

# Lattice calculation of the HVP contribution to the anomalous magnetic moment of muon

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&  
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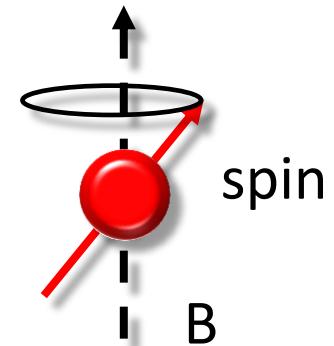
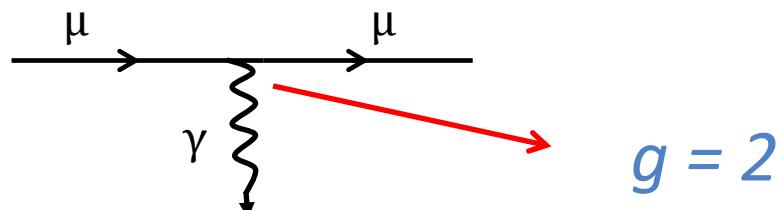
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# Relevance and Timeliness

$$\vec{\mu} = g \frac{e}{2m} \vec{S}$$

$$a_\mu = \frac{g - 2}{2}$$



$$a_\mu^{exp} = 11\ 659\ 208.9\ (63) \times 10^{-10} \text{ (0.54 ppm) (BNL E821)}$$

$$a_\mu^{SM} = 11\ 659\ 182.8\ (49) \times 10^{-10} \text{ (0.42 ppm)}$$

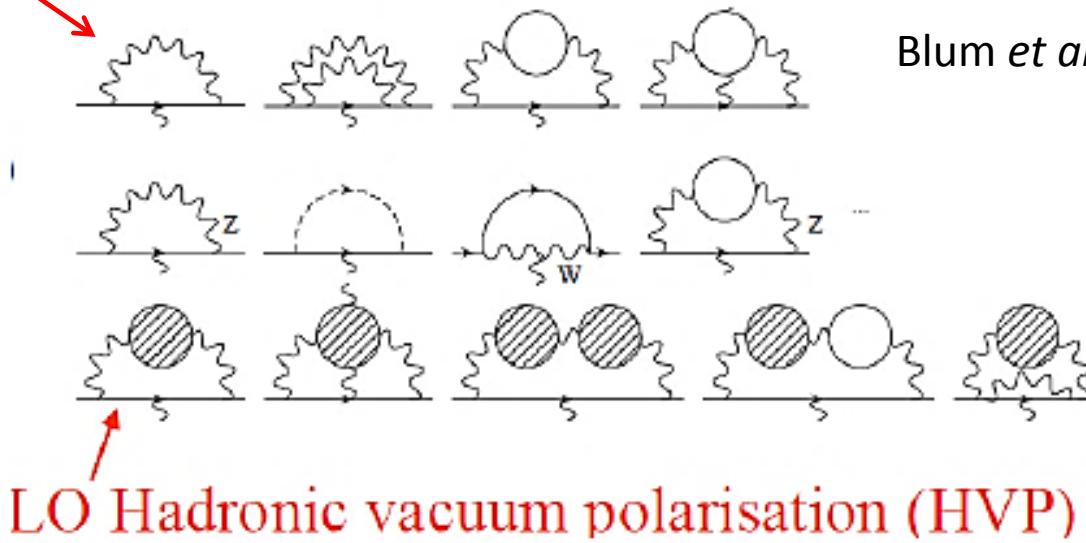
$$a_\mu^{exp} - a_\mu^{SM} = 26.1(8) \times 10^{-10} \quad 3+\sigma !$$

Expt. at FNAL E989 & J-PARC will reduce uncertainty to **0.16 ppm** starting 2017



# Current theoretical status of muon g-2

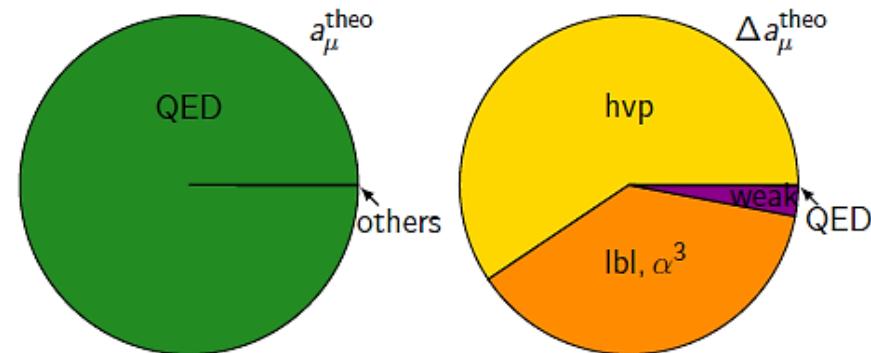
$$\frac{\alpha_{QED}}{2\pi} = 0.00116$$



Blum *et al.* 1301.2607

QED, EW & QCD  
Contributes;  
QED dominates

Contribution	Result ( $\times 10^{-10}$ )	Error
QED (leptons)	11658471.8	0.00 ppm
HVP(lo) [1]	692.3	0.36 ppm
HVP(ho)	-9.8	0.01 ppm
HLbL [2]	10.5	0.22 ppm
EW	15.4	0.02 ppm
Total SM	11659180.2	0.42 ppm



[1] Davier *et al.* Eur. Phys. J. C71 (2011) 1515

[2] Prades, de Rafael, Vainshtein, 0901.0306

# Hadronic contributions

$$\begin{aligned} a_{\mu}^{expt} - a_{\mu}^{QED} - a_{\mu}^{EW} &= 721.7(6.3) \times 10^{-10} \\ &= a_{\mu}^{HVP,LO} + a_{\mu}^{HVP,HO} + a_{\mu}^{HLBL} + a_{\mu}^{new\ physics} \end{aligned}$$

Focus on lowest order hadronic vacuum polarisation, so assume:

$$a_{\mu}^{HLBL} = 10.5(2.6) \times 10^{-10}$$

$$a_{\mu}^{HVP,HO} = -8.85(9) \times 10^{-10}$$

NLO+NNLO  
Kurz *et al.*, 1403.6400

$$a_{\mu}^{HVP,no\ new\ physics} = 719.8(6.8) \times 10^{-10}$$

# HVP from lattice QCD

Target : better than 1% in HVP

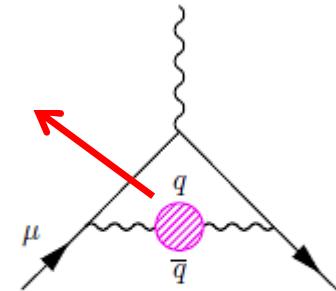
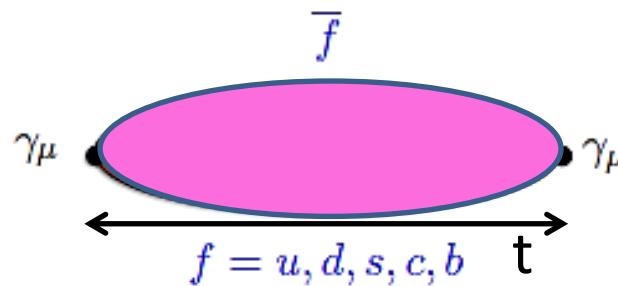
In Euclidean space,

connected contribution for flavour  $i$

$$a_{\mu}^{HVP,i} = \frac{\alpha}{\pi} \int_0^{\infty} dq^2 f(q^2) (4\pi\alpha e_i^2) \hat{\Pi}_i(q^2)$$

On lattice

$f(q^2)$  known, divergent function of  $q^2$

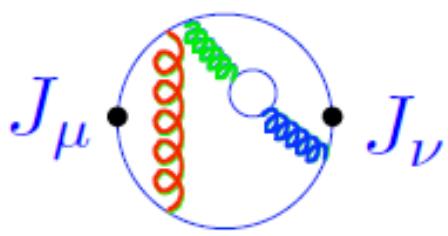


Blum, hep-lat/  
0212018

Renormalised Vacuum Polarisation function:  $\hat{\Pi}(q^2) = \Pi(q^2) - \Pi(0)$

Integrand strongly peaked at  $q^2 \sim \frac{m_\mu^2}{4} \sim 0.003 \text{ GeV}^2$

# HPQCD method for HVP calculation



Calculate correlation function on lattice  
 $\langle j^j(\vec{x}, t) j^j(0) \rangle$

Time moments of lattice current-current correlators

$$\begin{aligned} G_{2n} &\equiv a^4 \sum_t \sum_{\vec{x}} t^{2n} Z_V^2 \langle j^j(\vec{x}, t) j^j(0) \rangle \\ &= (-1)^n \frac{\partial^{2n}}{\partial q^{2n}} q^2 \widehat{\Pi}(q^2) \Big|_{q^2=0} \end{aligned}$$

$$\widehat{\Pi}(q^2) = \sum_{j=1}^{\infty} q^{2j} \Pi_j \quad \text{with} \quad \Pi_j = (-1)^{j+1} \frac{G_{2j+2}}{(2j+2)!}$$

Use Padé approximant (ratio of m/n polynomials) for better  $q^2$  behaviour

Allows us to reconstruct  $\widehat{\Pi}(q^2)$  and integrate

# Analysis Ingredients

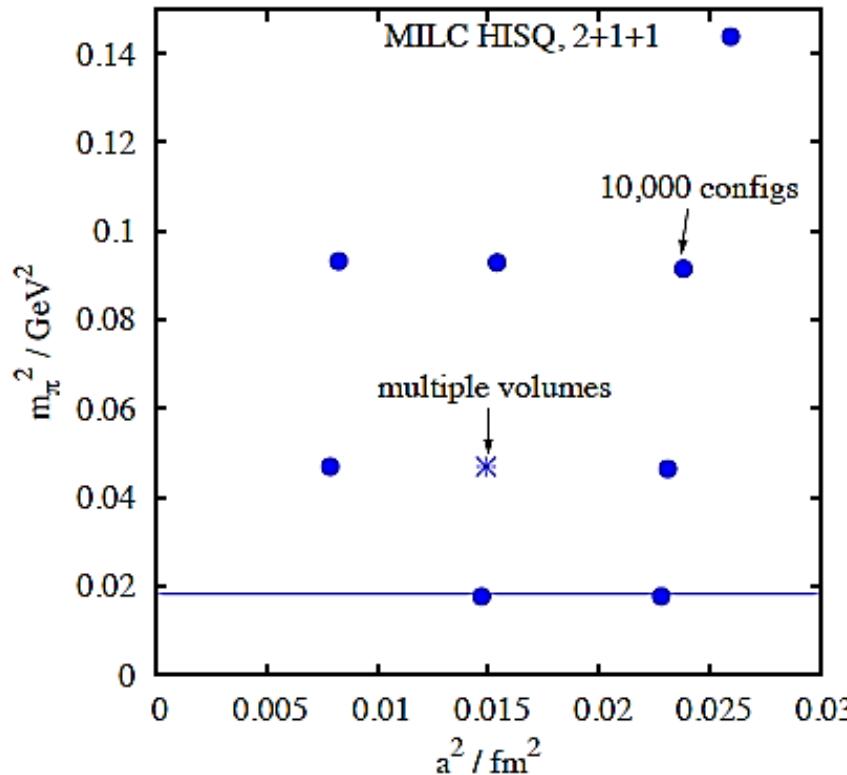
MILC configurations : up/down, strange, charm quarks in the sea

$m_u = m_d$

Multiple  $m_{u/d}$   
Including physical

Valence quark  
Masses tuned  
accurately

High statistics:  
**10,000** configurations  
on coarsest lattice

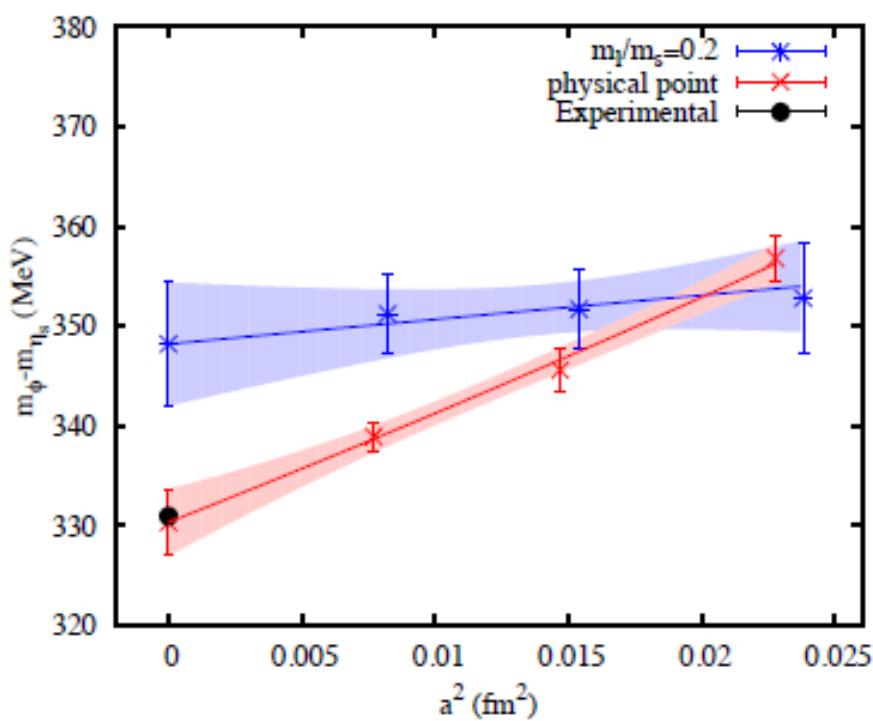


Lattice spacing  
 $a \sim 0.09 - 0.15 \text{ fm}$

Multiple volumes  
 $\sim (5-6 \text{ fm})^3$

**Vector current renormalisation  $Z_V$**   
calculated nonperturbatively  
with high precision

# Check strange-strange vector correlators

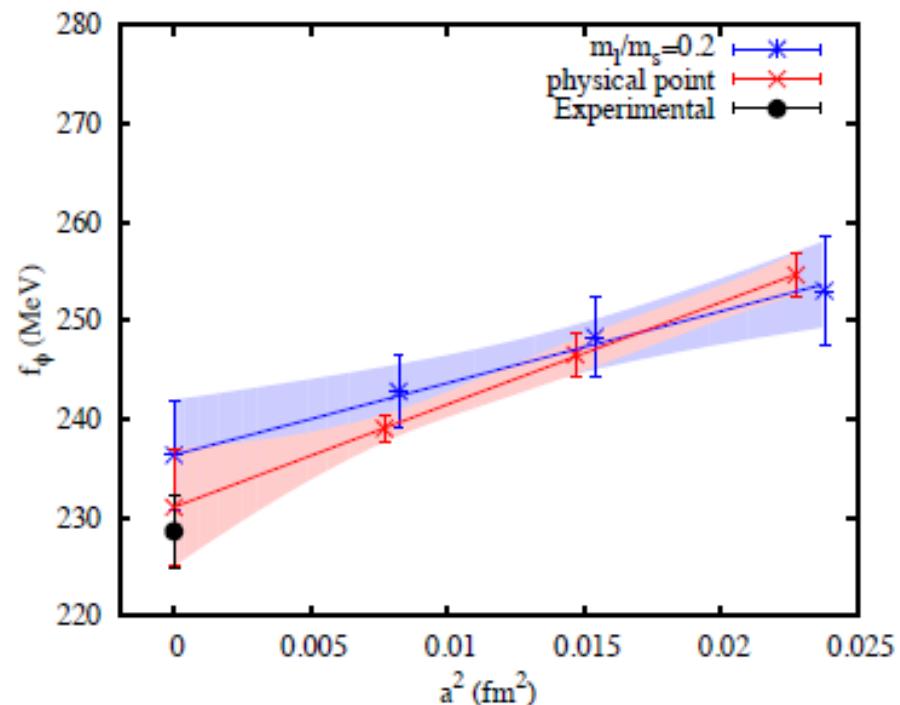


Experimental  $\Gamma(\phi \rightarrow e^+ + e^-)$   
used for  $f_\phi$

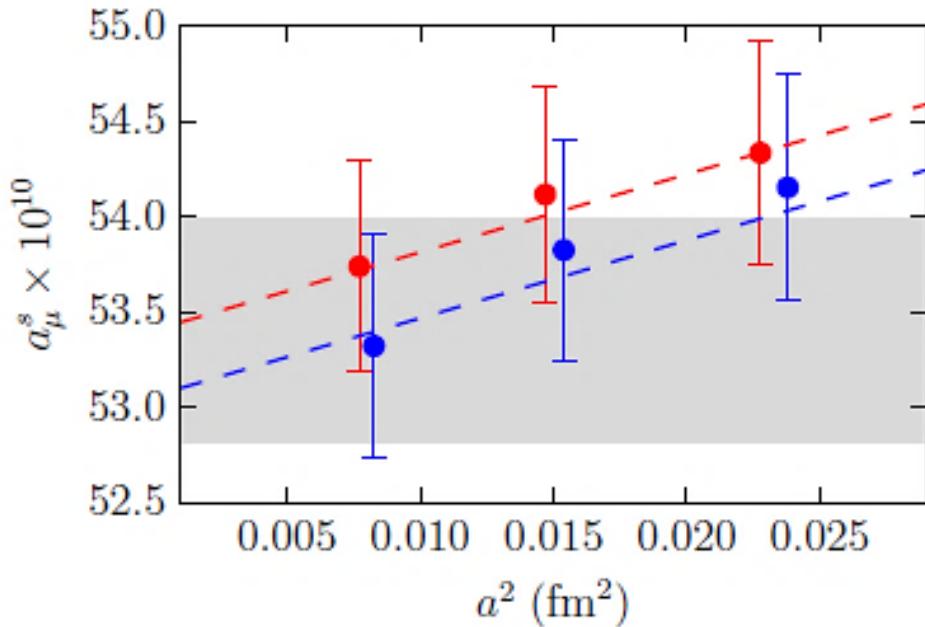
At large time : Correlator  $\sim Ae^{-mt}$

$\phi$ -meson mass

Related to decay constant  
(annihilation amplitude)



# Strange quark contribution to HVP



$a_\mu^s$	
Uncertainty in lattice spacing ( $w_0, r_1$ ):	1.0%
Uncertainty in $Z_V$ :	0.4%
Monte Carlo statistics:	0.1%
$a^2 \rightarrow 0$ extrapolation:	0.1%
QED corrections:	0.1%
Quark mass tuning:	0.0%
Finite lattice volume:	< 0.1%
Padé approximants:	< 0.1%
Total:	1.1%

$$a_{\mu, lat}^s = a_\mu^s \times (1 + c_{a^2} \left( \frac{a \Lambda_{QCD}}{\pi} \right)^2 + c_{sea} \delta x_{sea} + c_{val} \delta x_{val})$$

With  $m_{u/d}^{lat} = m_{u/d}^{phys}$ , after extrapolation to  $a = 0$

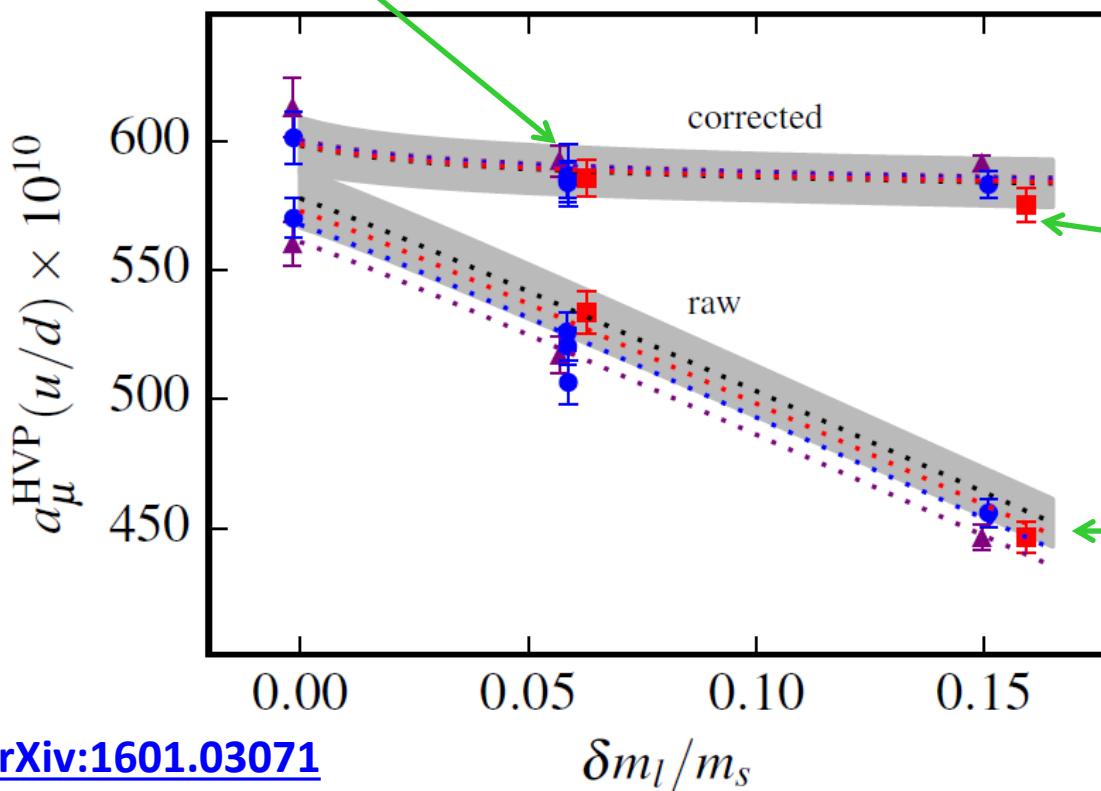
$$a_\mu^s = 53.41(59) \times 10^{-10}$$

# Light quark (connected) contribution to HVP

Additional challenges:

1. Noisy data
2. Finite volume error
3.  $\pi\pi$  contribution

Multiple volumes



New ingredient to handle noisy data:

$$G(t) = \begin{cases} G_{\text{data}}(t), & t \leq t^* \\ G_{\text{fit}}(t), & t > t^* \end{cases}$$

← From Monte Carlo

$t^* = 1.5 \text{ fm} = 6/\rho$  ← From multi-exponential fit

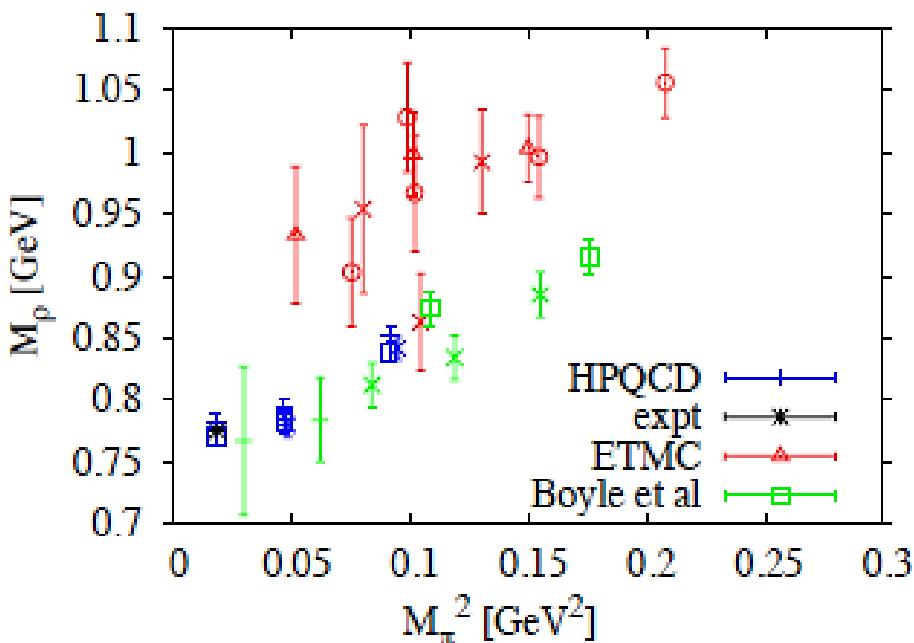
+ Smearings at source-sink

$m_u = m_d$

Ensemble size  
10,000

Finite volume  
Error, no  
corrections for  
Pion mass

# Analysis of $\rho$ meson parameters



Comparison with  
ETMC (1308.4327)  
And Boyle et al. (1107.1497)

HPQCD : a 0.09-0.15 fm

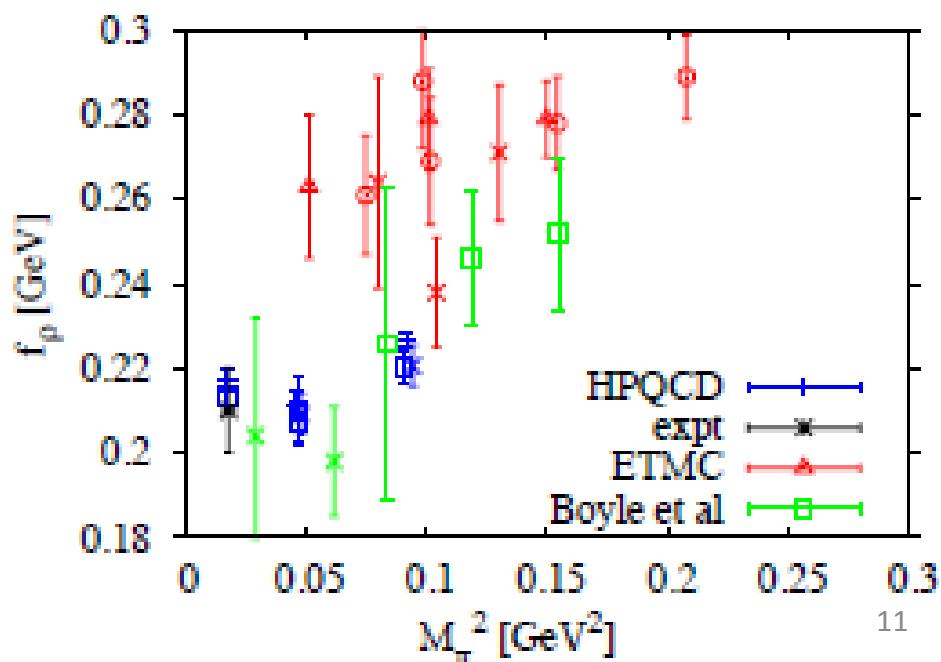
L 2.5-5.8 fm

ETMC : a 0.06-0.08 fm

L 2.5-2.9fm

Boyle et al : a 0.09-0.14 fm

L 2.7-4.6fm



## Our approach: To correct Taylor coefficients

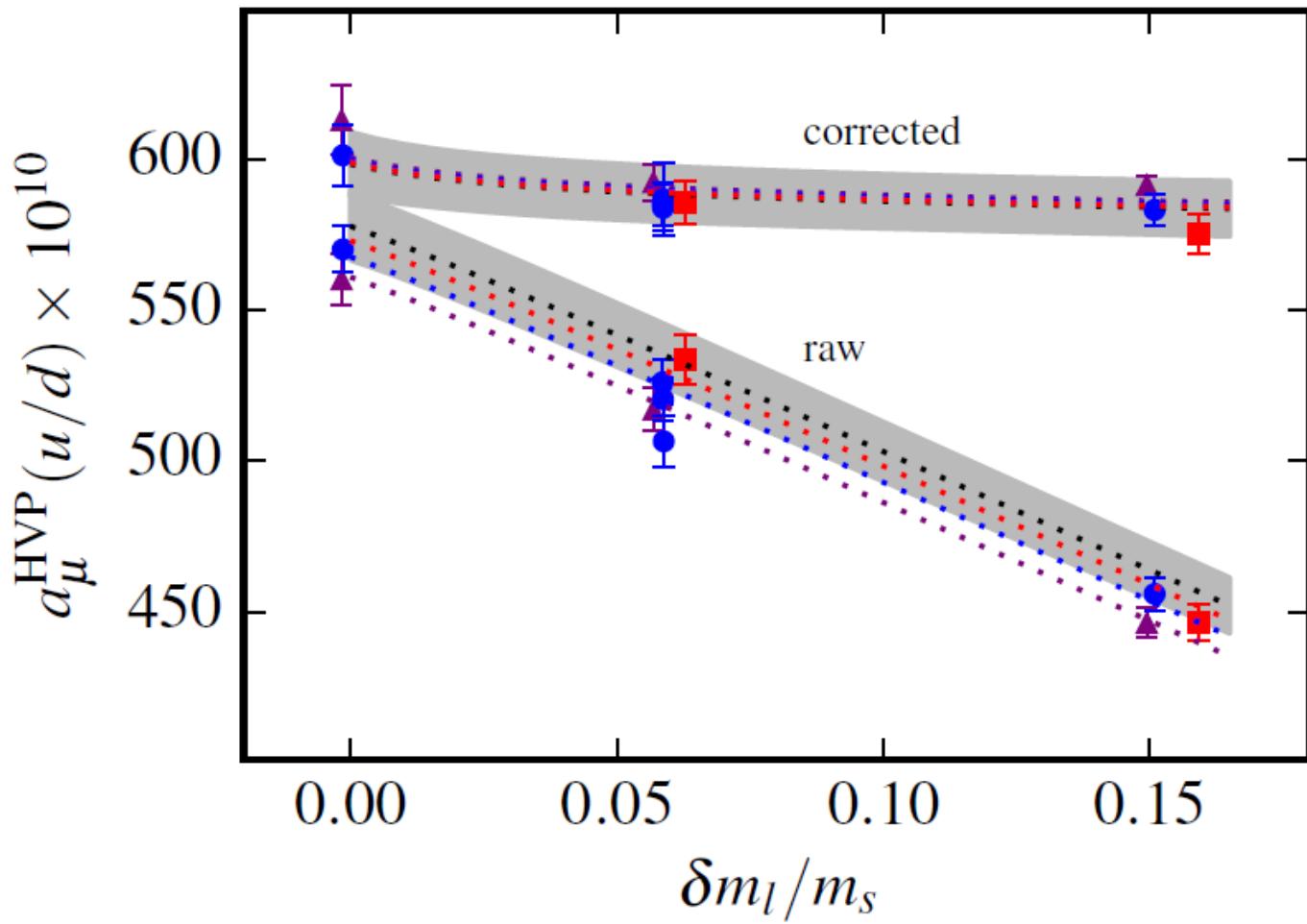
$$\hat{\Pi}_j^{latt} \rightarrow \left( \hat{\Pi}_j^{latt} - \hat{\Pi}_j^{latt}(\pi\pi) \right) \left[ \frac{m_\rho^{2j,latt}}{m_\rho^{2j,expt}} \right] + \hat{\Pi}_j^{cont}(\pi\pi)$$

Remove  $\pi\pi$  contribution on lattice using 1-loop ChPT; remove finite Volume error, taste-effect  
(Jegerlehner + Szafron 1101.2872)

Rescale with experimental  $m_\rho$  (elaboration of ETMC 1308.4327); removes light quark mass systematics

Restore  $\pi\pi$  Contribution from continuum ChPT

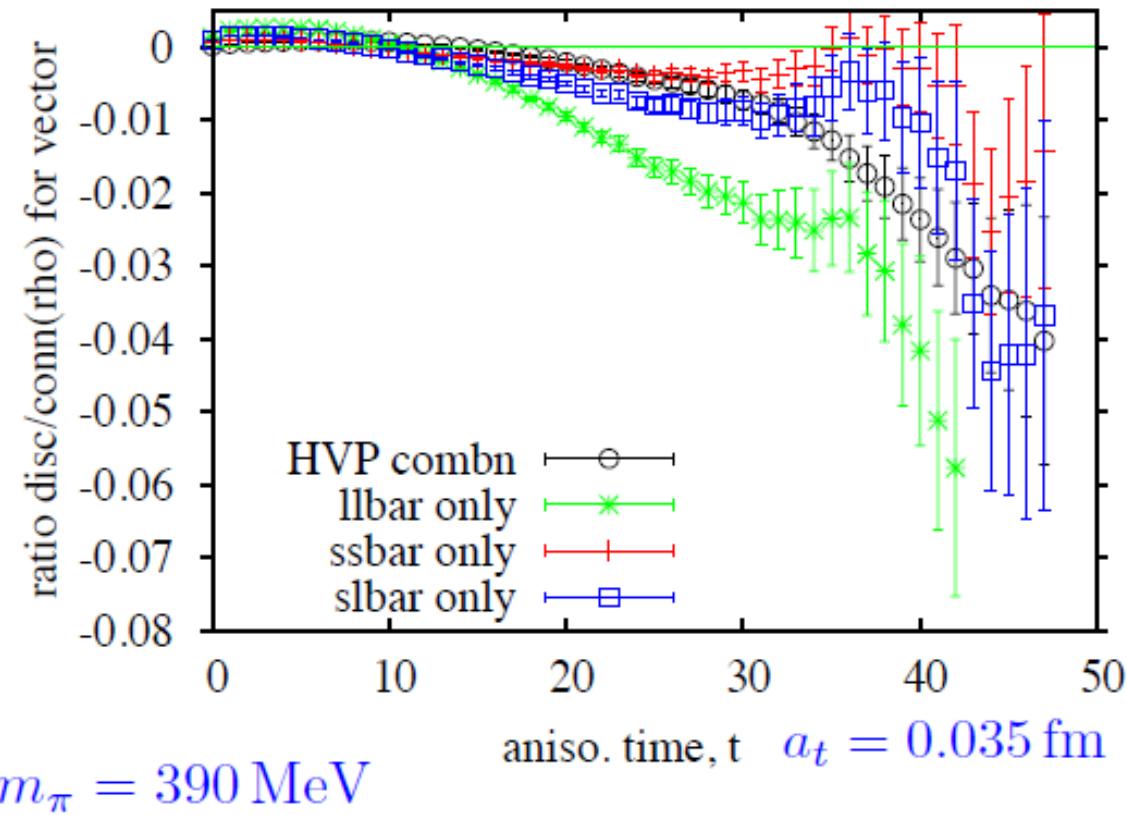
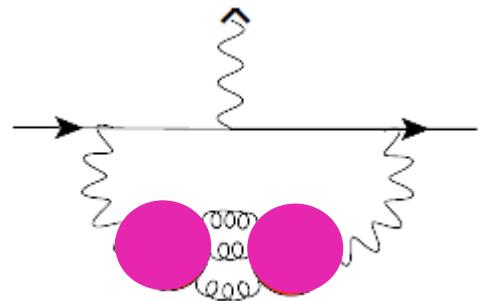
7 fm lattice reduces finite volume error by 10% in continuum  $\pi\pi$  contribution



# Quark-line disconnected contribution

Vanishes if  $m_u = m_d = m_s$

since,  $\sum_i e_i = 0$



HVP disconnected contribution :

- Negative
- 0.2(1.5)% of the connected

Hadron Spectrum + HPQCD

# HPQCD estimate for total $a_\mu HVP, LO$

ETMC  
1308.4327

**HPQCD:**  $\times 10^{-10}$

567(11)stat

light, connected	<b>598(11)</b>
strange connected	<b>53.4(6)</b>
charm connected	<b>14.4(4)</b>
bottom connected	<b>0.27(4)</b>
disconn. (estimate)	<b>0(9)</b>
<b>TOTAL</b>	<b>666(6)(12)</b>

preliminary.  
inc. 1% QED and  
1% isospin uncty

1403.1778

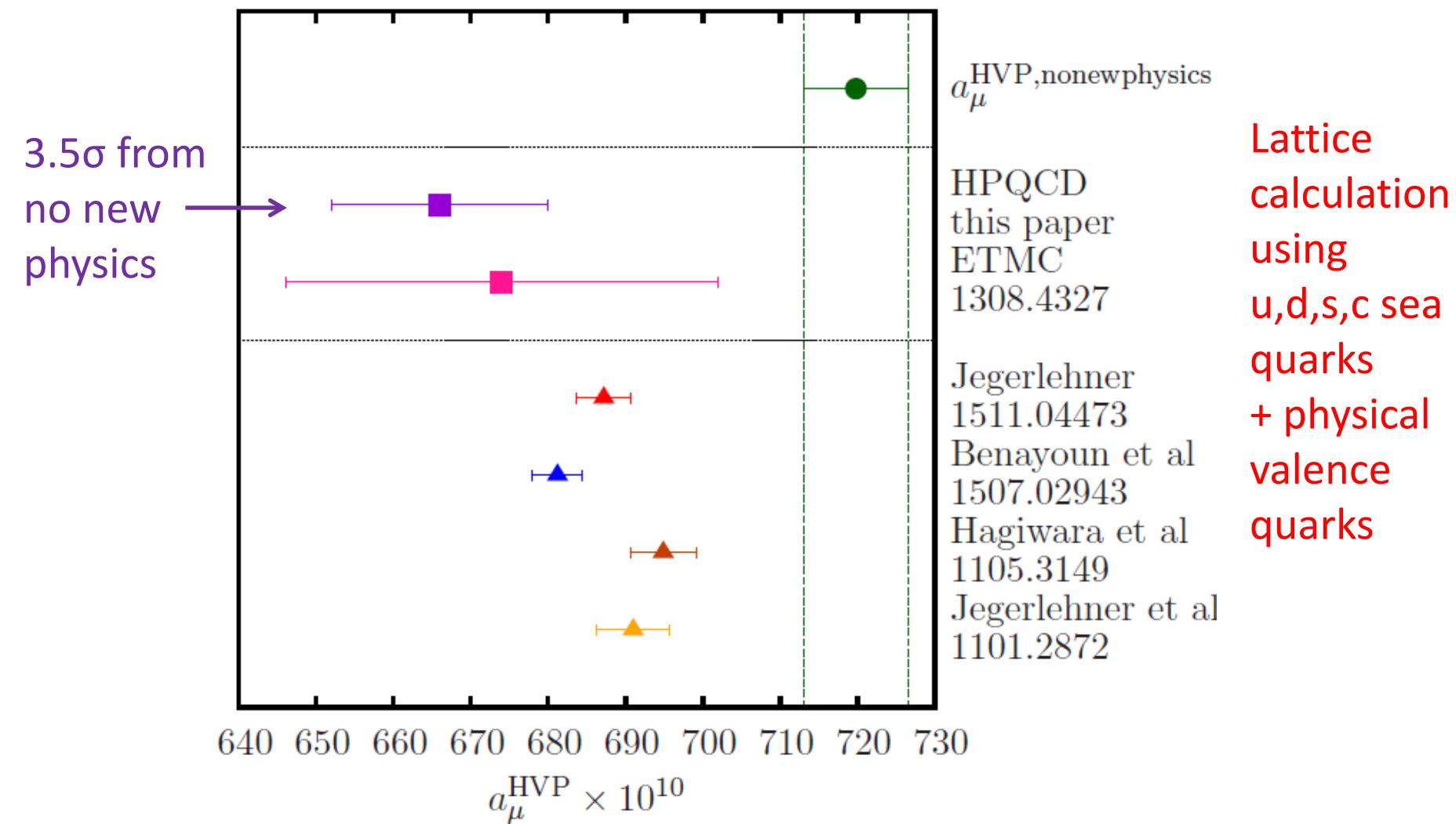
1403.1778,  
1208.2855

1408.5768

Take 1.5% as uncty.  
Contrbn negative

674(28)

# Lattice – Continuum comparison

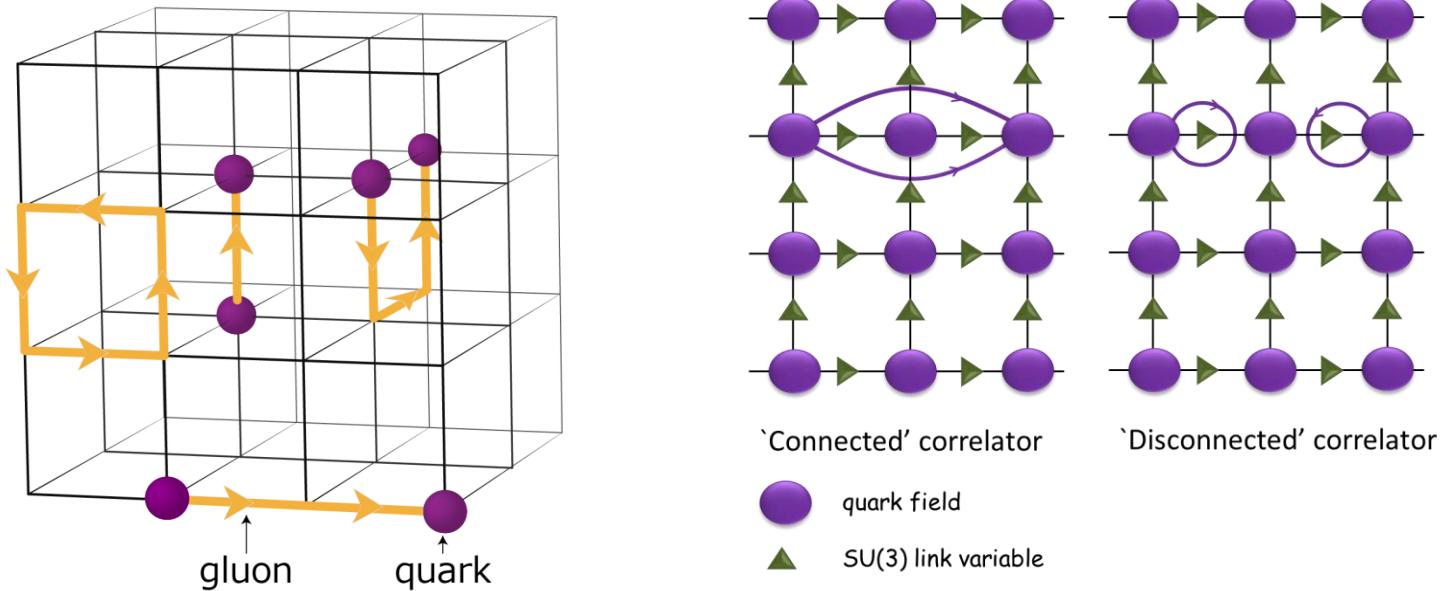


In future : Increase statistics, QED and isospin effects, disconnected piece  
More Calculations underway (Mainz, BMW, RBC/UKQCD, ...)

# Back up

# Lattice recipe for meson correlators

- Expectation values of observables :  $\int DUD\bar{\psi}D\bar{\psi}\exp(-\int L_{QCD}d^4x)$
- 4-D space-time lattice
- Gauge configurations : gluons + sea quarks



- Discretise :  $L_q \equiv \bar{\psi}(\gamma_\mu D^\mu + m)\psi \rightarrow \bar{\psi}(\gamma \cdot \Delta + ma)\psi$
- Inversion of Dirac matrix : propagator
- 2-point correlation function : extract meson properties
- Corrections for lattice calculation

# Computation at Darwin@Cambridge

Part of STFC's HPC facility

9600 Intel Sandybridge cores,  
Infiniband interconnect,  
fast switch, 2Pbytes storage



*DiRAC*

Under **Dirac 2** : Approx. 4 million  
core hours used

## Fit function

- The result for  $a_\mu^s$  converge to within errors, by the [1,1] Padé approximant, and no spurious poles appear up to and including [2,2].
- we fit the [2,2] results from each configuration set to a function of the form

$$a_{\mu, \text{lat}}^s = a_\mu^s \times \left( 1 + c_{a^2} \left( \frac{a \Lambda_{\text{QCD}}}{\pi} \right)^2 + c_{\text{sea}} \delta \chi_{\text{sea}} + c_{\text{val}} \delta \chi_{\text{val}} \right)$$

where  $\Lambda_{\text{QCD}} = 0.5 \text{ GeV}$  and

$$\delta \chi_{\text{sea}} \equiv \sum_{q=u,d,s} \frac{m_q^{\text{sea}} - m_q^{\text{phys}}}{m_s^{\text{phys}}}$$

$$\delta \chi_s \equiv \frac{m_s^{\text{val}} - m_s^{\text{phys}}}{m_s^{\text{phys}}}.$$

- Discretization effects are handled by  $c_{a^2}$ .