Optical Bifurcation in the Limit of Photon Statistics
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Overview

- Motivation for Axions and Axion Like Particles
- Methods to search for Axions
- Induced splitting creating Bifurcation
- Shifting in a cavity
- Limit of photon statistics – random walking starts
- Summary/Future
QCD Vacuum

- QCD Lagrangian contains CP Violating term. However strong interactions conserve CP symmetry:

\[ L = \Theta \frac{\alpha_s}{8\pi} A^{uva} \tilde{A}^a_{uv} \]

\[ L = (\Theta - \frac{\phi_f}{f_A}) \frac{\alpha_s}{8\pi} A^{uva} \tilde{A}^a_{uv} \]

- Peccei & Quinn proposed an axion Field
- QCD ground state (A is the color field strength tensor and \(\tilde{A}\) is its dual)

The simulation shown is the work of:
Photon Coupling to $B_{\text{ext}}$

- Previous searches focused on Primokoff Decays
- Diagrams: a) QED Vacuum Polarization b.) Photon Splitting c.) axion real production and d.) axion virtual production
Searches To Date:

- Helioscope
- Haloscope
- Cavity Regeneration & Vacuum Birefringence
Axion Coupling to Photons

- In an inhomogeneous magnetic field axions-photon coupling leads to the formation of a particle/antiparticle state which causes the beam to split in two
Mirror Cavity Experiments

- Consider the mirror cavity shown below.
- To counter the natural divergence of the beam a stable cavity can be constructed using concave mirrors.
In A Mirror Cavity

- In a mirror cavity reflection destroys the coupled state
- The returning beams re-couple to axions and continue to split with each entry into the field (each reflection)
Modeling Splitting

- Jones matrices were used to track the photons through the cavity they are defined as follows:

  Focusing:
  \[
  \begin{bmatrix}
  1 & 0 \\
  -1 & 1 \\
  \frac{f}{1} & 1 \\
  \end{bmatrix}
  \]

  Propagating:
  \[
  \begin{bmatrix}
  1 & d \\
  0 & 1 \\
  \end{bmatrix}
  \]

  Splitting:
  \[
  \begin{bmatrix}
  1 & 0 \\
  \pm \frac{\theta_{split}}{X_0} & 1 \\
  \end{bmatrix}
  \]
Intensity Changes

- Assuming an initial gaussian distribution, the changes in the beam profile due to splitting as a function of position relative to the center of the beam can be calculated:

\[
P_D = Ae^{\frac{-1}{2} \cdot \frac{x^2}{r^2}}
\]

\[
P_D'' + P_D' \approx A \cdot r - \varepsilon e^{\frac{1}{2} \cdot \frac{x^2}{r^2}} \cdot e^{\frac{x^2 \cdot \varepsilon}{r^2 \cdot r}} \cdot e^{\frac{-\alpha^2}{r^2}} \cdot \cosh\left(\frac{x^2 \cdot \alpha}{r^2 \cdot x}\right)
\]

\[
P_D - (P_D' + P_D'') \approx Ae^{\frac{-1}{2} \cdot \frac{x^2}{r^2}} \left[1 - \frac{r - \varepsilon}{r} e^{\frac{x^2 \cdot \varepsilon}{r^2 \cdot r}} e^{\frac{-\alpha^2}{r^2}} \cdot \cosh\left(\frac{x^2 \cdot \alpha}{r^2 \cdot x}\right)\right]
\]
Central Depletion

- The splitting leads to a drop in the central intensity accompanying an increase in the intensity of the sidebands
Real Parameters

- For actual experiments involving cavities there are stability conditions.

- As well, there are limits on the strength of magnetic fields and gradients that subsequently limit the splitting angle for a given coupling of matter to axion particles.

<table>
<thead>
<tr>
<th>Cavity Type</th>
<th>Cavity Length</th>
<th>Magnetic Field Length</th>
<th>VB Strength</th>
<th>Laser Wavelength</th>
<th>Laser Energy</th>
<th>Mirror Radius</th>
<th>Number of Bounces</th>
<th>θ_{split} ~ 10^{-10} (g_a = 10^{-6})</th>
<th>Injection Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Confocal</td>
<td>14 m</td>
<td>10 m</td>
<td>200 T/m</td>
<td>1064 nm</td>
<td>1 W</td>
<td>25 m</td>
<td>1.2 \cdot 10^4</td>
<td>4 \cdot 10^{-10}</td>
<td>1.5 \cdot 10^{-2} rad</td>
</tr>
<tr>
<td>Convex-Concaved</td>
<td>14 m</td>
<td>10 m</td>
<td>200 T/m</td>
<td>1064 nm</td>
<td>1 W</td>
<td>25 m, -11 m</td>
<td>1.2 \cdot 10^4</td>
<td>4 \cdot 10^{-10}</td>
<td>1.5 \cdot 10^{-2} rad</td>
</tr>
<tr>
<td>Planear-Planear</td>
<td>14 m</td>
<td>10 m</td>
<td>200 T/m</td>
<td>1064 nm</td>
<td>1 W</td>
<td>∞</td>
<td>1.2 \cdot 10^4</td>
<td>4 \cdot 10^{-10}</td>
<td>1.5 \cdot 10^{-2} rad</td>
</tr>
</tbody>
</table>
Each Traversal

- With each traversal the splitting occurs
- The simulations allow for a numerical solution to the intensity change as a function of traversal

\[ \alpha = \theta_{\text{split}} \cdot d \cdot f(n) \]

- Where \( \theta_{\text{split}} \) gives the angle of splitting
- Where \( d \) gives the length of the cavity
- Where \( f(n) \) must be extracted
60 Traversals for a 1Watt Beam

- A 1 Watt Beam has ~ 6.25 x 10^{-18} photons
- As the beam returns with the same polarization, with each pass splitting of a single beam into two beams of roughly the same intensity must occur
- After some 60 traversals (2^{+60} distributions) the limit of splitting is reached and no new distributions can be created
- Now when a photon gains transverse momentum and moves left there is no photon moving right from the same position – the intensity changes no longer scale as a sinh(\sim \theta_1\theta_2^{\text{split}}) function
The End Of Splitting

- To capture what happens when splitting no longer occurs, a limit was imposed on the total number of rays tracked following splitting.
- Some specific scenarios were considered first:
  - +/- directed tracks loose/gain momentum
    - Each track couples as a particle – shifts left
    - Each track couples as an antiparticle – shifts right
- Each ray was given a random positive or negative shift.
- Samples of 1000 randomly propagated sets of rays analyzed to understand behavior.
Random Change at n=11

- When the tracks are given randomly either + or – momentum at n=11
From Predicted To Random

- The continuous pattern is broken when the splitting behavior ends leading to rapidly changes
Varying Coupling

- The effect becomes linear once the limit of splitting is reached
Depletion vs Shifting

- Comparing Depletion to Shifting, when splitting stops, shows that shifting is more appropriate as coupling decreases.
Numerical Observations

- For scenarios that shift tracks by one unit of momentum such that equal distributions of tracks gain (+) or loose (-) momentum, the overall effect is quadratic as with the splitting case.
- For some random scenarios the energy shifting departs from the splitting case and gives a significant increase in left-right movement.
- The large changes observed show linear behavior with respect to coupling strength.
Birefringent Microscopy

- BNL Summer 2016 – Detector Measurement
- Null measurement
- Detector Measurement + Birefringent Sample 1
- Birefringent Samples – Stress induced effects
Birefringence & Noise

Background vs Birefringence Data

\[ y = 0.0004x - 1.023 \quad R^2 = 0.989 \]

\[ y = 0.0121x - 0.528 \quad R^2 = 0.9365 \]

\[ y = 0.0003x - 0.988 \quad R^2 = 0.8033 \]

\[ y = 0.0009x^{-0.793} \quad R^2 = 0.9571 \]

\[ y = 0.0004x^{-0.023} \quad R^2 = 0.989 \]
Summary

- Splitting leads to a quadratic depletion of the central intensity as well as a quadratic shifting of the beam’s center
- The limit of photon statistics ends the process of splitting and gives rise to an energy shifting that has a better than 95.2% probability of behaving linearly
- The energy shifting grows chaotically
- Experiments seeking to observe exotic particles may benefit from such an enhancement in signal strength
Axion Mass

- The calculations for splitting assumed “maximum mixing” or an axion mass at the resonance condition $ma = 0$
- For non-resonance, there is a correction factor:

$$\tan(2 \cdot \varphi) = \frac{2 \cdot Q_M}{Q_\gamma - Q_a}$$

Where the parameters are defined:

$$Q_M \approx \omega g_a B^e \approx 1 ev \cdot 10^{-21} ev^{-1} \cdot 1T \cdot 195 \frac{ev^2}{T} \approx 10^{-19} ev^2$$

$$Q_\gamma = \omega^2 \frac{7 \alpha}{45\pi} \left( \frac{B^e}{B_{crit}} \right)^2 \approx 3.19 \cdot 10^{-23} ev^2$$

$$Q_a = -m_a^2$$
Non-resonance mass factor

- The resonance axion mass is just $m_a = 5.7 \times 10^{-10}$
- Accounting for non-resonance, signal strength drops:
Figure 8: Axion space thus far explored by experimentation and astro-physical observation along with predictions (double solid and dashed lines) for magnetar observations.