

Current status of ε_K in lattice QCD

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ε_K and \hat{B}_K , V_{cb} |

- Definition of ε_K

$$\varepsilon_K = \frac{A[K_L \rightarrow (\pi\pi)_{I=0}]}{A[K_S \rightarrow (\pi\pi)_{I=0}]}$$

- Relation between ε_K and \hat{B}_K in standard model.

$$\begin{aligned} \varepsilon_K &= \exp(i\phi_\varepsilon) \sqrt{2} \sin(\phi_\varepsilon) [C_\varepsilon X_{\text{SD}} \hat{B}_K + \frac{\xi_0}{\sqrt{2}} + \xi_{\text{LD}}] \\ &\quad + \mathcal{O}(\omega\varepsilon') + \mathcal{O}(\xi_0 \Gamma_2/\Gamma_1) \end{aligned}$$

$$\begin{aligned} X_{\text{SD}} &= \text{Im} \lambda_t \left[\text{Re} \lambda_c \eta_{cc} S_0(x_c) - \text{Re} \lambda_t \eta_{tt} S_0(x_t) \right. \\ &\quad \left. - (\text{Re} \lambda_c - \text{Re} \lambda_t) \eta_{ct} S_0(x_c, x_t) \right] \end{aligned}$$

ε_K and \hat{B}_K , $V_{cb} \parallel$

$$\lambda_i = V_{is}^* V_{id}, \quad x_i = m_i^2/M_W^2, \quad C_\varepsilon = \frac{G_F^2 F_K^2 m_K M_W^2}{6\sqrt{2} \pi^2 \Delta M_K}$$

$$\frac{\xi_0}{\sqrt{2}} = \frac{1}{\sqrt{2}} \frac{\text{Im} A_0}{\text{Re} A_0} \approx -5\%$$

$\xi_{\text{LD}} = \text{Long Distance Effect} \approx 2\% \longrightarrow \text{a systematic error.}$

- Inami-Lim functions:

$$S_0(x_i) = x_i \left[\frac{1}{4} + \frac{9}{4(1-x_i)} - \frac{3}{2(1-x_i)^2} - \frac{3x_i^2 \ln x_i}{(1-x_i)^3} \right],$$

$$S_0(x_i, x_j) = \left\{ \begin{aligned} & \frac{x_i x_j}{x_i - x_j} \left[\frac{1}{4} + \frac{3}{2(1-x_i)} - \frac{3}{4(1-x_i)^2} \right] \ln x_i \\ & - (i \leftrightarrow j) \end{aligned} \right\} - \frac{3x_i x_j}{4(1-x_i)(1-x_j)}$$

ε_K and \hat{B}_K , V_{cb} III

$$S_0(x_t) \longrightarrow + 70\%$$

$$S_0(x_c, x_t) \longrightarrow + 44\%$$

$$S_0(x_c) \longrightarrow - 14\%$$

- Dominant contribution ($\approx 70\%$) comes with $|V_{cb}|^4$.

$$\text{Im}\lambda_t \cdot \text{Re}\lambda_t = \bar{\eta} \lambda^2 |V_{cb}|^4 (1 - \bar{\rho})$$

$$\text{Re}\lambda_c = -\lambda \left(1 - \frac{\lambda^2}{2}\right) + \mathcal{O}(\lambda^5)$$

$$\text{Re}\lambda_t = -\left(1 - \frac{\lambda^2}{2}\right) A^2 \lambda^5 (1 - \bar{\rho}) + \mathcal{O}(\lambda^7)$$

$$\text{Im}\lambda_t = \eta A^2 \lambda^5 + \mathcal{O}(\lambda^7)$$

ε_K and \hat{B}_K , V_{cb} IV

- Definition of B_K in standard model.

$$B_K = \frac{\langle \bar{K}_0 | [\bar{s}\gamma_\mu(1-\gamma_5)d][\bar{s}\gamma_\mu(1-\gamma_5)d] | K_0 \rangle}{\frac{8}{3} \langle \bar{K}_0 | \bar{s}\gamma_\mu\gamma_5 d | 0 \rangle \langle 0 | \bar{s}\gamma_\mu\gamma_5 d | K_0 \rangle}$$

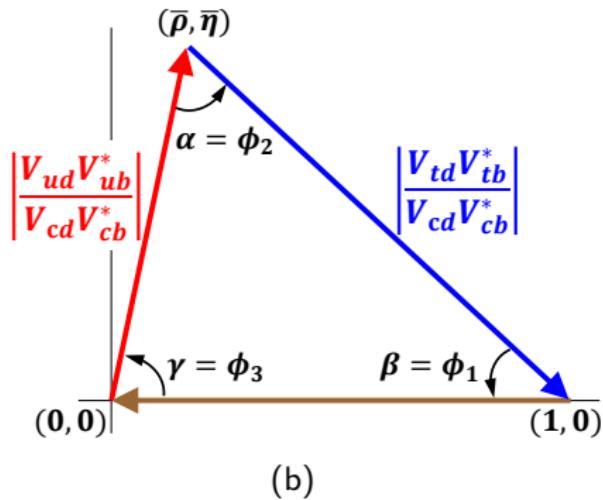
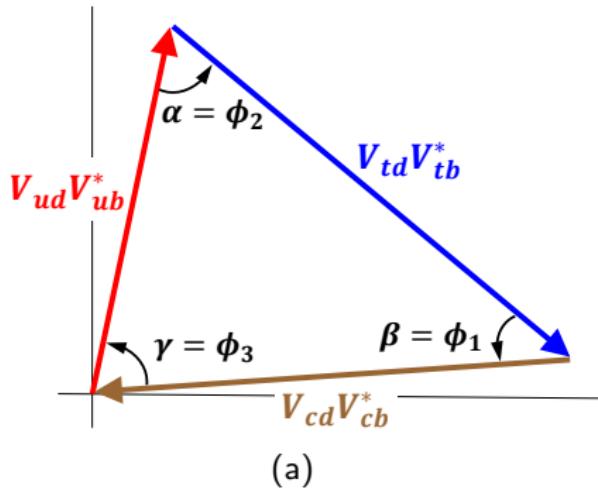
$$\hat{B}_K = C(\mu) B_K(\mu), \quad C(\mu) = \alpha_s(\mu)^{-\frac{\gamma_0}{2b_0}} [1 + \alpha_s(\mu) J_3]$$

- Experiment:

$$\varepsilon_K = (2.228 \pm 0.011) \times 10^{-3} \times e^{i\phi_\varepsilon}$$

$$\phi_\varepsilon = 43.52(5)^\circ$$

ε_K on the lattice

Unitarity Triangle $\rightarrow (\bar{\rho}, \bar{\eta})$ 

Global UT Fit and Angle-Only-Fit (AOF)

Global UT Fit

- Input: $|V_{ub}|/|V_{cb}|$, Δm_d , $\Delta m_s/\Delta m_d$, ε_K , and $\sin(2\beta)$.
- Determine the UT apex $(\bar{\rho}, \bar{\eta})$.
- Take λ from

$$|V_{us}| = \lambda + \mathcal{O}(\lambda^7),$$

which comes from K_{l3} and $K_{\mu 2}$.

- Disadvantage: **unwanted correlation** between $(\bar{\rho}, \bar{\eta})$ and ε_K .

AOF

- Input: $\sin(2\beta)$, $\cos(2\beta)$, $\sin(\gamma)$, $\cos(\gamma)$, $\sin(2\beta + \gamma)$, $\cos(2\beta + \gamma)$, and $\sin(2\alpha)$.
- Determine the UT apex $(\bar{\rho}, \bar{\eta})$.
- Take λ from $|V_{us}| = \lambda + \mathcal{O}(\lambda^7)$, which comes from K_{l3} and $K_{\mu 2}$.
- Use $|V_{cb}|$ to determine A .

$$|V_{cb}| = A\lambda^2 + \mathcal{O}(\lambda^7)$$

- Advantage: **NO correlation** between $(\bar{\rho}, \bar{\eta})$ and ε_K .

Inputs of Angle-Only-Fit (AOF)

- $A_{CP}(J/\psi K_s) \rightarrow S_{\psi K_s} = \sin(2\beta)$ with assumption of $S_{\psi K_s} \ggg C_{\psi K_s}$.
- $(B \rightarrow DK) + (B \rightarrow [K\pi]_D K)$ + (Dalitz method) give $\sin(\gamma)$ and $\cos(\gamma)$.
- $S(D^-\pi^+)$ and $S(D^+\pi^-)$ give $\sin(2\beta + \gamma)$ and $\cos(2\beta + \gamma)$.
- $(B^0 \rightarrow \pi^+\pi^-) + (B^0 \rightarrow \rho^+\rho^-) + (B^0 \rightarrow (\rho\pi)^0)$ give $\sin(2\alpha)$.
- Combining all of these gives β , γ , and α , which leads to the UT apex $(\bar{\rho}, \bar{\eta})$.

Wolfenstein Parameters

Input Parameters for Angle-Only-Fit (AOF)

- ϵ_K , \hat{B}_K , and $|V_{cb}|$ are used as inputs to determine the UT angles in the global fit of UTfit and CKMfitter.
- Instead, we can use angle-only-fit result for the UT apex ($\bar{\rho}$, $\bar{\eta}$).
- Then, we can take λ independently from

$$|V_{us}| = \lambda + \mathcal{O}(\lambda^7),$$

which comes from K_{l3} and $K_{\mu 2}$.

- Use $|V_{cb}|$ instead of A .

$$|V_{cb}| = A\lambda^2 + \mathcal{O}(\lambda^7)$$

| | | |
|--------------|-------------|----------------------|
| λ | 0.22537(61) | [1] CKMfitter |
| | 0.2255(6) | [1] UTfit |
| | 0.2253(8) | [1] $ V_{us} $ (AOF) |
| $\bar{\rho}$ | 0.117(21) | [1] CKMfitter |
| | 0.124(24) | [1] UTfit |
| | 0.139(29) | [2] UTfit (AOF) |
| $\bar{\eta}$ | 0.353(13) | [1] CKMfitter |
| | 0.354(15) | [1] UTfit |
| | 0.337(16) | [2] UTfit (AOF) |

Input Parameters of B_K , V_{cb} and others

 B_K

| | | |
|-------------|-------------------|-----------------|
| \hat{B}_K | $0.7625(97)$ | [3] FLAG-2016 |
| | $0.7379(47)(365)$ | [4] SWME-2014 |
| | $0.7499(24)(150)$ | [5] RBC-UK-2014 |

 V_{cb}

| | | |
|-------------------------|---------------------|-----------|
| $V_{cb} \times 10^{-3}$ | $42.00(64)$ | [6] Incl. |
| | $39.04(49)(53)(19)$ | [7] Excl. |

Others

| | | |
|--------------|--|------|
| G_F | $1.1663787(6) \times 10^{-5} \text{ GeV}^{-2}$ | [1] |
| M_W | $80.385(15) \text{ GeV}$ | [1] |
| $m_c(m_c)$ | $1.2733(76) \text{ GeV}$ | [8] |
| $m_t(m_t)$ | $163.3(2.7) \text{ GeV}$ | [9] |
| η_{cc} | $1.72(27)$ | [10] |
| η_{tt} | $0.5765(65)$ | [11] |
| η_{ct} | $0.496(47)$ | [12] |
| θ | $43.52(5)^\circ$ | [1] |
| m_{K^0} | $497.614(24) \text{ MeV}$ | [1] |
| ΔM_K | $3.484(6) \times 10^{-12} \text{ MeV}$ | [1] |
| F_K | $156.2(7) \text{ MeV}$ | [1] |

ξ_0

Indirect Method

$$\xi_0 = \frac{\text{Re}A_0}{\text{Im}A_0}$$

| | | |
|---------|----------------------------|------------------|
| ξ_0 | $-1.63(19) \times 10^{-4}$ | RBC-UK-2015 [13] |
|---------|----------------------------|------------------|

- RBC-UKQCD calculated $\text{Im}A_2$. $\text{Im}A_2 \rightarrow \xi_0$

$$\text{Re}\left(\frac{\epsilon'_K}{\epsilon_K}\right) = \frac{1}{\sqrt{2}|\epsilon_K|} \omega \left(\frac{\text{Im}A_2}{\text{Re}A_2} - \xi_0 \right).$$

Other inputs ω , ϵ_K and ϵ'_K/ϵ_K are taken from the experimental values.

- Here, we choose an approximation of $\cos(\phi_{\epsilon'} - \phi_\epsilon) \approx 1$.
- $\phi_\epsilon = 43.52(5)$, $\phi_{\epsilon'} = 42.3(1.5)$
- Isospin breaking effect: (at most 20% of ξ_0) \rightarrow (1% in ε_K) \rightarrow neglected!

ξ_0

Direct Method

- RBC-UKQCD calculated $\text{Im}A_0$. $\text{Im}A_0 \rightarrow \xi_0$.

$$\xi_0 = \frac{\text{Im}A_0}{\text{Re}A_0} = -0.57(49) \times 10^{-4}$$

Other input $\text{Re}A_0$ is taken from the experimental value.

- RBC-UKQCD also calculated δ_0

$$\delta_0 = 23.8(49)(12)^\circ$$

This value is 3.5σ away from the experimental value.

- This indicates that this method belongs to the category of exploratory study rather than precision measurement.
- Hence, we use the **indirect method** to determine ξ_0 .

ξ_0

Comparison

Input Parameters: ξ_0

| Method | Value | Reference |
|----------|----------------------------|------------------|
| Indirect | $-1.63(19) \times 10^{-4}$ | RBC-UK-2015 [13] |
| Direct | $-0.57(49) \times 10^{-4}$ | RBC-UK-2015 [14] |

ξ_{LD}

Indirect Method

- Definition:

$$\xi_{\text{LD}} = \frac{m'_{\text{LD}}}{\sqrt{2} \Delta M_K}$$

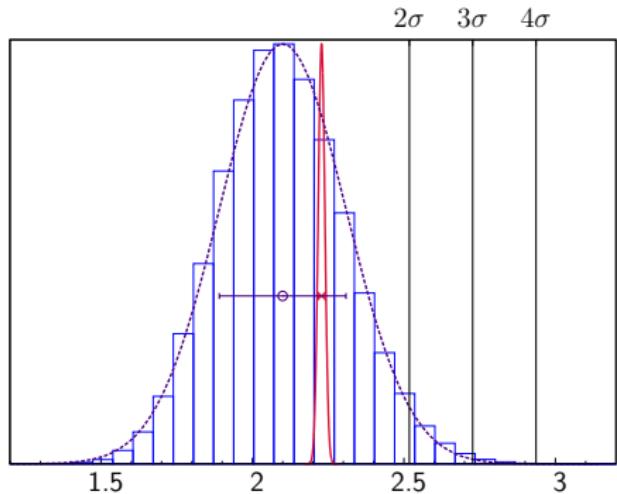
$$m'_{\text{LD}} = -\text{Im} \left[\mathcal{P} \sum_C \frac{\langle \bar{K}^0 | H_w | C \rangle \langle C | H_w | K^0 \rangle}{m_{K^0} - E_C} \right]$$

- Rough estimate in [PRD 88, 014508] gives

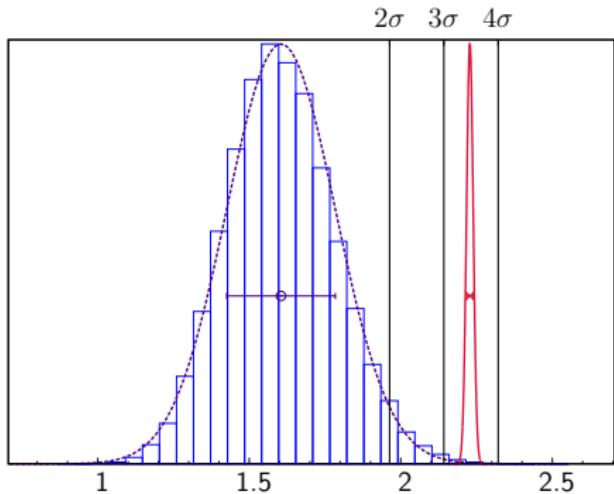
$$\xi_{\text{LD}} = (0 \pm 1.6)\%$$

- Precise lattice QCD calculation is not available yet.

ϵ_K : FLAG \hat{B}_K , AOF of $(\bar{\rho}, \bar{\eta})$, V_{us}



Inclusive V_{cb}



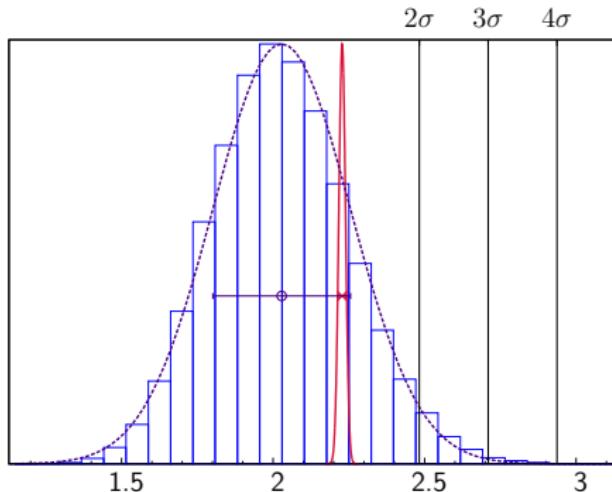
Exclusive V_{cb}

- With exclusive V_{cb} , it shows 3.5σ tension.

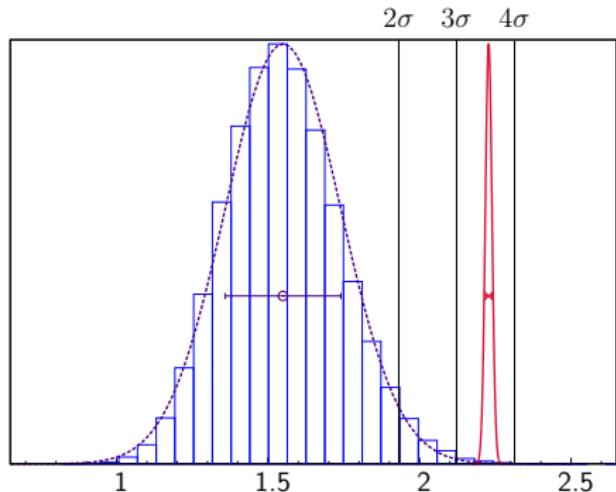
$$\epsilon_K^{Exp} = 2.228(11) \times 10^{-3}$$

$$\epsilon_K^{SM} = 1.60(18) \times 10^{-3}$$

ϵ_K : SWME \hat{B}_K , AOF of $(\bar{\rho}, \bar{\eta})$, V_{us}



Inclusive V_{cb}



Exclusive V_{cb}

- With exclusive V_{cb} , it shows 3.6σ tension.

$$\epsilon_K^{Exp} = 2.228(11) \times 10^{-3}$$

$$\epsilon_K^{SM} = 1.55(19) \times 10^{-3}$$

Current Status of ε_K

- FLAG 2016: (in units of 1.0×10^{-3} , AOF)

$$\varepsilon_K = 1.60 \pm 0.18 \quad \text{for Exclusive } V_{cb} \text{ (Lattice QCD)}$$

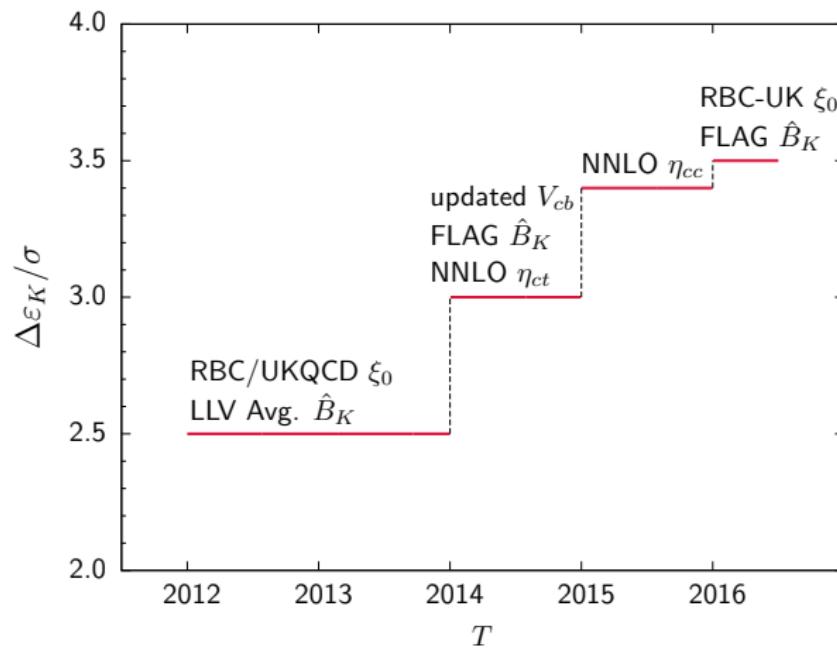
$$\varepsilon_K = 2.11 \pm 0.21 \quad \text{for Inclusive } V_{cb} \text{ (QCD Sum Rule)}$$

- Experiments:

$$\varepsilon_K = 2.228 \pm 0.011$$

- Hence, we observe 3.5σ difference between the SM theory (Lattice QCD) and experiments.
- What does this mean? → Breakdown of SM ?

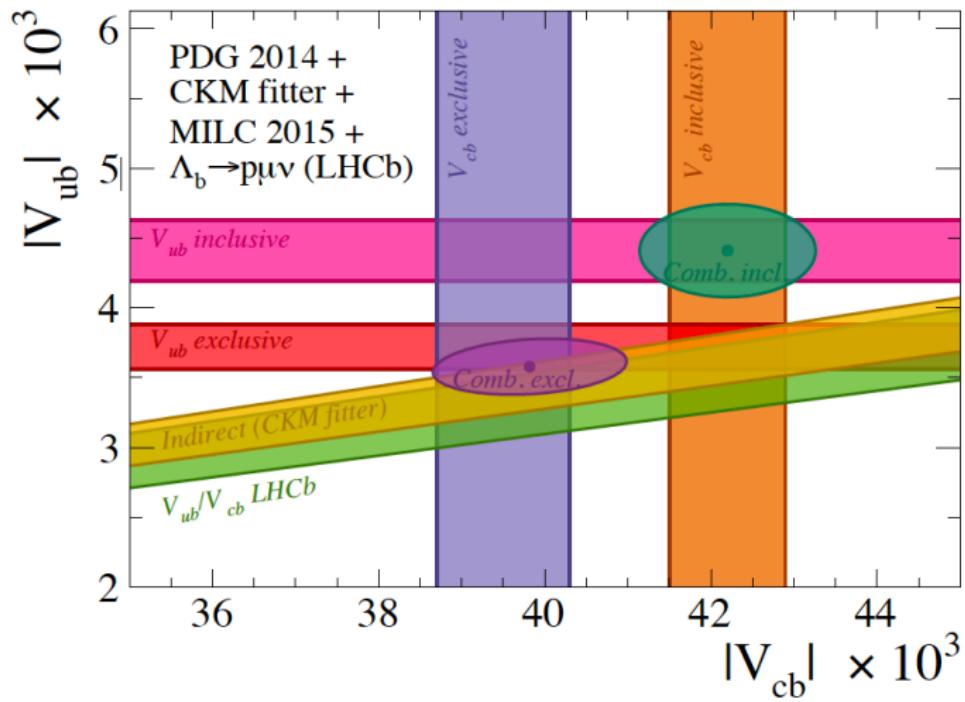
Time Evolution of $\Delta\varepsilon_K$ on the Lattice



- $\Delta\varepsilon_K \equiv \varepsilon_K^{\text{exp}} - \varepsilon_K^{\text{SM}}$

Error Budget of Exclusive ε_K

| source | error (%) | memo |
|--------------|-----------|---------------------------------|
| V_{cb} | 40.2 | Exclusive (FNAL/MILC) |
| $\bar{\eta}$ | 20.9 | AOF |
| η_{ct} | 17.1 | $c - t$ Box |
| η_{cc} | 7.2 | $c - c$ Box |
| $\bar{\rho}$ | 5.5 | AOF |
| m_t | 2.5 | top quark mass |
| ξ_{LD} | 2.1 | Long-distance |
| \hat{B}_K | 1.5 | FLAG |
| ξ_0 | 1.1 | $\text{Im}(A_0)/\text{Re}(A_0)$ |
| : | : | |

Current Status of $|V_{cb}|$ in KEK-FF 2015 (M. Rotondo)

Conclusion and Future Outlook

- ➊ Lattice determination of ε_K from the standard model with the exclusive V_{cb} channel determined from lattice QCD shows **3.5 σ tension** compared with the experiment.
- ➋ However, in the inclusive V_{cb} channel determined from the QCD sum rules, we observe no tension.
- ➌ The dominant systematic error in ε_K comes from V_{cb} in the exclusive channel.
- ➍ Hence, it becomes crucial to reduce the theoretical error of V_{cb} down to $\approx 1.1\%$ level: \longleftrightarrow the OK action.
- ➎ **Thank THE LORD very much for your help!!!**

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