Unitarity Triangle analysis beyond the Standard Model from UTfit

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Unitarity Triangle analysis beyond the SM

SM UT analysis:
- provide the best determination of CKM parameters
- test the consistency of the SM ("direct" vs "indirect" determinations)
- provide predictions (from data...) for SM observables

New Physics UT analysis:
- model-independent analysis
- provides limits on the allowed deviation from the SM
- translate into a NP scale in the different sectors
Other UT analyses exist, by:
CKMfitter (http://ckmfitter.in2p3.fr/),
Laiho&Lunghi&Van de Water (http://latticeaverages.org/)
Lunghi&Soni (1010.6069)
the method and the inputs:

\[ f(\bar{\rho}, \bar{\eta}, X | c_1, \ldots, c_m) \sim \prod_{j=1,m} f_j(C | \bar{\rho}, \bar{\eta}, X)^* \prod_{i=1,N} f_i(x_i) f_0(\bar{\rho}, \bar{\eta}) \]

**Bayes Theorem**

**X** \( \equiv x_1, \ldots, x_n = m_t, B_K, F_B, \ldots \)

**C** \( \equiv c_1, \ldots, c_m = \epsilon, \Delta m_d/\Delta m_s, A_{CP}(J/\psi K_S), \ldots \)

\[ (b \to u)/(b \to c) \]
\[ \begin{array}{|c|}
\hline 
\epsilon_K \\
\hline 
\Delta m_d \\
\hline 
\Delta m_d/\Delta m_s \\
\hline 
A_{CP}(J/\psi K_S) \\
\hline 
\end{array} \]

\[ \begin{array}{|c|}
\hline 
\rho^2 + \bar{\eta}^2 \\
\hline 
\bar{\eta}[(1 - \rho) + P] \\
\hline 
(1 - \rho)^2 + \bar{\eta}^2 \\
\hline 
\sin 2\beta \\
\hline 
\end{array} \]

\[ \begin{array}{|c|}
\hline 
\bar{\Lambda}, \lambda_1, F(1), \ldots \\
\hline 
B_K \\
\hline 
f_B^2 B_B \\
\hline 
\xi \\
\hline 
\end{array} \]

Standard Model + OPE/HQET/Lattice QCD to go from quarks to hadrons

M. Bona et al. (UTfit Collaboration) JHEP 0507:028,2005 hep-ph/0501199

M. Bona et al. (UTfit Collaboration) JHEP 0603:080,2006 hep-ph/0509219
$V_{cb}$ and $V_{ub}$

2D average inspired by D'Agostini skeptical procedure (hep-ex/9910036) with $\sigma=1$. Very similar results obtained from a 2D a la PDG procedure.

$V_{cb} = (41.7 \pm 1.0) \times 10^{-3}$
uncertainty $\sim 2.4\%$

$V_{ub} = (3.74 \pm 0.21) \times 10^{-3}$
uncertainty $\sim 5.6\%$

$V_{cb} = (42.6 \pm 0.7) \times 10^{-3}$

$V_{ub} = (3.66 \pm 0.13) \times 10^{-3}$
Unitarity Triangle analysis:

- $|V_{cb}/V_{ub}|$
  - $\bar{\rho}^2 + \bar{\eta}^2$
- $\epsilon_K$
  - $\bar{\eta}[(1 - \bar{\rho}) + P]$
- $\Delta m_d$
  - $(1 - \bar{\rho})^2 + \bar{\eta}^2$
- $\Delta m_s/\Delta m_d$
- $\beta$
- $\alpha$
- $\gamma$
- $2\beta + \gamma$
- $B \rightarrow \tau \nu$
Unitarity Triangle analysis in the SM:

$$\bar{\rho} = 0.154 \pm 0.015$$
$$\bar{\eta} = 0.344 \pm 0.013$$
Unitarity Triangle analysis in the SM:

<table>
<thead>
<tr>
<th>Observables</th>
<th>Measurement</th>
<th>Prediction</th>
<th>Pull (#$\sigma$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sin 2\beta$</td>
<td>$0.680 \pm 0.023$</td>
<td>$0.725 \pm 0.030$</td>
<td>~ 1.2</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>$70.5 \pm 5.7$</td>
<td>$65.4 \pm 2.1$</td>
<td>&lt; 1</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>$94.2 \pm 4.5$</td>
<td>$90.9 \pm 2.5$</td>
<td>&lt; 1</td>
</tr>
<tr>
<td>$</td>
<td>V_{ub}</td>
<td>\cdot 10^3$</td>
<td>$3.74 \pm 0.21$</td>
</tr>
<tr>
<td>$</td>
<td>V_{ub}</td>
<td>\cdot 10^3$ (incl)</td>
<td>$4.41 \pm 0.22$</td>
</tr>
<tr>
<td>$</td>
<td>V_{ub}</td>
<td>\cdot 10^3$ (excl)</td>
<td>$3.62 \pm 0.14$</td>
</tr>
<tr>
<td>$</td>
<td>V_{cb}</td>
<td>\cdot 10^3$</td>
<td>$41.7 \pm 1.0$</td>
</tr>
<tr>
<td>$\beta_s$</td>
<td>$0.97 \pm 0.94$</td>
<td>$1.05 \pm 0.04$</td>
<td>&lt; 1</td>
</tr>
<tr>
<td>BR($B \to \tau\nu)$[$10^{-4}$]</td>
<td>$1.06 \pm 0.20$</td>
<td>$0.81 \pm 0.06$</td>
<td>~ 1.2</td>
</tr>
<tr>
<td>$A_{SL}^d \cdot 10^3$</td>
<td>$0.2 \pm 2.0$</td>
<td>$-0.283 \pm 0.024$</td>
<td>&lt; 1</td>
</tr>
<tr>
<td>$A_{SL}^s \cdot 10^3$</td>
<td>$1.7 \pm 3.0$</td>
<td>$0.013 \pm 0.001$</td>
<td>&lt; 1</td>
</tr>
</tbody>
</table>
fit simultaneously for the CKM and the NP parameters (generalized UT fit)

- add most general loop NP to all sectors
- use all available experimental info
- find out NP contributions to $\Delta F=2$ transitions

**$B_d$ and $B_s$ mixing amplitudes**

(2+2 real parameters):

$$A_q = C_{B_q} e^{2i \phi_{B_q}} A_q^{SM} e^{2i \phi_{SM}} = \left(1 + \frac{A_q^{NP}}{A_q^{SM}} e^{2i(\phi_{NP} - \phi_{SM})} \right) A_q^{SM} e^{2i \phi_{SM}}$$

$$\Delta m_{q/K} = C_{B_q/\Delta m_K} (\Delta m_{q/K})^{SM}$$

$$A_{CP}^{B_d \to J/\psi K_s} = \sin 2(\beta + \phi_{B_d})$$

$$A_{SL}^q = \text{Im} \left( \frac{\Gamma_{12}^q}{A_q} \right)$$

$$\Delta \Gamma^q/\Delta m_q = \text{Re} \left( \frac{\Gamma_{12}^q}{A_q} \right)$$

$$\varepsilon_K = C_{\varepsilon_{SM}}$$

$$A_{CP}^{B_s \to J/\psi \phi} \sim \sin 2(-\beta_s + \phi_{B_s})$$

$$\text{UT analysis including new physics}$$
NP analysis results

$\rho = 0.150 \pm 0.027$
$\eta = 0.363 \pm 0.025$

$\rho = 0.154 \pm 0.015$
$\eta = 0.344 \pm 0.013$
NP parameter results

dark: 68%
light: 95%
SM: red cross

\[ A_q = C_{Bq} e^{2i\phi_{Bq}} A_q^{SM} e^{2i\phi_{q}^{SM}} \]

- \( C_{Bq} = 1.05 \pm 0.11 \)
- \( \phi_{B_q} = (1.8 \pm 1.7)^\circ \)
- \( C_{B_s} = 1.07 \pm 0.09 \)
- \( \phi_{B_s} = (0.1 \pm 1.0)^\circ \)
The ratio of NP/SM amplitudes is:

< 15% @68% prob. (30% @95%) in $B_d$ mixing
< 15% @68% prob. (25% @95%) in $B_s$ mixing

see also Lunghi & Soni, Buras et al., Ligeti et al.
testing the new-physics scale

At the high scale
new physics enters according to its specific features

At the low scale
use OPE to write the most general effective Hamiltonian.
the operators have different chiralities than the SM
NP effects are in the Wilson Coefficients C

NP effects are enhanced
- up to a factor 10 by the values of the matrix elements especially for transitions among quarks of different chiralities
- up to a factor 8 by RGE

\[ H_{\Delta B=2} = \sum_{i=1}^{5} C_i Q_i^{bq} + \sum_{i=1}^{3} \tilde{C}_i \tilde{Q}_i^{bq} \]

\begin{align*}
Q_1^{q_i q_j} &= \bar{q}_{iL}^\alpha \gamma_\mu q_{iL}^\alpha \bar{q}_{iL}^\beta \gamma^\mu q_{iL}^\beta , \\
Q_2^{q_i q_j} &= \bar{q}_{iL}^\alpha q_{iL}^{\alpha} \bar{q}_{iR}^{\beta} q_{iL}^{B} , \\
Q_3^{q_i q_j} &= \bar{q}_{iR}^\alpha q_{iL}^{\beta} \bar{q}_{iR}^{\beta} q_{iL}^{B} , \\
Q_4^{q_i q_j} &= \bar{q}_{iR}^\alpha q_{iL}^{\alpha} \bar{q}_{iL}^{\beta} q_{iR}^{B} , \\
Q_5^{q_i q_j} &= \bar{q}_{iR}^\alpha \bar{q}_{iL}^{\beta} \bar{q}_{iL}^{\beta} q_{iR}^{B} .
\end{align*}
The Wilson coefficients $C_i$ have in general the form

$$C_i(\Lambda) = \frac{F_i}{\Lambda^2} L_i$$

- $F_i$: function of the NP flavour couplings
- $L_i$: loop factor (in NP models with no tree-level FCNC)
- $\Lambda$: NP scale (typical mass of new particles mediating $\Delta F=2$ transitions)

Putting bounds on the Wilson coefficients give insights into the NP scale in different NP scenarios that enter through $F_i$ and $L_i$. Effective BSM Hamiltonian for $\Delta F=2$ transitions
The dependence of $C$ on $\Lambda$ changes depending on the flavour structure. We can consider different flavour scenarios:

- **Generic**: $C(\Lambda) = \alpha/\Lambda^2$ for $F_1 \sim 1$, arbitrary phase
- **NMFV**: $C(\Lambda) = \alpha \times |F_{SM}|/\Lambda^2$ for $F_i \sim |F_{SM}|$, arbitrary phase
- **MFV**: $C(\Lambda) = \alpha \times |F_{SM}|/\Lambda^2$ for $F_1 \sim |F_{SM}|$, $F_{\neq 1} \sim 0$, SM phase

$\alpha (L_i)$ is the coupling among NP and SM

- $\alpha \sim 1$ for strongly coupled NP
- $\alpha \sim \alpha_W (\alpha_S)$ in case of loop coupling through weak (strong) interactions

If no NP effect is seen, lower bound on NP scale $\Lambda$ if NP is seen, upper bound on NP scale $\Lambda$
results from the Wilson coefficients

**Generic:** $C(\Lambda) = \alpha/\Lambda^2$, $F_i \sim 1$, arbitrary phase

$\alpha \sim 1$ for strongly coupled NP

To obtain the lower bound for loop-mediated contributions, one simply multiplies the bounds by $\alpha_s (\sim 0.1)$ or by $\alpha_W (\sim 0.03)$.

$\alpha \sim \alpha_W$ in case of loop coupling through *weak* interactions

NP in $\alpha_W$ loops

$\Lambda > 1.5 \times 10^4$ TeV

Best bound from $\epsilon_K$

dominated by CKM error

CPV in charm mixing follows,

exp error dominant

Best CP conserving from $\Delta m_K$,

dominated by long distance

$B_d$ and $B_s$ behind,

errors from both CKM

and B-parameters
Non-perturbative NP

\[ \Lambda > 114 \text{ TeV} \]

results from the Wilson coefficients

\[ C(\Lambda) = \alpha \times |F_{\text{SM}}|/\Lambda^2, \quad F_i \sim |F_{\text{SM}}|, \text{ arbitrary phase} \]

\[ \alpha \sim 1 \] for strongly coupled NP

To obtain the lower bound for loop-mediated contributions, one simply multiplies the bounds by \( \alpha_s (\sim 0.1) \) or by \( \alpha_W (\sim 0.03) \).

\[ \alpha \sim \alpha_W \] in case of loop coupling through weak interactions

NP in \( \alpha_W \) loops

\[ \Lambda > 3.4 \text{ TeV} \]

If new chiral structures present, \( \varepsilon_K \) still leading

\( B(s) \) mixing provides very stringent constraints, especially if no new chiral structures are present

Constraining power of the various sectors depends on unknown NP flavour structure.
Look at the future

In the next decade, Belle-II and LHCb upgrade will push down the exp. error on sin 2\beta(s) to less than 0.01

Theory error can be kept below 0.01 using control channels as S(B → J/ψπ)

B-parameters will go below the % level, new ideas to attack long-distance in K and D

Improving γ, α and |V_{cb}| & |V_{ub}| crucial!

[Silvestrini@Pisa]
Look at the future

errors from tree-only fit on $\rho$ and $\eta$:
- $\sigma(\rho) = 0.008 \text{ [currently 0.050]}$
- $\sigma(\eta) = 0.010 \text{ [currently 0.035]}$

errors from 5-constraint fit on $\rho$ and $\eta$:
- $\sigma(\rho) = 0.005 \text{ [currently 0.015]}$
- $\sigma(\eta) = 0.004 \text{ [currently 0.013]}$
Look at the future

errors predicted from
Belle II + LHCb upgrade

well-tuned
central values

not-so-well-tuned
central values

errors on general NP parameters:

\[ \sigma(C_{Bd}) = 0.03 \text{ [currently 0.12]} \]
\[ \sigma(\phi_{Bd}) = 0.7 \text{ [currently 1.7]} \]

\[ \sigma(C_{Bs}) = 0.03 \text{ [currently 0.09]} \]
\[ \sigma(\phi_{Bs}) = 0.3 \text{ [currently 1.0]} \]
SM analysis displays very good overall consistency

Still open discussion on semileptonic inclusive vs exclusive

UTA provides determination of NP contributions to $\Delta F=2$ amplitudes. It currently leaves space for NP at the level of 25-30%

So the scale analysis points to high scales for the generic scenario and at the limit of LHC reach for weak coupling. Indirect searches are complementary to direct searches.

Even if we don't see relevant deviations in the down sector, we might still find them in the up sector.
Back up slides
new-physics-specific constraints

B meson mixing matrix element NLO calculation

\[
\frac{\Gamma^q_{12}}{A^\text{full}_q} = -2 \kappa \left\{ \frac{2\phi_{Bq}}{C_{Bq}} \left( n_1 + \frac{n_6 B_2 + n_{11}}{B_1} \right) - \frac{e^{(\phi_q^{\text{SM}} + 2\phi_{Bq})}}{R_t^q} \left( n_2 + \frac{n_7 B_2 + n_{12}}{B_1} \right) \right. \\
+ \left. \frac{e^{2(\phi_q^{\text{SM}} + \phi_{Bq})}}{R_t^{q^2}} \left( n_3 + \frac{n_8 B_2 + n_{13}}{B_1} \right) + e^{(\phi_{\text{Pen}} + 2\phi_{Bq})} C_{\text{Pen}} \left( n_4 + \frac{n_9 B_2}{B_1} \right) \right\}
\]

ϕ_s = 2β_s vs ΔΓ_s from B_s → J/ψφ
angular analysis as a function of proper time
and b-tagging
additional sensitivity from the ΔΓ_s terms

C_{\text{Pen}} and φ_{\text{pen}} are
parameterize possible NP contributions from
b → s penguins

ϕ_s and ΔΓ_s:
from HFAG
**contribution to the mixing amplitudes**

Analytic expression for the contribution to the mixing amplitudes

\[
\langle \bar{B}_q | \mathcal{H}_{\text{eff}}^{\Delta B=2} | B_q \rangle_i = \sum_{j=1}^{5} \sum_{r=1}^{5} \left( b_{j}^{(r,i)} + \eta c_{j}^{(r,i)} \right) \eta^{a_j} C_i(\Lambda) \langle \bar{B}_q | Q_r^{b_q} | B_q \rangle
\]

*arXiv:0707.0636: for ”magic numbers” a, b and c, \( \eta = \alpha_s(\Lambda)/\alpha_s(m_t) \)*

(numerical values updated last in summer’12)

analogously for the K system

\[
\langle \bar{K}^0 | \mathcal{H}_{\text{eff}}^{\Delta S=2} | K^0 \rangle_i = \sum_{j=1}^{5} \sum_{r=1}^{5} \left( b_{j}^{(r,i)} + \eta c_{j}^{(r,i)} \right) \eta^{a_j} C_i(\Lambda) R_r \langle \bar{K}^0 | Q_1^{s_d} | K^0 \rangle
\]

To obtain the p.d.f. for the Wilson coefficients \( C_i(\Lambda) \) at the new-physics scale, we switch on one coefficient at a time in each sector and calculate its value from the result of the NP analysis.
the results obtained for the flavour scenarios:
In deriving the lower bounds on the NP scale, we assume $L_i = 1$, corresponding to strongly-interacting and/or tree-level NP.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>95% allowed range</th>
<th>Lower limit on $\Lambda$ (TeV) for arbitrary NP</th>
<th>Lower limit on $\Lambda$ (TeV) for NMFV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Re$C^1_K$</td>
<td>$[-9.6, 9.6] \cdot 10^{-13}$</td>
<td>$1.0 \cdot 10^{3}$</td>
<td>0.35</td>
</tr>
<tr>
<td>Re$C^2_K$</td>
<td>$[-1.8, 1.9] \cdot 10^{-14}$</td>
<td>$7.3 \cdot 10^{3}$</td>
<td>2.0</td>
</tr>
<tr>
<td>Re$C^3_K$</td>
<td>$[-6.0, 5.6] \cdot 10^{-14}$</td>
<td>$4.1 \cdot 10^{3}$</td>
<td>1.1</td>
</tr>
<tr>
<td>Re$C^4_K$</td>
<td>$[-3.6, 3.6] \cdot 10^{-15}$</td>
<td>$17 \cdot 10^{5}$</td>
<td>4.0</td>
</tr>
<tr>
<td>Re$C^5_K$</td>
<td>$[-1.0, 1.0] \cdot 10^{-14}$</td>
<td>$10 \cdot 10^{3}$</td>
<td>2.4</td>
</tr>
<tr>
<td>Im$C^1_K$</td>
<td>$[-4.4, 2.8] \cdot 10^{-15}$</td>
<td>$1.5 \cdot 10^{4}$</td>
<td>5.6</td>
</tr>
<tr>
<td>Im$C^2_K$</td>
<td>$[-5.1, 9.3] \cdot 10^{-17}$</td>
<td>$10 \cdot 10^{4}$</td>
<td>28</td>
</tr>
<tr>
<td>Im$C^3_K$</td>
<td>$[-3.1, 1.7] \cdot 10^{-16}$</td>
<td>$5.7 \cdot 10^{4}$</td>
<td>19</td>
</tr>
<tr>
<td>Im$C^4_K$</td>
<td>$[-1.8, 0.9] \cdot 10^{-17}$</td>
<td>$24 \cdot 10^{4}$</td>
<td>62</td>
</tr>
<tr>
<td>Im$C^5_K$</td>
<td>$[-5.2, 2.8] \cdot 10^{-17}$</td>
<td>$14 \cdot 10^{4}$</td>
<td>37</td>
</tr>
</tbody>
</table>

To obtain the lower bound for loop-mediated contributions, one simply multiplies the bounds by $\alpha_s \sim 0.1$ or by $\alpha_w \sim 0.03$.  

![Graph showing distribution of parameters](image.png)
$\sin 2\alpha \ (\phi_2)$ from charmless B decays: $\pi\pi$, $\rho\rho$, $\pi\rho$

$\pi^0\pi^0$ from Belle at CKM14 to be updated soon (?)

$$\text{BR}(\pi^0\pi^0) = (1.17 \pm 0.13) \times 10^{-6}$$

HFAG 2014

$a \ à \ la \ PDG$ average would give an inflated uncertainty of 0.41

$\rho^+\rho^-$ average updated including Belle arXiv:1510.01245

$\rho^0\rho^0$ average updated including LHCb arXiv:1503.07770

$\alpha$ from $\pi\pi$, $\rho\rho$, $\pi\rho$ decays:

- combined: $(94.2 \pm 4.5)^\circ$
- UTfit prediction: $(90.9 \pm 2.5)^\circ$
After a decade of analyses and almost 50 papers published, the world average uncertainty has decreased by a factor 3 combined: $(70.5 \pm 5.7)^\circ$

UTfit prediction: $(65.4 \pm 2.1)^\circ$
tensions? not really.. still that $V_{ub}$ inclusive

$V_{ub} (excl) = (3.62 \pm 0.14) \times 10^{-3}$

$V_{ub} (incl) = (4.41 \pm 0.22) \times 10^{-3}$

$V_{ubexp} = (3.74 \pm 0.21) \times 10^{-3}$

$V_{ubUTfit} = (3.66 \pm 0.11) \times 10^{-3}$

$\sin2\beta_{exp} = 0.680 \pm 0.023$

$\sin2\beta_{UTfit} = 0.725 \pm 0.030$