}$	$a_{\parallel}^{pp,p}$
0.266(16)	0.279(14)	0.277(9)	0.272(9)	0.275(14)
$a_{\perp}(B=0)$	$a_{\perp}^{tt,p}$	$a_{\perp}^{tp,t}$	$a_{\perp}^{tp,p}$	$a_{\perp}^{pp,t}$
0.929(16)	0.94(3)	0.873(20)	0.913(23)	0.88(3)

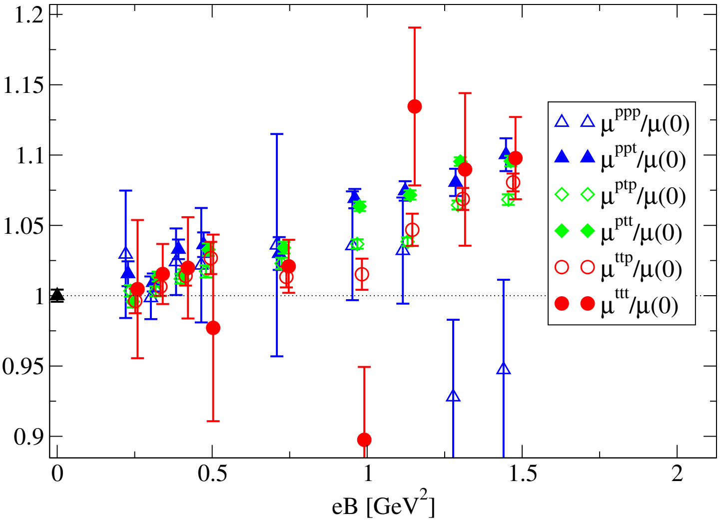


Figure : The ratio $\mu^{\text{class}}(|e|B)/\mu(0)$ vs. $|e|B$ ($\mu(0) = 0.721(3)$ GeV).

Gluon condensate vs. the magnetic field

The *gluon condensate* is defined as [Shifman, Vainshtein & Zakharov (SVZ), 1979]

$$G_2 = \frac{g^2}{4\pi^2} \sum_{\mu\nu,a} \langle G^a_{\mu\nu} G^a_{\mu\nu} \rangle \ .$$

In the case of $B \neq 0$ we can distinguish three contributions, coming from different sets of plaquettes in the sum:

$$G_2 = G_2^{tt} + G_2^{tp} + G_2^{pp} \ .$$

One can extract G_2 from the small-distance limit of the NP part of the correlator, obtaining, using our parametrization:

$$G_2 = \frac{1}{\pi^2} \left[A_0^{tt} + A_1^{tt} + 4 \left(A_0^{tp} + A_1^{tp} \right) + A_0^{pp} + A_1^{pp} \right].$$

We have found that G_2, as a function of $|e|B$, increases as $G_2(|e|B)/G_2(0) \simeq 1 + K(|e|B)^2$, with $K = 0.164(7)\ \mathrm{GeV}^{-4}$.

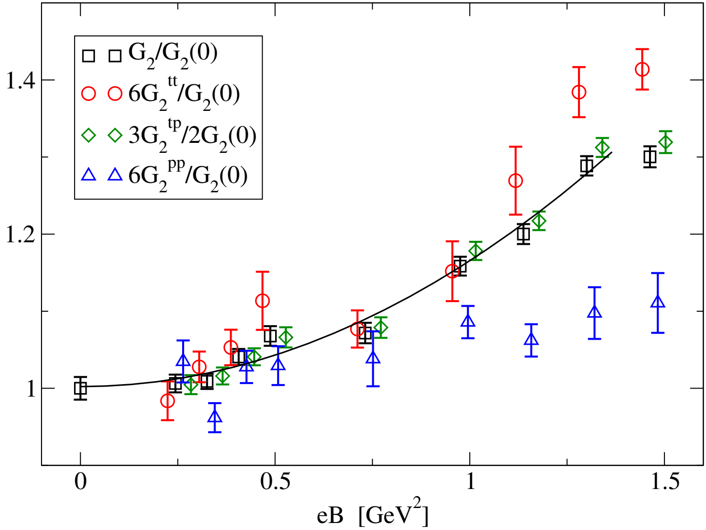


Figure : Effect of the magnetic field on the *gluon condensate G_2*.

Conclusions

We have found evidence of a significant effect of the magnetic field on the nonperturbative part of the field-strength correlators.
In particular, we have observed that the *gluon condensate* itself increases as a function of B, with the increase being of the order of 20% for $|e|B \sim 1$ GeV^2. (A similar behaviour for G_2 has been also predicted making use of QCD sum rules [Ayala *et al.*, 2015] and has been also found in [Ozaki, 2014].)
Relative differences between the different contributions are of the same order of magnitude, meaning that anisotropies induced by B are significant and comparable to those observed in other pure-gauge quantities.
The increase of the *gluon condensate* provides evidence of the phenomenon known as *gluon catalysis*, which had been previously observed based on the magnetic-field effects on plaquette expectation values [Ilgenfritz *et al.*, 2012 & 2014; Bali *et al.*, 2013].

Perspectives

- To repeat the present exploratory study by adopting a discretization of QCD at the *physical point*, i.e., with quark masses tuned at their phenomenological values ...
- To study, within the *Stochastic Vacuum Model*, which takes the correlators as an input, the effect of the magnetic background field on the static quark-antiquark potential, i.e., on the *string tension*, in order to obtain another confirmation of the anisotropy which has been already observed by direct lattice measurements [Bonati *et al.*, 2014 & 2016] ...
- To adopt, as a regulator for the measure of the correlators, a different prescription for fixing the amount of *cooling* or a different smoothing procedure, such as the so-called *gradient flow* [Lüscher, 2010 & 2014] ...