

# *Implications from $B \rightarrow K^* \ell^+ \ell^-$ observables using $3 \text{ fb}^{-1}$ of LHCb data*

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on arXiv: 1506:04535, 1603:04355

with Rahul Sinha, Thomas E. Browder,  
Abinash Kr. Nayak & Anirban Karan.



# Outline

- Introduction

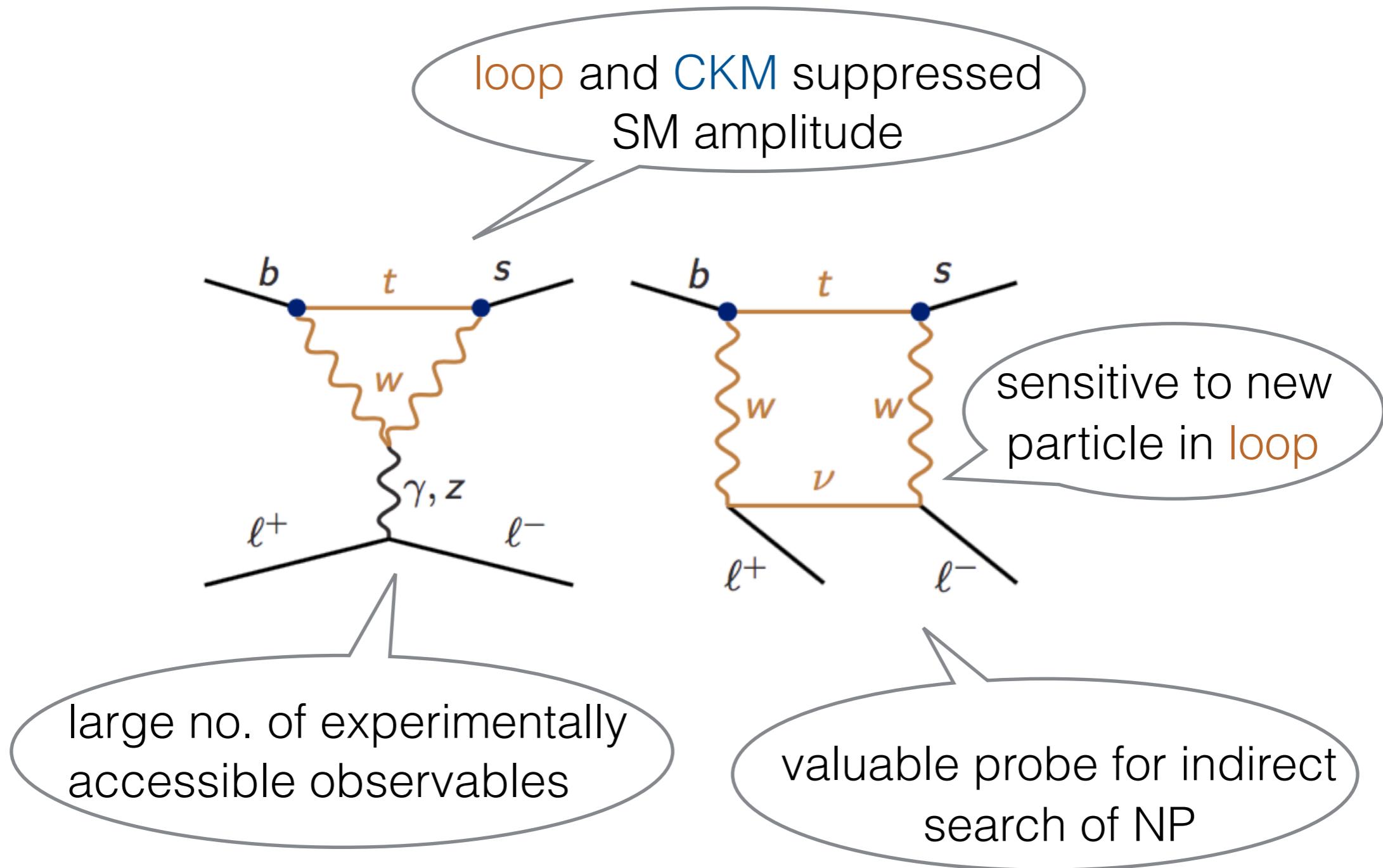
- Model Independent Framework

- ▶ Hadronic parameter extraction
- ▶ Kinematic endpoint analysis

- Evidence of New Physics

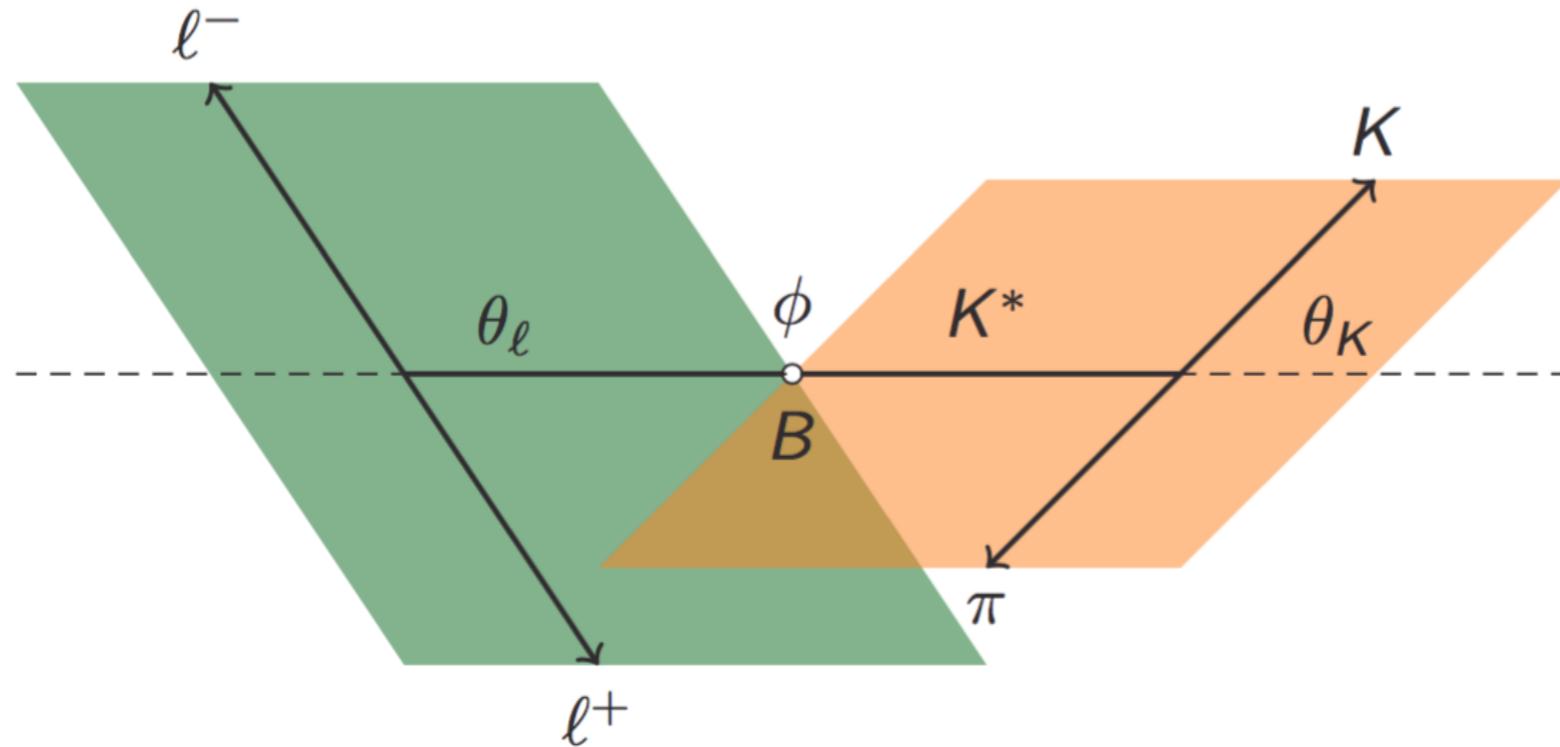
- Summary

# Introduction



# Introduction

Angular analysis in well known helicity frame [Kruger, Sehgal, Sinha, Sinha '99]



The differential distribution

$$\frac{d^4\Gamma(B \rightarrow K^* \ell^+ \ell^-)}{dq^2 d\cos\theta_l d\cos\theta_k d\phi}$$

$$\begin{aligned} &= \frac{9}{32\pi} \left[ I_1^s \sin^2 \theta_K + I_1^c \cos^2 \theta_K + (I_2^s \sin^2 \theta_K + I_2^c \cos^2 \theta_K) \cos 2\theta_l + I_3 \sin^2 \theta_K \sin^2 \theta_l \cos 2\phi \right. \\ &\quad + I_4 \sin 2\theta_K \sin 2\theta_l \cos \phi + I_5 \sin 2\theta_K \sin \theta_l \cos \phi + I_6^s \sin^2 \theta_K \cos \theta_l \\ &\quad \left. + I_7 \sin 2\theta_K \sin \theta_l \sin \phi + I_8 \sin 2\theta_K \sin 2\theta_l \sin \phi + I_9 \sin^2 \theta_K \sin^2 \theta_l \sin 2\phi \right] \end{aligned}$$

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- ▶  $I_i = \text{short distance} + \text{long distance}$

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*tremendous effort since past*

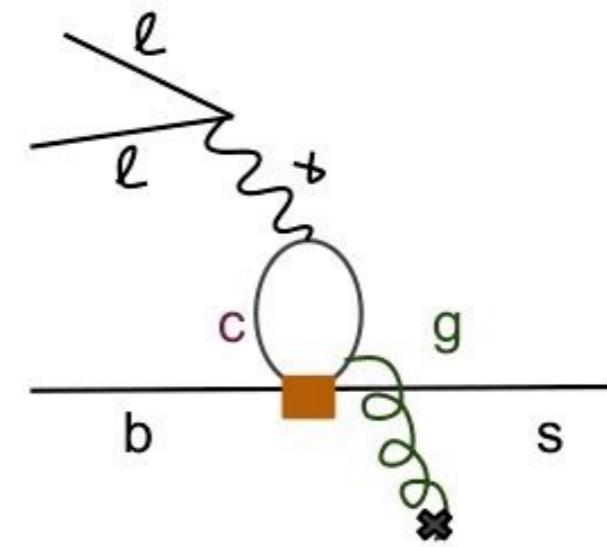
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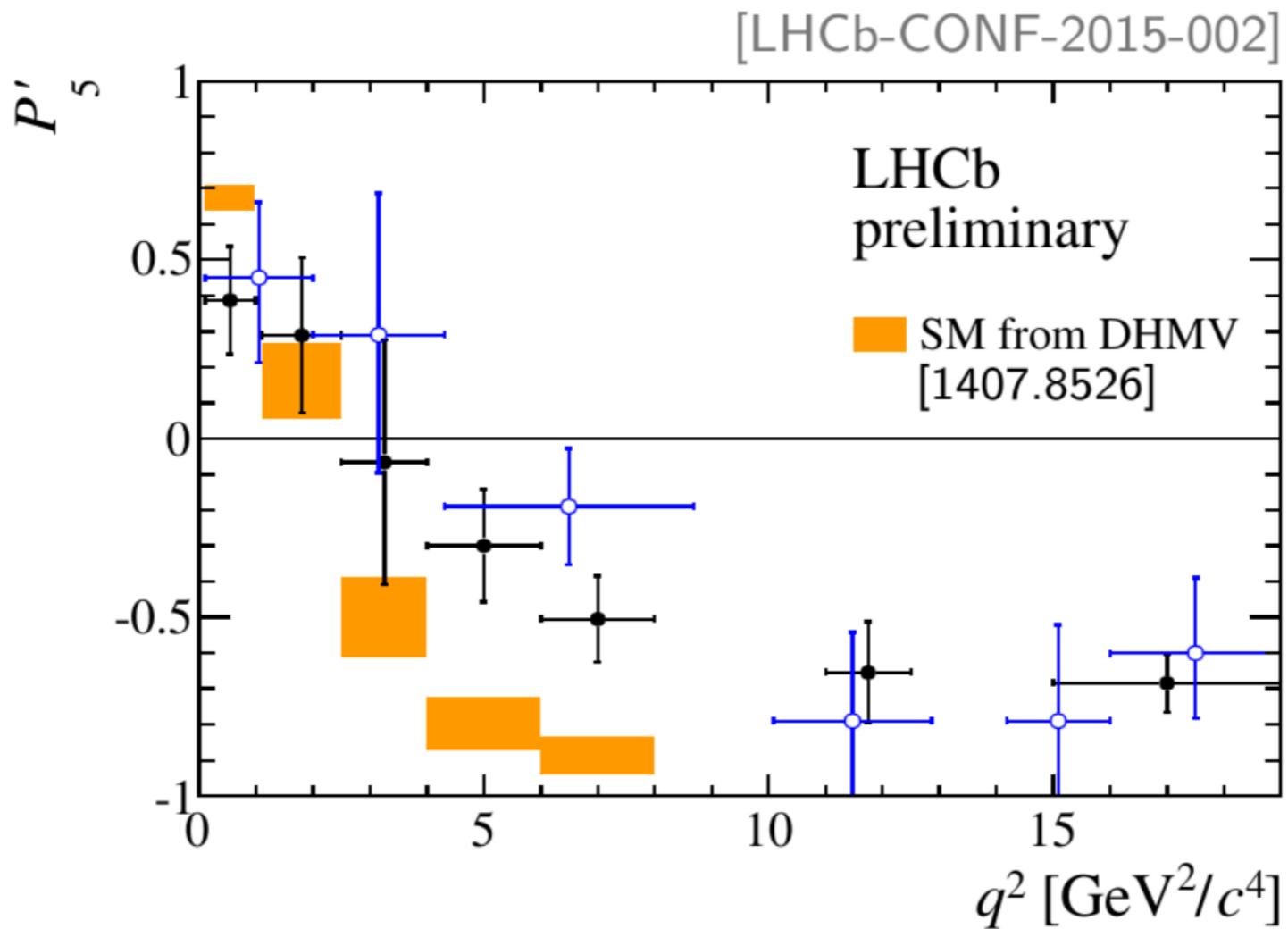
Non-factorizable  
contributions:



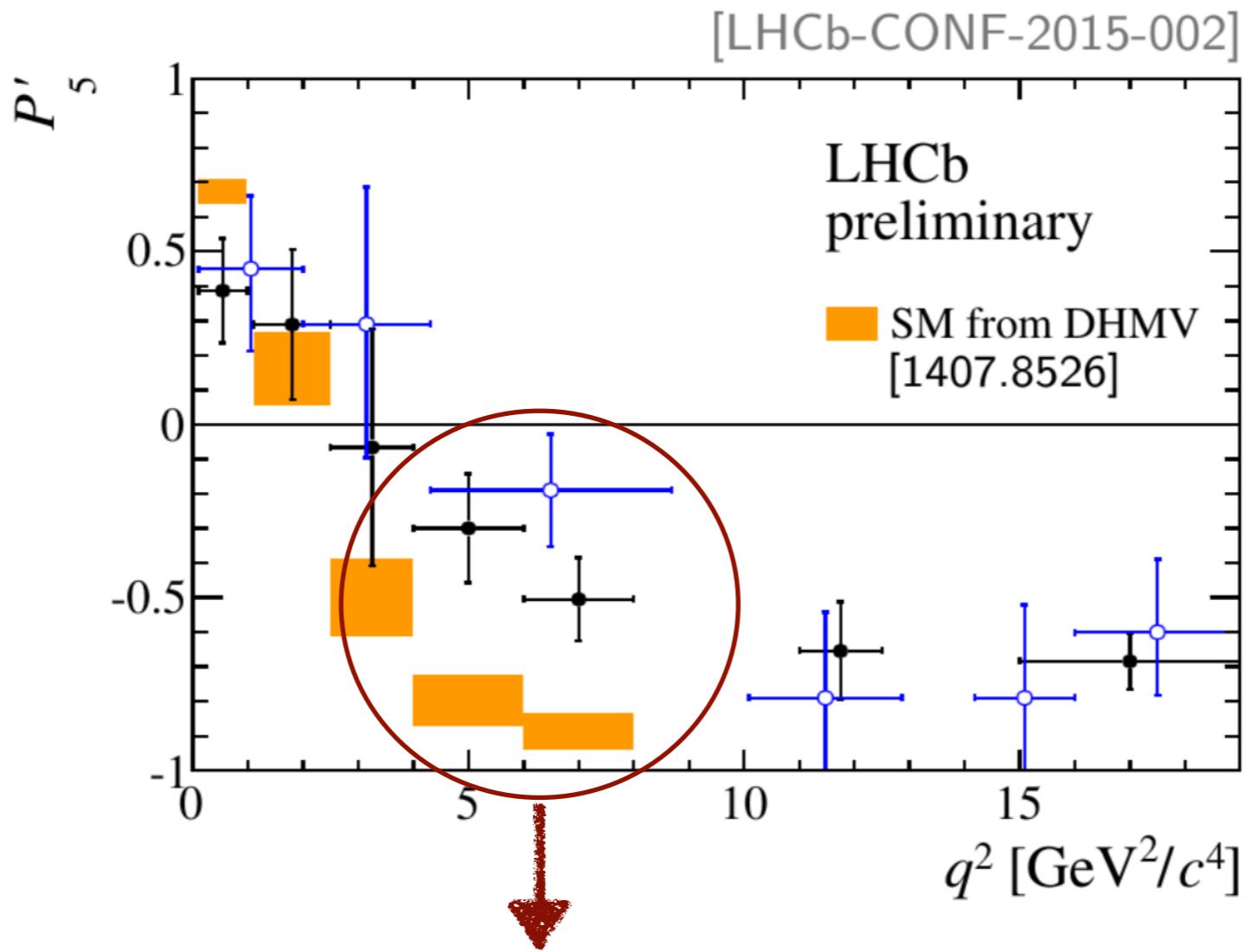
no quantitative computation

► Challenge: either estimate accurately or eliminate

# The discrepancy



# The discrepancy



$2.9\sigma$  discrepancy in each bin

$$P'_5 = \frac{I_5}{F_L(1 - F_L)}$$

# Model Independent Framework

- The amplitude  $\mathcal{A}(B(p) \rightarrow K^*(k)\ell^+\ell^-)$  [RM, Sinha, Das '14]

$$= \frac{G_F \alpha}{\sqrt{2}\pi} V_{tb} V_{ts}^* \left[ \left\{ C_9 \langle K^* | \bar{s} \gamma^\mu P_L b | \bar{B} \rangle - \frac{2C_7}{q^2} \langle K^* | \bar{s} i \sigma^{\mu\nu} q_\nu (m_b P_R + m_s P_L) b | \bar{B} \rangle \right. \right.$$
$$\left. \left. - \frac{16\pi^2}{q^2} \sum_{i=\{1-6,8\}} C_i \mathcal{H}_i^\mu \right\} \bar{\ell} \gamma_\mu \ell + C_{10} \langle K^* | \bar{s} \gamma^\mu P_L b | \bar{B} \rangle \bar{\ell} \gamma_\mu \gamma_5 \ell \right]$$

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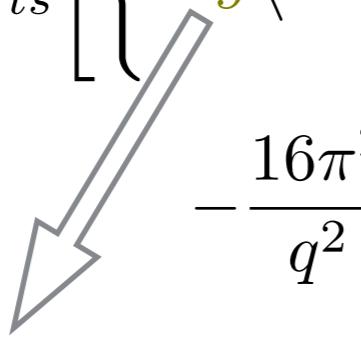
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Wilson coefficients

lorentz & gauge invariance  
allow general parametrization  
with form-factors  $\mathcal{X}_j, \mathcal{Y}_j$

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Wilson coefficients

non-local operator

for non factorization contributions



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$$\mathcal{H}_i^\mu \sim \left\langle K^* | i \int d^4x e^{iq \cdot x} T\{ j_{em}^\mu(x), \mathcal{O}_i(0) \} | \bar{B} \right\rangle \Rightarrow \text{parametrize with 'new' form-factors } \mathcal{Z}_j^i$$

[Khodjamirian *et. al* '10]

# Model Independent Framework

- ▶ Absorbing factorizable & non-factorizable contributions into

$$C_9 \rightarrow \tilde{C}_9^{(j)} = C_9 + \Delta C_9^{(\text{fac})}(q^2) + \Delta C_9^{(j),(\text{non-fac})}(q^2)$$

$$\underbrace{\phantom{\sum_i C_i z_j^i / \chi_j}}_{\sim \sum_i C_i z_j^i / \chi_j}$$

$$\frac{2(m_b+m_s)}{q^2} C_7 \mathcal{Y}_j \rightarrow \tilde{\mathcal{Y}}_j = \frac{2(m_b+m_s)}{q^2} C_7 \mathcal{Y}_j + \dots$$

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$$\underbrace{\Delta C_9^{(\text{fac})}(q^2) + \Delta C_9^{(j),(\text{non-fac})}(q^2)}_{\sim \sum_i \mathcal{C}_i \mathcal{Z}_j^i / \mathcal{X}_j}$$

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- ▶ Most general parametric form of amplitude in SM

$$\mathcal{A}_\lambda^{L,R} = (\tilde{C}_9^\lambda \mp C_{10}) \mathcal{F}_\lambda - \tilde{\mathcal{G}}_\lambda \quad \quad \mathcal{A}_t|_{m_\ell=0} = 0$$

Form-factors:  $\mathcal{F}_\lambda \equiv \mathcal{F}_\lambda(\mathcal{X}_j)$  and  $\tilde{\mathcal{G}}_\lambda \equiv \tilde{\mathcal{G}}_\lambda(\tilde{\mathcal{Y}}_j)$

# Hadronic parameter extraction

- ▶ SM amplitude  $\mathcal{A}_\lambda^{L,R} = (\tilde{C}_9^\lambda \mp C_{10})\mathcal{F}_\lambda - \tilde{\mathcal{G}}_\lambda$
- ▶ Notation  $P_1 = \frac{\mathcal{F}_\perp}{\mathcal{F}_\parallel}$ ,  $P_2 = \frac{\mathcal{F}_\perp}{\mathcal{F}_0}$ ,  $\zeta = \frac{\mathcal{F}_\perp^2 C_{10}^2}{\Gamma_f}$ ,  $u_\lambda^2 = \frac{2}{\Gamma_f} \left( \text{Re}(\tilde{\mathcal{G}}_\lambda) - \text{Re}(\tilde{C}_9^\lambda) \mathcal{F}_\lambda \right)^2$ .
- ▶ 5 independent **parameters** in 5 equations of observables

$$\left. \begin{array}{l} F_\perp = u_\perp^2 + 2\zeta \\ F_L P_2^2 = u_0^2 + 2\zeta \\ A_{FB}^2 = \frac{9\zeta}{2P_1^2} (u_\parallel \pm u_\perp)^2 \\ A_5^2 = \frac{9\zeta}{4P_2^2} (u_0 \pm u_\perp)^2 \\ A_4 = \frac{\sqrt{2}}{\pi P_1 P_2} (2\zeta \pm u_0 u_\parallel) \end{array} \right\} \text{Solution for LHCb data:}$$

$\zeta \rightarrow \mathcal{F}_\perp$  assuming  $C_{10}$  & using BR  $\Gamma_f$   
 $\rightarrow$  conventional form-factor  $V$   
 $P_1 \rightarrow \mathcal{F}_\parallel \rightarrow A_1$   
 $P_2 \rightarrow \mathcal{F}_0 \rightarrow A_{12}$

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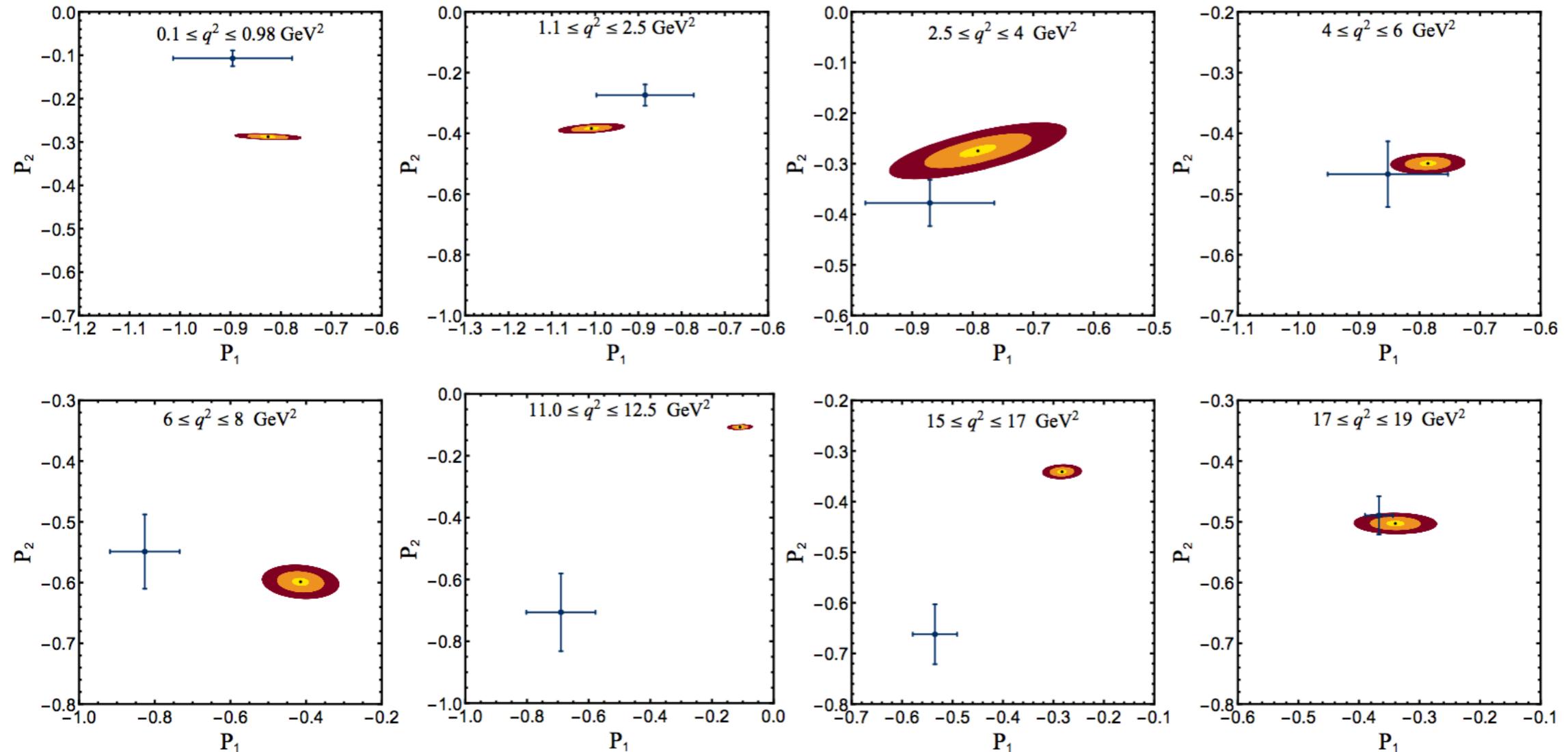
$\rightarrow$  conventional form-factor  $V$

$P_1 \rightarrow \mathcal{F}_\parallel \rightarrow A_1$

$P_2 \rightarrow \mathcal{F}_0 \rightarrow A_{12}$

Unaffected by resonances  
as independent of  $\tilde{C}_9^\lambda$

# Hadronic parameter extraction



form-factors from LCSR [Bharucha et. al '15] & Lattice [Horgan et. al '15]

★ *Significant deviations in several bins*

# Hadronic parameter extraction

$q^2$ range in $\text{GeV}^2$	$V(q^2)$	$A_1(q^2)$	$A_{12}(q^2)$
$0.1 \leq q^2 \leq 0.98$	$0.677 \pm 0.092$ $3.05\sigma$	$0.570 \pm 0.077$ $3.40\sigma$	$0.246 \pm 0.034$ $0.88\sigma$
$1.1 \leq q^2 \leq 2.5$	$0.625 \pm 0.071$ $2.78\sigma$	$0.409 \pm 0.046$ $2.00\sigma$	$0.326 \pm 0.047$ $0.69\sigma$
$6.0 \leq q^2 \leq 8.0$	$0.485 \pm 0.045$ $1.27\sigma$	$0.598 \pm 0.073$ $3.18\sigma$	$0.252 \pm 0.025$ $1.78\sigma$
$11.0 \leq q^2 \leq 12.5$	$0.166 \pm 0.018$ $5.64\sigma$	$0.560 \pm 0.065$ $1.76\sigma$	$0.450 \pm 0.054$ $1.81\sigma$
$15.0 \leq q^2 \leq 17.0$	$0.828 \pm 0.12$ $2.79\sigma$	$0.649 \pm 0.098$ $1.38\sigma$	$0.496 \pm 0.074$ $1.51\sigma$

form-factor values from LCSR  $q^2 \leq 8 \text{ GeV}^2$  & Lattice  $q^2 \geq 11 \text{ GeV}^2$

★ *Significant deviations in several bins*

## Statistical fluctuation

Only resonances

In which  $q^2$  region  
to look at

How to be  
completely sure  
from theory!



Is it new physics

If it is, what kind of

Which variables are  
sensitive

# Right-Handed Current

► Amplitudes  $\mathcal{A}_{\perp}^{L,R} = ((\tilde{C}_9^{\perp} + C'_9) \mp (C_{10} + C'_{10})) \mathcal{F}_{\perp} - \tilde{\mathcal{G}}_{\perp}$

$$\mathcal{A}_{\parallel,0}^{L,R} = ((\tilde{C}_9^{\parallel,0} - C'_9) \mp (C_{10} - C'_{10})) \mathcal{F}_{\parallel,0} - \tilde{\mathcal{G}}_{\parallel,0}$$

► Notation  $r_{\lambda} = \frac{\text{Re}(\tilde{\mathcal{G}}_{\lambda})}{\mathcal{F}_{\lambda}} - \text{Re}(\tilde{C}_9^{\lambda}) \quad \xi = \frac{C'_{10}}{C_{10}} \quad \xi' = \frac{C'_9}{C_{10}}$

► Variables  $R_{\perp} = \frac{\frac{r_{\perp}}{C_{10}} - \xi'}{1 + \xi}, \quad R_{\parallel} = \frac{\frac{r_{\parallel}}{C_{10}} + \xi'}{1 - \xi}, \quad R_0 = \frac{\frac{r_0}{C_{10}} + \xi'}{1 - \xi}.$

► HQET limit  $\frac{\tilde{\mathcal{G}}_{\parallel}}{\mathcal{F}_{\parallel}} = \frac{\tilde{\mathcal{G}}_{\perp}}{\mathcal{F}_{\perp}} = \frac{\tilde{\mathcal{G}}_0}{\mathcal{F}_0} = -\kappa \frac{2m_b m_B C_7}{q^2}, \quad \begin{matrix} [\text{Grinstein, Prijol '04}] \\ [\text{Bobeth et. al '10}] \end{matrix}$



$r_0 = r_{\parallel} = r_{\perp} \equiv r$  ignoring non-factorisable corrections



$R_0 = R_{\parallel} \neq R_{\perp}$  *in presence of RH currents*

# RH Current

At kinematic endpoint



- exact HQET limit
- polarization independent non-factorisable correction

► Observables  $F_L(q_{\max}^2) = \frac{1}{3}$ ,  $F_{\parallel}(q_{\max}^2) = \frac{2}{3}$ ,  $A_4(q_{\max}^2) = \frac{2}{3\pi}$ ,  
 $F_{\perp}(q_{\max}^2) = 0$ ,  $A_{\text{FB}}(q_{\max}^2) = 0$ ,  $A_{5,7,8,9}(q_{\max}^2) = 0$ .

[Hiller, Zwicky '14]

► Taylor series expansion around  $\delta \equiv q_{\max}^2 - q^2$

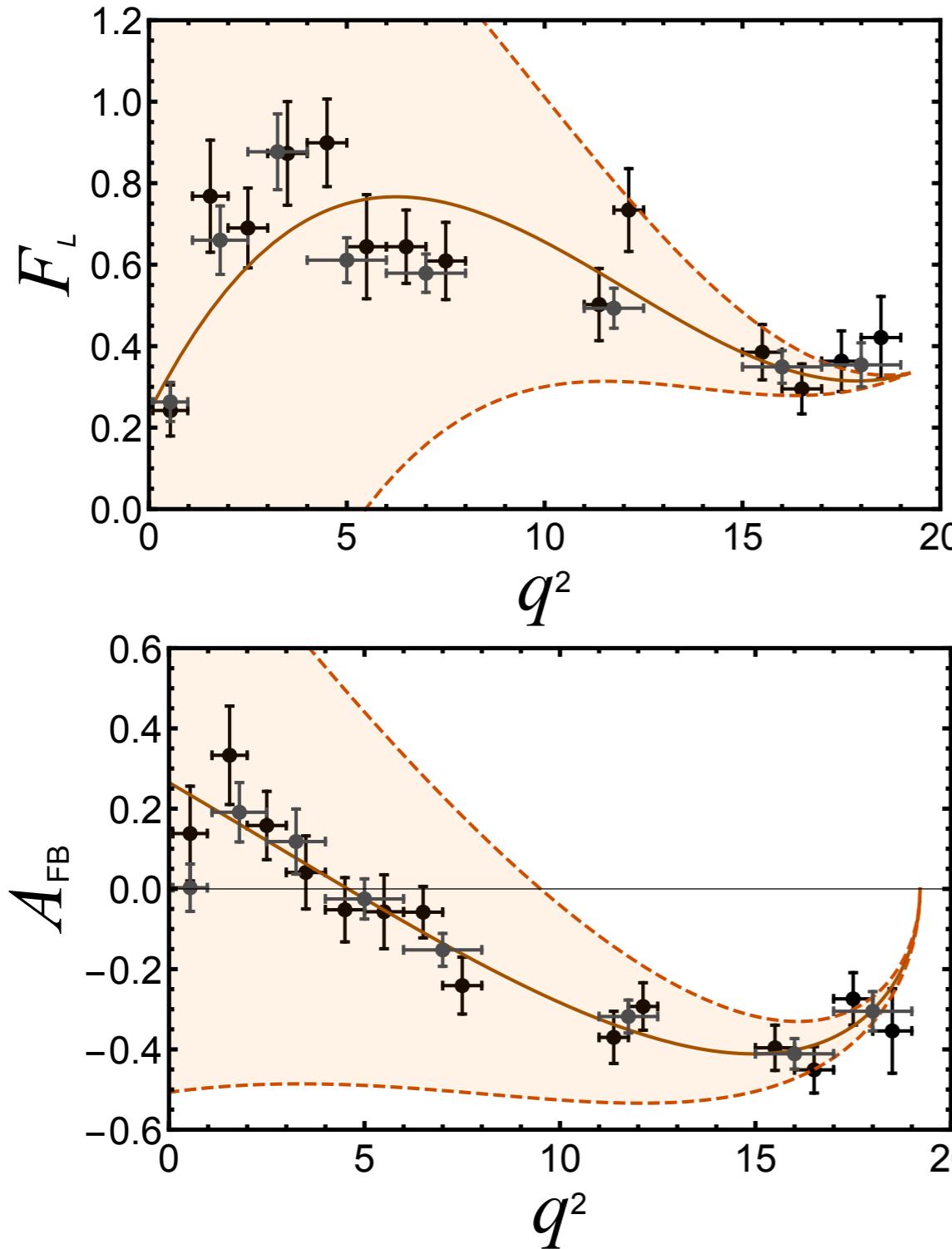
$$F_L = \frac{1}{3} + F_L^{(1)}\delta + F_L^{(2)}\delta^2 + F_L^{(3)}\delta^3$$

$$F_{\perp} = F_{\perp}^{(1)}\delta + F_{\perp}^{(2)}\delta^2 + F_{\perp}^{(3)}\delta^3$$

$$A_{\text{FB}} = A_{\text{FB}}^{(1)}\delta^{\frac{1}{2}} + A_{\text{FB}}^{(2)}\delta^{\frac{3}{2}} + A_{\text{FB}}^{(3)}\delta^{\frac{5}{2}}$$

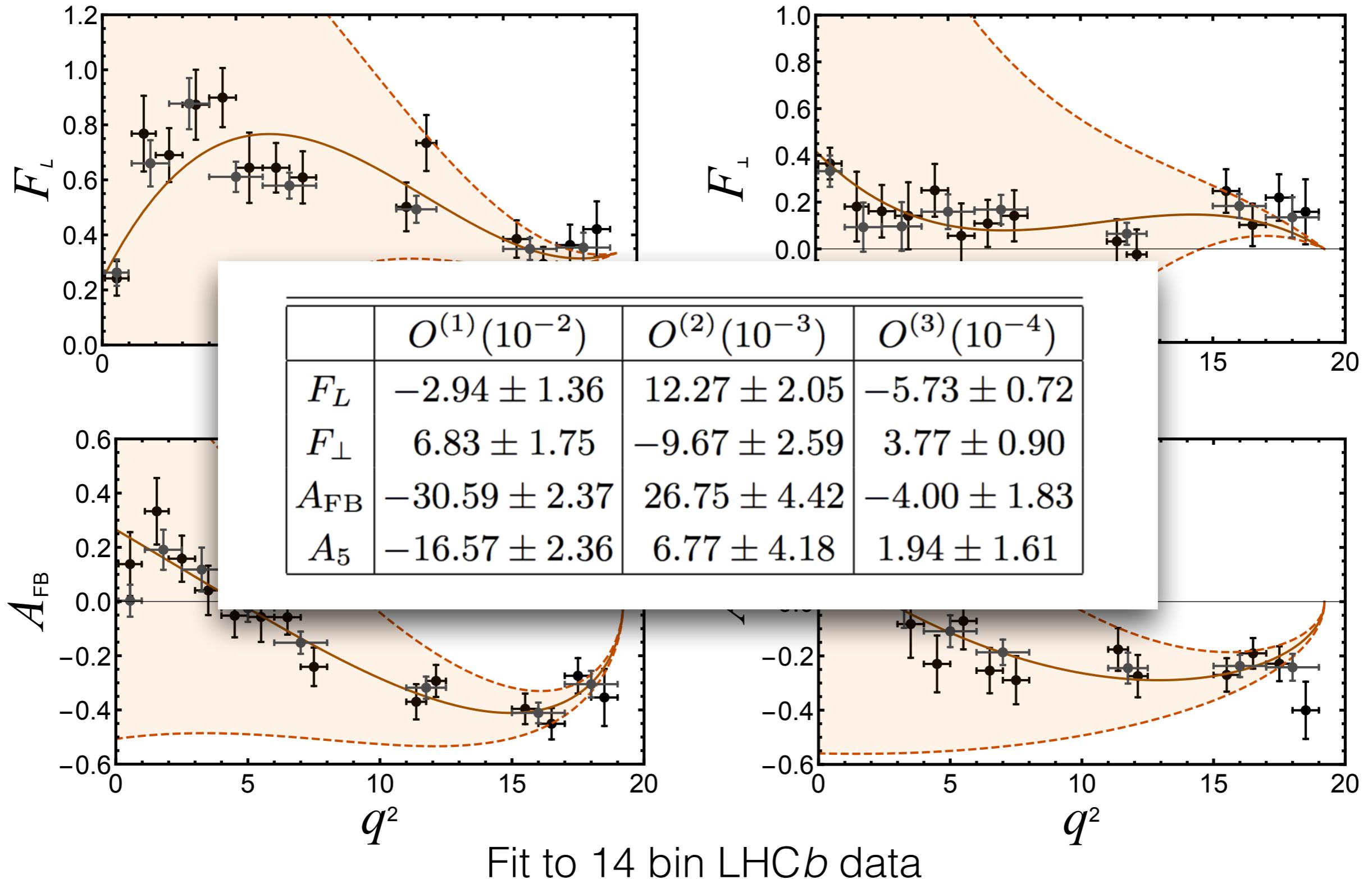
$$A_5 = A_5^{(1)}\delta^{\frac{1}{2}} + A_5^{(2)}\delta^{\frac{3}{2}} + A_5^{(3)}\delta^{\frac{5}{2}},$$

# RH Current



Fit to 14 bin LHC $b$  data

# RH Current

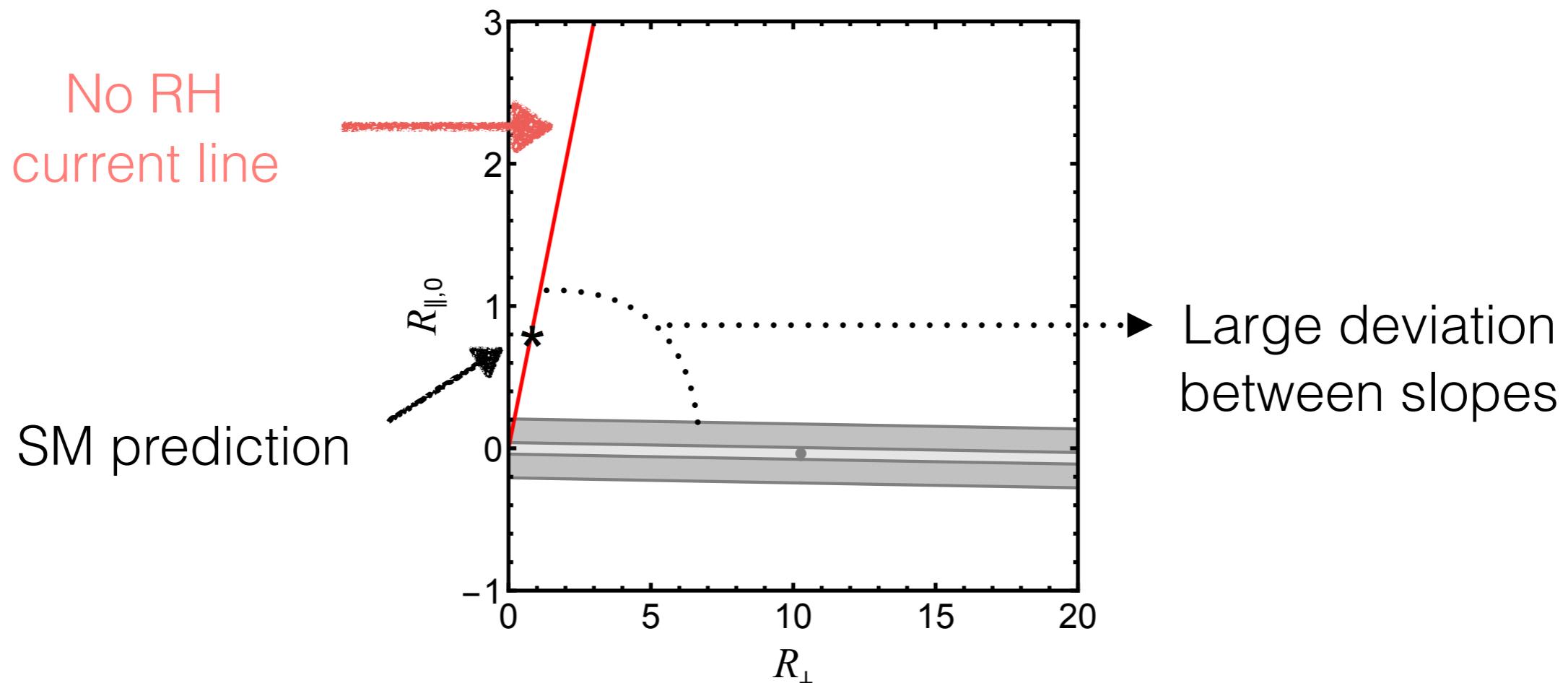


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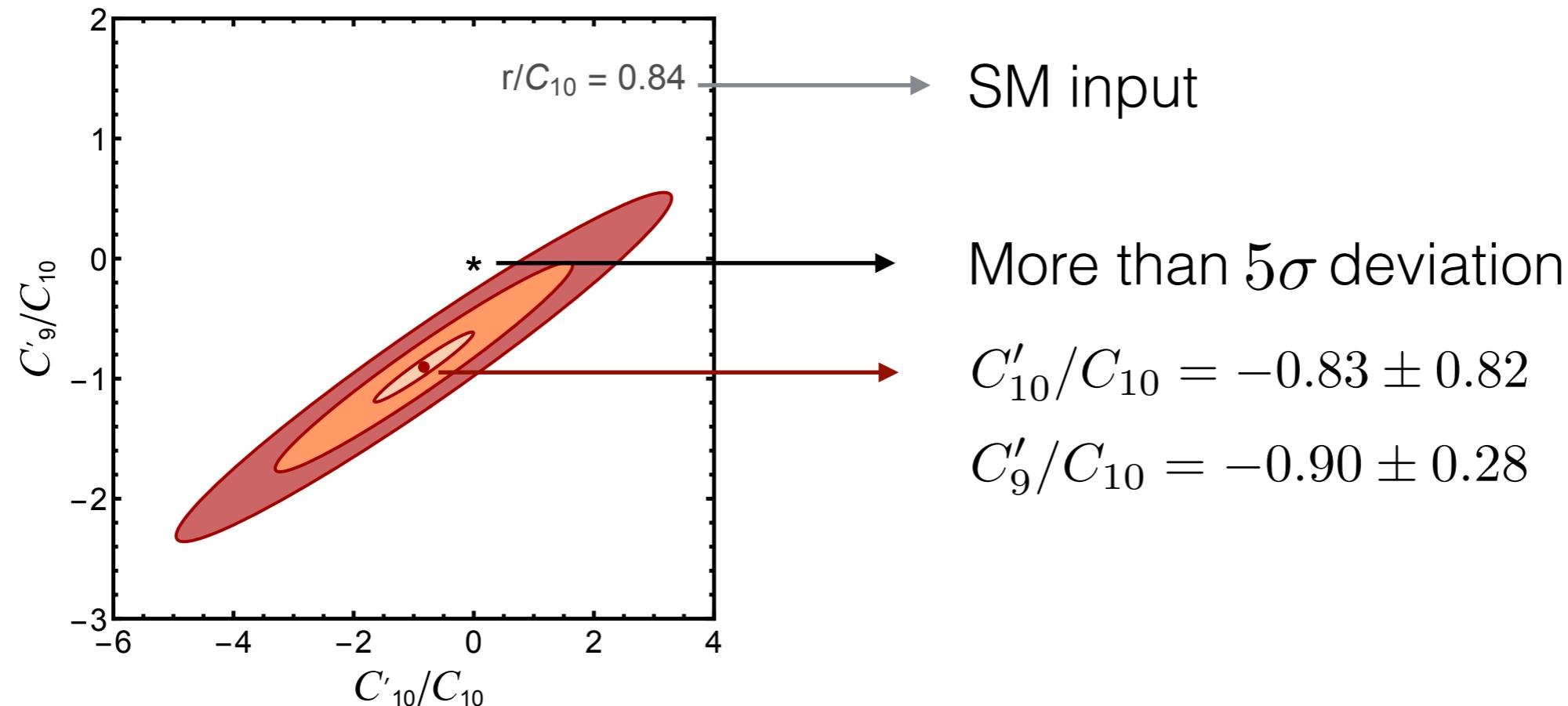
► Limiting analytic expressions

$$R_{\perp}(q_{\max}^2) = \frac{\omega_2 - \omega_1}{\omega_2 \sqrt{\omega_1 - 1}}, \quad R_{\parallel}(q_{\max}^2) = \frac{\sqrt{\omega_1 - 1}}{\omega_2 - 1} = R_0(q_{\max}^2)$$

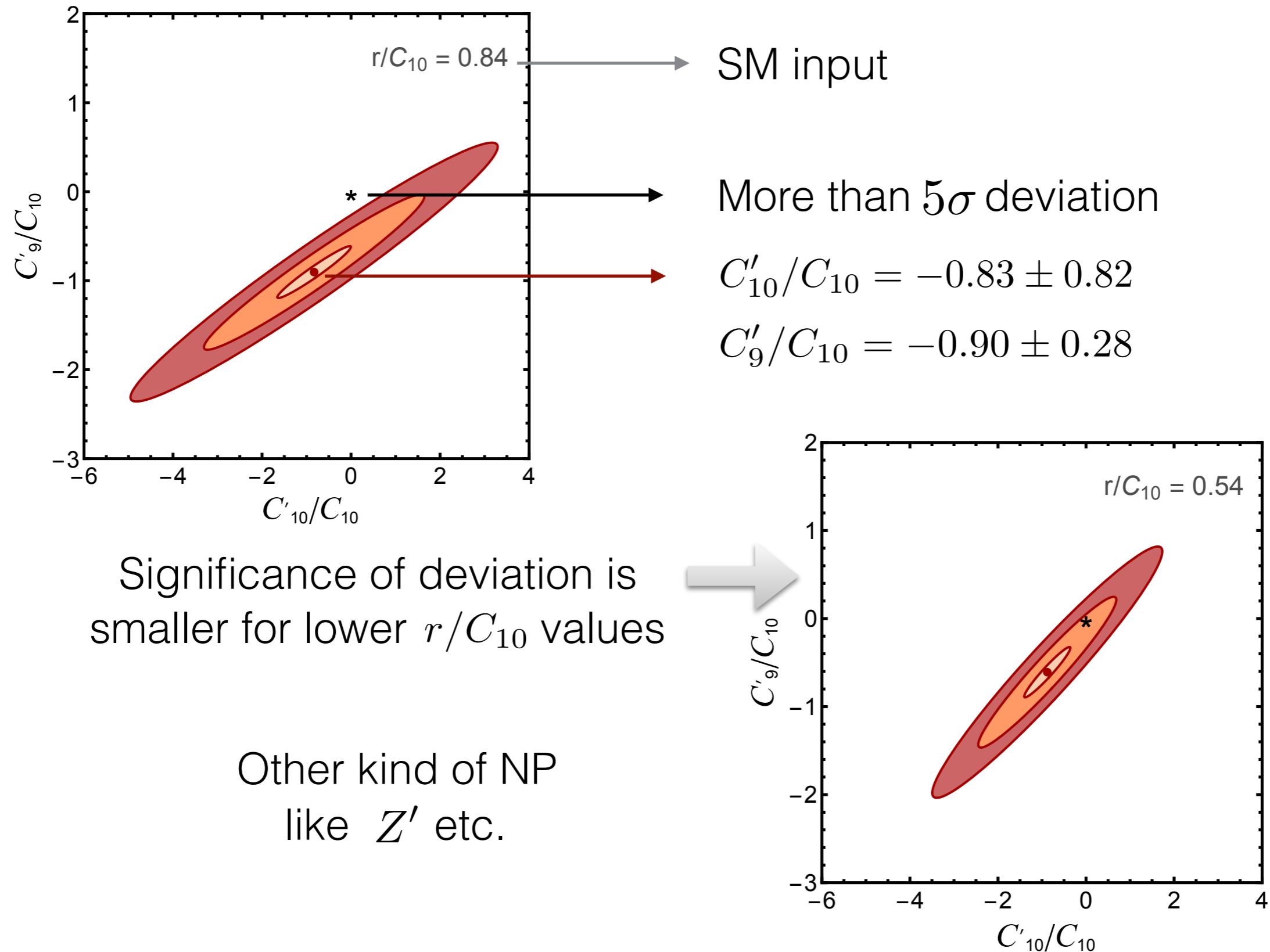
with  $\omega_1 = \frac{3}{2} \frac{F_{\perp}^{(1)}}{A_{\text{FB}}^{(1)2}}$  and  $\omega_2 = \frac{4(2A_5^{(2)} - A_{\text{FB}}^{(2)})}{3A_{\text{FB}}^{(1)}(3F_L^{(1)} + F_{\perp}^{(1)})}$ .



# Results in $C'_{10}/C_{10} - C'_9/C_{10}$



# Results in $C'_{10}/C_{10} - C'_9/C_{10}$



# Summary

- ☒ Formalism developed to include all possible effects within SM
- ☒ Approach differs due to no/minimal assumption on hadronic estimates
- ☒ Discrepancies in form-factors extracted from data —
  - ▶ complex contributions of the amplitude are included
  - ▶ systematics added for bin-bias
  - ▶ resonances can't affect all of them by definition
- ☒ Strong evidence of RH currents derived at endpoint limit —
  - ▶ systematics studied by varying polynomial order & bin no.
  - ▶ finite  $K^*$  width effect considered
  - ▶ resonance systematics & experimental correlation can reduce significance of deviation
- ☒ Fluctuation? Wait for more data to be accumulated!

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*Thank you!*

Back Up

# Hadronic parameter extraction

- 3 independent form-factors at low recoil of  $K^*$

[Grinstein, Prijol '04]  
[Bobeth *et. al* '10]

$$\frac{V(q^2)}{T_1(q^2)} = \frac{A_1(q^2)}{T_2(q^2)} = \frac{A_2(q^2)}{T_3(q^2)} \frac{m_B^2}{q^2}$$



$$u_0 = u_{\parallel} = u_{\perp}$$

ignoring non-factorisable contributions

$q^2$ range in $\text{GeV}^2$	$u_0$	$u_{\parallel}$	$u_{\perp}$
$15 \leq q^2 \leq 17$	$0.000 \pm 0.016$	$0.013 \pm 0.153$	$0.367 \pm 0.025$
$17 \leq q^2 \leq 19$	$0.166 \pm 0.014$	$0.000 \pm 4.579$	$0.260 \pm 0.048$
$15 \leq q^2 \leq 19$	$0.120 \pm 0.007$	$0.004 \pm 0.441$	$0.244 \pm 0.026$

★ not satisfied with  $3 \text{ fb}^{-1}$  LHCb data

# Complex part of amplitudes

- ▶ SM amplitude  $\mathcal{A}_\lambda^{L,R} = (\tilde{C}_9^\lambda \mp C_{10})\mathcal{F}_\lambda - \tilde{\mathcal{G}}_\lambda$
- ▶ Complex part  $\varepsilon_\lambda \equiv \text{Im}(\tilde{C}_9^\lambda)\mathcal{F}_\lambda - \text{Im}(\tilde{\mathcal{G}}_\lambda)$
- ▶ Iterative solutions

$$\varepsilon_\perp = \frac{\sqrt{2}\pi\Gamma_f}{(r_0 - r_\parallel)\mathcal{F}_\perp} \left[ \frac{A_9\mathsf{P}_1}{3\sqrt{2}} + \frac{A_8\mathsf{P}_2}{4} - \frac{A_7\mathsf{P}_1\mathsf{P}_2 r_\perp}{3\pi\hat{C}_{10}} \right],$$

$$\varepsilon_\parallel = \frac{\sqrt{2}\pi\Gamma_f}{(r_0 - r_\parallel)\mathcal{F}_\perp} \left[ \frac{A_9 r_0}{3\sqrt{2}r_\perp} + \frac{A_8\mathsf{P}_2 r_\parallel}{4\mathsf{P}_1 r_\perp} - \frac{A_7\mathsf{P}_2 r_\parallel}{3\pi\hat{C}_{10}} \right],$$

$$\varepsilon_0 = \frac{\sqrt{2}\pi\Gamma_f}{(r_0 - r_\parallel)\mathcal{F}_\perp} \left[ \frac{A_9\mathsf{P}_1 r_0}{3\sqrt{2}\mathsf{P}_2 r_\perp} + \frac{A_8 r_\parallel}{4r_\perp} - \frac{A_7\mathsf{P}_1 r_0}{3\pi\hat{C}_{10}} \right].$$

# Complex part of amplitudes

$q^2$ range in $\text{GeV}^2$	$\varepsilon_{\perp}/\sqrt{\Gamma_f}$	$\varepsilon_{\parallel}/\sqrt{\Gamma_f}$	$\varepsilon_0/\sqrt{\Gamma_f}$
$0.1 \leq q^2 \leq 0.98$	$-0.048 \pm 0.116$	$-0.047 \pm 0.103$	$0.020 \pm 0.111$
$1.1 \leq q^2 \leq 2.5$	$-0.010 \pm 0.078$	$-0.010 \pm 0.078$	$0.078 \pm 0.172$
$2.5 \leq q^2 \leq 4.0$	$-0.009 \pm 0.079$	$-0.008 \pm 0.080$	$-0.025 \pm 0.212$
$4.0 \leq q^2 \leq 6.0$	$-0.026 \pm 0.097$	$0.014 \pm 0.093$	$0.032 \pm 0.234$
$6.0 \leq q^2 \leq 8.0$	$-0.011 \pm 0.088$	$-0.046 \pm 0.078$	$-0.132 \pm 0.129$
$11.0 \leq q^2 \leq 12.5$	$-0.011 \pm 0.050$	$0.038 \pm 0.074$	$-0.078 \pm 0.114$
$15.0 \leq q^2 \leq 17.0$	$-0.0003 \pm 0.067$	$-0.027 \pm 0.071$	$0.020 \pm 0.072$
$17.0 \leq q^2 \leq 19.0$	$0.006 \pm 0.076$	$-0.090 \pm 0.090$	$-0.040 \pm 0.088$

$\frac{\varepsilon_{\lambda}}{\sqrt{\Gamma_f}}$  values with errors are consistent with zero

# Chi-square

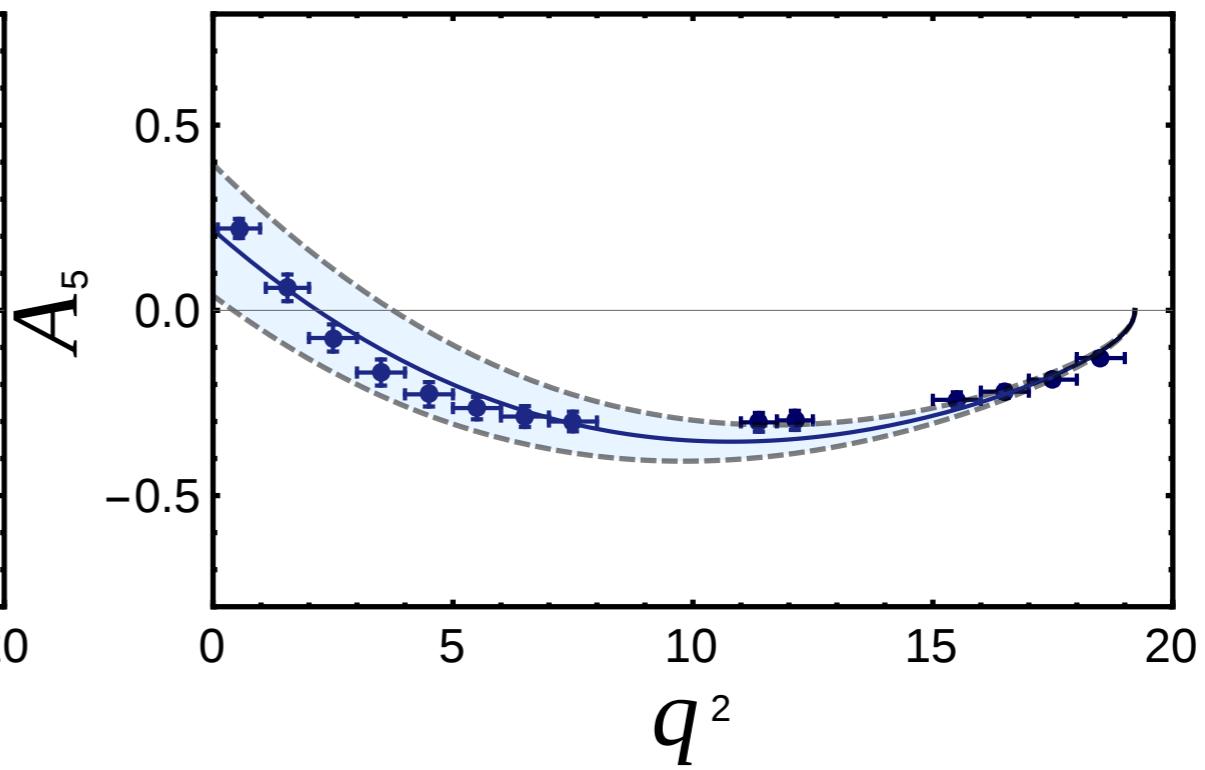
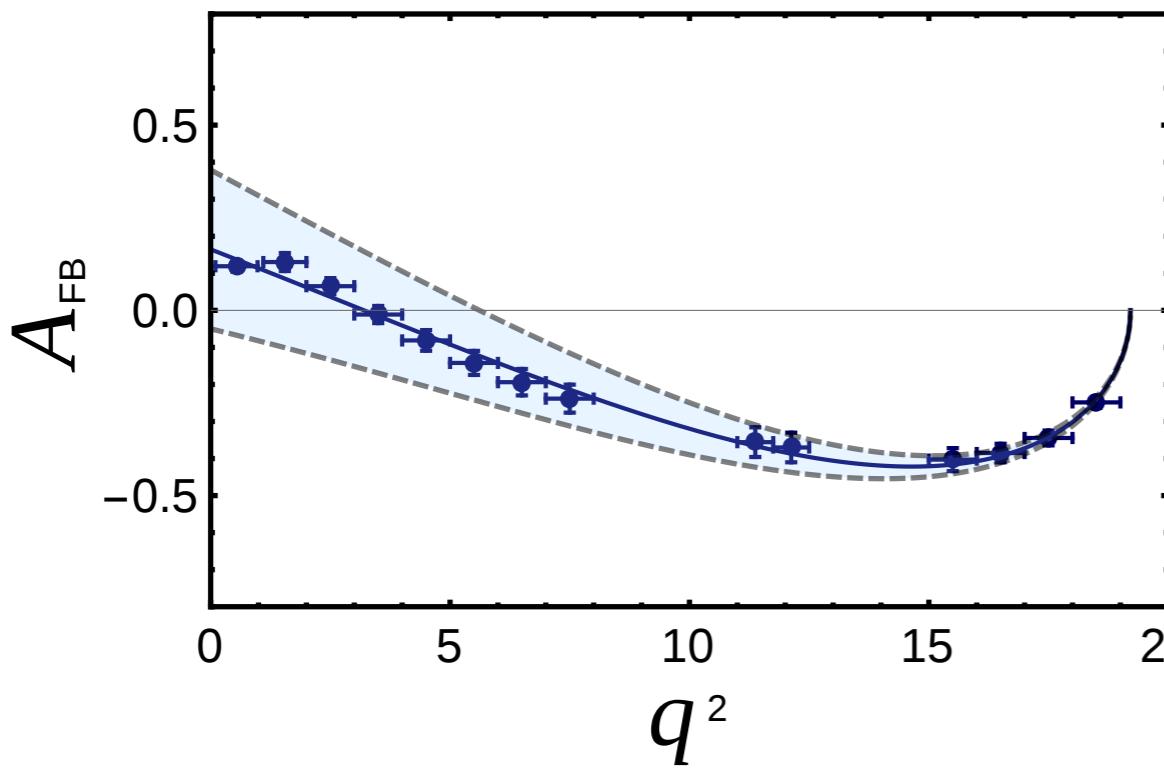
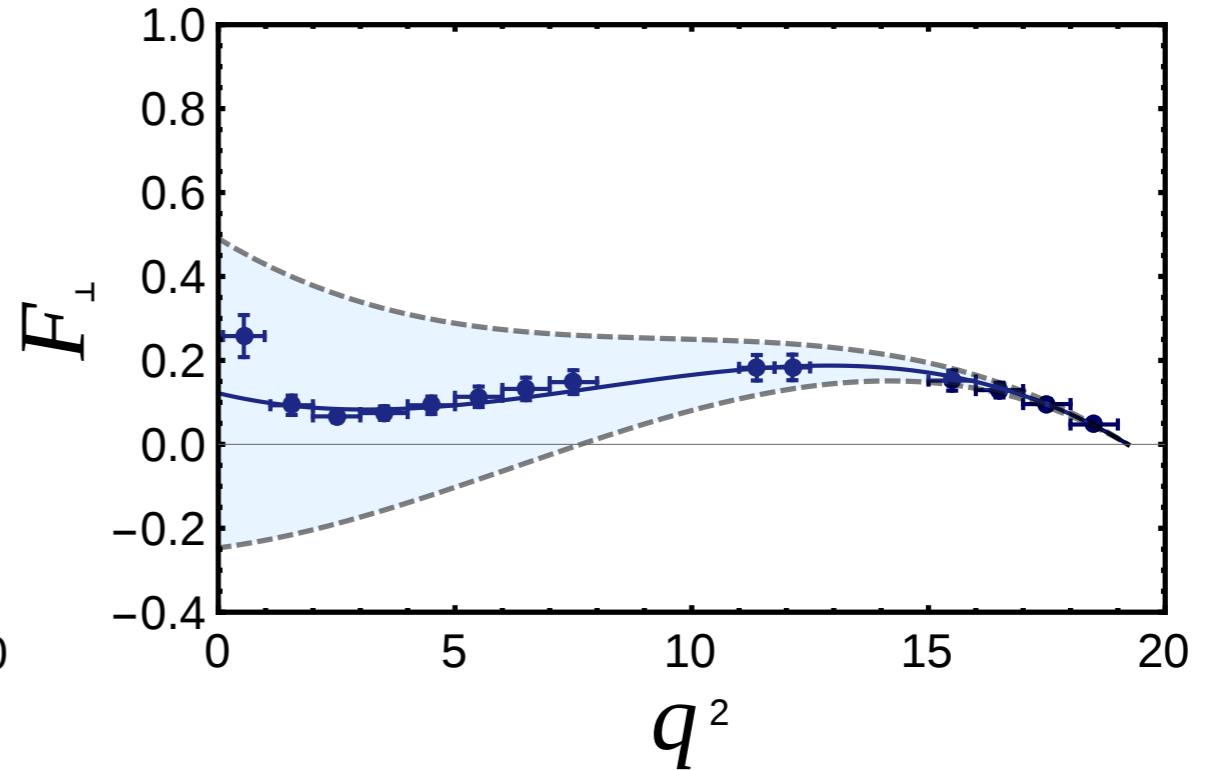
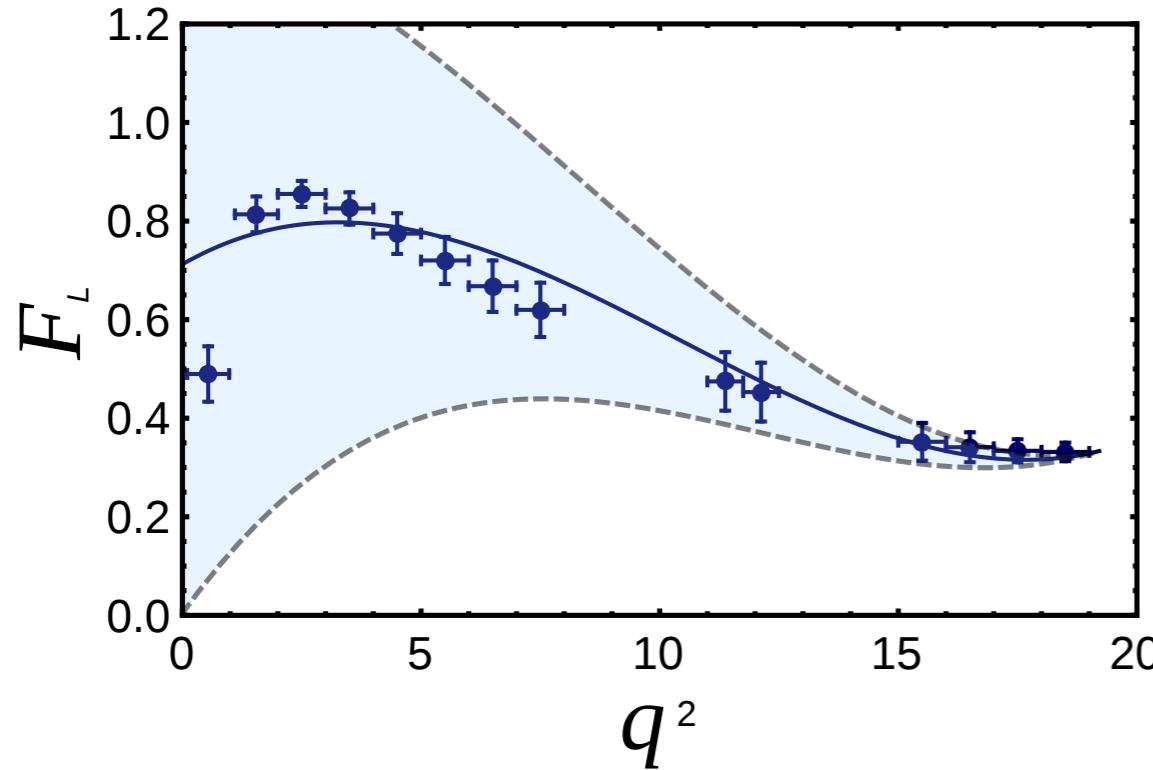
$$\chi^2 = \left( \frac{F_{\perp}^{\text{ex}} - F_{\perp}}{\Delta F_{\perp}^{\text{ex}}} \right)^2 + \left( \frac{F_L^{\text{ex}} - F_L}{\Delta F_L^{\text{ex}}} \right)^2 + \left( \frac{A_4^{\text{ex}} - A_4}{\Delta A_4^{\text{ex}}} \right)^2 + \left( \frac{A_{\text{FB}}^{2\text{ex}} - A_{\text{FB}}^2}{2A_{\text{FB}}^{\text{ex}} \Delta A_{\text{FB}}^{\text{ex}}} \right)^2 + \left( \frac{A_5^{2\text{ex}} - A_5^2}{2A_5^{\text{ex}} \Delta A_5^{\text{ex}}} \right)^2$$

- Contributions from complex part of amplitudes are considered.
- Systematics are added for each observables due to bin average effect with the introduction of new parameter  $\beta$  (nuisance).

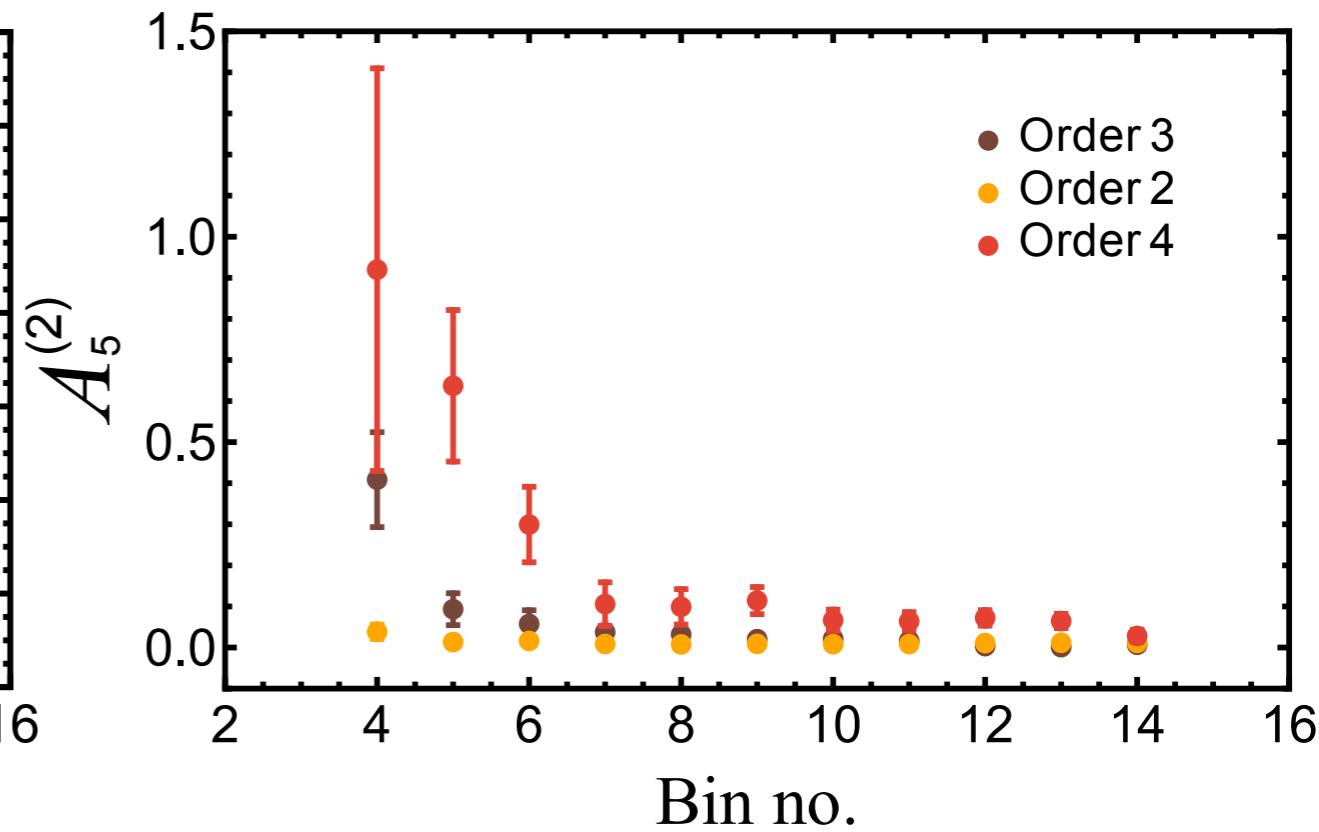
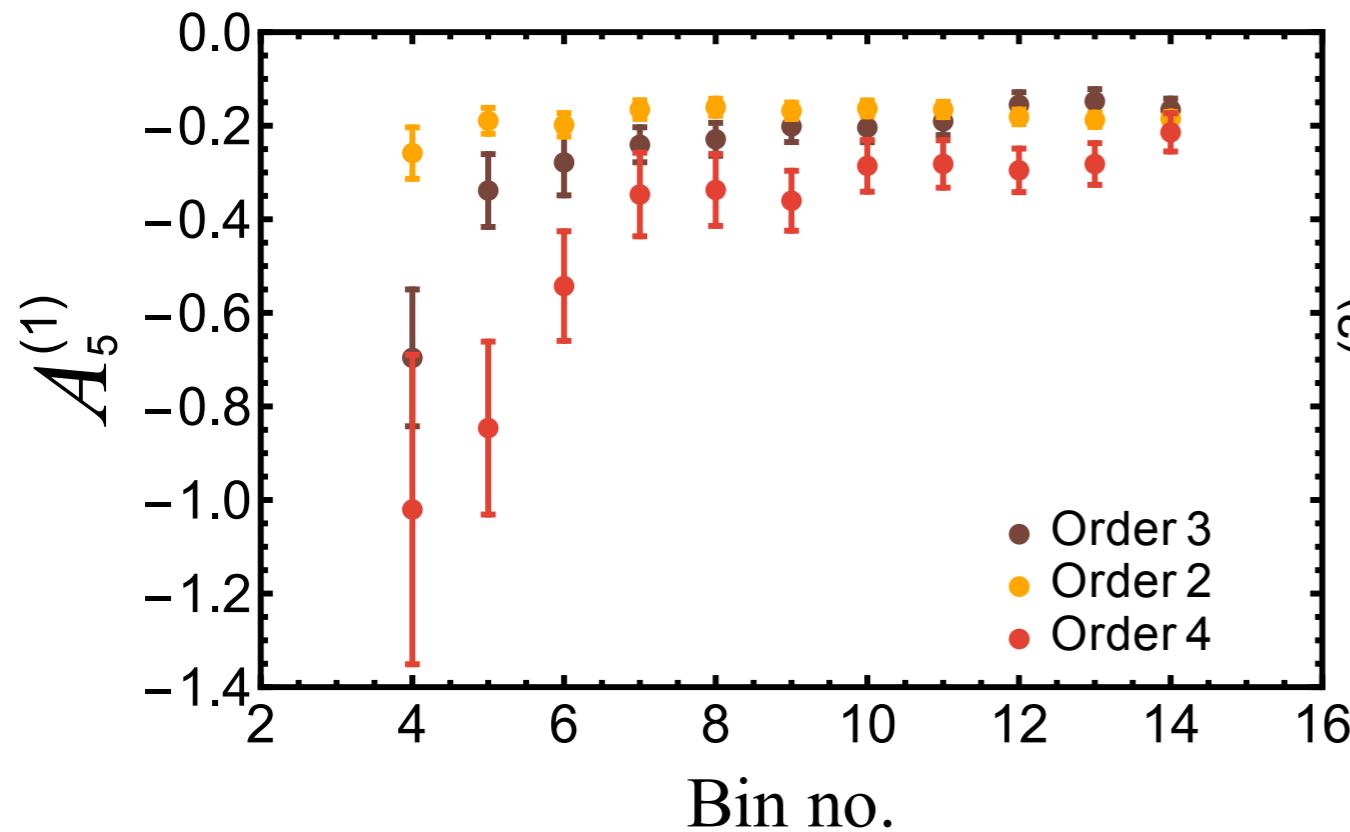
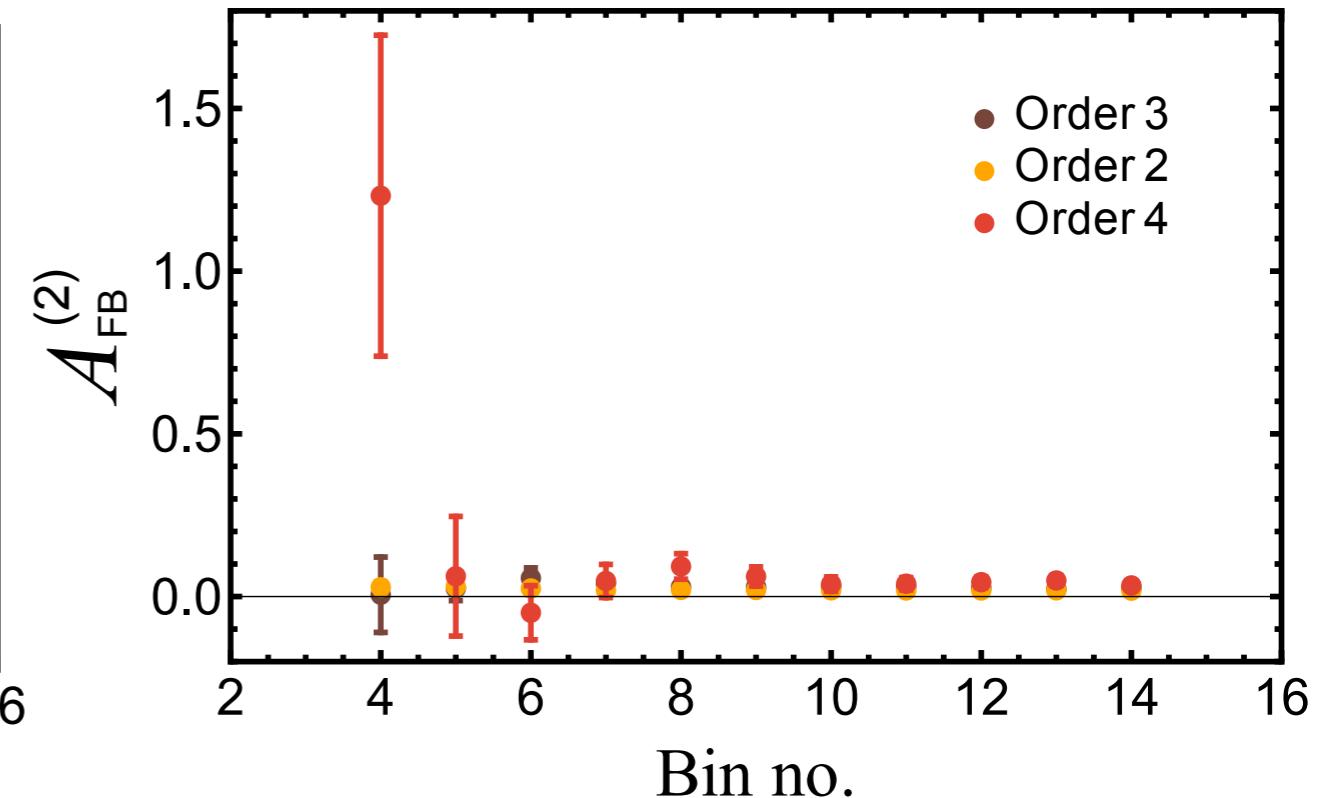
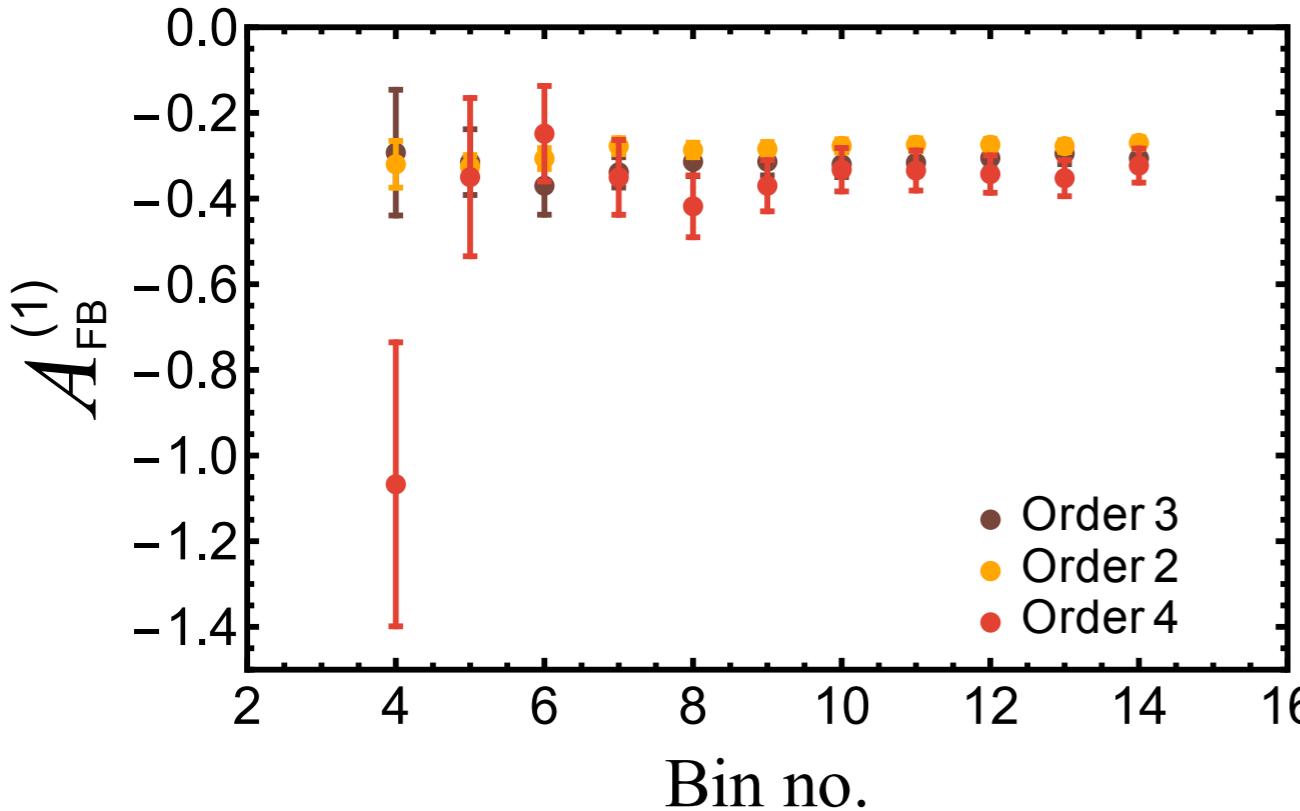
$$\mathcal{O} \rightarrow \mathcal{O} + \beta \mathcal{O}_{\text{shift}}$$

$\mathcal{O}_{\text{shift}}$ : maximum deviation of bin-average value with fitted  $q^2$  function

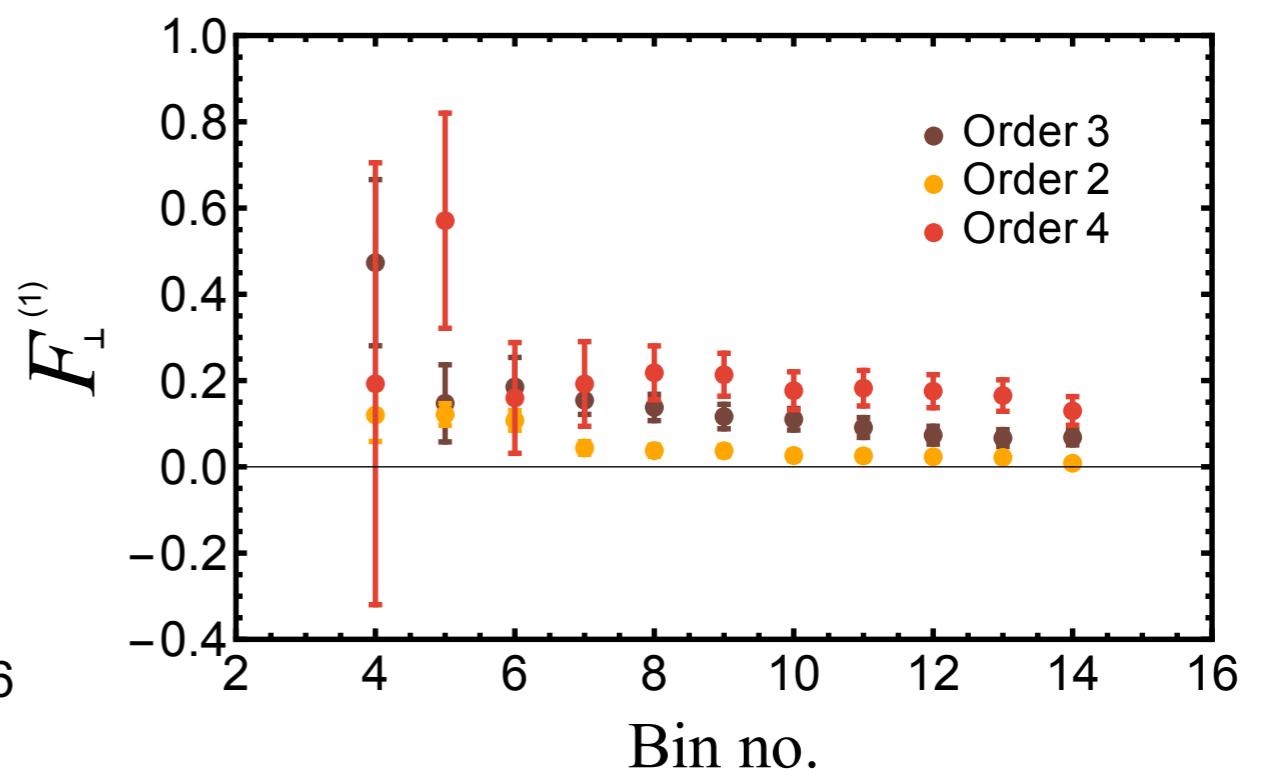
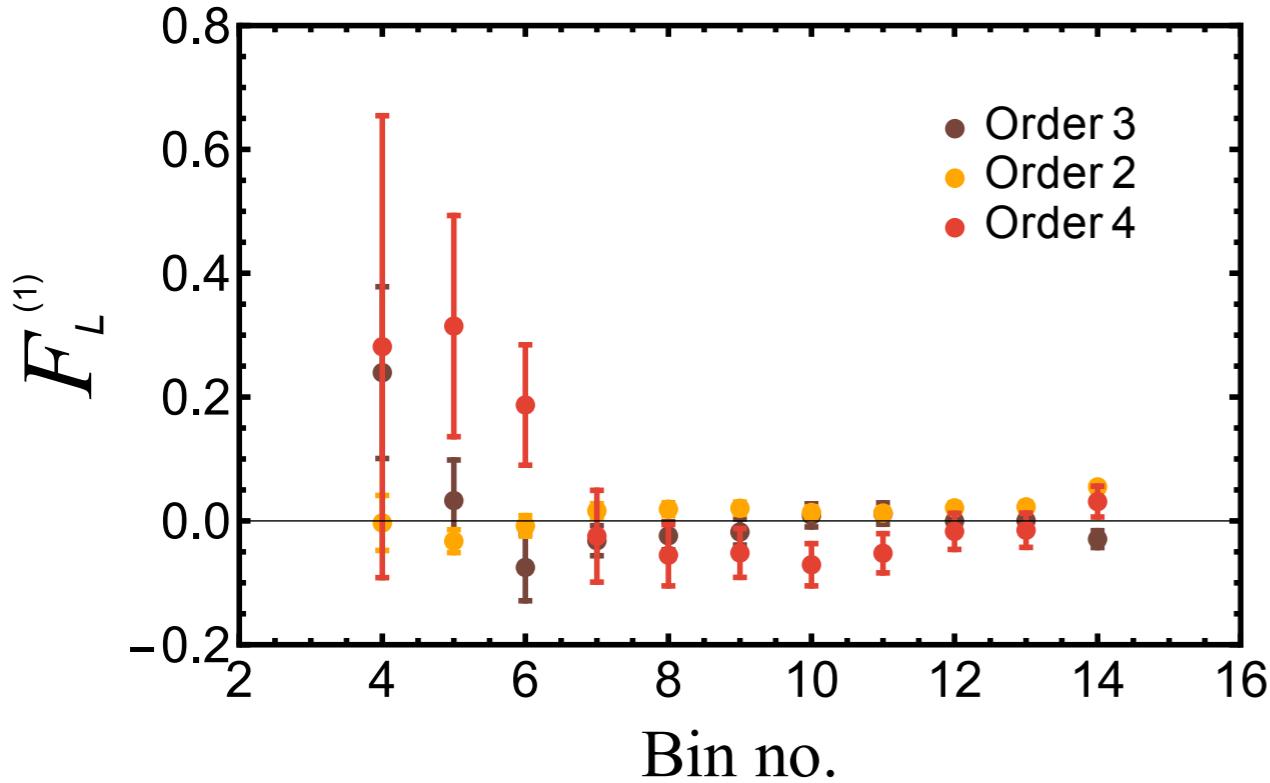
# Fit to form factor observables



# Convergence of coefficients



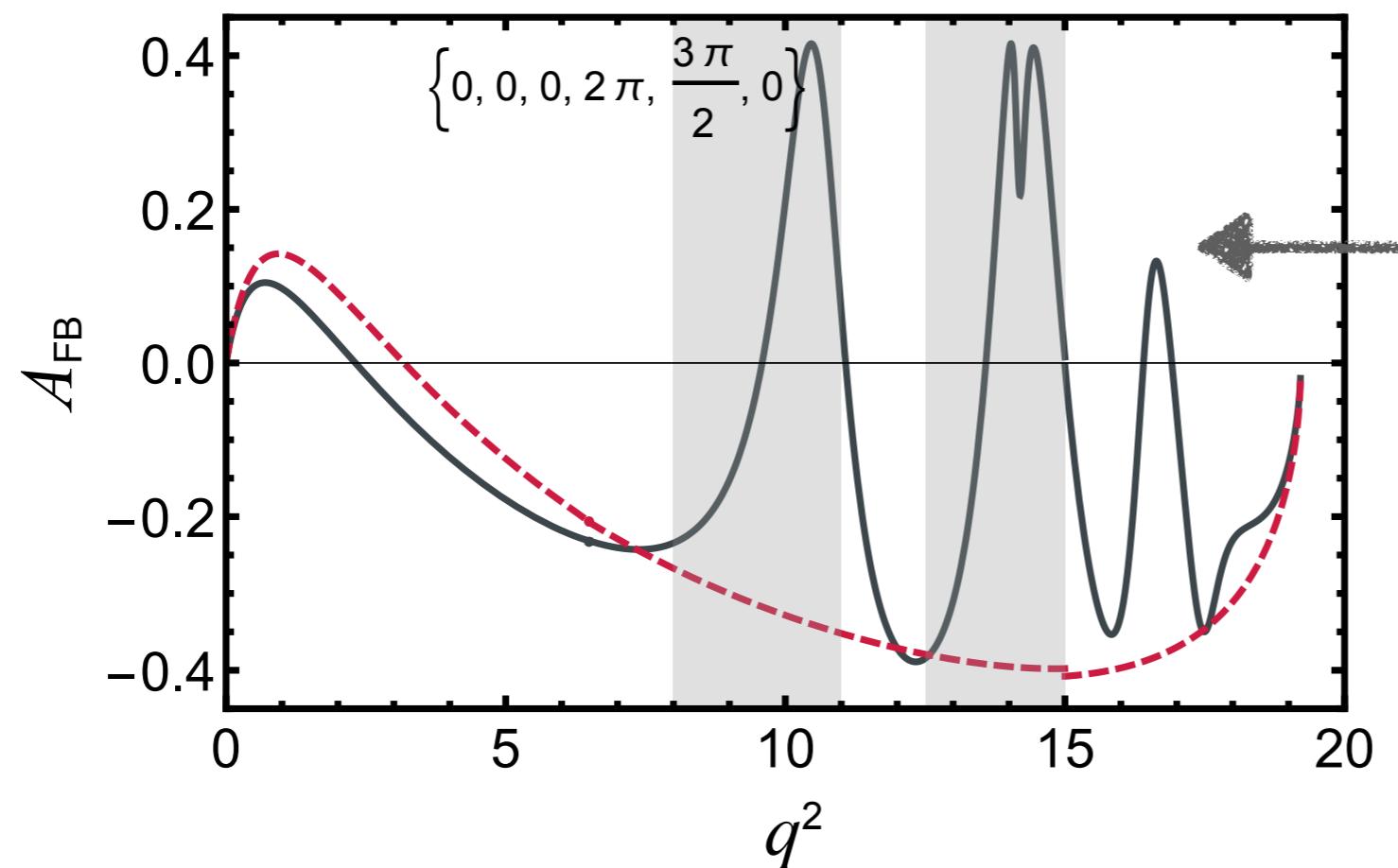
# Convergence of coefficients



# Resonances

$c\bar{c}$  bound states added:  $J/\psi, \psi(2S), \psi(3770), \psi(4040), \psi(4160), \psi(4415)$ .

Observable = Form-factors + Kruger & Sehgal parametrization



Asymmetries decrease  
in high  $q^2$  region

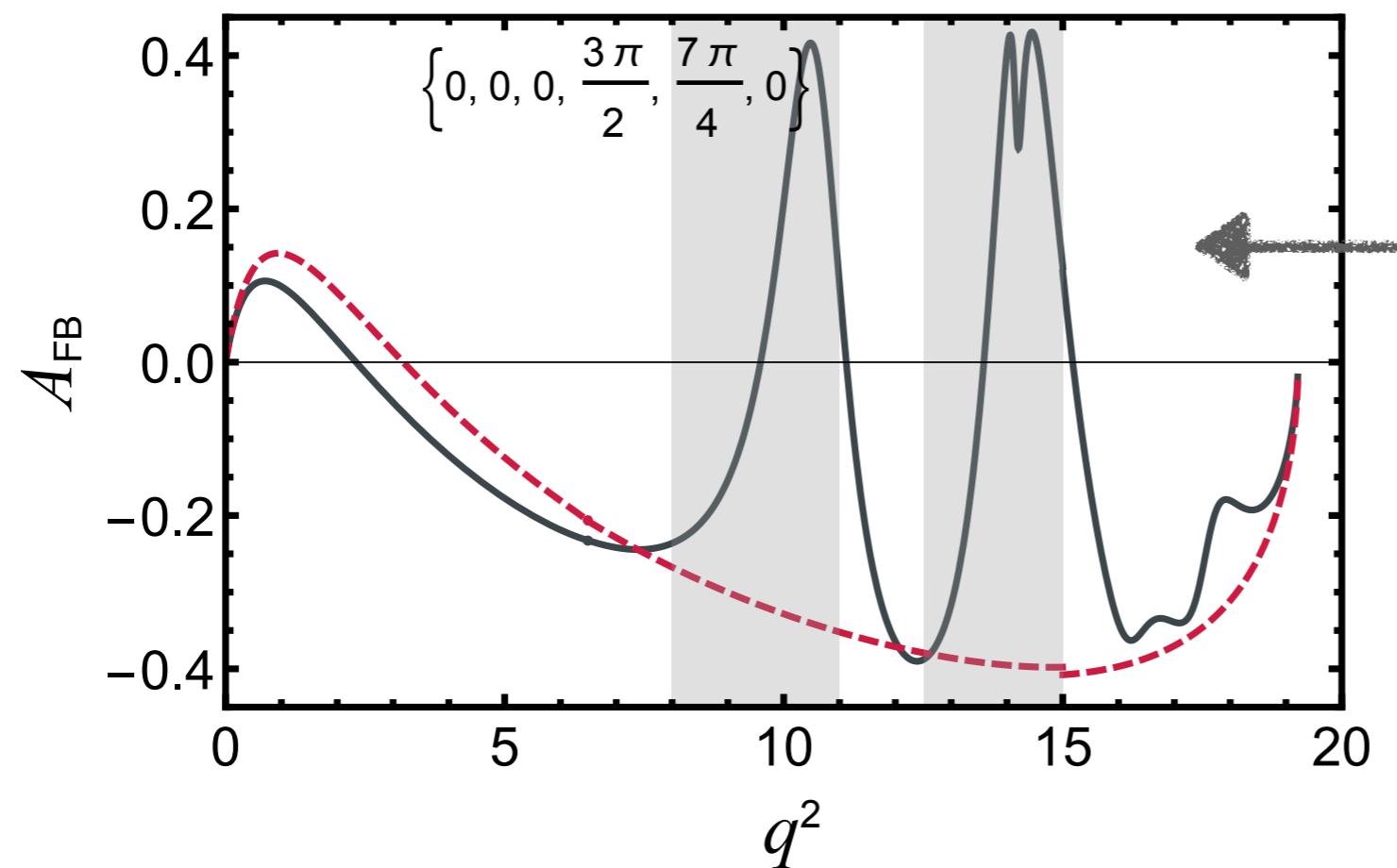
makes observable  
 $\omega_1$  unphysical

Random variation of each strong phases

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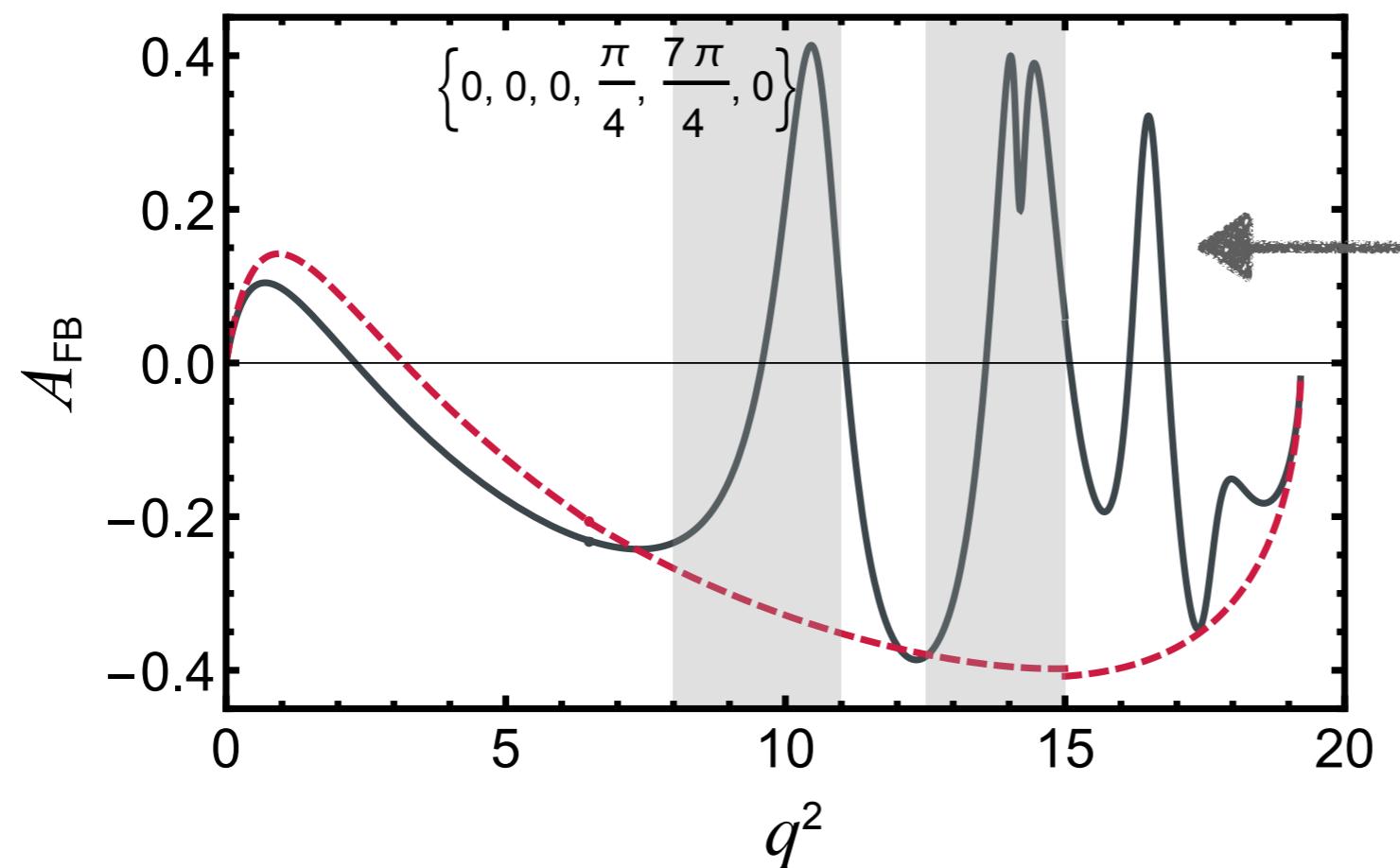
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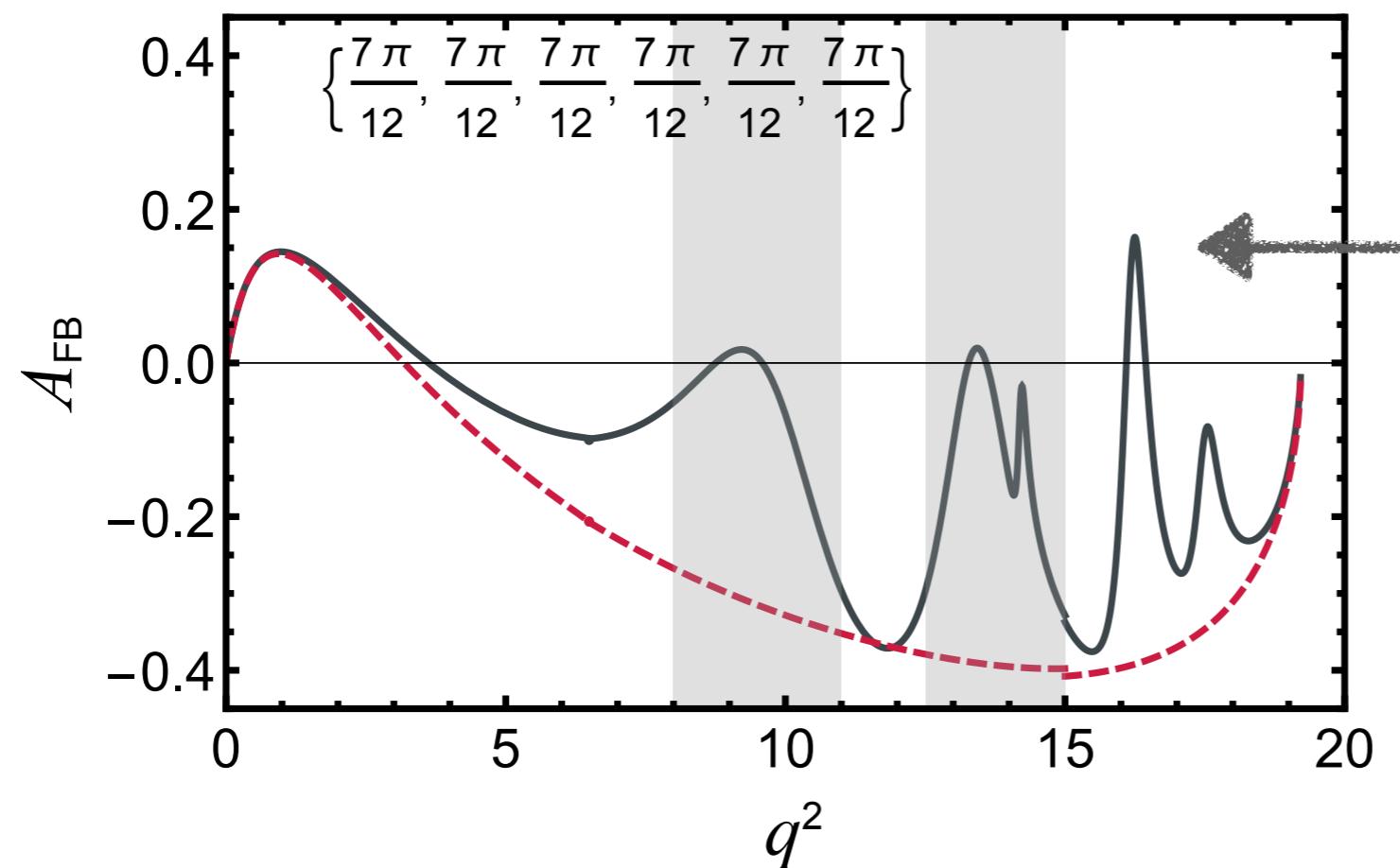
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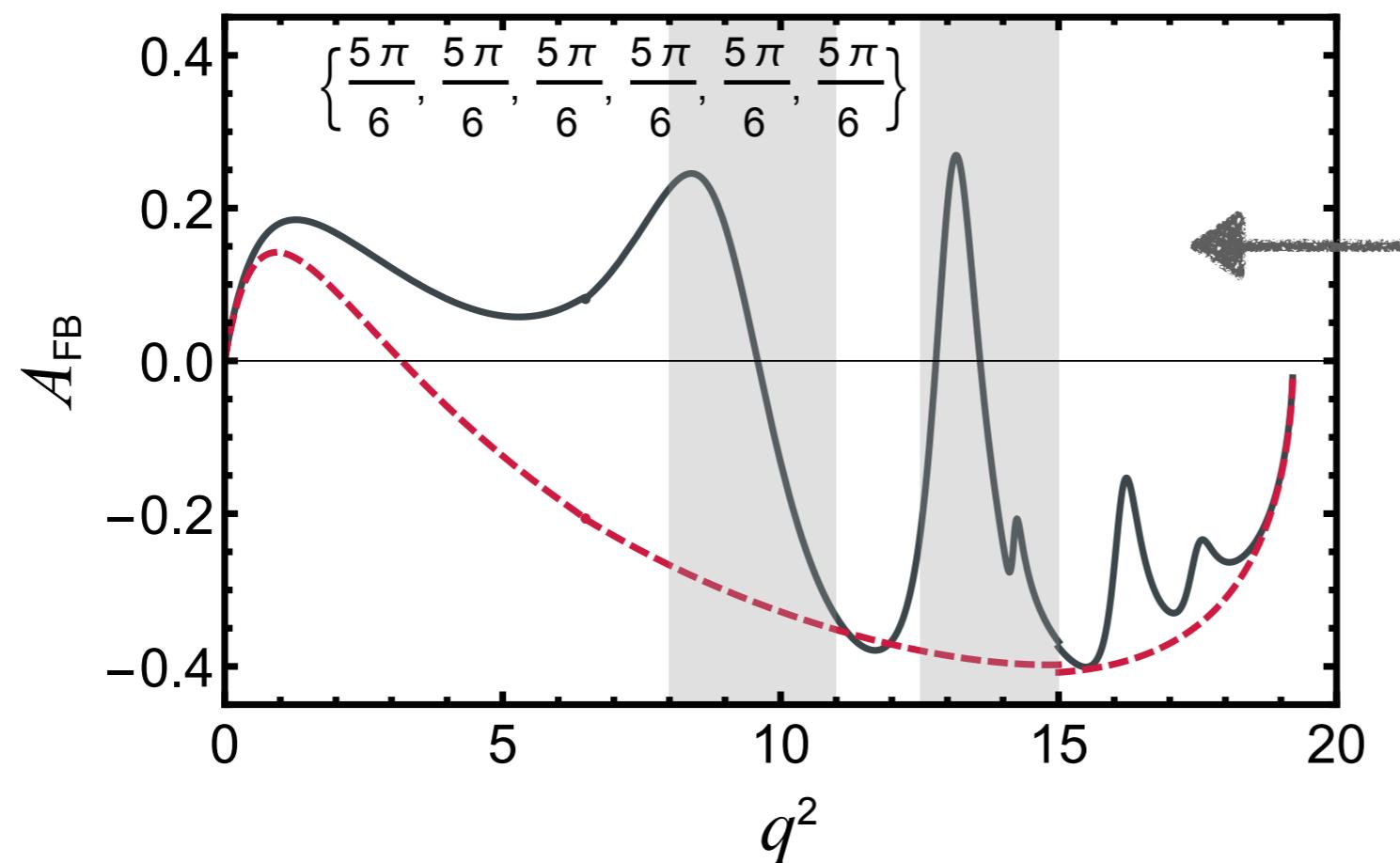
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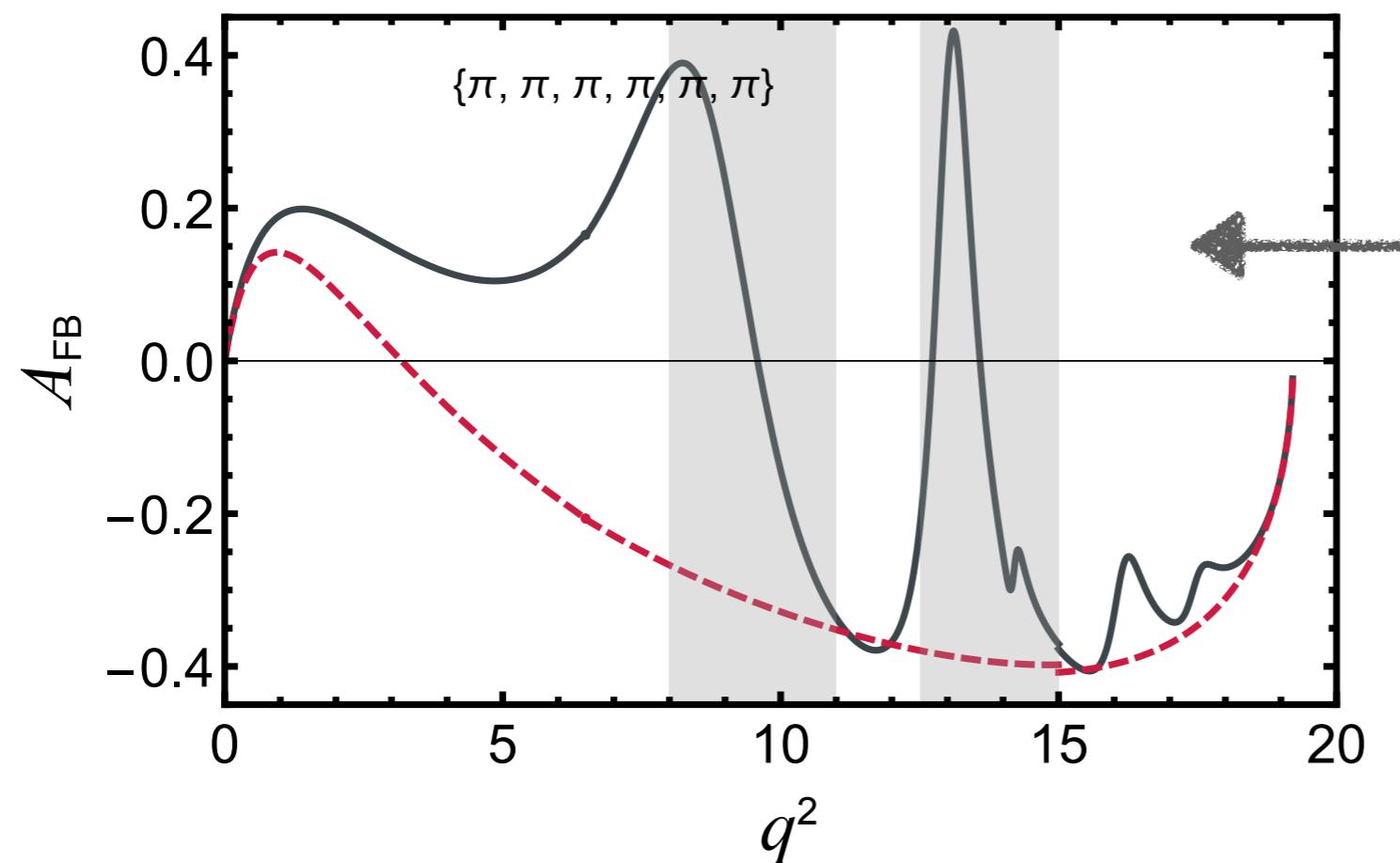
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Random variation of each strong phases

# $\omega_1$ - $\omega_2$ values

$$\omega_1 = \frac{3}{2} \frac{F_{\perp}^{(1)}}{A_{\text{FB}}^{(1)2}} \quad \text{or} \quad \frac{3}{8} \frac{F_{\perp}^{(1)}}{A_5^{(1)2}} \quad \text{and} \quad \omega_2 = \frac{4 \left( 2A_5^{(2)} - A_{\text{FB}}^{(2)} \right)}{3 A_{\text{FB}}^{(1)} \left( 3F_L^{(1)} + F_{\perp}^{(1)} \right)} \quad \text{or} \quad \frac{4 \left( 2A_5^{(2)} - A_{\text{FB}}^{(2)} \right)}{6 A_5^{(1)} \left( 3F_L^{(1)} + F_{\perp}^{(1)} \right)}$$

	Real limit	Complex limit	Adding finite $K^*$ width
$\omega_1$	$1.09 \pm 0.33$	$0.98 \pm 0.33$	$1.18 \pm 0.35$
	$0.93 \pm 0.36$	$0.85 \pm 0.30$	$1.02 \pm 0.40$
$\omega_2$	$-2.87 \pm 6.69$	$-2.85 \pm 12.54$	$-2.48 \pm 5.95$
	$-2.65 \pm 6.18$	$-2.59 \pm 6.22$	$-2.30 \pm 5.51$

# Observables & Relations

Our observables  $\longleftrightarrow$  LHCb

$$F_{\perp} = \frac{1}{2}(1 - F_L + 2S_3), \quad A_4 = -\frac{2}{\pi}S_4, \quad A_5 = \frac{3}{4}S_5,$$
$$A_{FB} = -A_{FB}^{\text{LHCb}}, \quad A_7 = \frac{3}{4}S_7, \quad A_8 = -\frac{2}{\pi}S_8, \quad A_9 = \frac{3}{2\pi}S_9.$$

$$A_4 = \frac{2\sqrt{2}\varepsilon_{\parallel}\varepsilon_0}{\pi\Gamma_f} + \frac{8A_5A_{FB}}{9\pi\left(F_{\perp} - \frac{2\varepsilon_{\perp}^2}{\Gamma_f}\right)}$$
$$+ \sqrt{2} \frac{\sqrt{\left(F_L - \frac{2\varepsilon_0^2}{\Gamma_f}\right)\left(F_{\perp} - \frac{2\varepsilon_{\perp}^2}{\Gamma_f}\right) - \frac{8}{9}A_5^2} \sqrt{\left(F_{\parallel} - \frac{2\varepsilon_{\parallel}^2}{\Gamma_f}\right)\left(F_{\perp} - \frac{2\varepsilon_{\perp}^2}{\Gamma_f}\right) - \frac{4}{9}A_{FB}^2}}{\pi\left(F_{\perp} - \frac{2\varepsilon_{\perp}^2}{\Gamma_f}\right)}$$

- ★ Valid for entire  $q^2$  range, only assumes SM gauge structure,
- ★ almost free from hadronic assumptions except one!

# Observables & Relations

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# Observables & Relations

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Imaginary part of amplitude  $\sim$  asymmetries  $A_7, A_8, A_9$   
 $\sim 0$  for  $3 \text{ fb}^{-1}$  LHCb data

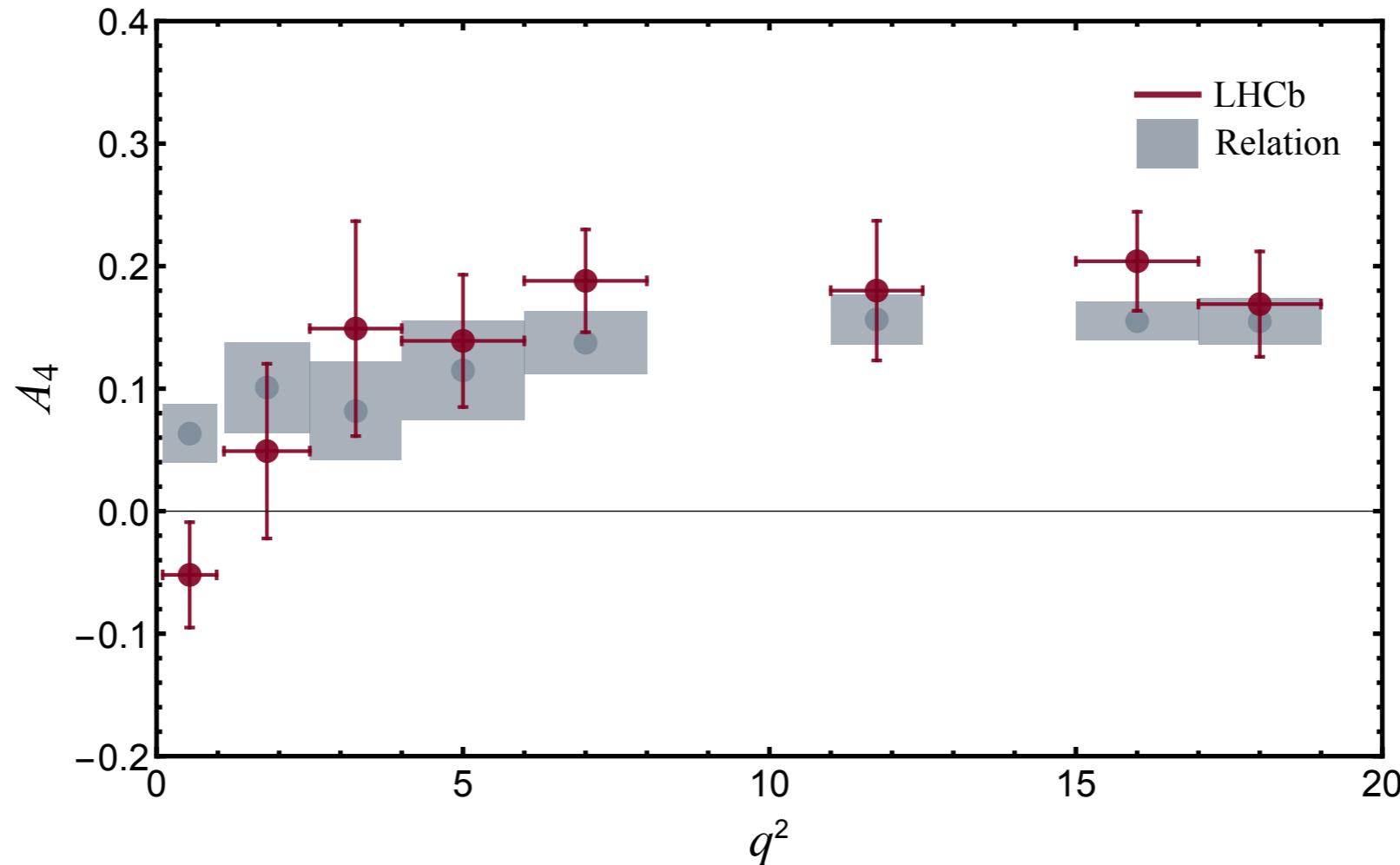
depends on form-factor ratio  $P_1 \equiv \mathcal{F}_{\perp}/\mathcal{F}_{||}$

$$P_1 = -\frac{\lambda^{1/2}(m_B^2, m_{K^*}^2, q^2)}{2E_{K^*}m_B} \text{ in leading } 1/m_B$$

[Beneke, Feldmann '00]

free from  $\mathcal{O}(\alpha_s)$  correction — *reliable theoretical input*

# Observables & Relations



$A_4$  deviates by  $\sim 2\sigma$  only at  $0.1 \leq q^2 \leq 0.98$   $\text{GeV}^2$