Standard Model Prediction for Direct CP-violation in K-decays, and Long-Distance Contributions to Kaonic Amplitudes

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Introduction

- Lattice QCD directly simulates low-energy hadronic interactions on supercomputers.
- Only known *ab initio, systematically improvable* technique for studying non-perturbative QCD.
- The simulations have now reached levels of precision with which it can make a significant impact on the search for BSM physics.
- In particular, with recent advances in theoretical and computational techniques, we are now able to directly compute matrix elements involving multi-particle states.
- In this talk we focus upon lattice calculations of kaonic matrix elements relating to quantities that are sensitive to BSM physics:
K→ππ and \( \varepsilon'/\varepsilon \)
Motivation for studying $K \to \pi \pi$ Decays

- Likely explanation for matter/antimatter asymmetry in Universe, baryogenesis, requires violation of CP.

- Amount of CPV in Standard Model appears too low to describe measured M/AM asymmetry: tantalizing hint of new physics.

- Direct CPV first observed in late 90s at CERN (NA31/NA48) and Fermilab (KTeV) in $K^0 \to \pi \pi$:

\[ \eta_{00} = \frac{A(K_L \to \pi^0 \pi^0)}{A(K_S \to \pi^0 \pi^0)}, \quad \eta_{+-} = \frac{A(K_L \to \pi^+ \pi^-)}{A(K_S \to \pi^+ \pi^-)}. \]

\[ \text{Re}(\epsilon'/\epsilon) \approx \frac{1}{6} \left(1 - \left| \frac{\eta_{00}}{\eta_{\pm}} \right|^2 \right) = 16.6(2.3) \times 10^{-4} \quad \text{(experiment)} \]

- In terms of isospin states: $\Delta I = 3/2$ decay to $I = 2$ final state, amplitude $A_2$

$\Delta I = 1/2$ decay to $I = 0$ final state, amplitude $A_0$

\[ \omega = \frac{\text{Re}A_2}{\text{Re}A_0} \]

\[ A(K^0 \to \pi^+ \pi^-) = \sqrt{\frac{2}{3}} A_0 e^{i\delta_0} + \sqrt{\frac{1}{3}} A_2 e^{i\delta_2}, \]

\[ A(K^0 \to \pi^0 \pi^0) = \sqrt{\frac{2}{3}} A_0 e^{i\delta_0} - 2 \sqrt{\frac{1}{3}} A_2 e^{i\delta_2}. \]

- Small size of $\epsilon'$ makes it particularly sensitive to new direct-CPV introduced by most BSM models.
Overview of calculation

- Hadronic energy scale $<< M_W$ - use weak effective theory.
- $K \rightarrow \pi\pi$ decays require single insertion of $\Delta S=1$ Hamiltonian:

$$H^{\Delta S=1}_W = \frac{G_F}{\sqrt{2}} V_{ud}^* V_{us} \sum_{j=1}^{10} [z_j(\mu) + \tau y_j(\mu)] Q_j$$

10 effective four-quark operators

perturbative Wilson coeffs.

$$\tau = -\frac{V_{ts}^* V_{td}}{V_{us}^* V_{ud}} = 0.0014606 + 0.00060408i$$

Imaginary part solely responsible for CPV (everything else is pure-real)

LL finite-volume correction

$$A^I = F \frac{G_F}{\sqrt{2}} V_{ud} V_{us} \sum_{i=1}^{10} \sum_{j=1}^{7} \left[ (z_i(\mu) + \tau y_i(\mu)) Z^\text{lat}\to\text{MS}_{i,j} \bar{M}_{j}^{I,\text{lat}} \right]$$

$$M_{j}^{I,\text{lat}} = \langle (\pi\pi)_I | Q_j | K \rangle \text{(lattice)}$$

- Operators must be renormalized into same scheme as Wilson coeffs: Use RI-(S)MOM NPR and perturbatively match to MSbar at high scale.
Key challenges of lattice calculation

- Primary challenge is to assure physical kinematics. Signal dominated by amplitude with 2 stationary pions in final state.

\[ 2m_\pi \approx 260 \text{ MeV} \ll m_K \approx 500 \text{ MeV} \]

Solution: Remove stationary pion state from system by manipulating lattice spatial boundary conditions.

- Antiperiodic BCs on down-quark for \( A_2 \)
- G-parity BCs on both quarks for \( A_0 \)

\[ p_\pi = 0 \rightarrow \pi / L \]

For \( A_0 \) serious noise issue due to “disconnected diagrams”

\[
\text{type4}
\]

Solution: Use “all-to-all” propagators to tune source to minimize overlap with vacuum and maximize sampling for every configuration.
Summary of RBC/UKQCD calculations


- $A_2$ computed on RBC/UKQCD $64^3 \times 128$ and $48^3 \times 96$ 2+1f Mobius DWF ensembles with the Iwasaki gauge action.
- $a^{-1} = 2.36$ GeV and 1.73 GeV resp - continuum limit taken.
- 10% and 12% total errors on Re($A_2$) and Im($A_2$) resp.
- Statistical errors sub-percent, dominant systematic errors due to truncation of PT series in computation of renormalization and Wilson coefficients.


- $A_0$ computed on $32^3 \times 64$ Mobius DWF ensemble with Iwasaki+DSDR gauge action. G-parity BCs in 3 directions to give physical kinematics.
- Single, coarse lattice with $a^{-1} = 1.38$ GeV but large physical volume to control FV errors.
- 21% and 65% stat errors on Re($A_0$) and Im($A_0$) due to disconn. diagrams and, for Im($A_0$) a strong cancellation between $Q_4$ and $Q_6$.
- Dominant, 15% systematic error is due again to PT truncation errors exacerbated by low renormalization scale 1.53 GeV.
\[ \Delta I = 1/2 \text{ rule} \]

- In experiment kaons approx 450x (!) more likely to decay into I=0 pi-pi states than I=2.
  \[ \frac{\text{Re}A_0}{\text{Re}A_2} \approx 22.5 \] (the \( \Delta I = 1/2 \) rule)

- Perturbative running to charm scale accounts for about a factor of 2. Is the remaining 10x non-perturbative or New Physics?


Strong cancellation between the two dominant contractions:

\[
\text{Re}(A_2) \sim \text{①} + \text{②}
\]

find \( \text{②} \approx -0.7 \text{①} \) heavily suppressing \( \text{Re}(A_2) \).

Pure-lattice calculation

\[
\frac{\text{Re}(A_0)}{\text{Re}(A_2)} = 31.1(11.2)
\]

[\text{Re}(A_0) \text{ agrees with expt.}]
Results for $\varepsilon'$

- $\text{Re}(A_0)$ and $\text{Re}(A_2)$ from expt.
- Lattice values for $\text{Im}(A_0)$, $\text{Im}(A_2)$ and the phase shifts,

$$\text{Re} \left( \frac{\varepsilon'}{\varepsilon} \right) = \text{Re} \left\{ \frac{i\omega e^{i(\delta_2 - \delta_0)}}{\sqrt{2\varepsilon}} \left[ \frac{\text{Im}A_2}{\text{Re}A_2} - \frac{\text{Im}A_0}{\text{Re}A_0} \right] \right\}$$

$$= 1.38(5.15)(4.43) \times 10^{-4}, \quad \text{(this work)}$$

$$16.6(2.3) \times 10^{-4}, \quad \text{(experiment)}$$

- Total error on $\text{Re}(\varepsilon'/\varepsilon)$ is $\sim 3\times$ the experimental error.
- Find reasonable consistency with Standard Model (at 2.1$\sigma$ level).
- Tantalizing hint of discrepancy strong motivation for continued study!
Outlook

- Error is dominated by those on $A_0$.

- Main priority is to increase statistics on $A_0$ calculation, enabling improved precision and better sys. error estimation. Aim for 4x increase in stats within ~1 yr.

- Reduce dominant NPR (15%) and Wilson coefficient (12%) systematic error arising from perturbative MSbar matching by increasing renormalization scale. Currently close to completion.

- On longer timescale:
  - Add second lattice spacing and perform continuum limit.
  - Include dynamical charm rather than relying on PT.
  - Investigate (expected %-level) EM and isospin corrections.
$K_L - K_S$ mass difference
Introduction

- Neglecting small CP-violation effects, mixing of neutral kaons induced by 2\textsuperscript{nd} order weak processes gives rise to mass difference between CP eigenstates $K_1 \sim K_L$ and $K_2 \sim K_S$

$$\Delta M_K = 2 \sum_n \frac{\langle \overline{K}^0 | H_W | n \rangle \langle n | H_W | K^0 \rangle}{M_K - E_n}$$

- Arises due to FCNC and therefore highly suppressed in SM due to GIM mechanism: $\Delta m_K = 3.483(6) \times 10^{-12}$ MeV small and highly sensitive to new BSM FCNC.

- PT calc using weak EFT with $\Delta S=2$ eff. Hamiltonian (charm integrated out) dominated by $p \sim m_c$: poor PT convergence at charm scale $\rightarrow \sim 36\%$ PT sys error.

- PT calc neglects long-distance effects arising when 2 weak operators separated by distance $\sim 1/\Lambda_{QCD}$.

- Use lattice to evaluate matrix element of product of $H_W^{\Delta S=1, \text{eff}}$ directly:

including charm quark.
Key challenges of lattice calculation

\[ \mathcal{A} = N_K^2 e^{-M_K(t_f-t_i)} \sum_n \frac{\langle \overline{K}^0 | H_W | n \rangle \langle n | H_W | K^0 \rangle}{M_K - E_n} \left( -T - \frac{1}{M_K - E_n} + \frac{e^{(M_K-E_n)T}}{M_K - E_n} \right) \]

- Vary integration window T to extract desired matrix element.
- Require explicit subtraction of exponentially-growing term when \( E_n < m_K \)
- Disconnected diagrams make the calculation noisy. Requires large statistics.

- Divergence when operators approach removed by GIM – requires (valence) charm on lattice. Need fine lattice to control discretization errors.
- Finite-volume corrections due to multi-particle intermediate states may be large, requiring explicit Lellouch-Luscher correction.
Calculation status

- Most recent calculation on one $24^3 \times 64$ DWF+I lattice with $a^{-1} = 1.73$ GeV.
- Unphysically heavy pions (330 MeV) - no multi-particle intermediate states.
- Unphysically light charm (949 MeV) for controlled discretization error on this somewhat coarse lattice.
- Large statistics (800 configs) with maximal use of each configuration give good signal for disconnected diagrams.

- Disconnected diagram contribution large, appears to violate OZI “rule”.
- Results in good agreement with expt:

\[ \Delta M_K = 3.19(41)(96) \times 10^{-12} \text{ MeV} \quad \text{lattice} \]
\[ 3.483(6) \times 10^{-12} \text{ MeV} \quad \text{experiment} \]

- Dominant error expected to be charm disc. effects $\sim 30\%$. Need finer lattices.
Rare kaon decays $K \rightarrow \pi l\bar{l}$
Additional FCNC processes “rare kaon decays”: $K^+ \rightarrow \pi^+ l^+ l^-$, $K_S \rightarrow \pi^0 l^+ l^-$

- Amplitude is long-distance dominated: Compute $K \rightarrow \pi \gamma^*$ on lattice.
- Lattice approach very similar to $\Delta m_K$ but with EM-current insertion:

Unphysical initial calculation presently underway on $24^3 \times 64$ ensemble with 430 MeV pions, 620 MeV kaons and $m_c \sim 533$ MeV.

- Multiple pion momenta allow extraction of form factor.
Preliminary results from Lattice 2016 talk by A.Lawson (July 28\textsuperscript{th}):

\[ A_\mu (q^2) \equiv G_F \frac{V(z)}{(4\pi)^2} \left( q^2 (k+p)_\mu - (M_K^2 - M_\pi^2) q_\mu \right), \quad z = q^2 / M_K \]

- Future steps are physical pion and kaon masses.
- No statistically significant charm mass dependence observed – is going to physical charm necessary?
Rare kaon decays $K \rightarrow \pi \nu \bar{\nu}$
• Another FCNC thus far not particularly well experimentally measured.
• NA62 expt expected to provide \(\sim 10\%\) error on 2-3 yr timescale.
• Short-distance dominated but expect \(\sim 5\%\) LD effect: Lattice!
• Again 2x operator insertions:

Effective operators representing both W and Z required

Use lattice QCD

Known \(\langle \pi^+ | V_{\mu} | K^+ \rangle\)

• Short-distance divergence requires NPR “matching” to point operator.
• Exploratory calculation on $16^3 \times 32$ $a^{-1}=1.73$ GeV ensemble complete.

• Unphysical masses similar to previous slides.

• Preliminary results presented by N. Christ at Lattice 2016 (July 28th):

  - Unphysical kinematics but demonstration of technique:
    \[
    P_C = P^{SD}_C + \delta P_{cu}
    \]
    total charm contrib.  short-distance part from PT: 0.365(12)  bilocal contrib.

  \[
  PT \quad \delta P_{cu} = 0.040(20) \quad \text{[Isodori et al hep-ph/0503107]}
  \]

  \[
  \Delta P_{cu}(\overline{MS}, 2 \text{ GeV}) = -0.007(2) \begin{pmatrix} +7 \\ -11 \end{pmatrix}_{\text{RI}} \begin{pmatrix} +5 \\ -21 \end{pmatrix}_{\overline{MS}}
  \]

  \[
  WW : -0.032(1) \quad + \quad Z : 0.025(1)
  \]

• Strong cancellation of $WW$ and $Z$ contribs.

• Wait to observe if situation stays true for more physical kinematics (underway).
Conclusions

- Lattice techniques now sufficiently advanced to significantly impact search for BSM physics.
- RBC & UKQCD collaboration are leading the way in computing experimentally relevant kaonic amplitudes.
- Complete, physical calculation of $\varepsilon'/\varepsilon$ performed with hint of tension with experiment.
- Lattice calculation demonstrates $\Delta I=1/2$ rule arises due to non-perturbative suppression of $\text{Re}(A_2)$.
- Matrix elements with 2 operator insertions now possible with controlled systematics, particularly finite-volume errors.
- Early results for FCNC suppressed $\Delta m_K$ and $K \to \pi l\bar{l}$.
- $K \to \pi \nu\bar{\nu}$ long-distance contributions computable by similar technique. Preliminary calculation complete.

We hope that these results will spur the experimental community to improve their determinations, particularly for $\varepsilon'/\varepsilon$. 